



Loading optimization of palletized products on trucks

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Abstract

In this paper we analyze the application of an optimization model to solve problems of arranging products (packed in boxes) on pallets, and arranging loaded pallets on trucks. Initially the model is applied to solve thousands of randomly generated experiments. Then, in order to assess the effectiveness of the solutions in practice, the model is applied to two Brazilian case studies: a food company distribution center and a large wholesale distribution center. We also discuss the use of this approach for optimizing the sizes of packages, pallets and trucks. In particular, we analyze the performance of the Brazilian standard pallet (PBR), adopted by the Brazilian Association of Supermarkets (ABRAS) and recommended by the Brazilian Logistics Association (ASLOG), in comparison with other standard pallets. By examining not only the loading of products on pallets, but also the loading of pallets on trucks, we can obtain global utilization indices which are useful to evaluate the economical performance of unit load systems in the logistics chain of a company. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Business logistics is an essential area of management that has been described variously as physical distribution, materials handling, and transportation management. Its associated activities can include transportation, inventory maintenance, order processing, purchasing, warehousing, materials handling, packaging, and customer service standards. As pointed out by several authors, the mission of logistics is basically to get the right goods or services to the right place, at the right time, and in the desired condition, while making the greatest contribution to the company. This goal can be attained by means of effective coordinated management of all logistics activities.

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Many analysts have been suggesting that logistics is one of the main current sources of opportunities to increase the firm's efficiency and obtain competitive advantages. Surveys show that the average physical distribution costs in percent of sales dollars are 7.53 in the United States, 11.6 in the United Kingdom, and 21.1 in Australia (Ballou, 1992). In Brazil, the logistics activities can absorb more than 30% of sales and consume up to 80% of the total time to produce an item (Morales, 1995). A factor that directly affects Brazilian logistics costs is the "ant operation" approach, that is, a series of operations which handle the cargo load, item by item, through the logistics chain. In order to improve the performance of these operations, unit load devices have been utilized to reduce transportation and inventory costs, facilitate material handling, and increase the integration among different transportation modals.

This situation has focussed the attention of professionals, practitioners and researchers toward the importance of optimizing cargo palletization, in particular, to the well-known pallet loading problem which consists of arranging products (packed in boxes) on pallets in the most effective way. This problem appears in the "move-and-store" operations, and depending on the scale of the supply/distribution channel, a small increase in the number of products loaded on each pallet may result in substantial savings. In this paper, we analyze the results obtained by applying an optimization model to solve the pallet loading problem in thousands of random experiments as well as in two Brazilian case studies: a food industry distribution center and a large wholesale distribution center.

We also discuss the use of the model for the optimal sizing of packages, pallets and trucks. In particular, we analyze the performance of the Brazilian standard pallet (PBR), adopted by the Brazilian Association of Supermarkets (ABRAS) and recommended by the Brazilian Logistics Association (ASLOG), in comparison with other standard pallets. By examining not only the loading of products on pallets, but also the loading of pallets on trucks, we can obtain global utilization indices which are useful to evaluate the economical performance of unit load systems in the logistics chain of a company.

In summary, our aim is to illustrate how this simple approach can be useful for supporting decisions in: (i) the loading of packed products on pallets, (ii) the loading of pallets on trucks, and (iii) the design of packages, pallets and trucks. In the next section we briefly discuss the cargo palletization in Brazil, the manufacturer's pallet loading problem and an optimization model to solve it. In Section 3, we present the computational results obtained by applying the model to thousands of randomly generated examples. Then, in Section 4, in order to assess the effectiveness of the model in practice, we analyze the obtained results of loading products on pallets (Section 4.2) and loading pallets on trucks (Section 4.2) in the two case studies. The concluding remarks are presented in Section 5.

2. Cargo palletization and the manufacturers' pallet loading problem

Probably the two main advantages of cargo palletization in Brazil are: (i) cost reduction of transportation and inventories, and (ii) time decrease of the loading and unloading operations. Other advantages include the facilitation of cargo handling, vehicle loading and intermodal transfer, and the reduction of the number of damaged, lost and stolen items. Nevertheless, palletized systems can have some disadvantages, such as: the cost of palletization, the equipment

requirements (pallets and special handling equipment), the lack of well-accepted standard pallets and compatible trucks designed for these standards, and the undesirable empty space of the cargo loading.

In Brazil, the experience of companies that adopted cargo palletization, such as: Braspelco, Cofesa, Gessy Lever, Martins and Nestlé, has shown that the advantages greatly outweigh the disadvantages. Distribution managers have reported that the average times of truck unloading dropped from as much as 2 h to less than 15 min, while the mean numbers of daily truck deliveries significantly increased (Tecnológica, 1996). In spite of these encouraging results, Brazilian operations apparently have not yet developed an effective culture of palletized distribution. Research financed by the Brazilian Association of Supermarkets (ABRAS) was performed a few years ago in order to define a standard pallet for this sector (Superhiper, 1990). This study proposed a double face non-reversible wooden pallet with four entrances and size 1200×1000 mm². It is worth noting that this is the ISO series 2 pallet recommended by the International Standards Organization since 1980. In Brazil this pallet has been called PBR and is likely to become the Brazilian standard pallet.

A cargo palletization design for a company should include: (i) the optimal pallet sizing (or simply a selection of the best among a number of standard pallets), and (ii) the optimal pallet loading. It is usual to classify the latter problem as either the manufacturer's pallet loading (MPL) or the distributor's pallet loading (DPL) (Hodgson, 1982). The MPL is the problem of a manufacturer that produces goods packaged in identical boxes of size (l, w, h) , which are then arranged in horizontal layers on pallets of size (L, W, H) (where H is the maximum height of the loading). It is assumed that the boxes are available in large quantities and are orthogonally loaded on each pallet (i.e., with their sides parallel to the pallet sides). When boxes are not of the same size, we have the DPL. Different optimization approaches are found in the literature for the DPL, for example, in Hodgson (1982), Beasley (1985), Dowsland (1993), Abdou and Yang (1994) and Arenales and Morabito (1995). Under some conditions, the DPL problem can be seen as a container loading problem, as pointed out in Bischoff and Ratcliff (1995) and Morabito and Arenales (1994).

In this paper we are concerned with only the MPL problem. We assume that all boxes are identical and a vertical orientation is fixed. In this way, the problem consists of packing the maximum number of rectangles (l, w) and (w, l) orthogonally into a larger rectangle (L, W) without overlapping, yielding a layer of height h . We also assume that either there are no constraints related to the weight, density, fragility, etc., of the cargo loading, or, if there are, they are considered only after the determination of the packing pattern of the layer. The MPL has been extensively treated in the operations research literature and can be classified as 2/B/O/C according to Dyckhoff's typology of cutting and packing problems (Dyckhoff, 1990).

Although apparently easy to be optimally solved, the MPL is claimed to be NP-hard (not solvable by a polynomial bounded algorithm; Nelissen, 1995). It can be formulated as an integer linear programming, for example, specializing the 0–1-model in Beasley (1985), based on overlap restraints, or adapting the 0–1 model in Tsai et al. (1993) with disjunctive constraints. Exact methods of branch-and-bound type can be defined using bounds derived from the surrogate and Lagrangean relaxation of the model, as in Beasley (1985) and Tsai et al. (1993). Alternatively, Dowsland (1987) presented an interesting approach which basically consists of finding the maximum stable set from a particular finite graph, where the nodes correspond to the possible

positioning of the boxes on the pallet and the arcs connecting two nodes represent an overlap between them. However, because of the size of the practical problems, both the 0–1 models mentioned above and this graph approach are, in general, too large to be computationally treated. Therefore, several authors have proposed heuristic approaches, such as Steudel (1979), Smith and De Cani (1980), Bischoff and Dowsland (1982), Nelissen (1995), Scheithauer and Terno (1996) and Morabito and Morales (1998).

In this paper, we utilized the heuristic algorithm recently presented in Morabito and Morales (1998), which is based on a recursive procedure and seems to be very effective for the MPL (it was able to find the optimal solution of 99.9% of more than 20 000 examples of the literature, requiring moderate computational effort). To solve the examples of the next sections, the algorithm was implemented in Pascal (Turbo-Pascal compiler v.5.5) and run on a Pentium 100 microcomputer.

3. Random experiments

Initially we show that, besides optimizing the loading patterns, the MPL solutions can be useful for either the optimal pallet sizing or the selection of the best among a set of standard pallets. We generated 30 independent samples of 1000 random examples (a total of 30 000 examples) and, for each example, we applied the algorithm to obtain the utilization of the pallet area (as a percentage). The dimensions (l, w) of each example were uniformly sorted from the intervals suggested by Wright (1984)

$$200 \leq l \leq 600 \quad \text{and} \quad 150 \leq w \leq 450,$$

which are typical in the United States as well as in Brazil (Morales, 1995). Indeed, the data collected in the two companies (see Section 4) also belong to these intervals.

Table 1 presents the average results of the 30 samples for four different pallets (P₁–P₄). Pallet P₁, of size 1200 × 1000 mm², is the PBR and the ISO series 2 pallet recommended since 1980 (see Section 2). Pallet P₂, of size 1100 × 1100 mm², is the ISO series 1 pallet. Pallet P₃, of size 1200 × 800 mm², is the Europallet adopted by UIC (Union International des Chemins de Fer) since 1961. Pallet P₄ is a hypothetical pallet of size 1200 × 1200 mm². For each sample we computed the mean area utilization. The last three columns in Table 1 present, respectively, the average of the 30 mean utilization, the standard deviation of mean utilization, and the 95% confidence interval of mean utilization.

Table 1
Computational performance of pallets P₁–P₄ in 30 samples of 1000 random examples

Pallet	L (mm)	W (mm)	L/W	Area (m ²)	Average of mean area utilization (%)	Standard deviation of mean area utilization	95% confidence interval of mean area utilization
P ₁	1200	1000	1.2	1.20	84.3	0.2	[84.2, 84.4]
P ₂	1100	1100	1.0	1.21	80.9	0.3	[80.8, 81.1]
P ₃	1200	800	1.5	0.96	78.9	0.4	[78.6, 79.2]
P ₄	1200	1200	1.0	1.44	83.5	0.3	[83.3, 83.7]

Pallet P_1 yielded the highest average of mean utilization (84.3%), followed by pallets P_4 , P_2 and P_3 (note in Table 1 that the 95% confidence intervals do not overlap). Moreover, pallet P_1 had the smallest standard deviation of mean utilization, which means that the variability of the box lengths and widths had less impact on the mean utilization of pallet P_1 , in comparison with the other pallets.

Table 1 presents the average results of one of the 30 samples of Table 1. The numbers in the penultimate column correspond to the mean and standard deviation of utilization (not to be confused with the standard deviation of mean utilization of Table 1). As expected, for all pallets P_1 – P_4 , the mean utilization is near the average of the 30 mean utilization of Table 1, while the standard deviation (of utilization) is much higher than the standard deviation of mean utilization of Table 1. Besides pallets P_1 – P_4 , Table 2 also includes pallets P_5 – P_8 (pallets P_9 – P_{12} are discussed later) with the same area as the square pallet P_2 (i.e., 1.21 m²), but different ratios L/W varying from 1.25 to 2. Pallets P_5 – P_8 can be seen as “rectangularizations” of pallet P_2 . The last column of Table 2 presents the mean number of boxes per pallet layer and the standard deviation (in parenthesis). Observe that the standard deviation is large in comparison with the mean, since the sample contains different box sizes from (200, 150) to (600, 450) mm.

Comparing the average results for pallets P_1 and P_2 in Table 2, we verify that the mean number of boxes per layer is almost the same (10.2 and 10.0, respectively); however, the first pallet yielded a higher mean area utilization (84.4% against 81.0%). Pallet P_3 has the smallest mean number of boxes per layer (7.7), but it also has the smallest area (0.96 m²). Moreover, pallet P_3 yielded the lowest mean utilization (78.4%). On the other hand, pallet P_4 , the largest one (1.44 m²), had the largest mean number of boxes per layer (12.2); nevertheless, it yielded a mean utilization (83.32%) lower than pallet P_1 . All these results are compatible with those of Table 1.

Among the pallets of the same area (P_2 and P_5 – P_8), pallet P_5 ($L/W = 1.25$) had the best performance (84.1%), almost as good as pallet P_1 ($L/W = 1.2$). Suspecting that the optimal performance for this sample would be obtained by a pallet with a ratio close to 1.2, we defined four other pallets, P_9 – P_{12} , with the same area as pallet P_2 (1.21 m²) but different ratios L/W varying from 1.1 to 1.3, as shown in Table 2. Note that pallet P_{11} , with $L/W = 1.2$, had the highest mean

Table 2
Computational performance of pallets P_1 – P_{12} in a sample of 1000 random examples^a

Pallet	L (mm)	W (mm)	L/W	Area (m ²)	Mean area utilization (%)	Mean number of boxes per layer
P_1	1200	1000	1.2	1.20	84.4 (8.7)	10.2 (5.3)
P_2	1100	1100	1.0	1.21	81.0 (11.5)	10.0 (5.5)
P_3	1200	800	1.5	0.96	78.4 (13.2)	7.7 (4.3)
P_4	1200	1200	1.0	1.44	83.3 (10.6)	12.2 (6.5)
P_5	1230	984	1.25	1.21	84.1 (8.6)	10.3 (5.4)
P_6	1347	898	1.5	1.21	83.0 (8.9)	10.1 (5.5)
P_7	1455	831	1.75	1.21	82.6 (10.9)	10.1 (5.5)
P_8	1556	778	2.0	1.21	80.9 (13.0)	10.0 (5.5)
P_9	1154	1049	1.10	1.21	83.2 (9.7)	10.3 (5.6)
P_{10}	1180	1026	1.15	1.21	83.8 (9.0)	10.3 (5.4)
P_{11}	1205	1004	1.20	1.21	84.3 (8.7)	10.3 (5.3)
P_{12}	1255	965	1.30	1.21	83.6 (9.1)	10.3 (5.5)

^a Standard deviation in parentheses, mean l and w and corresponding standard deviations: $\bar{l} = 400$ (118), $\bar{w} = 300$ (86).

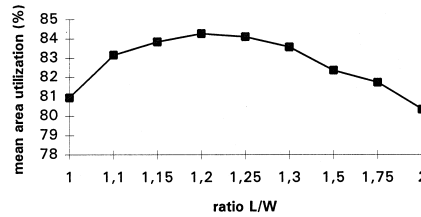


Fig. 1. Performance of pallets P_2 and P_5 – P_{12} , with area 1.21 m^2 .

utilization (84.3%) among pallets P_5 – P_{12} , which is very close to pallet P_1 's utilization (84.4%). Fig. 1 depicts the performance of the pallets of equal area, suggesting that the best pallet for the present sample data ($\bar{l} = 400$, $\bar{w} = 300$) is very close to the PBR ratio, $L/W = 1.2$. Observe that the optimal ratio need not be equal to the expected box length/width ratio of the data, $\bar{l}/\bar{w} = 1.33$.

In order to show that this result is not strongly related to the fact that the box ratio used in the experiments averaged close to 1.2, we generated another set of 1000 examples with both l and w uniformly sorted from the interval $[150, 600]$, which yields $\bar{l}/\bar{w} = 1.0$. The mean area utilization for pallets P_2 ($L/W = 1.0$), P_9 ($L/W = 1.1$), P_{10} ($L/W = 1.15$), P_{11} ($L/W = 1.2$), P_5 ($L/W = 1.25$) and P_{12} ($L/W = 1.3$) were 79.7%, 82.2%, 82.4%, 82.5%, 81.7% and 80.9%, respectively, indicating that the optimal ratio for this data set ($\bar{l} = 375$, $\bar{w} = 375$) is also very close to $L/W = 1.2$.

It should be remarked that this simple approach can be useful for supporting decisions in either the design of pallets as a function of the product mix of the company, or the selection of the most appropriate pallet (among the set of candidate pallets) for that product mix.

4. Case studies

4.1. Loading packed products on pallets

In this section we illustrate the effectiveness of the approach in practice: the model is applied to the loading of packed products on pallets in two Brazilian case studies. The first was performed in one of the distribution centers of a large food company (company A), and the second refers to a large wholesale distribution center (company B). A sample of 148 and 78 products, together with their corresponding loading patterns, was randomly collected in each company, respectively. Tables 3 and 4 present the average results obtained by the algorithm for pallets P_1 – P_4 with the data of companies A and B, respectively.

It is worth noting that the mean area utilization of Tables 3 and 4 are higher than those of Table 2. This suggests that the random samples of Section 2 represent more unfavorable instances than those found in practice. However, the relative performance between pallets P_1 – P_4 in Table 2 was approximately maintained in Tables 3 and 4. Observe that pallet P_1 again had the best performance (i.e., 86.0% and 89.5%, respectively), followed by pallets P_2 (Table 3) or P_4 (Table 4), and pallet P_3 . In fact, both companies were utilizing pallet P_1 before this study, but with loading patterns worse than those of Tables 3 and 4.

In order to compare the results obtained by the companies with those obtained by the algorithm, we separated the samples into two disjoint sets: (i) the set of products whose loading patterns had sizes less than or equal to the pallet size $1200 \times 1000 \text{ mm}^2$, and (ii) the set of products whose loading patterns had sizes greater than $1200 \times 1000 \text{ mm}^2$. In the latter case, the usual tolerance of the companies was 25 mm in each border of the pallet. Therefore, let us define pallet P_{13} as pallet P_1 with a border tolerance of 25 mm, that is, a pallet of size $1250 \times 1050 \text{ mm}^2$. Tables 5–8 compare the average results of the two companies with the average results obtained by the algorithm: Tables 5 and 6 present the results for pallet P_1 with the set of products of each company without border tolerance, whereas Tables 7 and 8 present the results for pallet P_{13} with the set of products of each company with a border tolerance of 25 mm.

As shown in Table 5, the algorithm produced a mean utilization (88.1%) slightly higher than the company (87.5%). This difference (0.6%) corresponds to only seven products (out of 71) for which the algorithm found better loading patterns (i.e., patterns with a larger number of boxes per layer). Fig. 2 illustrates the corresponding patterns of one of these products.

Table 3
Computational performance of pallets P_1 – P_4 in a sample of 148 products of company A^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)	Mean area utilization (%)	Mean number of boxes per layer
P_1	1200	1000	1.2	1.20	86.0 (7.3)	9.9 (5.7)
P_2	1100	1100	1.0	1.21	84.7 (9.6)	9.8 (5.9)
P_3	1200	800	1.5	0.96	80.1 (10.1)	7.4 (4.7)
P_4	1200	1200	1.0	1.44	84.4 (8.1)	11.7 (6.9)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 436$ (85), $\bar{w} = 279$ (61).

Table 4
Computational performance of pallets P_1 – P_4 in a sample of 78 products of company B^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)	Mean area utilization (%)	Mean number of boxes per layer
P_1	1200	1000	1.2	1.20	89.5 (6.2)	15.1 (8.2)
P_2	1100	1100	1.0	1.21	86.6 (8.7)	14.9 (8.3)
P_3	1200	800	1.5	0.96	86.0 (8.5)	11.7 (6.5)
P_4	1200	1200	1.0	1.44	88.3 (7.3)	17.9 (9.8)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 330$ (107), $\bar{w} = 270$ (86).

Table 5
Comparison between results obtained by algorithm and company A for pallet P_1 in a sample of 71 products (without border tolerance)^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)		Mean area utilization (%)	Mean number of boxes per layer
P_1	1200	1000	1.2	1.20	Algorithm	88.1 (7.3)	10.6 (6.7)
					Company A	87.5 (7.1)	10.5 (6.5)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 435$ (85), $\bar{w} = 272$ (65).

Table 6

Comparison between results obtained by algorithm and company B for pallet P₁ in a sample of 56 products (without border tolerance)^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)		Mean area utilization (%)	Mean number of boxes per layer
P ₁	1200	1000	1.2	1.20	Algorithm	91.0 (4.8)	15.9 (8.5)
					Company B	87.4 (6.2)	15.1 (7.9)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 361$ (101), $\bar{w} = 233$ (62).

Table 7

Comparison between results obtained by algorithm and company A for pallet P₁₃ in a sample of 77 products (with a border tolerance of 25 mm)^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)		Mean area utilization (%)	Mean number of boxes per layer
P ₁	1200	1000	1.2	1.20	Algorithm	84.0 (6.8)	9.2 (4.5)
P ₁₃	1250	1050	1.19	1.31	Algorithm	87.3 (6.0)	10.5 (5.3)
					Company A	85.9 (4.6)	10.6 (4.9)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 437$ (84), $\bar{w} = 285$ (57).

Table 8

Comparison between results obtained by algorithm and company B for pallet P₁₃ in a sample of 22 products (with a border tolerance of 25 mm)^a

Pallet	<i>L</i> (mm)	<i>W</i> (mm)	<i>L/W</i>	Area (m ²)		Mean area utilization (%)	Mean number of boxes per layer
P ₁	1200	1000	1.20	1.20	Algorithm	85.8 (7.2)	13.4 (7.5)
P ₁₃	1250	1050	1.19	1.31	Algorithm	90.5 (3.6)	15.2 (7.7)
					Company B	87.9 (5.9)	14.8 (7.6)

^a Standard deviation in parentheses, mean *l* and *w* and corresponding standard deviations: $\bar{l} = 320$ (104), $\bar{w} = 292$ (80).

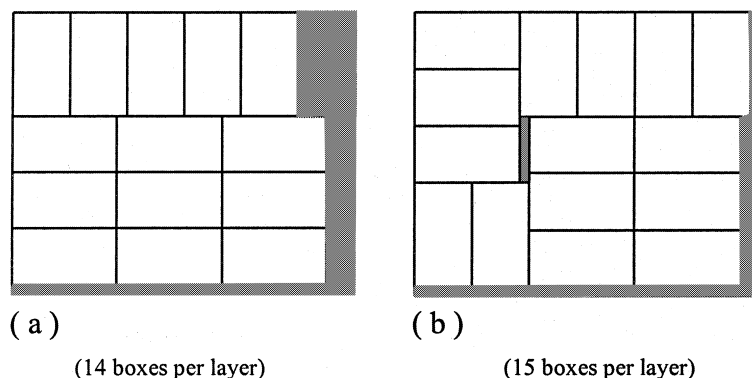


Fig. 2. Pallet (1200, 1000) and boxes (348, 208): (a) pattern utilized by company A; (b) pattern found by algorithm.

In the same way, note in Table 6 that the algorithm also yielded a mean utilization (91.0%) higher than the company (87.4%). This substantial difference (3.6%) corresponds to 22 products (out of 56) for which the algorithm found better loading patterns. Similar remarks can be made with respect to Tables 7 and 8. In Table 7, the difference between the algorithm (for pallet P_{13}) and the company results was 1.4% (87.3–85.9%), corresponding to 22 products out of 77. In Table 8, that difference was 2.6% (90.5–87.9%), corresponding to nine products out of 22.

It is interesting to note that there are patterns found by the algorithm for pallet P_1 (i.e., without border tolerance) which load the same number of boxes per layer as patterns used by the companies, requiring border tolerances. Fig. 3 illustrates an example found in Table 8 – observe that both patterns arrange the same number of boxes (i.e., 21); however, the pattern used by company B has a larger size ($1200 \times 1035 \text{ mm}^2$) than the one found by the algorithm.

The computer runtime required to solve each example of Tables 1–8 was on average less than 1 s. It is worth remarking that the sizes of the boxes sampled in companies A and B belong to the intervals suggested by Wright (1984) (see the means and standard deviations for the box lengths and widths at the bottom of the tables).

4.2. Loading pallets on trucks

Examining not only the loading of products on pallets, but also the loading of pallets on trucks, allows us to obtain global utilization indices which are useful for evaluating the economic performance of unit load systems in the logistics chain of a company. In this section we initially analyze the application of the algorithm to load standard pallets on standard trucks – note that this can be useful to support pallet and truck design decisions. An analogous situation occurs in the design of product packaging.

Now the input data of the MPL is the pallet size (L, W) and the truck size (A, B), where A and B are, respectively, the effective internal length and width of the trucks. As before, the problem consists of arranging the maximum number of rectangles (L, W) and (W, L) in the larger rectangle (A, B). Table 9 presents, for each pallet P_1 – P_4 , the area utilization (in percentage) and the number of pallets per layer obtained by the algorithm for trucks T_1 – T_3 . Trucks T_1 – T_3 are semi-trailers commonly used in Brazilian trucking industry (Widmer and Morales, 1994).

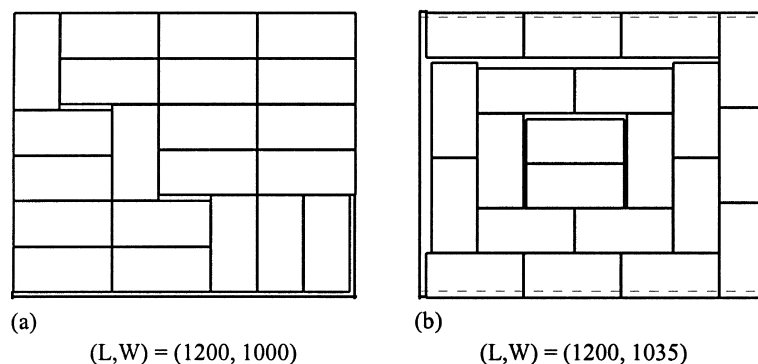


Fig. 3. Loading patterns for boxes (345, 160): (a) pattern found by algorithm; (b) pattern utilized by company B.

Note in Table 9 that, for all vehicles, pallet P₁ resulted in a utilization of the truck area better than pallet P₂. The difference between their utilization can be substantial; see for example the first row of the table where this difference is 6% (i.e., 93.9–87.9%).

Based on the results of Table 9, we can calculate global utilization indices for the palletized cargo of each company. These indices are computed by simply multiplying the mean utilization indices of the pallet area (Tables 5–8 of Section 4.1) by the utilization indices of the truck area (Table 9). Tables 10 and 11 present these results for companies A and B, respectively.

Tables 10 and 11 show that the global performance of pallet P₁ dominates the other pallets in both companies. The difference in the performances varied from 5% to 9%. From the point of view of loading pallets on trucks, note in Table 9 that pallet P₃ had the best utilization indices. However, as we consider its performance with respect to the box loading, the global performance of pallet P₃ becomes worse than pallet P₁ (Tables 10 and 11).

It is worth noting that, besides optimizing the loading of pallets on trucks, this simple approach can also be useful in the selection of the most appropriate trucks as a function of the adopted pallets or, vice versa, in the selection of the most adequate pallets as a function of the available trucks. Similarly, the MPL solutions can be applied to help the packaging design as a function of

Table 9
Area utilization and number of pallets per layer for trucks T₁–T₃

	Truck		Pallet							
	A (mm)	B (mm)	P ₁ (%)	P ₂ (%)	P ₃ (%)	P ₄ (%)				
T ₁	14 370	2480	93.9	28	87.9	26	93.9	35	88.5	22
T ₂	13 370	2480	93.9	26	87.4	24	95.3	33	95.3	22
T ₃	12 470	2480	92.8	24	85.7	22	93.9	31	92.8	20

Table 10
Global performance of the palletized cargo for company A

	Truck		Pallet			
	A (mm)	B (mm)	P ₁ (%)	P ₂ (%)	P ₃ (%)	P ₄ (%)
T ₁	14 370	2480	80.7	74.5	75.2	74.7
T ₂	13 370	2480	80.7	74.0	76.4	80.5
T ₃	12 470	2480	79.8	72.6	75.2	78.3

Table 11
Global performance of the palletized cargo for company B

	Truck		Pallet			
	A (mm)	B (mm)	P ₁ (%)	P ₂ (%)	P ₃ (%)	P ₄ (%)
T ₁	14 370	2480	84.0	76.0	80.7	78.1
T ₂	13 370	2480	84.0	75.7	81.9	84.1
T ₃	12 470	2480	83.0	74.3	80.7	81.9

the pallets utilized along the logistics chain of the company. In this case, the short runtimes of the algorithm allow the evaluation of many alternative packaging sizes (l, w), eventually by means of an explicit enumeration of all possible technically feasible combinations of the values of l and w .

5. Concluding remarks

In this paper we analyzed the loading of palletized products on trucks. By means of random experiments and two case studies, we illustrated how a relatively simple approach can be useful for supporting decisions in: (i) the loading of products (packed in boxes) on pallets, (ii) the loading of pallets on trucks, and (iii) the design or selection of packages, pallets and trucks.

In particular, we studied the performance of the Brazilian standard pallet (pallet P_1 of size $1200 \times 1000 \text{ mm}^2$), adopted by the Brazilian Association of Supermarkets (ABRAS) and recommended by the Brazilian Logistics Association (ASLOG). Our results indicate that pallet P_1 has a very good performance in comparison with other standard pallets. For example, based on the data from the two case studies, the utilization indices of pallet P_1 were from 3% to 9% better than the indices of the ISO series 1 pallet (pallet P_2 of size $1100 \times 1100 \text{ mm}^2$).

We also emphasized the importance of an analysis based on global performance measures in order to compare different unit load devices. By examining not only the loading of products on pallets, but also the loading of pallets on trucks, we can obtain global utilization indices which are useful for evaluating the economic performance of unit load systems in the logistics chain of a company.

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