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Loan-Size Limits: A Simple Model

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Most banks limit the maximum size of a loan they will make to any given customer. This limit constitutes an important constraint on commercial lending activity and deserves careful consideration. In this article, a simple way of thinking about loan-size limits, risk, and profitability is presented. This article also discusses the possible effects of loan-size limits on personnel evaluations.

Any discussion of loan-size policies needs to begin with some consideration of the objectives of the policies. Limits on loan size appear to be used to control the risk of the loan portfolio. Risk, in this sense, means the likelihood that the loan portfolio will have returns in any given year that fall below a specified level.

Most of the analysis presented in this article focuses on the likelihood that a loan portfolio will have negative net earnings. However, other areas of interest include the likelihood of higher or lower levels of performance and the total variability of returns.

While this article discusses the implications of loan-size policies on risk, any decisions related to loan-size

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limits must be made in consideration of both risk and return. By their very nature, loan-size policies will require the rejection of some loans promising adequate returns.

Defining the Portfolio

A common and costly error is to ignore the effects of the definition of the portfolio on risk. Even a very risk-averse individual should act almost risk-neutral if the investment being considered is sufficiently small relative to the size of the portfolio. For example, consider a gamble with a 55% chance of winning. For high stakes of, say, $100,000, many of us would be unwilling to take the gamble. We would see a 45% chance of losing $100,000 and say no. On the other hand, if a portfolio consists of 1,000 gambles, each of $100, and there is a 55% probability of winning each independent gamble, it would be considered a very safe bet. The chances of losing money would be only .01% (one in 10,000), and the chances of gaining less than $2,800 would be around 1%. There would be only a 20% chance of winning less than $7,400, and, of course, the expected gain would be $10,000.[1] A portfolio of smaller gambles is much less risky than a single large gamble.

Similar to the differences between a single large gamble and many smaller gambles, the way in which a bank defines a portfolio is critical to understanding loan default probabilities. In some banks, the portfolio of interest may be the total portfolio of commercial loans, but in many banks, a far smaller portfolio is considered. Many large regional banks (greater than $1 billion in assets) will have loan-size limits in the $2 to $10-million range. From a perspective of corporate risk and return, such a loan-size limit seems very low. Given an asset size greater than $1 billion, possible defaults on loans of this size would not have much effect on total corporate earnings or assets. If a $10-million loan of acceptable risk will produce a $2-million return, there would be no reason to refuse it. But if this is so, why do banks maintain low loan-size limits relative to their size?

It might be that some banks believe they can judge the likelihood of default of smaller borrowers better than they can judge the likelihood of default of larger borrowers. But this seems unlikely, especially since the quality of information regarding larger borrowers is at least as good as that available for smaller borrowers. Similarly, fraud or deception is no more likely for larger companies. Perhaps the answer is that price competitiveness varies with the size of borrowers, with larger margins being available for smaller loans, but the sensible response would be better pricing policies rather than a ban on large loans.

[1] When something takes on only two values, and the probability of each value is constant across trials, and the trials are independent then the outcomes follow the binomial distribution used in this example. Any conventional probability text explains the construction of the binomial distribution. As a matter of practice, most standard statistical packages and spreadsheets provide the binomial distribution probability as a built-in function.

Another possible explanation is that for legitimate performance-incentive and control reasons, banks organize as sets of smaller banks. For example, a bank with $1 billion in assets may actually comprise a number of $50-million to $100-million divisions. Loan-size limits make more sense if they are considered at the division level rather than the corporate level. For example, it might make sense for divisions to have loan-loss limits that make losses extremely unlikely at the division level.

By organizing and thinking as a set of smaller banking operations, large banks create more individual responsibility for accomplishments and allow for specialization. But along with such responsibility is the danger that a bad year (which may be due to chance alone) could seriously damage the prospects for entire lines of businesses and the careers of the managers or loan officers associated with them. If managers or loan officers are responsible for losses and are allowed to make extremely large loans, then the chance occurrence of a single default on a loan could seriously hurt their careers.

A Simple Model

For an example of the effects of loan-size limits on default probabilities, assume that a bank is concerned about the likelihood of losses in a $100-million portfolio—whether this constitutes the total bank's portfolio or a division's. To develop a model of portfolio risk and return, a number of assumptions are necessary. The assumptions chosen for this example are reasonable but are not statistically derived from any single bank's experience. (The parameters will be different for each bank, and modification for a particular bank's situation is essential.)

Assumptions

Let us assume a $100-million portfolio is composed of equal-sized loans. (Assuming all the loans are of equal size simplifies the modeling, but this assumption can be easily replaced with the actual distribution of loan sizes in a specific application.) The assumption of equal-sized loans actually overestimates the risk of a loan-size limit, since relatively few loans will actually be at the limit and the average loan will probably be somewhat below the limit. Following are other assumptions used in this example:

- The default rate of the portfolio is 1%.
- Defaulted loans return 80% of principal.
- The likelihood that a loan will default is independent of the likelihood that another loan will default.
- Profits are 3% of the value of good loans minus the loss on defaulted loans and minus fixed administrative costs, which are assumed to be $500,000.

To understand the possible outcomes of a scenario based on these assumptions, two issues must be examined: the probability of a given outcome (for example, two defaults per portfolio) and the profitability associated with the portfolio.

Probability

Consider a $100-million portfolio, composed of 10 loans each of $10 million dollars. If the probability of default is 1%, then according to probability theory, the likelihood of no defaults is 90% and there is a 9% chance of one default and 0.4% chance of more than one default.

Profitability

Given a $100-million portfolio consisting of 10 loans, each of $10 million (and no defaults), the profitability of the portfolio is $2.5 million. If there is one defaulted loan (of $10 million) and 20% is not recovered, the profitability of the portfolio is $200,000. And if there are two defaulted loans, each of $10 million, the profitability of the portfolio becomes a negative $2.1 million. This outcome is expected about 4 times out of 1,000. See Figure 1 for the calculations supporting these conclusions.

Figure 2 presents the results of a number of different scenarios based on varying numbers of loans, loan size amounts, and default probabilities. To clarify Figure 2, if there is a portfolio of 10 $10-million loans, with 1% chance of default, the likelihood of no defaults is 90%, with expected profits of $2.5 million. The likelihood of one default is 9.1%, with expected profits of $200,000.

The fifth column, "Probability of More Defaults" shows the probability that more loans will default than listed in column 3. For example, for 10 loans, each of $10 million, with one default, there is a 0.4% chance of more defaults occurring.

Using the Model for Policy Setting

Setting a loan-size policy requires balancing expected profits and risks (the probability of different profit levels.) In this article, a significant factor has been ignored: Loan-size limits require the rejection of potentially profitable business. Although such an effect could be added to the model, this effect will be ignored in this article since it depends heavily on a bank's specific conditions. Suppose, for example, a 99% chance of positive profits is desired by an institution. According to the data in Figure 2, for an average loan size of $10 million, there is a 99% chance of 0 or 1 default. The return for this portfolio is $2.5 million if there are no defaulted loans and $200,000 if there is one default.

Substantial portfolio effects can be observed in portfolios with a small number of loans. Consider column 4 of Figure 2, "Probability of Defaults." For a portfolio of 10 loans, the chance of large negative returns (defined as a loss of $2.1 million or more) is 0.4%. However, with a portfolio of 20 loans, the chance of loss drops to 0.01% and the magnitude of loss is smaller (starting at $950,000).

While smaller loans in the portfolio rapidly reduce the likelihood of losses on the portfolio level, they substantially increase the likelihood of defaults. For example, as shown in

Figure 1. Calculating Profitability

All calculations are based on a $100-million portfolio consisting of 10 $10-million loans.

Assumption No. 1: No defaults.

Profits = Portfolio Size x 3% Rate of Return - Administrative Costs

= $100 million x 3% - $500,000

= $2.5 million

Assumption No. 2: One loan default and 20% of loan is not recovered.

Profits = (Portfolio Size - Defaulted Loans) x 3% Rate of Return - Loan Losses - Administrative Costs

= ($100 million - $10 million) x 3% - (20% x $10 million) - $500,000

= $200,000

Assumption No. 3: Two Loan Defaults

Profits = (Portfolio Size - Defaulted Loans) x 3% Rate of Return - Loan Losses - Administrative Costs

= ($100 million - $20 million) x 3% Rate of Return - (20% x $20 million) - $500,000

= ($2.1 million)

Figure 2. Results of Model with 1% Default Rate

(1) Number of Loans	(2) Loan Size	(3) Number of Defaults	(4) Probability of Defaults	(5) Probability of More Defaults	(6) Profit Before Bad Loans	(7) Losses on Defaulted Loans	(8) Profit After Bad Loans
10	10,000,000	0	0.90438	0.09562	2,500,000	0	2,500,000
10	10,000,000	1	0.09135	0.00427	2,200.000	2,000,000	200,000
10	10,000,000	2	0.00415	0.00011	1,900,000	4,000,000	-2,100,000
10	10,000,000	3	0.00011	0.00000	1,600,000	6,000,000	-4,400,000
10	10,000,000	4	0.00000	0.00000	1,300,000	8,000,000	-6,700,000
10	10,000,000	5	0.00000	0.00000	1,000,000	10,000,000	-9,000,000
10	10,000,000	6	0.00000	0.00000	700,000	12,000,000	-11,300,000
10	10,000,000	7	0.00000	0.00000	400,000	14,000,000	-13,600,000
20	5,000,000	0	0.81791	0.18209	2,500,000	0	2,500,000
20	5,000,000	1	0.16523	0.01686	2,350,000	1,000,000	1,350,000
20	5,000,000	2	0.01586	0.00100	2,200,000	2,000,000	200,000
20	5,000,000	3	0.00096	0.00004	2,050,000	3,000,000	-950,000
20	5,000,000	4	0.00004	0.00000	1,900,000	4,000,000	-2,100,000
20	5,000,000	5	0.00000	0.00000	1,750,000	5,000,000	-3,250,000
20	5,000,000	6	0.00000	0.00000	1,600,000	6,000,000	-4,400,000
20	5,000,000	7	0.00000	0.00000	1,450,000	7,000,000	-5,550,000
30	3,333,333	0	0.73970	0.26030	2,500,000	0	2,500,000
30	3,333,333	1	0.22415	0.03615	2,400,000	666,667	1,733,333
30	3,333,333	2	0.03283	0.00332	2,300,000	1,333,333	966,667
30	3,333,333	3	0.00310	0.00022	2,200,000	2,000,000	200,000
30	3,333,333	4	0.00021	0.00001	2,100,000	2,666,667	-566,667
30	3,333,333	5	0.00001	0.00000	2,000,000	3,333,333	-1,333,333
30	3,333,333	6	0.00000	0.00000	1,900,000	4,000,000	-2,100,000
30	3,333,333	7	0.00000	0.00000	1,800,000	4,666,667	-2,866,667
Number of Loans	Loan Size	Number of Defaults	Probability of Defaults	Probability of More Defaults	Profit Before Bad Loans	Losses on Defaulted Loans	Profit After Bad Loans
40	2,500,000	0	0.66897	0.33103	2,500,000	0	2,500,000
40	2,500,000	1	0.27029	0.06074	2,425,000	500,000	1,925,000
40	2,500,000	2	0.05324	0.00750	2,350,000	1,000,000	1,350,000
40	2,500,000	3	0.00681	0.00069	2,275,000	1,500,000	775,000
40	2,500,000	4	0.00064	0.00005	2,200,000	2,000,000	200,000
40	2,500,000	5	0.00005	0.00000	2,125,000	2,500,000	-375,000
40	2,500,000	6	0.00000	0.00000	2,050,000	3,000,000	-950,000
40	2,500,000	7	0.00000	0.00000	1,975,000	3,500,000	-1,525,000
50	2,000,000	0	0.60501	0.39499	2,500,000	0	2,500,000
50	2,000,000	1	0.30556	0.08944	2,440,000	400,000	2,040,000
50	2,000,000	2	0.07562	0.01382	2,380,000	800,000	1,580,000
50	2,000,000	3	0.01222	0.00160	2,320,000	1,200,000	1,120,000
50	2,000,000	4	0.00145	0.00015	2,260,000	1,600,000	660,000
50	2,000,000	5	0.00013	0.00001	2,200,000	2,000,000	200,000
50	2,000,000	6	0.00001	0.00000	2,140,000	2,400,000	-260,000
50	2,000,000	7	0.00000	0.00000	2,080,000	2,800,000	-720,000

Figure 2, a portfolio of 50 $2-million loans has only a 60% chance of no defaults; defaults should be expected in this portfolio. However, smaller defaults have significantly less effect on profitability. With a portfolio of 50 loans, the portfolio remains profitable with up to five defaults. A portfolio of smaller loans makes portfolio-level losses extremely unlikely.

This disparity creates a difficult set of incentives and constraints. By having many small loans, a division is almost guaranteed to have defaults, but the likelihood of a large default and negative earnings on the portfolio is reduced. However, if a manager's or a loan officer's career would be hurt by small defaults, then making only small loans can actually increase the likelihood of negative consequences for these individuals. (Of course, results will depend on the specific numerical assumptions used.)

Business Cycle Effects

The analysis presented in Figure 2 assumes a 1% loan default rate, but another primary concern of bankers may be a significant increase in loss rates during a bad year. From a profitability perspective, the limits on loan size with respect to catastrophic years should be set at levels to avoid corporate bankruptcy, but limits should not be set to simply avoid negative profits in a division or two. That is, a loan-size policy protecting a bank from a once-every-50-years calamity at the corporate level might be defensible, but a loan-size policy protecting a bank against any of its divisions having negative profits in a bad year that primarily affects one division of the bank would probably be considered excessively cautious.

Consider, for example, a bank that has a diversified portfolio that includes agricultural and industrial firms. The once-every-50-years drought may increase default rates in the agricultural division while not increasing them in the industrial division. Overall, the bank would show an adequate performance even though a significant portion of its business had a bad year. Even so, banks might want to look at the sensitivity of their profitability to changes in the business cycle.

Changes to reflect the business cycle can be incorporated into the model by changing the default parameters. For example, the recovery rate on bad loans can be decreased if a review indicates recovery rates decline during business cycle downturns. Figure 3 presents the results of a simulation using the model with a different default rate assumption. In Figure 3, the effect on various portfolios is shown if the default rate is increased from 1% to 5%. (The recovery rate of defaults was retained at 80%.)

The simulation in Figure 3 shows that the effects of portfolio size on risk may be more striking when the default rate is increased.

With a portfolio of 10 loans, the chance of positive profits is 91.4% and the chance of losses is 8.6%. The losses are also quite large, starting with $2.1 million. With the 20-loan portfolio, the likelihood of loss

Figure 3. Results of model with 5% Default Rate

(1) Number of Loans	(2) Loan Size	(3) Number of Defaults	(4) Probability of Defaults	(5) Probability of More Defaults	(6) Profit Before Bad Loans	(7) Losses on Defaulted Loans	(8) Profit After Bad Loans
10	10,000,000	0	0.59874	0.40126	2,500,000	0	2,500,000
10	10,000,000	1	0.31512	0.08614	2,200,000	2,000,000	200,000
10	10,000,000	2	0.07463	0.01150	1,900,000	4,000,000	-2,100,000
10	10,000,000	3	0.01048	0.00103	1,600,000	6,000,000	-4,400,000
10	10,000,000	4	0.00096	0.00006	1,300,000	8,000,000	-6,700,000
10	10,000,000	5	0.00006	0.00000	1,000,000	10,000,000	-9,000,000
10	10,000,000	6	0.00000	0.00000	700,000	12,000,000	-11,300,000
10	10,000,000	7	0.00000	0.00000	400,000	14,000,000	-13,600,000
20	5,000,000	0	0.35849	0.64151	2,500,000	0	2,500,000
20	5,000,000	1	0.37735	0.26416	2,350,000	1,000,000	1,350,000
20	5,000,000	2	0.18868	0.07548	2,200,000	2,000,000	200,000
20	5,000,000	3	0.05958	0.01590	2,050,000	3,000,000	-950,000
20	5,000,000	4	0.01333	0.00257	1,900,000	4,000,000	-2,100,000
20	5,000,000	5	0.00224	0.00033	1,750,000	5,000,000	-3,250,000
20	5,000,000	6	0.00030	0.00003	1,600,000	6,000,000	-4,400,000
20	5,000,000	7	0.00003	0.00000	1,450,000	7,000,000	-5,550,000
30	3,333,333	0	0.21464	0.78536	2,500,000	0	2,500,000
30	3,333,333	1	0.33890	0.44646	2,400,000	666,667	1,733,333
30	3,333,333	2	0.25864	0.18782	2,300,000	1,333,333	966,667
30	3,333,333	3	0.12705	0.06077	2,200,000	2,000,000	200,000
30	3,333,333	4	0.04514	0.01564	2,100,000	2,666,667	-566,667
30	3,333,333	5	0.01235	0.00328	2,000,000	3,333,333	-1,333,333
30	3,333,333	6	0.00271	0.00057	1,900,000	4,000,000	-2,100,000
30	3,333,333	7	0.00049	0.00008	1,800,000	4,666,667	-2,866,667

(1) Number of Loans	(2) Loan Size	(3) Number of Defaults	(4) Probability of Defaults	(5) Probability of More Defaults	(6) Profit Before Bad Loans	(7) Losses on Defaulted Loans	(8) Profit After Bad Loans
40	2,500,000	0	0.12851	0.87149	2,500,000	0	2,500,000
40	2,500,000	1	0.27055	0.60094	2,425,000	500,000	1,925,000
40	2,500,000	2	0.27767	0.32326	2,350,000	1,000,000	1,350,000
40	2,500,000	3	0.18511	0.13815	2,275,000	1,500,000	775,000
40	2,500,000	4	0.09012	0.04803	2,200,000	2,000,000	200,000
40	2,500,000	5	0.03415	0.01388	2,125,000	2,500,000	-375,000
40	2,500,000	6	0.01049	0.00339	2,050,000	3,000,000	-950,000
40	2,500,000	7	0.00268	0.00071	1,975,000	3,500,000	-1,525,000
50	2,000,000	0	0.07694	0.92306	2,500,000	0	2,500,000
50	2,000,000	1	0.20249	0.72057	2,440,000	400,000	2,040,000
50	2,000,000	2	0.26110	0.45947	2,380,000	800,000	1,580,000
50	2,000,000	3	0.21987	0.23959	2,320,000	1,200,000	1,120,000
50	2,000,000	4	0.13598	0.10362	2,260,000	1,600,000	660,000
50	2,000,000	5	0.06584	0.03778	2,200,000	2,000,000	200,000
50	2,000,000	6	0.02599	0.01179	2,140,000	2,400,000	-260,000
50	2,000,000	7	0.00860	0.00319	2,080,000	2,800,000	-720,000

declines to 7.5% and the amount of loss has only a 1.6% chance of being larger than $950,000. With the 50-loan portfolio, the chances of loss decline to 3.8% and the loss is small; however, some defaults are almost assured because there is only a 7.7% chance of no defaults.

Simplifications and Modifications

The statistical results described in this article simply illustrate the effects of portfolio size and loan-size limits. The results depend on a number of assumptions that would certainly be modified for any specific application. Naturally, the more precisely the parameters and the model are tailored to a given bank, the more accurate the predictions will be. To apply this model to a particular bank, a number of factors must be estimated. These factors include the following:

1. Size distribution of loans in the portfolio.
2. Size of the portfolio.
3. Recovery rate on defaulted loans.
4. Default rates—preferably estimated for specific industries and time periods.
5. Margins and administrative costs.

Two simple structural changes might be justified in the model: The current model has fixed administrative costs, but they actually would vary according to the number of loans in a portfolio. Also, the model assumes that the events related to the portfolio occur in a single year and that income and defaults are both recognized immediately. A more realistic model would recognize that returns on successful loans are recognized annually but defaults may occur over time, with costs or reserves being recognized over several years. It seems likely that this assumption reduces the likelihood that the portfolio would experience losses in any particular year by spreading the losses over time.

Although a more sophisticated model with the parameters selected for a particular bank may more accurately reflect the bank's position, the results presented in this article provide a useful, although rough, approximation of portfolio situations.

Conclusion

This article provides some indication of the influence of portfolio size and loan-size limits on potential losses. We do not suggest any bank take the numbers developed as a given; although they are derived from a set of reasonable assumptions, they do not necessarily fit a specific bank's portfolio. Furthermore, the results are likely to err on the conservative side, since we assumed that all loans would be at the maximum allowable loan-size limit.

Several lessons can be learned from these simulations. First, loan-size limits substantially affect the likelihood of negative profits—smaller loan-size limits both lower the likelihood and decrease the magnitude of negative profits. Second, loan-size limits have the opposite effect on defaults—many smaller loans dramatically increase the likelihood of defaults.

A more complex set of issues arises from the interaction of loan-size limits and managerial evaluations. Assume that a bank negatively evaluates managers or loan officers who have loan defaults and losses above a certain level. As stated earlier, if the default rate is 1% and a manager has a portfolio of 10 $10-million loans, there is a 90.4% chance of having no defaults and a 9.1% chance of a $2-million loss on a defaulted loan. However, if the portfolio is composed of 50 $10-million loans, the likelihood of zero defaults drops to 60%, and the likelihood of a $400,000 loss is 30.6%.

In many banks, either loss level would constitute a serious problem for a manager. Even if the manager has half the normal loss rate on defaulted loans, that is, only 10% of the principal is lost on defaulted loans, there is still a $1-million loss if the portfolio is composed of 10 loans, and a $200,000 loss if the portfolio is composed of 50 loans. In either case, this could be considered "bad management" in many banks.

The issue is further complicated by the fact that a manager with one loss or no losses in a 10-loan portfolio or with up to six losses in a 50-loan portfolio is still generating positive profits. Underlying this analysis is the assumption that the bank concerns itself with profitability, predictability of profitability, and loan losses. Although financial theorists might argue that profitability (return to stockholders) should be the dominant objective, in practice, bankers are criticized for the variability of earnings and for the actual amount of loan losses as well.

As a bank reduces its loan-size limit, it reduces expected profits by turning away profitable loans based on their size and increases its administrative costs per loan (which may not be fully compensated by higher margins). On the other hand, it also reduces the likelihood of large losses in the portfolio as a whole, while increasing the likelihood of small defaults. Lower loan-size limits increase the predictability of profits at some cost. Although the discussion of results in this article focuses on the likelihood of negative earnings, the model can just as easily be used to calculate other indicators of the predictability of earnings.

Finally, while the loan portfolio analysis methods described in this article do provide a framework for thinking about loan-size limits, actual decisions on loan-size limits involve a variety of other significant issues. ■