LOCAL ELECTRIC FIELD ENHANCEMENT

DURING NANO-FOCUSING OF PLASMONS BY A TAPERED GAP

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ABSTRACT

We demonstrate that during plasmon nano-focusing in a tapered gap (V-groove), local electric field experiences much stronger enhancement than the magnetic field. Two distinct asymptotic regimes are found near the tip of the groove: the electric field approaches either zero or infinity when dissipation is above or below a critical level (at a fixed taper angle), or taper angle is smaller or larger than a critical angle (at a fixed level of dissipation). Tapered gaps are shown to be the best option for achieving maximal field enhancement, compared to nano-wedges and tapered rods. An optimal taper angle is determined.

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1. Introduction

Nano-focusing of surface plasmons in metallic nano-structures is one of the major approaches for concentrating and delivering electromagnetic energy to the scale well below diffraction limit [1-15]. It offers unique opportunities for the development of near-field optical microscopy with sub-wavelength resolution [7-15], high-resolution lithography [16], coupling of light into and out of photonic nano-circuits [5,17], new sensors and detection techniques based on surface-enhanced Raman scattering [8,18-20], etc.

Different metal structures have been suggested for nano-focusing of plasmons. These include sharp metal tips [1,3,8], dielectric conical tips covered in metal film [7,9-11], pyramidal tips covered in metal film with a nano-aperture [12], nano-particle lenses [20], sharp V-grooves and nano-wedges [2,4-6], etc. One of the major features of these structures is the possibility of strong local field enhancement in regions that are much smaller than the wavelength [3-12,20]. This opens unique opportunities for observation of non-linear plasmonic effects and development of new sensors, for example, based on surface-enhanced Raman scattering in metal structures with nano-focusing [8,18-20].

At the same time, it is still not clear which of these structures will be most efficient in terms of achieving the largest possible local field enhancement. It is possible to think that tapered metal rods and metal wedges should provide lowest dissipation, and thus lead to the most efficient local field enhancement. This is simply because in these structures a smaller fraction of the plasmon energy can be expected to propagate in the metal. However, this expectation may not be correct, because local field enhancement is determined not only by dissipation, but also by plasmon coupling (e.g., across a nano-gap). So far, local enhancement of only the plasmon magnetic field has been investigated in metal wedges and V-grooves [4-6], which makes it difficult to compare these structures with, for example, the tapered rod [3] for which local enhancement of only the electric field was presented.

Therefore, the aim of this paper is to conduct detailed investigation of the local enhancement of electric field during nano-focusing of surface plasmons in tapered metallic gaps in the adiabatic and non-adiabatic approximations. In particular, it will be demonstrated that the electric field of the plasmon experiences much stronger local enhancement than the magnetic field. Conditions for the strong local electric field enhancement are determined and investigated, depending on the structural and wave parameters. Optimization of the taper angle is carried out using the rigorous numerical analysis in the non-adiabatic regime of nano-focusing. It is also demonstrated that contrary to the above expectations, it is the tapered gaps that appear to provide maximal possible local field enhancement, compared to all other considered structures including sharp tapered rods.

2. Adiabatic nano-focusing

The considered tapered gap (sharp V-groove) is presented in Fig. 1a. As was shown in [4,5], only plasmons with anti-symmetric (across the gap) charge distribution can experience slowing down and nano-focusing near the tip of the groove and this will be adiabatic only if the taper angle β is smaller than the critical angle [4]:

$$\beta < \beta_c = -2\varepsilon_1/e_1, \qquad (1)$$

where ε_1 is the permittivity of the dielectric filling the tapered gap, and e_1 is the real part of the metal permittivity $\varepsilon_2 = e_1 + ie_2$. In this case, as the plasmon approaches the tip of the groove (Fig. 1b), the magnetic field may experience significant enhancement of ~ 5 – 10 times, reaching a finite value at the tip of the groove, if there is no dissipation in the metal [4].

The analysis of the local enhancement of the electric field in a tapered gap is conducted similar to that of the magnetic field for adiabatic and non-adiabatic nano-focusing [4,5]. According to the Maxwell equations, the amplitude of the z-component of the electric field in the plasmon is proportional to the amplitude of the magnetic field and the propagation constant q, the latter tending to infinity as the plasmon approaches the tip of the groove [4]. Therefore, it is possible to expect that the local electric field in the plasmon should also tend to infinity at the tip. This is indeed demonstrated by Fig. 2a corresponding to nano-focusing of an anti-symmetric (with respect to the charge distribution across the gap) plasmon in the tapered silver-vacuum gap at the vacuum wavelength $\lambda = 0.6328$ μm (He-Ne laser). For comparison, Fig. 2b shows enhancement of the magnetic field in the same groove [4].

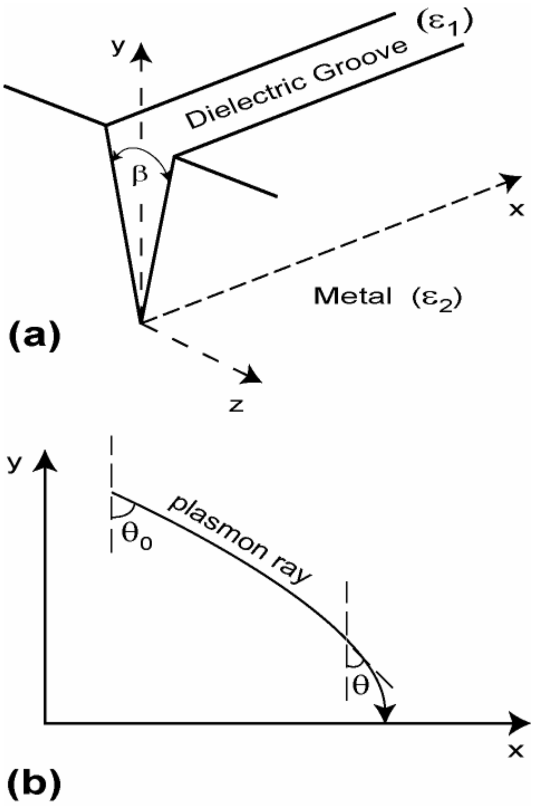

Figure 1. (a) A V-groove of angle β in a metal of permittivity ε_2, filled with dielectric of permittivity ε_1. (b) A ray of an anti-symmetric (with respect to the charge distribution across the groove) gap plasmon in the groove. θ_0 is the angle of incidence of the gap plasmon at large distances from the tip of the groove, where the coupling between the two surface plasmons representing the gap plasmon is negligible.

The presented dependencies (Fig. 2) are shown for the case of relatively low dissipation in the metal (silver with the permittivity $\varepsilon_m = -19.3 + 0.66i$). In order to investigate the dependence of the electric field enhancement on dissipation in the metal, we consider the fixed real part of metal permittivity ($e_1 = -19.3$) and change the imaginary part. The resultant dependencies of the electric field amplitude on distance from the tip of the groove for normal plasmon incidence ($\theta_0 = 0$) are shown in Fig. 3a.

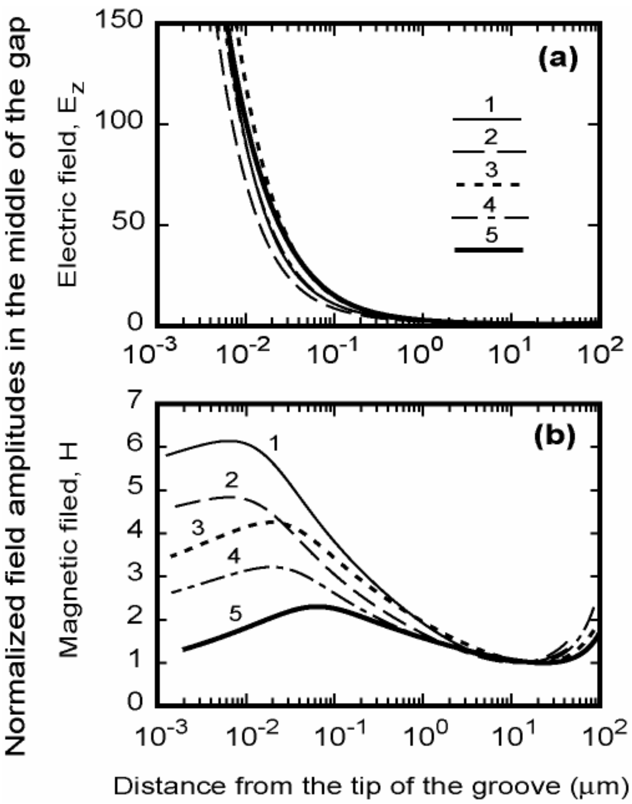

Figure 2. The *y*-dependencies of the amplitudes of the electric (a) and magnetic (b) fields in the middle of the gap for the anti-symmetric gap plasmons incident onto the tip of the vacuum V-groove in silver. All the amplitudes are normalized to the amplitudes at the local minimums (at ~ 10 – 30 μm). Metal permittivity: $\varepsilon_m = -19.3 + 0.66i$ [21], $\varepsilon_1 = 1$, $\lambda_{vac} = 0.6328$ μm, and (1) $\theta_0 = 0$, $\beta = 4^o$, (2) $\theta_0 = 45^o$, $\beta = 4^o$, (3) $\theta_0 = 0$, $\beta = 2^o$, (4) $\theta_0 = 45^o$, $\beta = 2^o$, (5) $\theta_0 = 0$, $\beta = 1^o$. (Curves 1 and 4 are nearly identical in (a)).

It can be seen that, if the dissipation is not too strong, the amplitude of the electric field in the anti-symmetric gap plasmon goes through a minimum and then monotonically increases to infinity as the plasmon approaches the tip of the groove (curves 1 – 4 in Fig. 3a). This is only correct in the approximation of continuous electrodynamics. In a real situation, spatial dispersion, Landau damping, and finite sharpness of the tip (due to fabrication and atomic structure of matter) will not allow the infinite increase of the electric field amplitude.

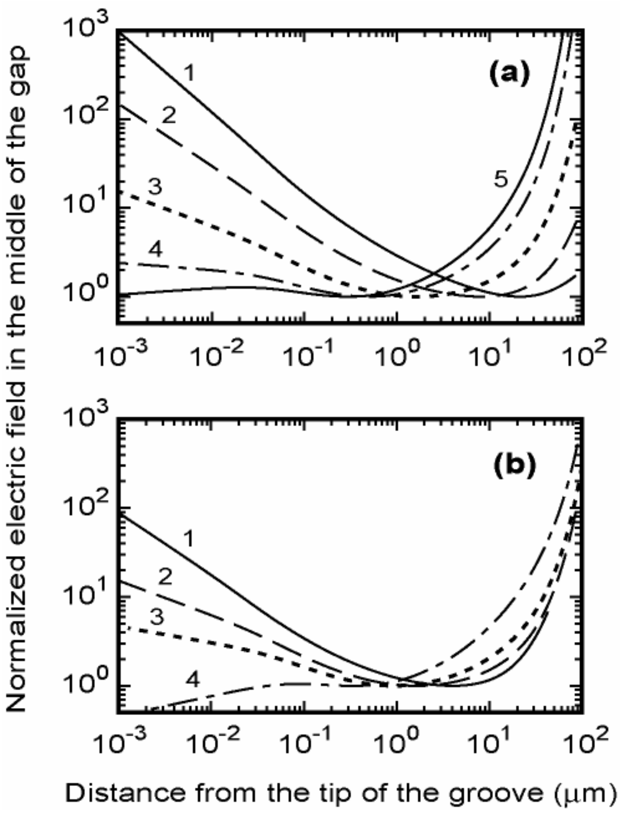

Figure 3. The y-dependencies of the magnitude of the electric field amplitudes $|E| = |E_z|$ of the anti-symmetric gap plasmon in the middle of the silver-vacuum V-groove groove with $\theta_0 = 0$, $\varepsilon_1 = 1$, $\lambda_{vac} = 0.6328$ μm, and $e_1 = -19.3$. (a) Fixed groove angle $\beta = 2^o$, but different imaginary parts of the metal permittivity: (1) $e_2 = 0.66$, (2) $e_2 = 2$, (3) $e_2 = 4$, (4) $e_2 = 6$, (5) $e_2 = 7$. (b) Fixed imaginary part of the metal permittivity $e_2 = 4$, but different groove angles: (1) $\beta = 4^o$, (2) $\beta = 2^o$, (3) $\beta = 1.5^o$, and (4) $\beta = 1^o$. (Curves 3 in (a) and 2 in (b) are identical).

However, the infinite (in the approximation of continuous electrodynamics) increase of the electric field amplitude occurs only if e_2 is smaller than a critical value e_{2c} (in Fig. 3a, $e_{2c} \approx 6.5$). If $e_2 > e_{2c}$, then the electric field amplitude near the tip monotonically decreases to zero (curve 5 in Fig. 3a). At the critical dissipation in the metal, the amplitude of the electric field near the tip asymptotically tends to a constant.

It is interesting that near the tip, the asymptotic behavior of the logarithm of the electric field amplitude is linear in logarithm of distance from the tip (Fig. 3a,b), which means that the asymptotic behavior of the electric field amplitude is the power law in distance from the tip:

$$E \approx Cy^{-\mu}, \qquad \text{if } y \to 0, \tag{2}$$

where C and μ are some constants depending on the structural parameters ($-\mu$ is the slope and ln(C) is the intercept of the linear asymptotic (at $z \to 0$) dependencies in Figs. 3a,b.). If $e_2 < e_{2c}$, then $\mu > 0$, and the amplitude of the electric field tends to infinity at the tip. If $e_2 > e_{2c}$, then $\mu < 0$, and the amplitude of the electric field tends to zero at the tip. If $e_2 = e_{2c}$, then $\mu = 0$ and the amplitude of the electric field at the tip is non-zero and finite (Fig. 3a).

A similar situation occurs if the imaginary part of the metal permittivity is fixed and the groove angle β is reduced (Fig. 3b). In this case, there exists a critical groove angle β_c. In the adiabatic approximation [3,4,6] and the assumption of continuous electrodynamics, if $\beta > \beta_c$, then the amplitude of the electric field in the plasmon increases to infinity as the plasmon propagates towards the tip (curves 1 – 3 in Fig. 3b). If $\beta < \beta_c$, then the plasmon amplitude tends to zero at the tip (curve 4 in Fig. 3b), and to a finite value if $\beta = \beta_c$.

There is a simple relationship between the critical imaginary part of the metal permittivity and the critical groove angle. To derive it, we calculate the Poynting vector in the gap plasmon at some point on the ray (Fig. 1b), average it over one period of the wave, and integrate over z from $-\infty$ to $+\infty$ to obtain the total energy flux S in the gap plasmon. For weak dissipation in the metal this gives [4]:

$$S = \frac{c^2 Q_1}{16\pi\omega}|H_{20}|^2 \exp(-2x_p Q_2)\left[\frac{2}{e_1\alpha_{20}} + \frac{\sinh(\alpha_{10}h)}{\varepsilon_1\alpha_{10}\cosh^2(\alpha_{10}h/2)} + \right.$$

$$\left. + \frac{h}{\varepsilon_1\cosh^2(\alpha_{10}h/2)}\right], \tag{3}$$

where $q = Q_1 + iQ_2$ is the wave number of the gap plasmon, the x_p-axis is parallel to the direction of propagation of the plasmon at the considered point on the ray (Fig. 1b), h is the local width of the gap, α_{10} and α_{20} are the real parts of the reciprocal penetration depths of the plasmon into the gap and the metal, respectively, ω is the angular frequency, c is the speed of light, and H_{20} is the amplitude of the magnetic field at either of the metal interfaces.

In the asymptotic region near the tip of the groove (i.e., where $h \to 0$), Eq. (3) is simplified as:

$$S \approx -\frac{h^2 e_1 \omega}{32\pi} |E_z|^2 \exp(2yQ_2), \tag{4}$$

where E_z is the z-component of the electric field in the plasmon in the middle of the gap. Here, we also used the asymptotic relationships $\alpha_{10} \approx \alpha_{20} \approx Q_1$, $Q_1 \approx -2\varepsilon_1/(he_1)$, $Q_2 \approx 2\varepsilon_1 e_2/(he_1^2)$, and $Q_1 h << 1$, if $h \to 0$. We also replaced the x_p coordinate (in the direction of plasmon propagation along the ray) by $-y$, because in the asymptotic region near the tip of the groove the plasmon ray is always normal to the tip, i.e., anti-parallel to the y-axis (see Fig. 1b and [4]).

From here, the amount of energy dissipated in the metal as the plasmon propagates the distance dy is given as

$$-dS_d \approx \frac{h\omega e_2 \varepsilon_1}{8\pi e_1} |E_z|^2 \exp(2yQ_2) dy, \tag{5}$$

Eqs. (4) and (5) give the energy flux S' in the plasmon at the point $y + dy$: $S' = S + dS_d$ (note that $dy < 0$ and $e_1 < 0$). On the other hand, the same flux S' at the point $y + dy$ can be obtained directly from Eq. (4) by using the field amplitude E'_z at that point and replacing h by $h + dh$, where dh is the variation of the local gap width within the distance dy: $dh \approx \beta dy$.

Comparing these two different equations for S', and taking into account that in the asymptotic regime at the critical dissipation (critical groove angle) the electric field amplitude must be constant: $E'_z = E_z$ (Figs. 3a,b), we obtain the condition relating the critical groove angle with the critical imaginary part of the metal permittivity:

$$\beta_c = \frac{2e_{2c}\varepsilon_1}{e_1^2}, \tag{6}$$

For each value of e_2, this equation determines the critical angle β_c, and vice versa. The reason for using both β and e_2 with the indices "*c*" is because if Eq. (6) is satisfied, then both these quantities are equal to their critical values.

If $\theta_0 \neq 0$, then in the asymptotic region near the tip $\theta \to 0$ (see Fig. 1b and [4]), and we again obtain Eq. (6). Therefore, Eq. (6) relating the critical values of the groove angle and imaginary part of the metal permittivity is independent of the angle of incidence θ_0.

Eq. (6) gives: $e_{2c} \approx 6.5$ for Fig. 3a, and $\beta_c \approx 1.23^{o}$ for Fig. 3b, which is in excellent agreement with the presented numerical results.

To compare the efficiency of adiabatic nano-focusing by a conical tip [3] and a tapered gap, we compare the local enhancement of the electric fields in both the structures at the same material and structural parameters (such as permittivities of the media in contact, taper angle, and frequency of the plasmons). The results are presented in Fig. 4. Curves 1 and 2 show the dependencies of the magnitude of the local electric field amplitude in the middle of the tapered gap ($E = E_z$) on distance from the tip in the vacuum-metal gap with $\beta = 1.5^{o}$ for the two different levels of dissipation: (1) $\varepsilon_m = -19.3 + 3i$ and (2) $\varepsilon_m = -19.3 + 3.5i$. Curves 3 and 4 show the similar dependencies for the amplitude of the electric field at surface of the tapered metal rod (considered in [3]) with the same taper angle (1.5^{o}) and metal permittivities as for curves 1 and 2, respectively.

In particular, it can be seen that while there is no local field enhancement for the tapered rod for both the considered dissipations in the metal (curves 3 and 4 in Fig. 4), nano-focusing of gap plasmons under the same conditions displays a substantial local field enhancement (curves 1 and 2 in Fig. 4). For example, for curve 1, the local field enhancement within the interval from ~ 2 μm from the tip to ~ 20 nm from the tip is ~ 5 times (in terms of the plasmon amplitude). At the same time, for the analogous conical tip, the magnitude of the amplitude of the local electric field at the rod surface drops ~ 2 times within the same interval (see curve 3 in Fig. 4). This is a clear demonstration of the superiority of tapered nano-gaps compared to conical tips, when strong local field enhancement is required.

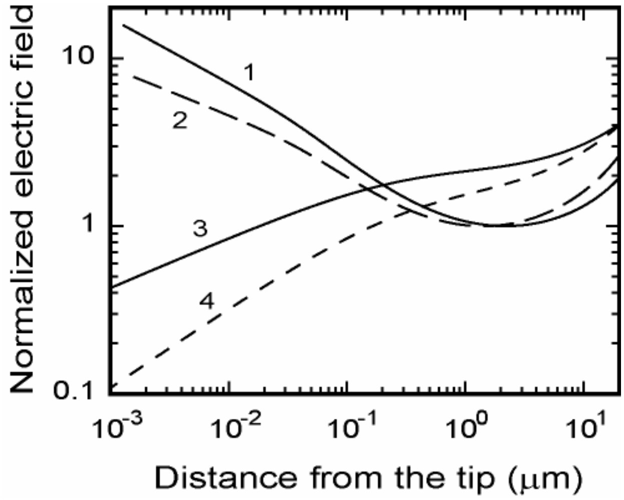

Figure 4. The adiabatic dependencies of the amplitudes of the electric field on distance from the tip for the gap plasmons (in the middle of the tapered metal-vacuum gaps – curves 1 and 2), and for the localized plasmons in the tapered rods (at the rod surface – curves 3 and 4). The taper angles for the gaps and the rods are the same and equal to 1.5°. (1) and (3): $\varepsilon_m = -19.3 + 3i$; (2) and (4): $\varepsilon_m = -19.3 + 3.5i$. $\lambda_{vac} = 0.6328$ μm, and the angle of incidence for the gap is $\theta_0 = 0$. Curves 1 and 2 are normalized to the amplitude at the local minimum, while curves 3 and 4 start from the same (arbitrary) value of the electric field amplitude.

It has also been shown that the local field enhancement during adiabatic nano-focusing in sharp metal wedges is typically several times smaller than in similar tapered gaps [6]. Thus, tapered gaps have been demonstrated to be the best option (out of the considered structures so far) for achieving maximal local field enhancement during adiabatic nano-focusing.

3. Non-adiabatic nano-focusing

The adiabatic approximation implies that variations of the wave number of the plasmon within one plasmon wavelength are negligible [3,4,6]. If the groove angle β is relatively large, then noticeable reflections of the plasmon at every distance from the tip may occur [5]. On the one hand, increasing groove angle results in increasing reflective losses, and thus reducing local field enhancement (due to non-adiabaticity of nano-focusing). On the other hand, increasing groove angle results in reducing distance that the plasmon propagates (in achieving the same reduction in gap width), and this leads to decreasing dissipative losses and increasing local field enhancement. Competition of these two opposing mechanisms results in an optimal taper angle at which maximal local field enhancement is achieved.

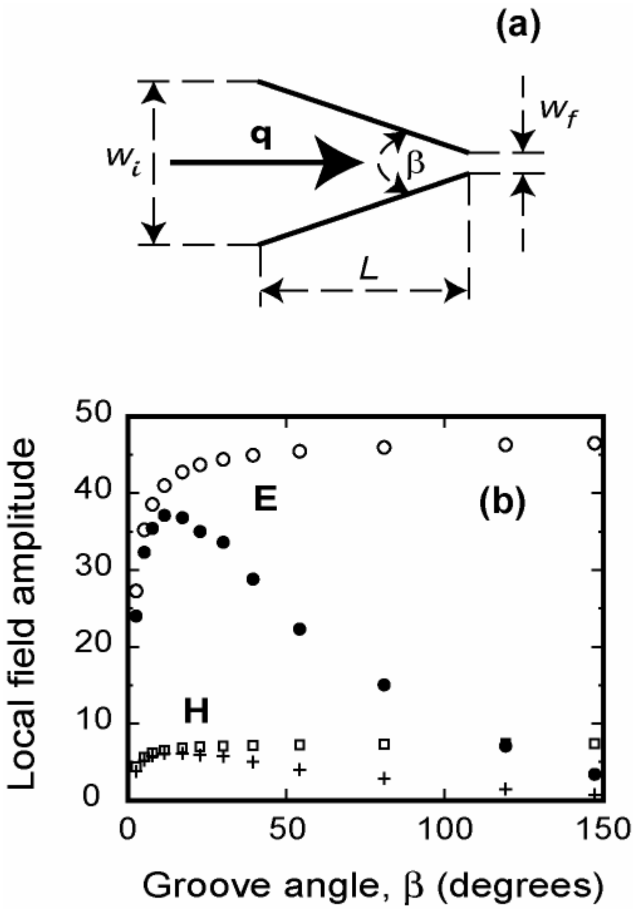

Figure 5. (a) A tapered gap with the entry width w_i, exit width w_f, taper angle β and the overall length L. (b) The dependencies of the magnetic and electric field amplitudes in the middle of the gap at the exit of the taper on angle β, calculated using the rigorous finite-element analysis (crosses and filled circles) and adiabatic approximation (squares and empty circles). The angle of incidence $\theta_0 = 0$, $\lambda_{vac} = 0.6328$ μm, $\varepsilon_m = -19.3 + 0.66i$, $w_i = 1.2$ μm, $w_f = 2$ nm. All the dependencies are normalized to the corresponding amplitude (of the electric and magnetic fields) at the entry of the taper.

For example, the optimal taper angle of the groove was determined in [5] for optimal enhancement of the local magnetic field in the gap plasmon. The results of the similar analysis for the electric field enhancement in a tapered gap are presented in Fig. 5.

In particular, it can be seen that both the electric and magnetic fields experience maximal enhancement at approximately the same optimal taper angle $\beta_{opt} \approx 14^o$ (see crosses and filled circles in Fig. 5b). At the same time, the optimal enhancement of the electric field appears to be much stronger than that of the magnetic field, which is in agreement with the previous analysis conducted in the adiabatic approximation (see above). It can also be seen that the adiabatic approximation gives a good agreement with the rigorous numerical dependencies for the taper angles that are smaller than the optimal angle β_{opt}. This is a demonstration that this approximation is sufficiently accurate in a relatively broad range of taper angles, even if formal adiabatic condition is not very well satisfied (in the considered structure, this condition gives $\beta < 7^o$ [4]).

4. Conclusions

In conclusion, this paper shows that the electric field experiences much stronger local enhancement, compared to the magnetic field, during nano-focusing of surface plasmons in metallic nano-structures. Critical structural and material parameters, such as dissipation in the metal and taper angle, for achieving significant local electric field enhancement were determined and discussed. Numerical analysis of non-adiabatic nano-focusing in tapered metallic gaps demonstrated the existence of an optimal taper angle that corresponds to a maximal possible field enhancement in a taper between fixed entry and exit widths.

Superiority of nano-gaps compared to tapered wedges and cones has been demonstrated for achieving maximal enhancement of the local plasmonic electric field. This feature of nano-focusing in gaps makes it especially promising for the development of applications in near-field microscopy, non-linear plasmonics, effective delivery of electromagnetic energy to the nano-scale, including nano-optical devices, quantum dots, single molecules, etc. Strong electric field enhancement will be especially useful for new optical sensors (e.g., based on surface-enhanced Raman spectroscopy combined with nano-focusing).

5. Acknowledgements

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