
Local Polynomial Modelling and Its Applications

J. Fan

*Department of Statistics
University of North Carolina
Chapel Hill, USA*

and

I. Gijbels

*Institute of Statistics
Catholic University of Louvain
Louvain-la-Neuve, Belgium*



CHAPMAN & HALL

London · Weinheim · New York · Tokyo · Melbourne · Madras

Contents

Preface	xiii
1 Introduction	1
1.1 From linear regression to nonlinear regression	1
1.2 Local modelling	4
1.3 Bandwidth selection and model complexity	7
1.4 Scope of the book	9
1.5 Implementation of nonparametric techniques	11
1.6 Further reading	12
2 Overview of existing methods	13
2.1 Introduction	13
2.2 Kernel estimators	14
2.2.1 Nadaraya-Watson estimator	14
2.2.2 Gasser-Müller estimator	15
2.2.3 Limitations of a local constant fit	17
2.3 Local polynomial fitting and derivative estimation	18
2.3.1 Local polynomial fitting	19
2.3.2 Derivative estimation	22
2.4 Locally weighted scatter plot smoothing	22
2.4.1 Robust locally weighted regression	24
2.4.2 An example	26
2.5 Wavelet thresholding	27
2.5.1 Orthogonal series based methods	28
2.5.2 Basic ingredient of multiresolution analysis	31
2.5.3 Wavelet shrinkage estimator	34
2.5.4 Discrete wavelet transform	35
2.6 Spline smoothing	39
2.6.1 Polynomial spline	40
2.6.2 Smoothing spline	43

2.7	Density estimation	46
2.7.1	Kernel density estimation	46
2.7.2	Regression view of density estimation	50
2.7.3	Wavelet estimators	52
2.7.4	Logspline method	54
2.8	Bibliographic notes	55
3	Framework for local polynomial regression	57
3.1	Introduction	57
3.2	Advantages of local polynomial fitting	60
3.2.1	Bias and variance	61
3.2.2	Equivalent kernels	63
3.2.3	Ideal choice of bandwidth	66
3.2.4	Design adaptation property	68
3.2.5	Automatic boundary carpentry	69
3.2.6	Universal optimal weighting scheme	74
3.3	Which order of polynomial fit to use?	76
3.3.1	Increases of variability	77
3.3.2	It is an odd world	79
3.3.3	Variable order approximation	80
3.4	Best linear smoothers	84
3.4.1	Best linear smoother at interior: optimal rates and constants	84
3.4.2	Best linear smoother at boundary	89
3.5	Minimax efficiency of local polynomial fitting	91
3.5.1	Modulus of continuity	92
3.5.2	Best rates and nearly best constant	94
3.6	Fast computing algorithms	94
3.6.1	Binning implementation	96
3.6.2	Updating algorithm	99
3.7	Complements	100
3.8	Bibliographic notes	105
4	Automatic determination of model complexity	109
4.1	Introduction	109
4.2	Rule of thumb for bandwidth selection	110
4.3	Estimated bias and variance	113
4.4	Confidence intervals	116
4.5	Residual squares criterion	118
4.5.1	Residual squares criterion	118
4.5.2	Constant bandwidth selection	119
4.5.3	Variable bandwidth selection	122

4.5.4	Computation and related issues	122
4.6	Refined bandwidth selection	123
4.6.1	Improving rates of convergence	123
4.6.2	Constant bandwidth selection	123
4.6.3	Variable bandwidth selection	124
4.7	Variable bandwidth and spatial adaptation	128
4.7.1	Qualification of spatial adaptation	128
4.7.2	Comparison with wavelets	129
4.8	Smoothing techniques in use	132
4.8.1	Example 1: modelling and model diagnostics	133
4.8.2	Example 2: comparing two treatments	136
4.8.3	Example 3: analyzing a longitudinal data set	137
4.9	A blueprint for local modelling	141
4.10	Other existing methods	148
4.10.1	Normal reference method	149
4.10.2	Cross-validation	149
4.10.3	Nearest neighbor bandwidth	151
4.10.4	Plug-in ideas	152
4.10.5	Sheather and Jones' bandwidth selector	153
4.11	Complements	154
4.12	Bibliographic notes	157
5	Applications of local polynomial modelling	159
5.1	Introduction	159
5.2	Censored regression	160
5.2.1	Preliminaries	160
5.2.2	Censoring unbiased transformation	165
5.2.3	Local polynomial regression	170
5.2.4	An asymptotic result	173
5.3	Proportional hazards model	175
5.3.1	Partial likelihood	175
5.3.2	Local partial likelihood	179
5.3.3	Determining model complexity	183
5.3.4	Complete likelihood	187
5.4	Generalized linear models	189
5.4.1	Exponential family models	190
5.4.2	Quasi-likelihood and deviance residuals	193
5.4.3	Local quasi-likelihood	194
5.4.4	Bias and variance	196
5.4.5	Bandwidth selection	197
5.5	Robust regression	199
5.5.1	Robust methods	199

5.5.2	Quantile regression	201
5.5.3	Simultaneous estimation of location and scale functions	207
5.6	Complements	208
5.7	Bibliographic notes	214
6	Applications in nonlinear time series	217
6.1	Introduction	217
6.2	Nonlinear prediction	218
6.2.1	Mixing conditions	218
6.2.2	Local polynomial fitting	220
6.2.3	Estimation of conditional densities	224
6.3	Percentile and expectile regression	228
6.3.1	Regression percentile	229
6.3.2	Expectile regression	230
6.4	Spectral density estimation	233
6.4.1	Smoothed log-periodogram	235
6.4.2	Maximum local likelihood method	237
6.4.3	Smoothed periodogram	242
6.5	Sensitivity measures and nonlinear prediction	243
6.5.1	Sensitivity measures	244
6.5.2	Noise amplification	247
6.5.3	Nonlinear prediction error	247
6.6	Complements	249
6.7	Bibliographic notes	260
7	Local polynomial regression for multivariate data	263
7.1	Introduction	263
7.2	Generalized additive models	265
7.3	Generalized partially linear single-index models	272
7.3.1	Partially linear models	273
7.3.2	Single-index models	274
7.3.3	Generalized partially linear single-index models	276
7.4	Modelling interactions	283
7.4.1	Interactions in generalized additive models	283
7.4.2	Interactions in generalized partially linear single-index models	287
7.4.3	Multivariate adaptive regression splines	289
7.5	Sliced inverse regression	290
7.6	Local polynomial regression as a building block	295
7.7	Robustness	296
7.8	Local linear regression in the multivariate setting	297

7.8.1	Multivariate local linear regression estimator	297
7.8.2	Bias and variance	301
7.8.3	Optimal weight function	302
7.8.4	Efficiency	303
7.9	Bibliographic notes	304
References		307
Author index		330
Subject index		336