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Local Quantum Mechanics and Lorentz Transformation

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The transformation property under the Poincaré group of the recently proposed local quantum mechanics is expressed by depending on a correspondence with the quantum field theory in terms of ten generators of infinitesimal transformations in the state-vector space for a simple example, in the continuum approximation which represents the finite momentum degree of freedom by a continuous density function. They do not generate the representation of the Poincaré group but seven of them generate the unitary representation of the group composed of the 4-dimensional translation and the 3-dimensional rotation around a universal timelike vector. The deviation from the Lorentz invariance is due to the finiteness of the momentum degree of freedom of the system and considered to have observable effects in ultrahigh energy phenomena.

§ 1. Introduction

The local quantum mechanics proposed recently[1),2)] is the quantum mechanics based on a theory of finite degree of freedom, of which the state-vector space describing a microscopic world is associated with each macroscopic space-time point which represents the space-time position of the observer, a measuring apparatus being macroscopic, for the microscopic world. The relation between different state-vector spaces associated with different space-time points is given by a dynamical principle. In our picture, the space-time description is valid only for the macroscopic world and we, the observer, cannot move in the microscopic world. In this sense, the microscopic world is non-spacio-temporal since the feature characterizing space-time in distinction from other abstract spaces introduced in physical theories is that we move in space-time.[3),5)]

In the quantum field theory being the space-time description of elementary particles, the degree of freedom of the dynamical system——a set of points specifying independent dynamical variables——is given by a spacelike surface extended unlimitedly in space-time. The infiniteness of the degree of freedom is required by the Lorentz invariance. In the local quantum mechanics, the degree of freedom of the system is finite and is given in a finite microscopic world associated with each space-time point. Thus, in the quantum field theory being Lorentz invariant, a microscopic object is described in an unlimitedly extended macroscopic world, while in the local quantum mechanics, the microscopic object is described in a finite local microscopic world, at the expense of the Lorentz invariance. The local quantum mechanics is shown to tend to the quantum field theory in an idealized limit of the infinite degree of freedom.

The descriptions of a phenomenon by two observers in different inertial systems are assumed to be connected by a Lorentz transformation also in the local quantum

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mechanics. In this sense, the local quantum mechanics is 'relativistic' though it is not Lorentz invariant. In order to be shown that the local quantum mechanics is 'relativistic', the unitary transformation containing ten parameters which corresponds to the Poincaré group should be given in the state-vector space. In this paper, ten generators for the unitary transformation are given by taking a simple example, the system of non-interacting neutral spin-zero particles in the continuum approximation which represents the finite degree of freedom by a continuous density function. They do not generate the representation of the Poincaré group but seven of them generate the representation of the group composed of the 4-dimensional translation and the 3-dimensional rotation around a universal timelike vector. The deviation from the Lorentz invariance is due to the finiteness of the momentum degree of freedom of the dynamical system. In the local quantum mechanics, the generators for the continuous group exists only in the continuum approximation due to the discreteness and the limited localizability of the degree of freedom. The Poincaré invariance of the physical theory is originally established in the macroscopic world and, in the local quantum mechanics, the macroscopic world and the microscopic world are separated so that the continuum approximation for the Poincaré transformation (the impossibility of the exact space-time description based on the macroscopic space-time) in the microscopic world does not lead immediately to inconsistencies,[3),4)] and it has a sufficient precision for practical problems.

The degree of freedom in the 'relativistic' theory should be given in a Minkowskian space. The Minkowskian space has no finite subspace (except for a point and the whole space). In order to specify a finite subspace in the Minkowskian space 'relativistically', a universal timelike vector N_μ should be introduced.[6)] The finite degree of freedom is specified in the N-system in which $N_\mu=(1,0,0,0)$. From a cosmological consideration, the relative velocity between our laboratory system on the earth and the N-system is inferred to be of the order of $10^{-3}c$ (c is the light velocity).[7)] Therefore, when thinned momenta in the density function of the momentum degree of freedom are taken to be sufficiently high, low energy phenomena are practically insensible to the deviation from the Lorentz invariance due to the existence of the universal vector N_μ.

It would be difficult to distinguish physically between a great number of things surpassing our counting ability and an infinite number of things. The motive of the theory of finite degree of freedom, on which the local quantum mechanics is based, is the inquiry whether an infinite number of variables are indispensable for the physical theory or not. A purpose of the local quantum mechanics is the investigation whether the finiteness principle is supported or excluded by experimental facts so that the possibility of the finite degree of freedom should be pursued until it is excluded. The trouble is that the mathematically complete unified formulation of the local quantum mechanics by itself is difficult for lack of symmetries caused by its finite degree of freedom. Consequently, the local quantum mechanics depends on a correspondence principle that the local quantum mechanics coincides with the quantum field theory in the idealized limit of the infinite degree of freedom which is considered as the continuum and no-boundary approximation of the finite degree of freedom.[1),2)] Models corresponding to established models or promising models (e.g., the Higgs model) in the quantum field theory are considered to be described in the local quantum mechanics by replacing the infinite degree of freedom in the field operators with the finite degree of freedom through a certain procedure.

In the Lorentz invariant theory, it is impossible to select a special Lorentz system, in other words, if a phenomenon is observed in a Lorentz system, all phenomena connected by Lorentz transformations with the phenomenon are possible to occur in the same Lorentz system. In the local quantum mechanics, a special Lorentz system, the N-system given by the finite degree of freedom, exists so that, in general, two phenomena in the same Lorentz system are not connected by a Lorentz transformation, because the coordinates of the universal vector N_μ are also tranformed by the Lorentz transformation and accordingly the Lorentz transformation is always the relation between different Lorentz systems. The deviation from the Lorentz invariance is considered to be possibly observed in the ultrahigh energy phenomena, for instance, by comparing phenomena observed in cosmic ray and colliding beam experiments.

In § 2, the gist of the local quantum mechanics is given. In § 3, the continuum approximation for the finite degree of freedom is introduced. In § 4, a correspondence principle between the local quantum mechanics in the continuum approximation and the quantum field theory as an idealized limit is mentioned. In § 5, the transformation property for the Poincaré group of the local quantum mechanics is expressed by depending on the correspondence principle in terms of the generators of infinitesimal transformations in the continuum approximation in the state-vector space. In § 6, the deviation from the Lorentz invariance is discussed.

§ 2. Local quantum mechanics of the system of neutral spin-zero particles

The dynamical variables are a finite number of creation and annihilation operators

$$a_k \text{ and } a_k{}^+, \qquad k=1, 2, \cdots, M, \tag{2·1}$$

satisfying the following commutation relations:

$$[a_k, a_{k'}^+]=\delta_{kk'}, \quad [a_k, a_{k'}]=[a_k{}^+, a_{k'}^+]=0. \tag{2·2}$$

They are specified by the wave functions in the $\underline{p}$-space, the momentum space mentioned below, representing the states of the particle being created or annihilated:

$$f_k(\underline{p}), \quad k=1, 2, \cdots, M, \quad \int f_k{}^*(\underline{p})f_{k'}(\underline{p})d^3\underline{p}=\delta_{kk'}, \tag{2·3}$$

which span the one-particle state-vector space.

The $\underline{p}$-space in which the functions (2·3) are given is the hyperplane perpendicular to a universal timelike unit vector N_μ in the Minkowskian p-space (4-dimensional momentum space), that is,

$$\underline{p}_\mu=p_\mu-(p\cdot N)N_\mu, \quad ((a\cdot b)=g^{\mu\nu}a_\mu b_\nu=a_0b_0-a_1b_1-a_2b_2-a_3b_3). \tag{2·4}$$

The p-space is transformed as

$$p_\mu'=a_\mu{}^\nu p_\nu\,, \tag{2·5}$$

under the inhomogeneous Lorentz (Poincaré) transformation of space-time

$$\xi_\mu'=a_\mu{}^\nu\xi_\nu+a_\mu\,. \tag{2·6}$$

The state-vector space of the local quantum mechanics is a vector space of a countable dimension (Hilbert space) spanned by (symbolically)

$$|0\rangle,\ a^+_{k_1}|0\rangle,\ a^+_{k_1}a^+_{k_2}|0\rangle,\cdots,\quad k_1,k_2,\cdots=1,2,\cdots,M\,, \tag{2·7}$$

where $|0\rangle$ is the normalized zero-particle state (vacuum) and $a^+_{k_1}a^+_{k_2}\cdots a^+_{k_n}|0\rangle$ is the n-particle state. The observable corresponding to a physical quantity is an Hermitic operator in the state-vector space composed of dynamical variables.

The function $f_k(\underline{p})$ in (2·3) is taken to have appreciable values only in the vicinity of a point $\underline{p}^k$ satisfying

$$\int(\underline{p}-\underline{p}^k)|f_k(\underline{p})|^2d^3\underline{p}=0\,, \tag{2·8}$$

and its extension is considered to represent the limits of localizability of the point $\underline{p}^k$. The distribution of points $\underline{p}^k$ forming the momentum degree of freedom of the dynamical system is given by

$$\rho(\underline{p})=|F(\underline{p})|^2\,,\quad F(\underline{p})=\sum_k f_k(\underline{p})\,,\quad \left(\int|F(\underline{p})|^2d^3\underline{p}=M\right), \tag{2·9}$$

that is, $\rho(\underline{p})d^3\underline{p}$ gives the number of points $\underline{p}^k$ in the volume $d^3\underline{p}$ at the point $\underline{p}$.

The wave functions (2·3) are expressed equivalently by their Fourier transforms

$$G_k(\underline{x-X})=(2\pi)^{-3/2}\int f_k(\underline{p})\exp[i\underline{p}\cdot(\underline{x-X})]d^3\underline{p},\quad k=1,2,\cdots,M,$$
$$(\underline{x-X})_\mu=(x-X)_\mu-((x-X)\cdot N)N_\mu\,, \tag{2·10}$$

being wave fnuctions on the spacelike plane, $\sigma_N(X)$, in a Minkowskian space, x_μ, containing a point X_μ and perpendicular to the universal vector N_μ. The unit $\hbar=c=1$ is used throughout this paper. The function $G_k(\underline{x-X})$ is the value for x on $\sigma_N(X)$ of

$$G_k(x-X)=(2\pi)^{-3/2}\int f_k(\underline{p})\exp[ip^*\cdot(x-X)]d^3\underline{p}$$
$$p_\mu^*=\underline{p}_\mu+(N\cdot p^*)N_\mu,\quad (N\cdot p^*)=(\underline{p}^2+m^2)^{1/2},\quad (p^{*2}=m^2)\,, \tag{2·11}$$

satisfying $(\Box_x-m^2)G_k(x-X)=0$.

The x-space is identified mathematically with space-time, by the dynamical principle mentioned in the following, in the approximation of neglecting the extension of the function $f_k(\underline{p})$ in (2·11),[2] although the physical meanings of mass points in the macroscopic space-time and those in the microscopic $(x-X)$-space are different.[1] The constant point X is interpreted to correspond to the space-time position of the observer, a measuring apparatus, which prepares the state. In this sense, the state-vector space is associated with each point of space-time. (State-vector spaces associated with different X are isomorphic, since they are identical in the p-representation.)

The $(x-X)$-space is transformed under the Poincaré transformation (2·6) as

$$(x'_\mu - X'_\mu) = a_\mu{}^\nu (x_\nu - X_\nu)\,. \tag{2·12}$$

The $(x-X)$-space is independent of the translation of space-time. This means that we do not move in the $(x-X)$-space as well as in the p-space (2·5). The $(x-X)$-space does not express the motion of the observer but expresses the degree of freedom of the dynamical system by giving in it the functions $G_k(\underline{x-X})$ equivalent to $f_k(\underline{p})$, $k=1, 2, \cdots, M$, specifying dynamical variables a_k and $a_k{}^+$ in (2·1). The function $G_k(\underline{x-X})$ is confined in a finite region of $(\underline{x-X})$-space of dimensions of the order of the inverse of the extension of the function $f_k(\underline{p})$.

The dynamical principle is as follows. When an observer (a measuring apparatus) at a space-time point ξ_1 measures a complete set of observables $\{A_m\}$ and gets a set of results $\{A'_m\}$, the expectation value $\langle A \rangle$ of an observable A in the ensuing measurement by an observer at a space-time point ξ is given by

$$\langle A \rangle = (\Psi(\xi), A\Psi(\xi))\,,$$
$$\Psi(\xi) = P(\xi)U(\xi, \xi_1)\Psi(\{A'_m\}; \xi_1)\,, \tag{2·13}$$

where (Ψ, Ψ') is the inner product of two state vectors Ψ and Ψ' in a local state-vector space associated with a space-time point, $P(\xi)$ is the projection operator to the state-vector space associated with ξ ($P(\xi)$ relates to the efficiency of the measurement.[2)]), $\Psi(\{A'_m\}; \xi_1)$ is the normalized eigenvector of $\{A_m\}$ corresponding to the eigenvalues $\{A'_m\}$ in the state-vector space associated with ξ_1 and $U(\xi, \xi_1)$ is a unitary operator given by

$$U(\xi, \xi_1) = \exp[i\theta(\xi, \xi_1)]\exp[iP\cdot(\xi - \xi_1)]\,, \tag{2·14}$$

where $\theta(\xi, \xi_1)$ is a possible relative phase of state-vectors $\Psi(\{A'_m\}; \xi_1)$ and $\Psi(\xi)$ in the state-vector spaces associated with ξ_1 and ξ, respectively, and P_μ are observables representing the energy and momentum (4-dimensional momentum) of the system being assumed to have the forms

$$P_\mu = \sum\sum_k p_\mu{}^{*k} a_k{}^+ a_k + N_\mu H\,, \quad ((p^{*k})^2 = m^2) \tag{2·15}$$

where the extra Σ means the summation over kinds of particles and H is an observable (a scalar for the rotation around the universal vector N_μ) representing the interaction. In this picture, the world changes along the universal time represented by the timelike vector N_μ.

§ 3. Continuum approximation for the finite degree of freedom

The following operator of the function of $(x-X)$ can be used as the dynamical variable of the system of neutral spin-zero particles, instead of a_k, $a_k{}^+$, $k=1, 2, \cdots, M$, in (2·1):

$$\phi(x-X)=\sum_{k=1}^{M}\frac{1}{\sqrt{2\omega_k}}\{a_kG_k(x-X)+a_k^+G_k^*(x-X)\}, \tag{3·1}$$

where $\omega_k=((\underline{p}^k)^2+m^2)^{1/2}$ and $G_k(x-X)$ is given by (2·11).

The continuum approximation for the finite degree of freedom is the replacement of the functions $f_k(\underline{p})$, $k=1, 2, \cdots, M$, in (2·3) specifying dynamical variables a_k and a_k^+ in (2·1) with the continuously distributed points $\underline{p}^k$ in the $\underline{p}$-space with the density $\rho(\underline{p})$ in (2·9). In this approximation, the functions $f_k(\underline{p})$ and $G_k(x-X)$ are replaced, apart from normalizations, by

$$f_k(\underline{p})\Rightarrow\delta(\underline{p}-\underline{p}^k) \qquad \text{and} \tag{3·2}$$

$$G_k(x-X)\Rightarrow(2\pi)^{-3/2}\exp[p^k\cdot(x-X)], \quad ((p^k)^2=m^2) \tag{3·3}$$

respectively, and the operator (3·1) is replaced, apart from the normalization, by

$$\phi(x-X)\Rightarrow\varphi_\rho(x-X)=(2\pi)^{-3/2}\int\rho(\underline{p})d^3\underline{p}\frac{1}{\sqrt{2\omega(\underline{p})}}$$
$$\times(a_\rho(\underline{p})e^{ip^*\cdot(x-X)}+a_\rho^+(\underline{p})e^{-ip^*\cdot(x-X)}), \tag{3·4}$$

where $\omega(\underline{p})=p_0^*=(\underline{p}^2+m^2)^{1/2}$, and, $a_\rho(\underline{p})$ and $a_\rho^+(\underline{p})$ are operators in the continuum approximation corresponding to a_k and a_k^+, and satisfy the following commutation relations:

$$[a_\rho(\underline{p}), a_\rho^+(\underline{p}')]=\frac{1}{\rho(\underline{p})}\delta(\underline{p}-\underline{p}'), \quad [a_\rho(\underline{p}), a_\rho(\underline{p}')]=[a_\rho^+(\underline{p}), a_\rho^+(\underline{p}')]=0, \tag{3·5}$$

where $(1/\rho(\underline{p}))\delta(\underline{p}-\underline{p}')$ satisfying $\int\rho(\underline{p})d^3\underline{p}[(1/\rho(\underline{p}))\delta(\underline{p}-\underline{p}')]=1$ is the continuum approximation of $\delta_{kk'}$ satisfying $\sum_{k'}\delta_{kk'}=1$.

§ 4. Idealized limit and a correspondence principle

The local quantum mechanics has an idealized limit of the quantum field theory and can utilize its Lagrangean formalism by means of a correspondence principle. In order to have the idealized limit, it is convenient to introduce the renormalized dynamical variable, $\varphi_g(x-X)$, obtained from the operator $\varphi_\rho(x-X)$ in (3·4) by

$$\varphi_g(x-X)=\frac{1}{\sqrt{\rho(0)}}\varphi_\rho(x-X)$$
$$=(2\pi)^{-3/2}\int g(\underline{p})d^3\underline{p}\frac{1}{\sqrt{2\omega(\underline{p})}}(a_g(\underline{p})e^{ip^*\cdot(x-X)}+a_g^+(\underline{p})e^{-ip^*\cdot(x-X)}), \tag{4·1}$$

where $g(\underline{p})=\rho(\underline{p})/\rho(0)$, ($\rho(0)$ is $\rho(\underline{p})$ at $\underline{p}=0$), $a_g(\underline{p})=\sqrt{\rho(0)}\,a_\rho(\underline{p})$, $a_g^+(\underline{p})=\sqrt{\rho(0)}\,a_\rho^+(\underline{p})$ and, $a_g(\underline{p})$ and $a_g^+(\underline{p})$ satisfy

$$[a_g(\underline{p}), a_g^+(\underline{p}')]=\frac{1}{g(\underline{p})}\delta(\underline{p}-\underline{p}'), \quad [a_g(\underline{p}), a_g(\underline{p}')]=[a_g^+(\underline{p}), a_g^+(\underline{p}')]=0. \tag{4·2}$$

The function $g(\underline{p})$ is unity at $\underline{p}=0$ and zero at $\underline{p}=\infty$. It is a cutoff function for the momentum degree of freedom.

The dynamical variable $\varphi_g(x-X)$ of the local quantum mechanics tends, in the limit

$$\rho(\underline{p})\to\infty \quad \text{with} \quad g(\underline{p})=\rho(\underline{p})/\rho(0)\to 1\,, \tag{4·3}$$

to the field operator in the quantum field theory given by

$$\varphi(x-X)=(2\pi)^{-3/2}\int d^3\underline{p}\,\frac{1}{\sqrt{2\omega(\underline{p})}}(a(\underline{p})e^{ip^*\cdot(x-X)}+a^+(\underline{p})e^{-ip^*\cdot(x-X)})\,, \tag{4·4}$$

where $a(\underline{p})$ and $a^+(\underline{p})$ satisfy the commutation relations

$$[a(\underline{p}),\, a^+(\underline{p}')]=\delta(\underline{p}-\underline{p}'),\quad [a(\underline{p}),\, a(\underline{p}')]=[a^+(\underline{p}),\, a^+(\underline{p}')]=0\,. \tag{4·5}$$

The operator $\varphi_g(x-X)$ contains the universal vector N_μ specifying the $\underline{p}$-space, (2·4), on which a finite number of points ($\int\rho(\underline{p})d^3\underline{p}=\int\rho(0)g(\underline{p})d^3\underline{p}=M$) representing the momentum degree of freedom are given. In the limit (4·3), the momentum degree of freedom is represented by the invariant hypersurface

$$(p^*)^2=(\omega(p))^2-\underline{p}^2=m^2\,, \tag{4·6}$$

so that, in $\varphi(x-X)$, the vector N_μ is an arbitrary timelike vector.

The density $\rho(\underline{p})$ of discrete points $\underline{p}^k$ in (2·9) is considered to be of the order of the inverse of the extension of the function $f_k(\underline{p})$ in the $\underline{p}$-space which gives the order of extension of the function $G_k(\underline{x-X})$, (2·10) and (3·1), in the $(\underline{x-X})$-space. Then $\rho(0)$ is considered to give the order of dimensions of the local microscopic world. In the operator $\varphi_g(x-X)$, (4·1), the points $\underline{p}^k$ are distributed continuously (the continuum approximation) but with the condition $\int\rho(\underline{p})d^3\underline{p}=M$ so that, also in this case, $\rho(0)$ is considered to give the order of dimensions of the microscopic world in which the dynamical variable $\varphi_g(\underline{x-X})$ is given. In the limit (4·3) of the operator $\varphi_g(x-X)$, the microscopic world extends over the whole $(\underline{x-X})$-space because of $\rho(0)\to\infty$. In this limit, all $(\underline{x-X})$-spaces associated with different $\underline{X}$ coincide with the whole $(\underline{x-X})$-space and, in this case, the vector N_μ which determines the $(\underline{x-X})$-space with X is an arbitrary timelike vector. Then the $(\underline{x-X})$-space in the field operator $\varphi(x-X)$ is an arbitrary unlimitedly extended spacelike plane and all state-vector spaces associated with different space-time points in the local quantum mechanics tend to a state-vector space in the quantum field theory.

In the next section, the transformation property of the local quantum mechanics for the Poincaré group is obtained with the aid of the correspondence between the operator $\varphi_g(x-X)$ in the local quantum mechanics and the field operator in the quantum field theory (a correspondence principle).

§ 5. Transformation property for the Poincaré group

The local quantum mechanics has the limit of the quantum field theory being invariant under the Poincaré group. In this limit, the dynamical variable $\varphi_g(x-X)$, (4·1), of the system of neutral spin-zero particles tends to the real scalar field operator $\varphi(x-X)\equiv\varphi(x)$, (4·4), of which the Lagrangean density is

$$L(x)=\frac{1}{2}(\partial^\mu\varphi\partial_\mu\varphi-m^2\varphi^2),\quad \partial^\mu=\partial/\partial x_\mu\,, \tag{5·1}$$

which derives the equation of motion

$$(\Box_x-m^2)\varphi(x)=0 \tag{5·2}$$

by the variational principle $\delta\int L(x)d^4x=0$.

The generators in the state-vector space of the Poincaré group,

$$P^\mu=i\partial^\mu \quad\text{and}\quad M^{\mu\nu}=i(x^\mu\partial^\nu-x^\nu\partial^\mu)\,, \tag{5·3}$$

are given by

$$P^\mu=\int d^3\underline{x}\Theta^{\mu 0}(x) \quad\text{and}\quad M^{\mu\nu}=\int d^3\underline{x}(x^\mu\Theta^{\nu 0}-x^\nu\Theta^{\mu 0})\,, \tag{5·4}$$

where $\Theta^{\mu\nu}$ is the energy-momentum tensor density defined by

$$\Theta^{\mu\nu}(x)=\frac{1}{2}(\partial^\mu\varphi\partial^\nu\varphi+\partial^\nu\varphi\partial^\mu\varphi)-g^{\mu\nu}L(x)\,. \tag{5·5}$$

The conservations of P^μ and $M^{\mu\nu}$, $\partial_0 P^\mu=0$ and $\partial_0 M^{\mu\nu}=0$, are shown by assuming that $\Theta^{\mu\nu}(x)=0$ at remote points in space.

By substituting $\varphi(x-X)$ in (4·4) into P^μ and $M^{\mu\nu}$ in (5·4), the momentum representations of the generators (5·3) are obtained. They are, except for c-numbers,

$$P^0=\int d^3\underline{p}\,\omega(\underline{p})a^+(\underline{p})a(\underline{p})\,,$$

$$P^k=\int d^3\underline{p}\,\underline{p}^k a^+(\underline{p})a(\underline{p})\,,$$

$$M^{0l}=\frac{1}{2}i\int d^3\underline{p}\,\omega(\underline{p})\left[a^+(\underline{p})\frac{\partial a(\underline{p})}{\partial \underline{p}_l}-\frac{\partial a^+(\underline{p})}{\partial \underline{p}_l}a(\underline{p})\right],$$

$$M^{kl}=\frac{1}{2}i\int d^3\underline{p}\left\{\underline{p}^k\left[a^+(\underline{p})\frac{\partial a(\underline{p})}{\partial \underline{p}_l}-\frac{\partial a^+(\underline{p})}{\partial \underline{p}_l}a(\underline{p})\right]-(k\leftrightarrow l)\right\}, \tag{5·6}$$

where k, $l=1, 2, 3$ and $(k\leftrightarrow l)$ represents the term obtained from the first term by interchanging k and l.

The generators P^μ and $M^{\mu\nu}$ give the infinitesimal transformations of the dynamical variable $\varphi(x)$,

$$i\partial^\mu\varphi(x)=[\varphi(x), P^\mu]\,,$$

$$i(x^\mu\partial^\nu-x^\nu\partial^\mu)\varphi(x)=[\varphi(x), M^{\mu\nu}]\,, \tag{5·7}$$

and satisfy the commutation relations

$$[P^\mu, P^\nu]=0\,,$$

$$[M^{\mu\nu}, P^{\rho}]=-i(g^{\mu\rho}P^{\nu}-g^{\nu\rho}P^{\mu}),$$

$$[M^{\mu\nu}, M^{\rho\sigma}]=-i(g^{\mu\rho}M^{\nu\sigma}-g^{\nu\rho}M^{\mu\sigma}+g^{\nu\sigma}M^{\mu\rho}-g^{\mu\sigma}M^{\nu\rho}), \tag{5·8}$$

which show that P^{μ} and $M^{\mu\nu}$ generate the unitary representation of the Poincaré group in the state-vector space.

The correspondence principle considers that the generators $P_g{}^{\mu}$ and $M_g{}^{\mu\nu}$ of the infinitesimal Poincaré transformation for the local quantum mechanics are obtained by substituting $\varphi_g(x-X)$ in (4·1) for $\varphi(x)$ in P^{μ} and $M^{\mu\nu}$ in (5·4). The operator $\varphi_g(x-X)$ satisfies $(\Box_x-m^2)\varphi_g(x-X)=0$ being the variational equation $\delta\int L(x)d^4x=0$ of the Lagrangean $L(x)$ in (5·1). Then $P_g{}^{\mu}$ and $M_g{}^{\mu\nu}$ are given, except for c-numbers, by

$$P_g{}^0=\int g(\underline{p})d^3\underline{p}\,\omega(\underline{p})a_g{}^+(\underline{p})a_g(\underline{p}),$$

$$P_g{}^k=\int g(\underline{p})d^3\underline{p}\,\underline{p}^k a_g{}^+(\underline{p})a_g(\underline{p}),$$

$$M_g{}^{0l}=\frac{1}{2}i\int g(\underline{p})d^3\underline{p}\,\omega(\underline{p})\left[a_g{}^+(\underline{p})\frac{\partial a_g(\underline{p})}{\partial \underline{p}_l}-\frac{\partial a_g{}^+(\underline{p})}{\partial \underline{p}_l}a_g(\underline{p})\right],$$

$$M_g{}^{kl}=\frac{1}{2}i\int g(\underline{p})d^3\underline{p}\left\{\underline{p}^k\left[a_g{}^+(\underline{p})\frac{\partial a_g(\underline{p})}{\partial \underline{p}_l}-\frac{\partial a_g{}^+(\underline{p})}{\partial \underline{p}_l}a_g(\underline{p})\right]-(k\leftrightarrow l)\right\}, \tag{5·9}$$

where k, $l=1, 2, 3$ and $(k\leftrightarrow l)$ is the term obtained from the first term by interchanging k and l. By these generators, the transformation property of the local quantum mechanics for the Poincaré transformation is defined.

The infinitesimal Poincaré transformation of the dynamical variable $\varphi_g(x-X)=\varphi_g(\eta)$ in the local quantum mechanics is given by

$$i\frac{\partial}{\partial \eta_{\mu}}\varphi_g(\eta)=[\varphi_g(\eta), P_g{}^{\mu}], \qquad (\eta=x-X) \tag{5·10}$$

$$i\left(\eta^{\mu}\frac{\partial}{\partial \eta_{\nu}}-\eta^{\nu}\frac{\partial}{\partial \eta_{\mu}}\right)\varphi_g(\eta)+(2\pi)^{-3/2}i\int\left[\frac{1}{2}\left(p^{\mu}\frac{\partial}{\partial p_{\nu}}-p^{\nu}\frac{\partial}{\partial p_{\mu}}\right)g(\underline{p})\right]d^3\underline{p}\frac{1}{\sqrt{2\omega(\underline{p})}}$$
$$\times(a_g(\underline{p})e^{ip^*\cdot(x-X)}+a_g{}^+(\underline{p})e^{-ip^*\cdot(x-X)})$$
$$=[\varphi_g(\eta), M_g{}^{\mu\nu}]. \tag{5·11}$$

From (2·12), $d\eta=d(x-X)=0$ under the translation a_{μ} in (2·6) so that in (5·10), $d\varphi_g(\eta)=(\partial\varphi_g(\eta)/\partial\eta_{\mu})d\eta_{\mu}=0$. The second term on the left-hand side in the equation (5·11) is due to the deviation from the Lorentz invariance of the cutoff function $g(\underline{p})$ and vanishes in the idealized limit $g(p)\to 1$.

The operator $\varphi_g(x-X)=\varphi_g(\eta)$ is rewritten as

$$\varphi_g(\eta)=(2\pi)^{-3/2}\int\frac{d^3\underline{p}}{\omega(\underline{p})}\sqrt{g(\underline{p})}\frac{1}{\sqrt{2}}(\alpha_g(\underline{p})e^{ip^*\cdot\eta}+\alpha_g{}^+(\underline{p})e^{-ip^*\cdot\eta}), \tag{5·12}$$

where $\alpha_g(\underline{p})$ and $\alpha_g{}^+(\underline{p})$ are

$$a_g(\underline{p})=\sqrt{g(\underline{p})\omega(\underline{p})}a_g(\underline{p})\,,\quad a_g{}^+(\underline{p})=\sqrt{g(\underline{p})\omega(\underline{p})}a_g{}^+(\underline{p})\,, \tag{5·13}$$

and satisfy the invariant commutation relations

$$[a_g(\underline{p}), a_g{}^+(\underline{p}')]=\omega(\underline{p})\delta(\underline{p}-\underline{p}')\,,\quad [a_g(\underline{p}), a_g(\underline{p}')]=[a_g{}^+(\underline{p}), a_g{}^+(\underline{p}')]=0\,. \tag{5·14}$$

Under the Lorentz transformation, p^μ and $\eta^\mu=x^\mu-X^\mu$ are transformed by (2·5) and (2·12), respectively. The left-hand side of the equation (5·11) gives the infinitesimal Lorentz transformation of $\varphi_g(\eta)$, the second term of which comes from the Lorentz transformation of the non-invariant factor $\sqrt{g(\underline{p})}$ in the integrand of $\varphi_g(\eta)$ in (5·12).

§ 6. Deviation from the Lorentz invariance

The symmetry property of the local quantum mechanics is determined by the cutoff function $g(\underline{p})$ of the dynamical variable $\varphi_g(x-X)$ in (4·1) or (5·12). The function $g(\underline{p})$ represents the finite momentum degree of freedom and gives the maximum symmetry, the Poincaré invariance, in the idealized limit $g(\underline{p})\to 1$ of the infinite degree of freedom (the quantum field theory).

The most realistic model for the function $g(\underline{p})$ is considered rotational invariant around the universal timelike vector N_μ introduced to define the finite degree of freedom, that is, in the N-system in which $N_\mu=(1, 0, 0, 0)$,

$$\left(\underline{p}^k\frac{\partial}{\partial \underline{p}_l}-\underline{p}^l\frac{\partial}{\partial \underline{p}_k}\right)g(\underline{p})=0, \quad k, l,=1, 2, 3. \tag{6·1}$$

In this case, the generators (5·9) for the infinitesimal Poincaré transformation in the state-vector space satisfy in the N-system the commutation relations

$$\begin{aligned}
&[P_g{}^\mu, P_g{}^\nu]=0\,,\\
&[M_g{}^{kl}, P_g{}^\rho]=-i(g^{k\rho}P_g{}^l-g^{l\rho}P_g{}^k)\,,\\
&[M_g{}^{kl}, M_g{}^{mn}]=-i(g^{km}M_g{}^{ln}-g^{lm}M_g{}^{kn}+g^{ln}M_g{}^{km}-g^{kn}M_g{}^{lm})\,,\\
&\mu, \nu, \rho=0, 1, 2, 3, \quad k, l, m, n=1, 2, 3\,,
\end{aligned} \tag{6·2}$$

and give, in the N-system, the infinitesimal transformations of the dynamical variable $\varphi_g(\eta)$,

$$\begin{aligned}
&i\frac{\partial}{\partial\eta_\mu}\varphi_g(\eta)=[\varphi_g(\eta), P_g{}^\mu]\,,\\
&i\left(\eta^k\frac{\partial}{\partial\eta_l}-\eta^l\frac{\partial}{\partial\eta_k}\right)\varphi_g(\eta)=[\varphi_g(\eta), M_g{}^{kl}]\,.
\end{aligned} \tag{6·3}$$

The relations (6·2) and (6·3) show that, in the case (6·1), the local quantum mechanics is invariant under the group composed of the 4-dimensional translation and the 3-dimensional rotation around the universal timelike vector N_μ being a subgroup of the Poincaré group.

The generators (5·9) generate the unitary representation of the above-mentioned group but do not generate the representation of the Poincaré group owing to the non-Lorentz-invariance due to the finiteness of the momentum degree of freedom represented by the functitn $g(\underline{p})$ as it can be seen from the equation (5·11).

In order that the momentum degree of freedom is finite, its high momentum part should be cut off. The magnitude of momentum depends on the Lorentz system so that the cutoff function $g(\underline{p})$ is specified by means of the introduction of the universal timelike vector N_μ.[6] In this case, there exist always different equivalent descriptions of a phenomenon in different Lorentz systems connected by a Lorentz transformation but, in the same Lorentz system, two phenomena are not, is general, connected by a Lorentz transformation contrary to the case of Lorentz invariant theories, because the universal timelike vector receives also the Lorentz transformation. The deviation from the Lorentz invariance due to the finite degree of freedom is considered to be possibly observed in ultrahigh energy phenomena, although low energy phenomena are insensible to the deviation from the Lorentz invariance when the thinned momenta in the cutoff $g(\underline{p})$ are taken to be sufficiently high.[7]

The nucleon-nucleon collision causing cosmic ray phenomena is the collision of a high energy primary proton with a nucleon approximately at rest in an atmospheric nucleus. The nucleon-nucleon collision in the colliding beam machine is the collision in the center of mass system. To each cosmic ray nucleon-nucleon collision, there corresponds a colliding beam nucleon-nucleon collision connected by a Lorentz transformation. In the Lorentz invariant theory, two nucleon-nucleon collisions connected by a Lorentz transformation cause the same phenomenon. In the local quantum mechanics, it is possible due to the deviation from the Lorentz invariance that, at energies beyond a certain value, two nucleon-nucleon collisions——a cosmic ray collision and a colliding beam collision——connected by a Lorentz transformation cause different phenomena.[8]

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References

1) T. Tati, Annals of the Japan Association of Philosophy of Science **7** (1986), 15.
2) T. Tati, Prog. Theor. Phys. **78** (1987), 996.
3) T. Tati, Prog. Theor. Phys. Suppl. No. 29 (1964), 1.
4) T. Tati, Prog. Theor. Phys. Suppl. No. 76 (1983), 186.
5) T. Tati, Ref. 4), p. 195.
6) T. Tati, Prog. Theor. Phys. **24** (1960), 1.
7) T. Tati, Ref. 4), § 8.
8) T. Tati, Ref. 4), p. 213.