

# **Local Spatiotemporal Modelling of House Prices: A Mixed Model Approach**

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## **Abstract:**

The real estate market has long provided an active application area for spatial-temporal modelling and analysis and it is well known that house prices tend to be not only spatially but also temporally correlated. In the spatial dimension, nearby properties tend to have similar values because they tend to share similar characteristics but house prices tend to vary over space due to differences in these characteristics. In the temporal dimension, current house prices tend to be based on property values from previous years and in the spatial-temporal dimension, the properties on which current prices are based tend to be in close spatial proximity. To date, however, most research work on house prices has adopted either a spatial perspective or a temporal one. Relative few efforts have been devoted to the situation where both spatial and temporal effects coexist. Even fewer analyses have allowed for both spatial and temporal variations in the determinants of house prices. Using 10-years of house price data in Fife, Scotland (2003-2012), this research applies a mixed model approach, semi-parametric geographically weighted regression (GWR), to explore, model and analyse the spatiotemporal variations in the relationships between house prices and associated determinants. The study demonstrates the mixed modelling technique provides better results than standard approaches to predicting house prices by accounting for spatiotemporal relationships at both global and local scales.

**Keywords:** House price; Semi-parametric GWR; Spatiotemporal modelling; GIS

The real estate market has provided an active application area for both spatial and spatial-temporal modelling and analysis (Meen 2001; Goodman and Thibodeau 2003; Bitter, Mulligan, and Dall'erba 2007; Huang, Wu, and Barry 2010; Helbich, Vaz, and Nijkamp 2014; Wu, Li, and Huang 2014). Unlike traditional hedonic price analysis, which usually attempts to explain house prices in terms of property attributes, neighbourhood characteristics and geographic locations through global models, spatial models in general explicitly account for two major spatial effects in housing prices typically ignored in global models: spatial dependency and spatial heterogeneity (Anselin 1988). The former refers to the similarity commonly observed in the values of nearby properties whilst the latter indicates that the processes generating house prices might vary over space, usually reflecting housing submarkets or variations in household preferences (Bitter, Mulligan, and Dall'erba 2007). Parameter estimates from traditional hedonic price models, which represent the relationships between property prices and associated characteristics, can be biased in the presence of spatial effects. As a result, extensive efforts have been devoted to incorporating such spatial effects into hedonic house price analysis and many spatial statistical techniques have been developed in the last a few decades (Anselin, Florax, and Rey 2004).

Several models applied in the spatial analysis of real estate data have been constructed to address spatial dependence and/or spatial heterogeneity. Examples include spatial lag and spatial error models (Anselin 1988; Can 1992; Dubin 1992), and geographically weighted regression (GWR) (Fotheringham, Brunson, and Charlton 2002). Recently, spatiotemporal models have been developed in order to incorporate the temporal dimension into hedonic house price analysis as housing price processes evolve not only over space but also over time (Case et al. 2004; Smith and Wu 2009; Huang, Wu, and Barry 2010; Wu, Li, and Huang 2014).

However, it is worth noting that both spatial effects (spatial dependence and spatial heterogeneity) frequently coexist in many spatial processes (Anselin 1999) and there is strong evidence indicating the presence of both spatial effects in the housing market (Goodman and Thibodeau 2003). In this case, accurate parameter estimates cannot be obtained from either global or local models which consider one effect in isolation from the other. Instead, models capable of addressing both spatial effects become a desirable option. To this end, of primary interest here is to understand both spatial effects in housing price processes through the application of a mixed model method, semi-parametric GWR (Fotheringham, Brunson, and Charlton 2002; Nakaya et al. 2005). Using a 10-year (2003 - 2012) house price dataset in Fife, Scotland, this research seeks to explore, model and analyse spatiotemporal variations in house prices and their relationships with associated determinants. Important in this study is the identification of which relationships tend to be globally stable and which tend to vary over space, and whether spatial variations in house price determinants are temporally stable.

The remainder of the paper is organized as follows. The subsequent section provides a review of spatial statistical approaches for housing price research. This is followed by a description of a 10-year house price dataset in Fife, Scotland. Then, the application of semi-parametric GWR to examine both temporal and spatial variations in the determinants of house prices is described. The paper concludes with the major findings and the significance of this research.

## **Spatial Hedonic Models for House Price Analysis**

Hedonic price analysis has long been widely applied in property assessment (Can 1992; Meen 2001; Fotheringham, Brunson, and Charlton 2002). In general, hedonic price modelling relates the value of goods to a set of their characteristics (Goodman 1998). In real estate studies, hedonic house price models aim to estimate the market value of properties based upon a set of associated characteristics which generally include structural attributes (e.g. number of rooms, floor area and dwelling age), neighbourhood attributes (e.g. quality of public education, unemployed rate and racial diversity) and locational attributes (e.g. proximity to workplaces, accessibility to pleasant landscapes and public facilities) (Basu and Thibodeau 1998). Then property prices can be defined as a function of the above three basic categories of characteristics, which can be expressed as:

$$p = f(S, N, L) \quad (1)$$

Where  $p$  represents property price;  $S$  represents a set of variables describing structural attributes of the property;  $N$  represents a set of neighbourhood characteristics; and  $L$  represents a set of location attributes. The function  $f$  is usually expressed in a traditional linear regression form and calibrated using ordinary least squares (OLS) technique.

However, as mentioned, the hedonic price model in (1) typically ignores the spatial effects commonly existing in housing prices. The awareness of limitations of traditional hedonic price analysis has led to a wide range of models accounting for spatial effects in residential datasets (Tse 2002; Farber and Yates 2006; Bitter, Mulligan, and Dall'erba 2007; Helbich, Vaz, and Nijkamp 2014). In general, such models can be considered global or local according to whether they deal with spatial dependency or spatial heterogeneity. The remainder of this section provides a brief review of two types of models as well as their applications in housing price analysis.

Global spatial models address spatial dependence or spatial autocorrelation in spatial processes. A comprehensive discussion of well-known such models can be found in Anselin (1988), Haining (1990), Anselin, Florax, and Rey (2004) and LeSage and Pace (2009). For example, the widely utilized specification provided by Anselin (1988) assumes spatial autocorrelation is in either the response variable or the error terms, and the corresponding models are usually calibrated by maximum likelihood (ML) rather than OLS technique as some OLS assumptions (e.g. independently and identically distributed residuals) are violated.

Unlike the spatial lag or error model, Tse (2002) specified spatial autocorrelation through the constant term using a stochastic approach.

In addition to the spatial dimension, dependence in time has also received much research interest. For instance, Pace et al. (2000) formulated a spatiotemporal autoregression model and applied it in a study of housing prices in Baton Rouge, Louisiana, by modelling both spatial and temporal dependence in the errors. Gelfand et al. (2004) proposed a class of spatiotemporal hedonic models under a Bayesian statistical framework and examined the spatiotemporal differences related to single versus multiple residential sales. Smith and Wu (2009) developed a spatiotemporal model allowing for both spatial and temporal lag effects, which was applied to study housing price trends in the Philadelphia area, USA.

Although these global spatial models represent a substantial improvement over traditional hedonic models, a major issue is that the housing price processes are assumed to be constant or stationary over space, which is not necessarily the case in reality. In order to capture spatial variations in housing price processes, numerous local spatial models have been proposed. Common local models include the spatial expansion method (Casetti 1972), moving window regression (MWR) (Farber and Yeats 2006), multilevel models (Duncan and Jones 2000) and geographically weighted regression (GWR) (Fotheringham, Brunson, and Charlton 2002). These can be considered to be generalizations of standard linear models, where parameter estimates are allowed to vary over space in order to better represent the processes generating house prices (Fotheringham, Brunson, and Charlton 2002). For example, Farber and Yeats (2006) found GWR outperformed other local modelling approaches with regard to explaining spatial variations in housing prices in a study of the real estate market in Toronto, Canada. Similarly, Bitter, Mulligan, and Dall'Erba (2007) compared two local models, the spatial expansion method and GWR, in a research of housing market in Tucson, Arizona, USA, concluding that GWR is superior in terms of the capability of identifying spatial heterogeneity in several housing attributes. Páez, Long, and Farber (2008) compared MWR, GWR, and the moving windows Kriging (MWK) approaches in relation to different spatial effects, highlighting the importance of market segmentation in housing price processes.

Beyond space, time has also been incorporated into local models to account for temporal effects on housing processes. For example, Crespo (2009) and Huang, Wu, and Barry (2010) extend traditional GWR to a spatiotemporal GWR (GTWR) by developing a spatiotemporal kernel function in local model calibration. Wu, Li, and Huang (2014) further extended GTWR by accounting for auto-correlated effects.

It can be seen, from the discussion above, that existing global and local spatial models are both extensive and diverse. Empirical applications in real estate markets in various spatial settings have shown their effectiveness in explaining spatial dependence or spatial heterogeneity. In contrast, mixed models dealing with both spatial effects have received less attention, particularly for hedonic house price analysis. This research will contribute to the

literature by utilizing a mixed model, semi-parametric GWR, to investigate both global and local relationships between house prices and associated influencing factors, as well as their variations over time.

## Data and Study Area

The data used in this research are provided by Registers of Scotland (ROS) and consist of sales prices for houses during 2003-2012 in Fife, Scotland. Fife is located in southeast Scotland as shown in Figure 1 and it covers an area about 1,325 km<sup>2</sup> with a population around 276/km<sup>2</sup> (estimated in 2012). St Andrews, a historic town renowned as the “home of golf”, is on the northeast coast. It is also home to the University of St Andrews which generates a high demand for accommodations and creates a distinctive local housing market.

Figure 1 about here

The geographical data are geo-referenced points defined by (x, y) coordinates representing the spatial location of houses. Each house has an associated attribute – property value – at the time of transaction. Figure 2 depicts the spatial distribution of house prices for 2012, where the points in Figure 2A represent the location of houses with the heights indicating the relative property values. Shown in Figure 2B is a continuous surface generated from those points, from which the general spatial pattern of house prices can be observed. Residences in the north east coast, mainly clustered around St Andrews, tend to be more expensive than those in rests of Fife. Another area having higher house prices is around Dunfermline, the 2<sup>nd</sup> largest town by population in Fife. All the other areas, in contrast, have relatively lower house prices.

Figure 2 about here

Table 1 summarizes the number of houses sold in each year, the mean house price and the inflation-adjusted mean house price. The number of houses sold peaked in 2006 and then declined rapidly, leading to and following from the economic crash in 2008. Inflation-adjusted house prices peaked in 2008 and declined each year thereafter. As after 2007 the average number of sales per zone drops to about 10, one concern here is about the potential impact of houses per zone on the validity of the zonal average. However, over the study time period (2003-2012), more than 90% of data zones contain <40 properties. Also, the average house prices on each data zone largely have similar distribution over time for the zones containing <40 properties. Thus, the number of houses per zone should not have great impacts in terms of model calibration.

Table 1 about here

Since the structural characteristics for each property are not available, relevant socio-economic data were obtained from the Scottish Neighbourhood Statistics (SNS)<sup>1</sup> in order to help understand the underlying housing price processes. These data are summarized statistics on small-area statistical geographies – data zones – which are nested within local authority boundaries and have a population of 500 – 1,000 household residents. In total there are 453 data zones in Fife. Accordingly, house prices are also aggregated to derive the average house prices for each data zone.

In the subsequent regression analysis, the dependent variable is the average house price within each of the 453 data zones and the definition of the covariates,  $X$ , are presented in Table 2. In addition to neighbourhood characteristics such as population density and crime rates, and property mix variables, two spatial variables, “distance to St Andrews” and “distance to coast”, are considered given the spatial context in the study area. The former accounts for the potential effects from the historic town St Andrews and the latter recognizes buyers’ preference for sea view properties. Also, given the spatial and temporal dependences commonly existing in house prices, a spatiotemporal lag variable is added to the model. A spatial lag can be calculated as the average house price of the neighbours (Anselin 1988). A spatiotemporal lag is therefore defined as the average house price of the neighbours in the previous year. In this case, the spatiotemporal lag is first calculated on neighboring houses and then aggregated at the data zone level.

Table 2 about here

## Methods

This research investigates spatiotemporal variations in housing prices using a mixed spatial model, semi-parametric GWR (Fotheringham, Brunson, and Charlton 2002; Nakaya et al. 2005), which is an extension of a local spatial modelling technique, traditional GWR (Fotheringham, Brunson, and Charlton 2002). In this case, semi-parametric GWR is utilized to examine both global and local spatial relationships between house prices and a set of associated attributes for each year and the temporal variations in the coefficients are obtained through a series of independent cross-sectional estimations. In addition, the performance of three models, the traditional global model, GWR and semi-parametric GWR is compared.

Before defining semi-parametric GWR, it is helpful to first describe a traditional global hedonic model and traditional GWR in the context of housing market studies. A global hedonic house price model can be formulated as in (2):

$$P_i = \sum_j \beta_j X_{ij} + \varepsilon_i \quad (2)$$

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<sup>1</sup> <http://www.sns.gov.uk/>

where  $i$  and  $j$  are index of observations and covariates, respectively;  $P$  represents house prices;  $X$  represents covariates; and  $\beta$  represents parameters associated with the various covariates. According to (2), the parameter  $\beta_j$  is constant across all observations. That is, the relationship between the  $j$ th covariate and house prices is considered invariant over space. In contrast, such relationships are allowed to vary across space in GWR which can be expressed in (3):

$$P_i = \sum_j \beta_{ij}(u_i, v_i)X_{ij} + \varepsilon_i \quad (3)$$

where  $(u_i, v_i)$  represents the geographic location of the  $i$ th observation. Thus, the parameter  $\beta_{ij}$  is a function of  $(u_i, v_i)$ , denoted as  $\beta_{ij}(u_i, v_i)$ . The local parameters  $\beta_{ij}$  are estimated with the aid of data in the neighbourhood, which is usually realized by a weight matrix. Commonly, the weights are defined by Gaussian or bi-square kernel functions where the size of neighbourhood is determined by an optimised bandwidth (e.g. distance or number of nearest neighbours) (Fotheringham, Brunson, and Charlton 2002). As a result, smaller bandwidths indicate more local processes whereas larger bandwidths indicate more regional processes with a bandwidth tending to infinity replicating a global model.

Built upon the above formulations, the semi-parametric GWR model can be defined as in (4):

$$P_i = \sum_k \gamma_k X_{ik} + \sum_j \beta_{ij}(u_i, v_i)X_{ij} + \varepsilon_i \quad (4)$$

where  $k$  is an index of covariates that have a global relationship with house prices and  $j$  is an index of covariates whose relationship to house prices varies spatially. Thus, semi-parametric GWR allows some parameters to be fixed over space and the other parameters to vary across space, representing stationary and non-stationary spatial relationships/processes simultaneously. The model in (4) is usually calibrated using an iterative procedure by estimating global and local parameters in turn repeatedly until some convergence condition is satisfied (Fotheringham, Brunson, and Charlton 2002; Nakaya et al. 2005).

In this research, the semi-parametric GWR model in (4) is used to study the 10-year house price dataset in Fife, Scotland. The aggregated house prices and all the values of covariates are transformed using the natural logarithm function to ensure the parameter estimation is free from scale effects. In other words, the particular mixed hedonic house price model in this research is defined as in (5):

$$\ln P_i = \sum_k \gamma_k \ln X_{ik} + \sum_j \beta_{ij}(u_i, v_i) \ln X_{ij} + \varepsilon_i \quad (5)$$

There are two important issues involved in the calibration of model (5). First, a bandwidth needs to be determined for the weight matrix construction. Also, variables need to be selected as global or local. In this research, a bi-square kernel function is used to define the weight matrix with the bandwidth specified by the number of nearest neighbours. The optimal bandwidth size is chosen such that the corresponding model has the smallest value



for the corrected Akaike information criterion (AICc) (Akaike 1974). AICc is widely applied in model selection with smaller values indicating better models. This procedure is repeated for annual house price data to ensure the best model is found for each year. The second issue is addressed by a local to global variable selection routine which can be summarized as follows:

- Step 1. Define two sets of variables,  $G$  and  $L$ , and initialize  $G = \emptyset$ , and  $L$  contains all the variables. That is,  $L = \{x_1, x_2, \dots, x_{13}\}$ . Construct a GWR model defined in (3) using variable sets  $G$  and  $L$ . Denote this model as *model\_old*;
- Step 2. Solve *model\_old* and get the corresponding AICc, recorded as *AICc\_old*;
- Step 3. Take a variable, e.g.  $x_i$ , out of set  $L$  and put it into set  $G$ . Construct a semi-parametric GWR using the variables defined by  $L$  and  $G$ . Denote this model as *model\_new*;
- Step 4. Solve *model\_new* and get the corresponding AICc, recorded as *AICc\_new*;
- Step 5. If  $AICc\_new < AICc\_old$ , keep  $x_i$  in  $G$  and let  $AICc\_old = AICc\_new$ ; otherwise, put  $x_i$  back to  $L$ ;
- Step 6. Repeat Step 3 – Step 5, until every variable in  $L$  is examined;
- Step 7. If there is at least one variable moved from  $L$  to  $G$  during Step 3 – Step 6, repeat Step 3 – Step 6; otherwise, stop.

It is worth noting that an optimal bandwidth search is implicitly contained in each model calibration procedure, which further complicates the above variable selection because more computation processing is required. The bandwidth search, variable selection and parameter estimation involved in model (5) are all carried out in GWR 4<sup>2</sup>, a package for local spatial modelling and analysis.

Once the parameter estimates are derived, it is critical to assess whether the measured relationships between house prices and associated determinants are intrinsically differences across space or simply caused by random sampling variations. This is carried out through stationarity tests of each local parameter in the semi-parametric GWR models. Specifically, two approaches are employed here: variability tests of local parameter estimates and Monte-Carlo (MC) tests (Fotheringham, Brunson, and Charlton 2002). The former is based on the inter-quartile range (IQR) of local estimates and the standard errors (SE) of global estimates. Empirically we can consider the  $2*SE$  as the expected variations in the values as it contains about 60% of all the estimates. Thus, it indicates a possible non-stationary process if IQR (which includes 50% of values) is larger than  $2*SE$ . The latter measures the variance of the local parameters, which can be defined as in (6):

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<sup>2</sup> <http://www.st-andrews.ac.uk/geoinformatics/gwr/gwr-software/>

$$V_j = \frac{1}{n} \sum_i \left( \hat{\beta}_{ij} - \frac{1}{n} \sum_i \hat{\beta}_{ij} \right)^2 \quad (6)$$

where  $V_j$  is the variance of the  $j$ th parameter;  $n$  is the total number of observations;  $\hat{\beta}_{ij}$  is the local estimate for observation  $i$  and parameter  $j$ . The MC tests are implemented in the following steps:

- Step 1. Obtain local parameter estimates and calculate  $V_j$  for each local parameter;
- Step 2. Rearrange data randomly across the zones (keeping  $Y_i$  and  $X_i$ ) together;
- Step 3. Compute a new set of local parameter estimates based on the rearranged data and calculate  $V_j$ ;
- Step 4. Repeat steps 2 and 3 for  $N$  times, each time computing the variance of the local estimates;
- Step 5. Compare the variance of local parameter estimates in step 1 with those from steps 2 and 3;
- Step 6. The  $p$  value associated with step 1 is then the proportion of variances that lie above that for step 1 in a list of variances sorted high to low.

If there is no significant pattern in the parameters, there should be no significant changes in the variations in the local estimates regardless of the permutation of the observations against their locations. As it is difficult to obtain the null distribution of the variance analytically, the MC method is commonly considered an effective option. Thus,  $N$  values of the variance of a parameter obtained from the MC test represent an experimental distribution, and a  $p$  value (experimental significance level) can be derived by comparing the actual value of the variance against that list of  $N$  values. Generally, IQR test is quite easy and straightforward but is more informal. In contrast, MC test is more rigorous but limited in repetitions due to computational time.

Finally, the coefficient of variation (CV) is employed to investigate the spatiotemporal variations of local parameter estimates. Specifically, a CV is calculated using the local estimates for each year, from which a set of CVs can be derived to demonstrate the spatial variations in the relationships between house prices and the covariates over time. Also, a CV is calculated for each data zone based on the local estimates across the study period (2003-2012), from which the temporal variations in the relationships between house prices and the covariates over space can be generated.

## Results

For the purpose of model comparison, global models, GWR models as well as semi-parametric GWR models are fitted in this research. First, a global model is calibrated to

explore the general relationships between house prices and the associated attributes. Then, results from the semi-parametric GWR model are presented, including estimates for fixed and local parameters with spatiotemporal variations in local parameter estimates highlighted. Finally, the performances of different models are compared using a common model selection criterion AICc.

Parameter estimates for the global model are given by Table 3. In total, nine models are calibrated using OLS for the years 2004 – 2012 because the spatiotemporal lag is not available for year 2003. It should be noted that the spatial lag in this case is temporal and excludes the properties from the current year on the RHS in (5). Thus, it is appropriate to use OLS for estimating the spatiotemporal lag parameter. According to the  $R^2$  (around 0.7), the overall model fit is quite satisfactory for every year except 2007 ( $R^2 = 0.38$ ). Seven variables have reasonably consistent significant effects on house prices:  $x_1$  (population density),  $x_7$  (% of dwellings with 1-3 rooms, i.e. small houses),  $x_8$  (% of dwellings with 7-9 rooms, i.e. big houses),  $x_9$  (% of household ownership),  $x_{11}$  (distance to St Andrews),  $x_{12}$  (distance to coast) and  $x_{13}$  (spatiotemporal lag). For example, the value of  $\hat{\beta}_1$  varies from -0.074 (2007) to -0.040 (2004), which indicates that house prices tend to be lower, *ceteris paribus*, in areas of higher population density. This effect strengthened up to 2007 and thereafter has weakened. Similarly, according to the values of  $\hat{\beta}_{11}$  (-0.088 ~ -0.157), the properties tend to have higher values the closer they are to St Andrews, *ceteris paribus*. In contrast, some variables almost have no significant effects on house prices at all except for a particular year, such as  $x_2$  (% of pensionable age population),  $x_3$  (% of working age population) and  $x_4$  (% of semi-detached properties). The impacts of the other variables are inconsistent.

Table 3 about here

Figure 3 describes the temporal variations in the seven consistently significant parameter estimates. It can be seen that the estimates are reassuringly consistent over time with the exception of the estimates associated with the variables  $x_9$  (% of household ownership) and  $x_{13}$  (spatiotemporal lag) which both show a pronounced spike in value in 2007 as the economic crisis loomed.

Figure 3 about here

The best semi-parametric GWR model is chosen for each year based on the variable selection procedure described in the previous section. This produces for each year a set of spatially varying parameter estimates for those variables whose effect on house prices varies over space and a set of spatially invariant estimates for those variables whose effect on house prices is constant over space. In each year only one or two variables appear to have a constant impact on house prices with the exception of 2006 and 2007 with 4 and 5 parameters being fixed over space. There was no consistency in which variables exhibited a fixed effect over time.

With regard to the local parameters, two tests were undertaken to identify if the spatial variation in their values was significant. For each year, tests based on IQR of local estimates and MC simulation (with repetition  $N=1000^3$ ) are implemented. The results are shown in Table 4. As can be seen, different sets of significant local parameters are found for each year. In general, the local parameters specified by the local to global variable selection routine described in Section 4 do not all have significant local variability. For example, for year 2004, the variable selection routine suggests that only  $x_1$  (population density) is fixed while the IQR test indicates the estimates for another three parameters do not significantly vary across space:  $x_5$  (% of terraced properties),  $x_6$  (% of flats) and  $x_8$  (% of dwellings with 7-9 rooms) and the MC test suggests only four sets of parameter estimates exhibit significant spatial variation –  $\beta_4, \beta_{10}, \beta_{11}$  and  $\beta_{13}$ .

Generally, the results of the IQR and the MC tests are reassuringly similar and where discrepancies exist, the MC test appears to be more rigorous. Three variables exhibit significant spatial variation in their impact on house prices throughout the 9 time periods. These are distance to St Andrews, distance to the coast and the spatial-temporal lag variable. Interestingly, crime rates appear to have a spatially varying impact on house prices up to 2008 but thereafter do not exhibit any significant spatial variation. The remaining variables exhibit no consistency in the significance of the spatial variation of their effects.

Table 4 about here

Accordingly, two variables  $x_{11}$  (distance to St Andrews) and  $x_{13}$  (spatiotemporal lag) are selected for further discussion here because they generally exhibit significant variation in the local estimates. For each variable and for each year, there are 453 local parameter estimates describing the impact of that variable on house prices in the vicinity of location  $i$  in time  $t$ . In Figure 4 the values of each of the 453 estimates across the 9 time periods is connected by a straight line across the 9 time periods. This is done separately for the parameter estimate associated with the variable “Distance to St Andrews” and the parameter estimate associated with the spatiotemporal lag variable. The local parameter estimates show remarkable consistency through time but the most noticeable feature is that for the year 2007, the local parameter estimates exhibit a vastly increased spatial variation. This is the year leading to the housing-led economic crisis when presumably the housing market was in the throes of impending turmoil. It is very interesting that this is exhibited in advance of the full-blown crisis being recognised and the turmoil in the determination of house prices is only for this one time period and not for the full period of the crisis.

Figure 4 about here

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<sup>3</sup> We use  $N=1000$  because of the computational complexity. We also run the MC test with  $N=100$  which gave similar results as those from  $N=1000$ , which indicates the MC test is rigorous.

To examine this effect further, the local parameter estimates for these two variables are mapped for each of the nine periods as shown in Figure 5. However, to aid clarity, only those local parameter estimates which were significant are depicted. In this case, significance is defined using the adjusted critical  $t$  value (Byrne, Charlton, and Fotheringham 2009) equated with an original significance level of 0.05, which addresses the issue of multiple hypothesis testing in GWR.

Figure 5 about here

Figure 5A depicts the significant local parameter estimates for the variable “Distance to St Andrews”. These are all negative indicating the region around St Andrews where house prices fall as distance to St Andrews increases, *ceteris paribus*. In effect, these maps indicate the spatial extent of the local St Andrews housing market in each time period. In general there is remarkable consistency over time in this housing market with the exception of 2007 when it disappears altogether and in 2012 when the southern portion disappears. In 2007, the location of St Andrews had no effect on house prices anywhere in Fife. In 2012 it had an impact only on those houses in an area to the north and west of the city. In other years the area of impact is consistently the whole of north-east Fife with a radius of approximately 20kms from St Andrews. This technique usefully quantifies the spatial extent of the housing market around an urban area and could easily be extended to other features deemed to have an impact on house prices such as airports or pollution sites.

Figure 5B depicts the spatial extent of neighbourhood effects in housing prices which are consistently significant only in the north-east half of Fife. The only exception to this again is the year 2007 when very little spatial lag effect is present anywhere in the county. The interpretation of this is that house prices are strongly related to neighbouring house prices only in the north-east of Fife – in the rest of the county, there is no spatial lag effect present in house prices. This may indicate again two very different housing regimes in the county and indeed, north-east Fife is quite different economically and socially from south-west Fife.

In addition, the extent to which the local estimates vary over space and time is also explored using the CV mentioned previously. Take the variable “spatiotemporal lag” as an example, Figure 6 shows the spatial variation of the CVs over time, where a CV is calculated for the 453 local estimates for every year. It can be seen that the value in 2007 is much higher than those in all the other years, indicating unusual spatial variation in the neighbourhood effects on house prices as observed in Figure 4B. Interestingly, in 2008 when the financial crisis occurred, the CV has the lowest value compared those in the other years. Further, the temporal variation of the CVs for the same variable is shown in Figure 7. In general, the values gradually decline towards southwest Fife, indicating decreasing temporal variations in neighbourhood effects. Also noteworthy is that north Fife has the relatively fewer temporal variations in the local estimates which, as shown in Figure 5B, are consistently significant across the study time period (except 2007).

Figure 6 about here

Figure 7 about here

Finally, a model comparison is undertaken out based on the AICc values as shown in Table 5. The AICc values cannot be compared across years but for each year, the lower the AICc, the better the model fit with differences of 3 or more generally deemed to indicate a significant difference. For all 9 time periods semi-parametric GWR consistently outperforms GWR which in turn is superior to the global model, implying improvements in modelling both global and local relationships underlying housing price processes.

Table 5 about here

## **Discussion and Conclusions**

It is well known that housing markets are characterized by both spatial dependence and spatial heterogeneity. The literature on spatial hedonic house price analysis so far has largely focused on either global models or local models and has either ignored both effects or treated them independently. This research accounts for both spatial effects in housing markets using a mixed model method, semi-parametric GWR. Particularly, spatiotemporal variations in neighbourhood effects (i.e. spatial dependence) on housing prices are investigated. The results demonstrate that semi-parametric GWR is capable of dealing with both global and local spatial relationships and therefore can produce more accurate estimates for parameters in hedonic house price models.

The most important contribution of this research is the specification of both global and local relationships between housing prices and the associated covariates, as well as their variations over both space and time. For example, the spatial/temporal lag is widely used in the global models and the extent of neighbourhood effects are traditionally considered invariant over space. This is reflected in Table 3 where the coefficients of spatiotemporal lag are all positive across the study time period. In fact, such neighbourhood effects can vary over space and time, which is revealed by Figure 5B. That is, only the prices of the residences in north eastern Fife are significantly affected by their neighbours' prices and such effects tend to increase from the northwest to the northeast. Meanwhile, the global parameters also have changed over time. For example, population density is found negatively related to housing prices in the global models (Table 3) but the mixed models suggest that this only holds for years 2004, 2007, 2008 and 2011, and the regression coefficients are only significant for year 2008.

Also worth noting is that this technique has quantified a local housing market effect – in this case on the basis the effect of distance to St Andrews has on house prices. St Andrews is the location of the University of St Andrews and has the unique features such as being the home of the Royal and Ancient Golf Club. Given the consistent high accommodation

demand, St Andrews has formed a distinct housing market and the housing prices are usually higher than those in other areas. Figure 5A describes the spatial extent of this effect. That is, generally only the houses up to 20km away from St Andrews are subject to such an effect in terms of housing prices. This technique could easily be expanded to other urban centres to compare their impact on house prices and to other features such as airports and landfill sites which are suspected to cause decreases in house prices but to what distance is largely unknown.

Another interesting finding is the distinction of spatiotemporal patterns before and after the financial crisis. As is well known, the residential real estate markets suffered greatly from the financial and economic crisis in 2008. This is reflected in Table 1 where the housing prices have an increasing trend before 2008 and a declining trend afterwards. Particularly, based on the unusual parameter estimates from both the global model (Table 3 and Figure 3) and the mixed model (Figures 4, 5 and 6), it would appear that the housing market in 2007 had detected some signs of the coming crisis and house prices in that year suddenly became much less predictable. Furthermore, Figures 5B suggest quite different spatial distributions of significant local estimates before and after the financial crisis. One concern here is that the “breakdown” of parameter estimates in 2007 might be caused by the covariates as different sets of coefficients are held constant in different years in the semi-parametric GWR models. This is further investigated by the variations in local estimates obtained from GWR, particularly for the two variables “Distance to St Andrews” and “Spatiotemporal lag”. The results from GWR are very similar to those shown in Figures 4 and 5, which suggest the covariates in semi-parametric GWR, particularly the fixed variables, have little impacts on the variations in local estimates.

One limitation of this research is that both housing prices and associated influencing characteristics are aggregated data based on data zones. It is well recognized that analyses using aggregated data are subject to the choice of geographic units, and the resulting conclusions might conceal the detailed information for underlying individual objects, which is known in geography as the modifiable area unit problem (MAUP) (Openshaw, 1984). Nevertheless, given that data zones are the only available geographic districts containing local statistics and they have reasonable sizes (covering 500 – 1,000 household residents), conclusions from this research still can provide useful insights regarding the general spatiotemporal patterns in housing prices and social-economic factors.

In summary, housing market remains a great concern of government, real estate developers and general population as it is a main component of macro-economy and also closely related to social equity. GIS based spatial analysis, particularly spatial statistics, offers a set of powerful tools to study housing price processes by explicitly accounting for spatial dependency and spatial heterogeneity. This research demonstrates the advantages of a mixed model method, semi-parametric GWR, in modelling both spatial effects as well as both global and local relationships in housing price processes.

Though semi-parametric GWR in this research has been used to study the spatiotemporal variations in the local housing market in Fife, Scotland, it can be employed in a wider field of hedonic price modelling wherever both global and local spatial relationships are of concern. Undoubtedly, semi-parametric GWR offers an effective way for spatial analysis and modelling by its capability of capturing both spatial stationary and non-stationary processes.

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Figure 1 Study area: Fife, Scotland

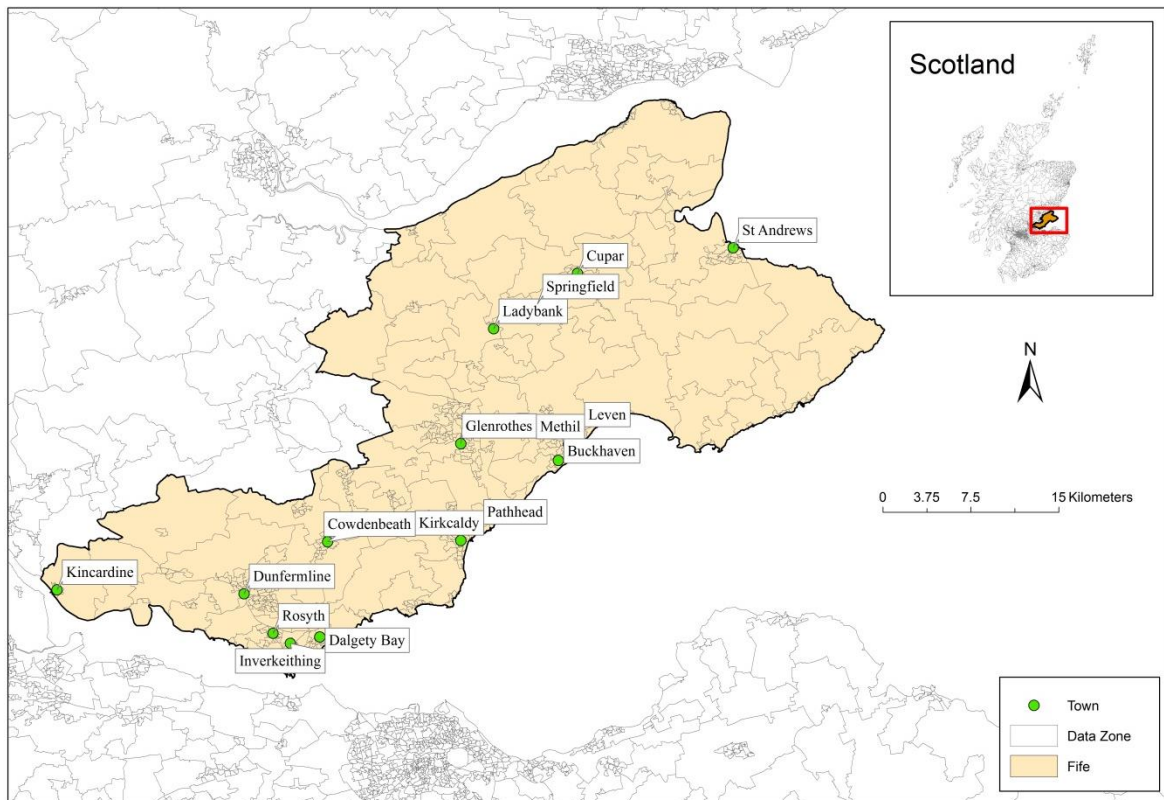
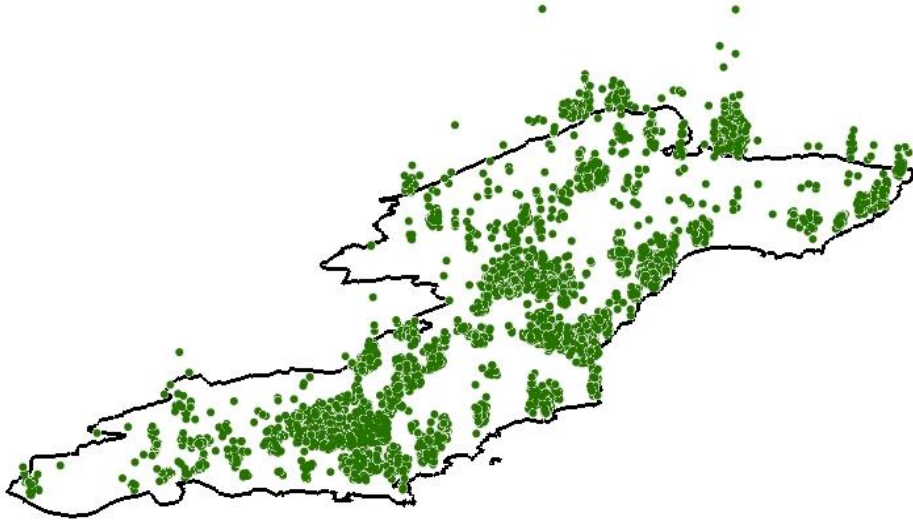


Figure 2 Spatial distributions of house prices in 2012. (A) Discrete point; (B) Continuous surface.

(A)



(B)

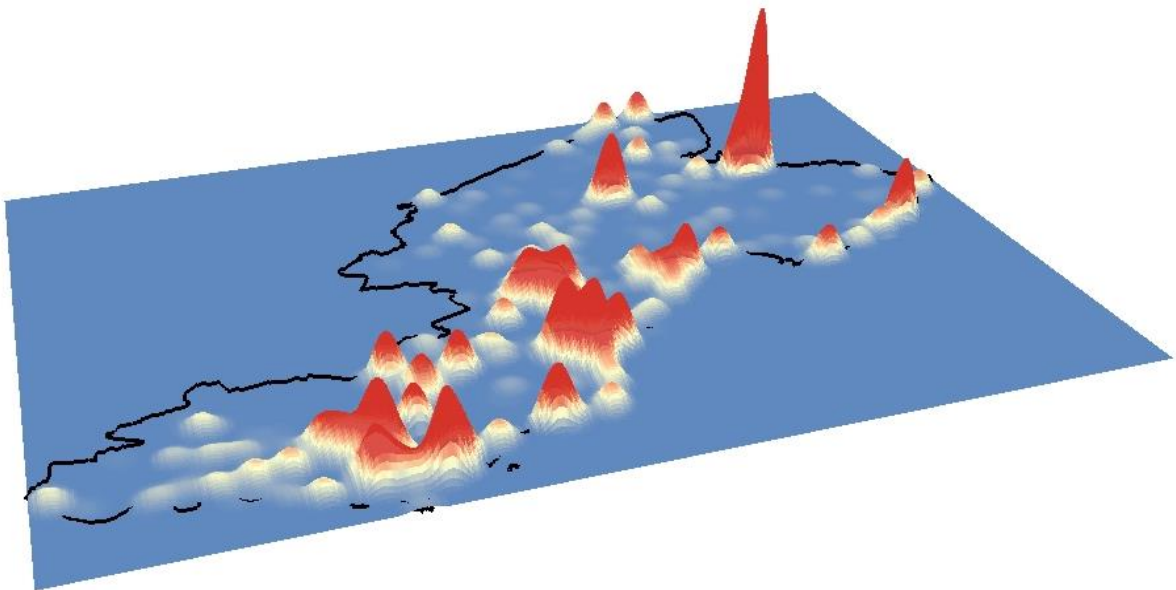


Figure 3 Temporal variations in global parameter estimates

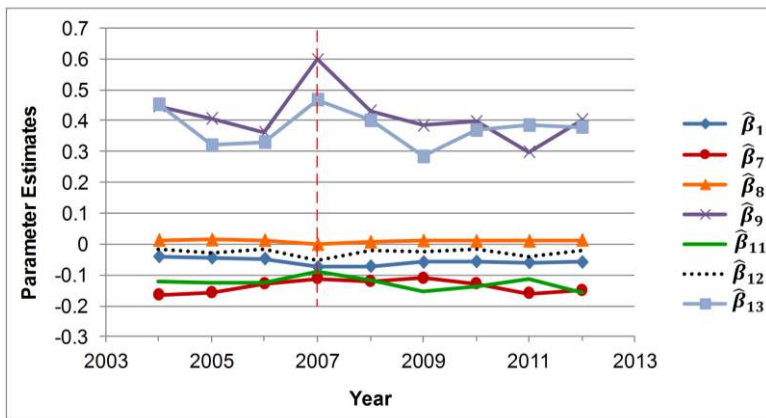
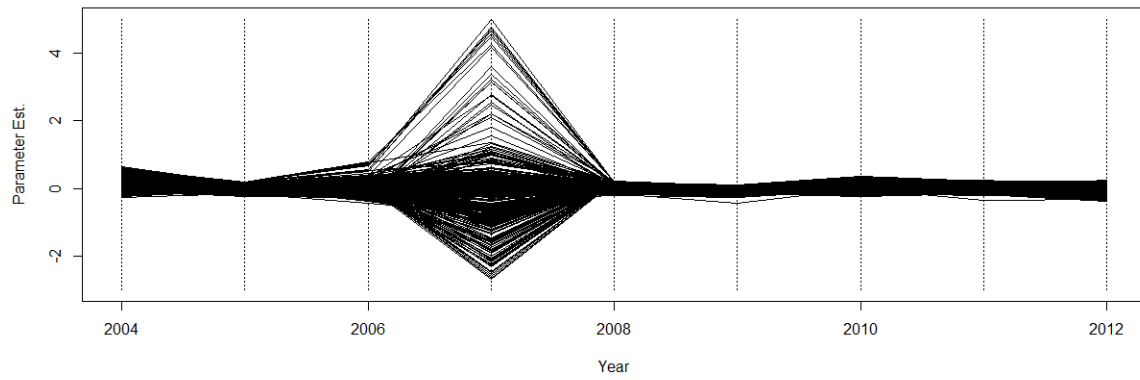


Figure 4 Temporal variations in local parameter estimates. (A) Distance to St Andrews; (B) Spatiotemporal lag.

(A)



(B)

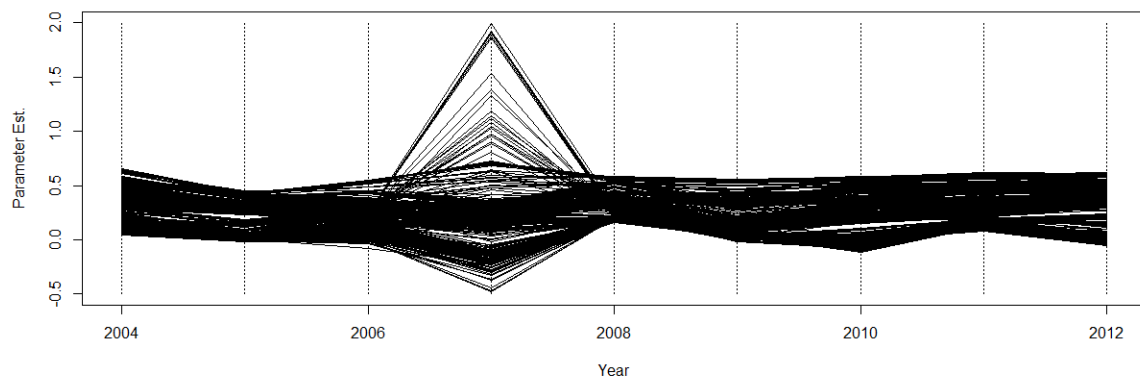
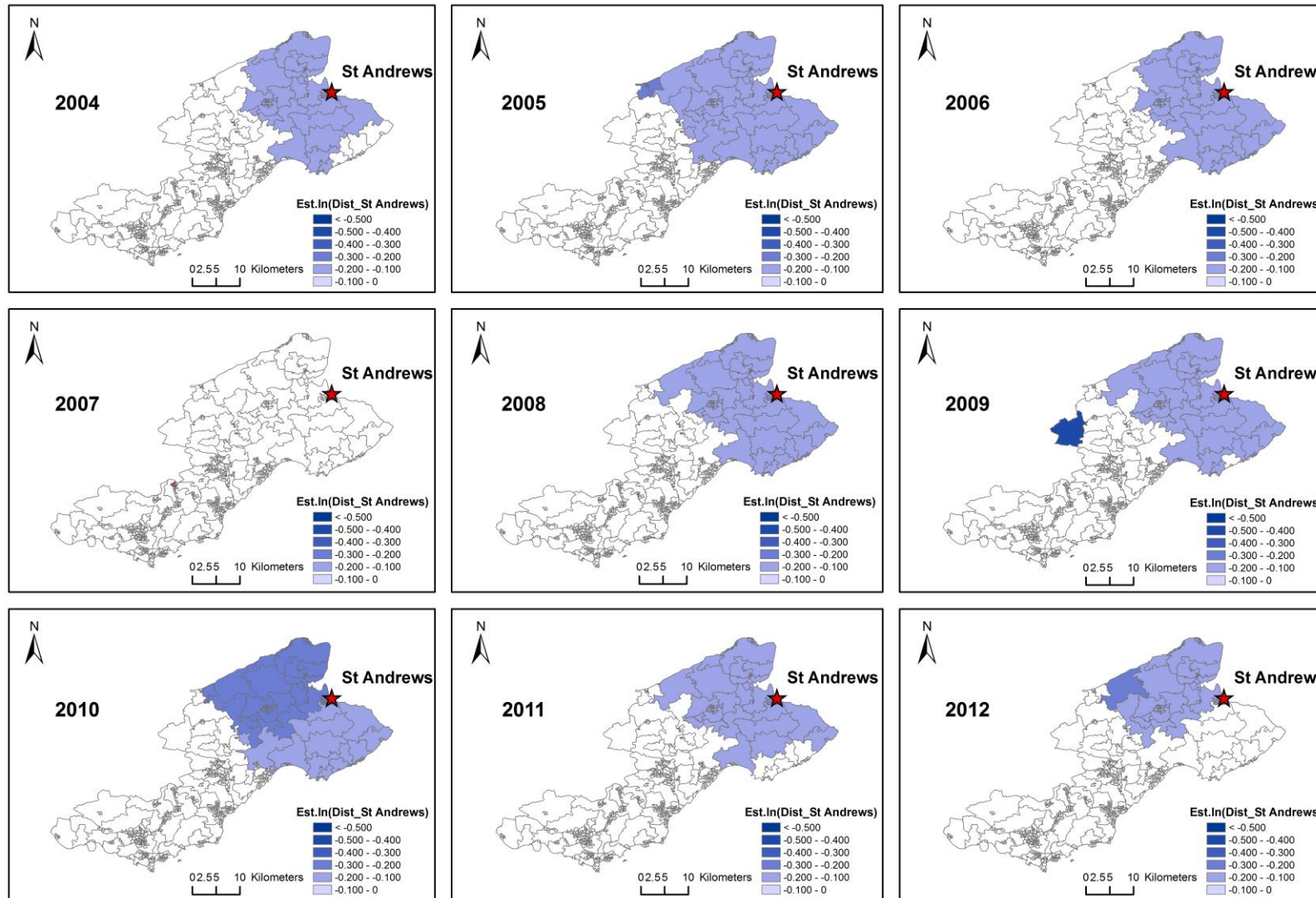


Figure 5 Spatial variations in significant local parameter estimates. (A) Distance to St Andrews; (B) Spatiotemporal lag.

(A)





(B)

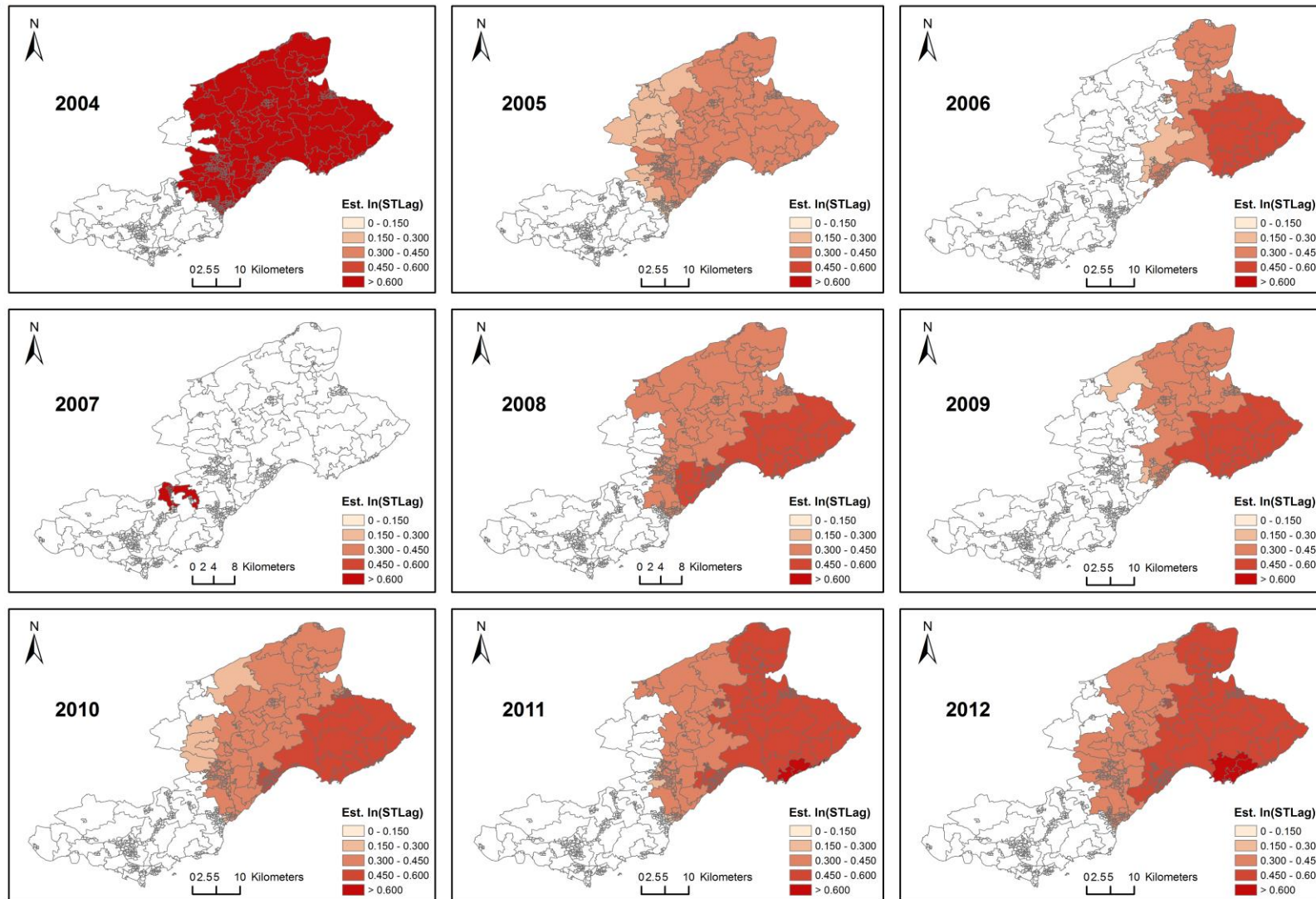




Figure 6 Spatial variations in local estimates of “Spatiotemporal lag” over time

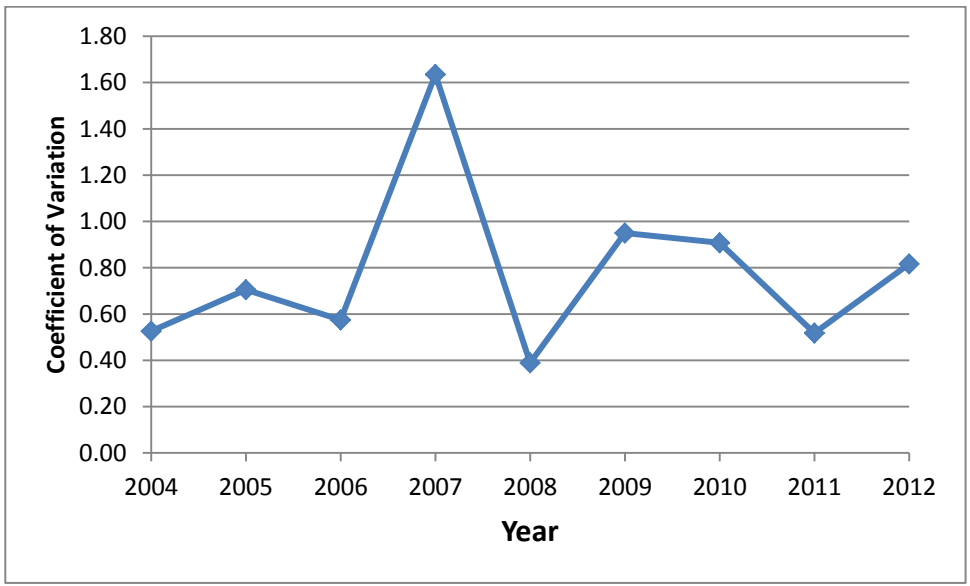


Figure 7 Temporal variations in local estimates of “Spatiotemporal lag” over space

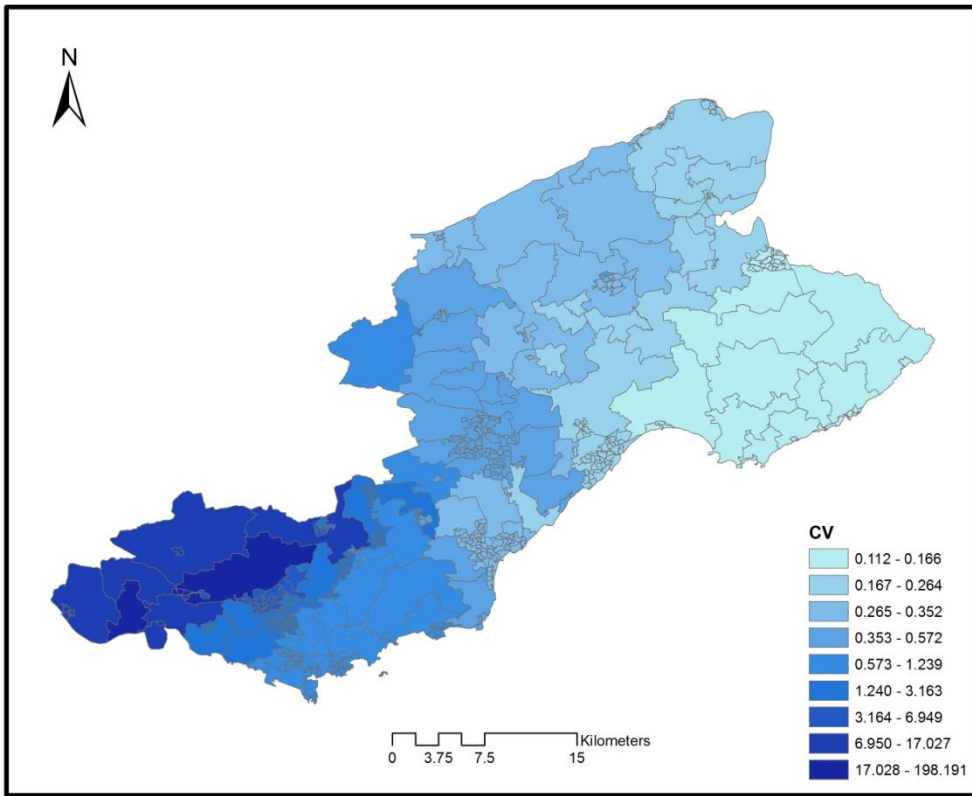


Table 1 Descriptive statistics of house prices

| Year | Sample Size | House price (£) |           |
|------|-------------|-----------------|-----------|
|      |             | Mean            | Adj. Mean |
| 2003 | 8,398       | 82,841          | 110,897   |
| 2004 | 9,579       | 98,211          | 127,668   |
| 2005 | 10,081      | 108,355         | 136,968   |
| 2006 | 10,901      | 121,667         | 149,059   |
| 2007 | 10,731      | 136,697         | 160,583   |
| 2008 | 6,422       | 143,039         | 161,618   |
| 2009 | 4,757       | 136,990         | 155,580   |
| 2010 | 5,006       | 137,466         | 149,237   |
| 2011 | 4,846       | 136,304         | 140,660   |
| 2012 | 4,939       | 133,494         | 133,494   |

Table 2 Definition of covariates

| <b>Covariates</b> | <b>Description</b>                                 |
|-------------------|--|
| $x_1$             | Population density                                 |
| $x_2$             | Percentage of pensionable age population           |
| $x_3$             | Percentage of working age population               |
| $x_4$             | Percentage of dwellings which are semi-detached    |
| $x_5$             | Percentage of dwellings which are terraced         |
| $x_6$             | Percentage of dwellings which are flats            |
| $x_7$             | Percentage of dwellings with 1 to 3 rooms          |
| $x_8$             | Percentage of dwellings with 7 to 9 rooms          |
| $x_9$             | Percentage of household ownership                  |
| $x_{10}$          | Number of SIMD crimes per 10,000 of the population |
| $x_{11}$          | Distance to St Andrews                             |
| $x_{12}$          | Distance to coast                                  |
| $x_{13}$          | Spatial-temporal lag                               |

Note: SIMD = Scottish Index of Multiple Deprivation.

Table 3 Parameter estimates for the global model

|                    | 2004    | 2005    | 2006    | 2007    | 2008    | 2009    | 2010    | 2011    | 2012    |
|--------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| constant           | 6.736*  | 8.488*  | 8.481*  | 7.560*  | 7.765*  | 8.919*  | 8.230*  | 7.791*  | 7.490*  |
| $\hat{\beta}_1$    | -0.040* | -0.044* | -0.047* | -0.074* | -0.072* | -0.058* | -0.056* | -0.060* | -0.058* |
| $\hat{\beta}_2$    | -0.087  | -0.032  | -0.019  | 0.098   | -0.023  | -0.095  | 0.006   | -0.042  | -0.223* |
| $\hat{\beta}_3$    | -0.018  | 0.076   | 0.090   | 0.910   | 0.156   | -0.207  | 0.247   | 0.209   | -0.392  |
| $\hat{\beta}_4$    | -0.001  | -0.001  | -0.001  | -0.003  | 0.000   | 0.011*  | 0.005   | -0.002  | 0.004   |
| $\hat{\beta}_5$    | -0.004  | -0.006  | -0.007* | -0.006  | -0.005  | -0.001  | -0.005  | -0.005  | -0.001  |
| $\hat{\beta}_6$    | 0.004   | 0.004   | 0.005   | 0.008   | 0.005   | 0.005   | 0.009*  | 0.006*  | 0.007*  |
| $\hat{\beta}_7$    | -0.165* | -0.158* | -0.129* | -0.112  | -0.121* | -0.111* | -0.129* | -0.160* | -0.151* |
| $\hat{\beta}_8$    | 0.013*  | 0.014*  | 0.013*  | -0.001  | 0.007*  | 0.011*  | 0.010*  | 0.010*  | 0.013*  |
| $\hat{\beta}_9$    | 0.447*  | 0.407*  | 0.361*  | 0.597*  | 0.432*  | 0.384*  | 0.397*  | 0.299*  | 0.403*  |
| $\hat{\beta}_{10}$ | 0.002   | -0.001  | -0.001  | -0.002  | -0.002  | -0.003  | -0.005* | -0.002  | -0.004  |
| $\hat{\beta}_{11}$ | -0.121* | -0.127* | -0.126* | -0.088* | -0.117* | -0.154* | -0.136* | -0.114* | -0.157* |
| $\hat{\beta}_{12}$ | -0.017  | -0.028* | -0.018* | -0.053* | -0.020* | -0.025* | -0.015  | -0.039* | -0.021* |
| $\hat{\beta}_{13}$ | 0.453*  | 0.320*  | 0.330*  | 0.468*  | 0.401*  | 0.284*  | 0.369*  | 0.385*  | 0.379*  |
| $R^2$              | 0.75    | 0.76    | 0.76    | 0.38    | 0.73    | 0.71    | 0.71    | 0.71    | 0.71    |

\*: p<0.05

Table 4 Stationarity test results for local parameters in semi-GWR models

| Parameter          | 2004 |    | 2005 |    | 2006 |    | 2007 |    | 2008 |    | 2009 |    | 2010 |    | 2011 |    | 2012 |    |
|--------------------|------|----|------|----|------|----|------|----|------|----|------|----|------|----|------|----|------|----|
|                    | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC | IQR  | MC |
| Constant           | √    | √  | √    | √  | √    | √  | √    |    | √    |    | √    | √  | √    | √  | √    | √  | √    | √  |
| $\hat{\beta}_1$    | F    | F  |      |    | √    | √  | F    | F  | F    | F  | √    |    |      |    | F    | F  |      |    |
| $\hat{\beta}_2$    | √    |    |      |    | √    | √  | √    |    | √    |    | √    |    | √    |    | √    |    |      |    |
| $\hat{\beta}_3$    | √    |    |      |    | √    | √  | √    |    | √    |    | √    | √  | √    |    |      |    | √    |    |
| $\hat{\beta}_4$    | √    | √  |      |    | F    | F  | F    | F  | √    |    |      |    | √    |    | √    |    |      |    |
| $\hat{\beta}_5$    |      |    |      |    | √    |    | F    | F  | √    |    | √    |    | F    | F  | √    |    | √    |    |
| $\hat{\beta}_6$    |      |    | F    | F  | F    | F  | F    | F  |      |    | √    |    |      |    |      |    |      |    |
| $\hat{\beta}_7$    | √    |    |      |    | √    |    |      | √  |      |    | F    | F  |      |    | √    |    | F    | F  |
| $\hat{\beta}_8$    |      |    |      |    | F    | F  |      |    |      |    |      |    |      |    |      |    |      |    |
| $\hat{\beta}_9$    | √    |    | √    | √  | F    | F  |      |    | √    |    | √    | √  | √    |    | √    |    | √    |    |
| $\hat{\beta}_{10}$ | √    | √  | √    | √  | √    | √  |      | √  | √    | √  | F    | F  | F    | F  |      |    |      |    |
| $\hat{\beta}_{11}$ | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  |
| $\hat{\beta}_{12}$ | √    |    | √    | √  | √    | √  | F    | F  |      |    | √    |    | √    | √  | √    | √  | √    | √  |
| $\hat{\beta}_{13}$ | √    | √  | √    | √  | √    | √  | √    |    | √    | √  | √    | √  | √    | √  | √    | √  | √    | √  |

Note: IQR D interquartile range; MC D Monte Carlo; F D fixed variable.

Table 5 AICc values for different models

| <b>Year</b> | <b>Global model</b> | <b>GWR</b> | <b>Semi-parametric<br/>GWR</b> |
|-------------|---------------------|------------|--------------------------------|
| 2004        | 86.454              | 55.704     | 45.166                         |
| 2005        | -6.959              | -48.246    | -50.566                        |
| 2006        | -81.502             | -90.672    | -115.836                       |
| 2007        | 757.513             | 660.499    | 644.008                        |
| 2008        | 6.544               | -19.860    | -33.796                        |
| 2009        | 15.175              | -1.883     | -5.416                         |
| 2010        | 88.658              | 35.989     | 31.993                         |
| 2011        | 58.043              | 44.235     | 37.442                         |
| 2012        | 122.913             | 108.591    | 107.477                        |

Note: GWR = geographically weighted regression