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Local stabilization for continuous-time Takagi-Sugeno fuzzy systems with time delay

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Abstract—This brief paper investigates the local stabilization for continues-time T-S fuzzy systems with constant time delay. In order to deal with the time delay, we design a Lyapunov-Krasovskii function which is dependent on the membership function. Based on the Lyapunov-Krasovskii function and the analysis of the time derivative of the membership function, less conservative results can be obtained, however, the Lyapunov-Krasovskii function is designed so complicated that the Lyapunov level set is hard to be measured directly. Alternatively, two sets are obtained to estimate the local stabilization. One set is for the time-varying initial conditions and the other is for the time-invariant initial conditions. The relationship between the two sets are also discussed. In the end, two examples are given to illustrate the effectiveness of the proposed approach.

Keywords: Takagi–Sugeno's fuzzy model, Time delay, Parallel distributed compensation law, Membership dependent Lyapunov-Krasovskii function.

I. INTRODUCTION

THe research of fuzzy control has been a popular topic in the past decades (see [1], [2], [29], [30], [31] and the references therein) especially for the Takagi-Sugeno model [3]. Many important issues such as stability have been studied via various kinds of methods (see [4], [5], [6] and the references therein). Generally speaking, the non-quadratic Lyapunov function [7] is a powerful tool in the stabilization analysis of discrete-time fuzzy systems but it is not often applied in continuous-time fuzzy systems because the derivatives of the membership function are hard to be dealt with. In the past, some restrictive assumptions on the time derivatives of the membership function are bounded [8], [9], but these assumptions are not reasonable for stabilization. Recently, the local stability and stabilization of continuous-time fuzzy systems were studied in [10], [11], [12]. In [11], the time derivatives of the membership function are explored by fixing the bounds of states instead of giving some restrictive assumptions. The local stabilization and observer design can be found in [10], [12] which show that the extension from local stability to stabilization or observer design is not easy.

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On the other hand, time delay is often found in real systems and has attracted attention from lots of researchers (see [13]-[28] and the references therein). Recently, in order to reduce the conservativeness, the fuzzy weighting-dependent Lyapunov–Krasovskii functional is applied in [23], [24] where the upper bound of the time-derivatives of membership functions is given but the fact is neglected that the future trajectories of the closed-loop system states should remain in the bounds defined a priori [10]. More recently, based on the Wirtinger inequality [32] and fuzzy line-integral Lyapunov function [33], new stability and stabilization results are proposed in [26]. The results in [26] are simple (do not introduce too many variables) and effective, however, the membership functions were not analyzed. Losing the information of membership function will lead to conservativeness, because the membership function is a very important factor that the fuzzy system is different from other systems.

Motivated by the above discussions, the problem of local fuzzy control with time delay is investigated in this technical paper. The focus is on how to deal with two issues. The first issue is that the designed controller should satisfy the upper bounds on the time derivatives of the membership functions defined a priori and the second is to determine the local stabilization region. Based on [10], the first issue is settled for the fuzzy systems obtained by Sector Nonlinearity method [34] and subsequently be extended to other fuzzy systems. For the second issue, the Lyapunov-Krasovskii level set is the first choice but it is too complicated to be measured directly. Alternatively, using different methods, we design two simple sets to estimate it. One set is for the time-varying initial conditions and the other is for the time-invariant initial conditions. Two examples are presented in this paper for verification. One example is used to show that less conservative results can be obtained and the other is to demonstrate the differences between the two sets mentioned above.

II. PRELIMINARIES AND BACKGROUNDS

Consider the following nonlinear model

$$
\begin{aligned}
\dot{x}(t) &= f_1(z(t)) x(t) + f_2(z(t)) x(t-\tau) \\
&\quad + f_3(z(t)) u(t), \qquad (1) \\
x(t) &= \phi(t), t \in \begin{bmatrix} -\tau & 0 \end{bmatrix}
\end{aligned}
$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $z(t) \in \mathbb{R}^p$ is known premise variables bounded and smooth in compact set $\mathbb{C}$, $\mathbb{C} = \bigcap_d \{x : |l_d x| \le \epsilon_d\}, d = 1, \cdots, n$, ($l_d \in \mathbb{R}^{1 \times n}$, only the element in $(1, q)$ is one and the others

are zeros, for example, if $n = 3$, we have $l_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $l_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, $l_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$). $f_1(\cdot)$, $f_2(\cdot)$ and $f_3(\cdot)$ are nonlinear functions or matrix functions with proper dimensions, $\phi(t)$ is the initial condition and the time delay τ is assumed to be constant. Applying the Sector Nonlinearity method or Local approximation method in [34], one has the following well known time delay T-S fuzzy model:

$$\dot{x}(t) = A_h x(t) + A_{\tau h} x(t - \tau) + B_h u(t), \quad (2)$$
$$x(t) = \phi(t), t \in \begin{bmatrix} -\tau & 0 \end{bmatrix},$$

where $A_h = \sum_{i=1}^{r} h_i(z(t)) A_i$, $A_{\tau h} = \sum_{i=1}^{r} h_i(z(t)) A_{\tau i}$, $B_h = \sum_{i=1}^{r} h_i(z(t)) B_i$. $A_i \in \mathbb{R}^{n \times n}$, $A_{\tau i} \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ are known matrices and $h_i(z(t))$ are membership functions. In this paper, we will design the following controller to stabilize (2)

$$u(t) = K_h x(t) + K_{\tau h} x(t - \tau). \quad (3)$$

To lighten the notation, we will drop the the time t, for instance, we will use x instead of $x(t)$. For simplicity, single and double sums are written similarly as [10] $\Upsilon_h = \sum_{i=1}^{r} h_i \Upsilon_i$, $\Upsilon_{hh} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \Upsilon_{ij}$ and for any matrix X, $\text{He}(X) = X + X^T$.

III. MAIN RESULTS

Theorem 1: For a given scalar $\tau > 0$, and parameters λ_1, λ_2, β_k, $k = 1, \cdots, p$, the closed-loop T-S fuzzy system (2) is asymptotically stabilized by the controller (3) in $\mathbb{M}$,

$$\mathbb{M} = \left\{ \phi : V_1(\phi(0)) + \int_{-\tau}^{0} \phi(s)^T Q_{h_{(s)}} \phi(s) ds \right.$$
$$\left. + \mathbb{P}(0) + \tau \int_{-\tau}^{0} \int_{\theta}^{0} \dot{\phi}(s)^T Z \dot{\phi}(s) ds d\theta \leq 1 \right\} \quad (4)$$

$$V_1(\phi(0)) = 2 \int_{\Gamma(0,\phi(0))} \hat{P}_h x d_x,$$

$$\mathbb{P}(0) = \rho(0)^T \begin{bmatrix} P_{11h_{(0)}} & P_{12h_{(0)}} \\ * & P_{22h_{(0)}} \end{bmatrix} \rho(0),$$

$$\rho(0)^T = \begin{bmatrix} \phi(0)^T & \int_{-\tau}^{0} \phi(s)^T ds \end{bmatrix},$$

if there exist matrices $\begin{bmatrix} \bar{P}_{11j} & \bar{P}_{12j} \\ * & \bar{P}_{22j} \end{bmatrix} > 0$, $\begin{bmatrix} W_{11kij} & W_{12kij} \\ * & W_{22kij} \end{bmatrix} > 0$, $\bar{P}_i > 0$, $\hat{\bar{P}}_i > 0$, $\bar{Q}_j > 0$, $\bar{Z} > 0$, $\bar{M}$, such that the following LMIs hold for $i, j, s = 1, \cdots, r$,

$$\tilde{\Omega}_{sij} + \tilde{\Omega}_{sji} \leq 0, \quad (5)$$

$$\Phi_{ij} + \Phi_{ji} \geq 0, \Psi_{ij} + \Psi_{ji} \geq 0, \quad (6)$$

$$\tilde{\Omega}_{sij} = \begin{bmatrix} \tilde{\Omega}_{11sij} & * \\ \tilde{\Omega}_{21sij} & \tilde{\Omega}_{22sij} \end{bmatrix},$$

$$\tilde{\Omega}_{11sij} = \begin{bmatrix} \tilde{\Omega}_{11ij} & \bar{\Omega}_{12ij} & \tilde{\Omega}_{13ij} & \bar{\Omega}_{14ij} \\ * & \bar{\Omega}_{22ij} & \bar{\Omega}_{23ij} & \bar{\Omega}_{24ij} \\ * & * & \tilde{\Omega}_{33ij} & \bar{P}_{12i} \\ * & * & * & \bar{\Omega}_{44ij} \end{bmatrix},$$

$$\tilde{\Omega}_{21sij} = \begin{bmatrix} \zeta_{11} & 0 & \zeta_{21} & 0 \\ \zeta_{21}^T & 0 & \zeta_{31} & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \zeta_{1p} & 0 & \zeta_{2p} & \vdots \\ \zeta_{2p}^T & 0 & \zeta_{3p} & \cdots \end{bmatrix},$$

$$\tilde{\Omega}_{22sij} = \begin{bmatrix} \omega_1 & * & * \\ 0 & \ddots & * \\ 0 & 0 & \omega_p \end{bmatrix},$$

$$\Phi_{ij} = \begin{bmatrix} \left(\frac{\delta_k \bar{\beta}_k}{\varrho \sqrt{\psi_k}} \right)^2 \bar{P}_i & * \\ \bar{\mathcal{A}} & I \end{bmatrix},$$

$$\Psi_{ij} = \begin{bmatrix} \left(\frac{(1-\delta_k) \bar{\beta}_k}{\varrho \sqrt{\psi_k}} \right)^2 \bar{P}_i & * \\ \bar{\mathcal{A}}_\tau & I \end{bmatrix},$$

where

$$\zeta_{1k} = \bar{P}_{11g_1(i,k)} - \bar{P}_{11g_2(i,k)},$$
$$\zeta_{2k} = \bar{P}_{12g_1(i,k)} - \bar{P}_{12g_2(i,k)},$$
$$\zeta_{3k} = \bar{P}_{22g_1(i,k)} - \bar{P}_{22g_2(i,k)},$$

$$\omega_k = \begin{bmatrix} -2W_{11kij} & * \\ -2W_{12kij}^T & -2W_{22kij} \end{bmatrix},$$
$$\bar{\mathcal{A}} = A_j \bar{M} + B_j \bar{K}_i, \bar{\mathcal{A}}_\tau = A_{\tau j} \bar{M} + B_j \bar{K}_{\tau i},$$
$$\tilde{\Omega}_{11ij} = \text{He}\left(\bar{P}_{12i} + \bar{\mathcal{A}}\right) + \bar{Q}_i - 4\bar{Z}_i + 0.5 \sum_{k=1}^{p} \beta_k^2 W_{11kij},$$

$$\tilde{\Omega}_{13ij} = \bar{P}_{22i} + 6\tau^{-1} \bar{Z}_i + 0.5 \sum_{k=1}^{p} \beta_k^2 W_{12kij},$$

$$\tilde{\Omega}_{33ij} = -12\tau^{-2} \bar{Z} + 0.5 \sum_{k=1}^{p} \beta_k^2 W_{22kij},$$

$$\bar{\Omega}_{14ij} = \hat{\bar{P}}_i + \bar{P}_{11i} - \bar{M} + \lambda_1 \bar{\mathcal{A}}^T,$$
$$\bar{\Omega}_{12ij} = -\bar{P}_{12i} - 2\bar{Z} + \bar{\mathcal{A}}_\tau + \lambda_2 \bar{\mathcal{A}}^T,$$
$$\bar{\Omega}_{22ij} = -\bar{Q}_s - 4\bar{Z} + \lambda_2 \text{He}\left(\bar{\mathcal{A}}_\tau\right),$$
$$\bar{\Omega}_{24ij} = \lambda_1 \bar{\mathcal{A}}_\tau^T - \lambda_2 \bar{M}, \bar{\Omega}_{23ij} = -\bar{P}_{22i} + 6\tau^{-1} \bar{Z},$$
$$\bar{\Omega}_{44ij} = \tau^2 \bar{Z} - \lambda_1 \bar{M} - \lambda_1 \bar{M}^T,$$
$$g_1(i,k) = \lfloor (i-1)/2^{p+1-k} \rfloor + 1 + (i-1) \bmod 2^{p-k},$$
$$g_2(i,k) = g_1(i,k) + 2^{p-k},$$
$$(i-1) \bmod 2^{p-k} = (i-1) - 2^{p-k} \times \lfloor (i-1)/2^{p-k} \rfloor,$$

and the controller gains are designed as $K_i = \bar{K}_i \bar{M}^{-1}$, $K_{\tau i} = \bar{K}_{\tau i} \bar{M}^{-1}$.

Proof: Based on [26], [32], we design the Lyapunov-Krasovskii functional as

$$V(x_t) = V_1(x_t) + \int_{t-\tau}^{t} x(s)^T Q_{h(s)} x(s) ds$$

$$+ \mathbb{P}(t) + \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}(s)^T Z \dot{x}(s) d_s d_\theta, \qquad (7)$$

$$V_1(x_t) = 2 \int_{\Gamma(0,x)} \hat{P}_h x d_x,$$

$$\mathbb{P}(t) = \rho(t)^T \begin{bmatrix} P_{11h} & P_{12h} \\ * & P_{22h} \end{bmatrix} \rho(t),$$

$$\rho(t)^T = \begin{bmatrix} x^T & \int_{t-\tau}^{t} x(s)^T ds \end{bmatrix}.$$

The LK function contains the line-integral Lyapunov function $V_1(x_t)$ which contains a special structure $\hat{P}_i$ and $\Gamma(0, x)$ is an integral path from the origin 0 to the current state $x(t)$. More details about $V_1(x_t)$ can be found in [26], [27], [33].

In the following, the proof has two parts. The first part is to ensure $\dot{V}(x_t) < 0$ and the second is to deal with the time derivatives of membership function. For the fist part, we have

$$\dot{V}(x_t) = 2x^T \hat{P}_h \dot{x} + \dot{\mathbb{P}}(t) + x^T Q_h x$$
$$- x(t-\tau)^T Q_{h(t-\tau)} x(t-\tau)$$
$$+ \tau^2 \dot{x}^T Z \dot{x} - \tau \int_{t-\tau}^{t} \dot{x}(s)^T Z \dot{x}(s) d_s.$$

Using the zero equation

$$2 \left(x^T M_1 + \lambda_1 \dot{x}^T M_1 + \lambda_2 x(t-\tau)^T M_1 \right)$$
$$\times \left(\mathcal{A}x + \mathcal{A}_\tau x(t-\tau) - \dot{x}(t) \right) = 0, \qquad (8)$$

where

$$\mathcal{A} = A_h + B_h K_h, \mathcal{A}_\tau = A_{\tau h} + B_h K_{\tau h},$$

and the results in [32] to deal with the term $-\tau \int_{t-\tau}^{t} \dot{x}(s)^T Z \dot{x}(s) d_s$, we have $\dot{V}(x_t) < \eta^T \Omega \eta$, where

$$\eta^T = \begin{bmatrix} x^T & x(t-\tau)^T & \int_{t-\tau}^{t} x(s)^T d_s & \dot{x}^T \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ * & * & \Omega_{33} & P_{12h}^T \\ * & * & * & \Omega_{44} \end{bmatrix}, \qquad (9)$$

$$\Omega_{11} = \text{He}\left(P_{12h} + M_1 \mathcal{A}\right) + Q_h - 4Z + \sum_{i=1}^{r} \dot{h}_i P_{11i},$$

$$\Omega_{12} = -P_{12h} - 2Z + M_1 \mathcal{A}_\tau + \lambda_2 \mathcal{A}^T M_1^T,$$

$$\Omega_{13} = P_{22h} + 6\tau^{-1} Z + \sum_{i=1}^{r} \dot{h}_i P_{12i},$$

$$\Omega_{14} = \hat{P}_h + P_{11h} - M_1 + \lambda_1 \mathcal{A}^T M_1^T,$$

$$\Omega_{22} = -Q_{h(t-\tau)} - 4Z + \text{He}\left(\lambda_2 M_1 \mathcal{A}_\tau\right),$$

$$\Omega_{23} = -P_{22h} + 6\tau^{-1} Z, \Omega_{24} = \lambda_1 \mathcal{A}_\tau^T M_1^T - \lambda_2 M_1,$$

$$\Omega_{33} = -12\tau^{-2} Z + \sum_{i=1}^{r} \dot{h}_i P_{22i},$$

$$\Omega_{44} = \tau^2 Z - \lambda_1 M_1 - \lambda_1 M_1^T.$$

The next part, we only need to deal with the time derivatives of the membership functions. Let $\begin{bmatrix} P_{11h} & P_{12h} \\ * & P_{22h} \end{bmatrix} = \tilde{P}_h$ and it's time derivative can be expressed as in [10],

$$\frac{d\tilde{P}_h}{dt} = \sum_{k=1}^{p} \frac{\partial w_0^k}{\partial z_k} \left(\frac{\partial z_k}{\partial x}\right)^T \left(\mathcal{A}x(t) + \mathcal{A}_\tau x(t-\tau)\right)$$

$$\times \left(\tilde{P}_{g_1(z,k)} - \tilde{P}_{g_2(z,k)}\right), \qquad (10)$$

where

$$\tilde{P}_{g_1(z,k)} = \sum_{i=1}^{r} h_i(z) \tilde{P}_{g_1(i,k)}, \tilde{P}_{g_2(z,k)} = \sum_{i=1}^{r} h_i(z) \tilde{P}_{g_2(i,k)}.$$

In order to ensure

$$\left| \frac{\partial w_0^k}{\partial z_k} \left(\frac{\partial z_k}{\partial x}\right)^T \left(\mathcal{A}x(t) + \mathcal{A}_\tau x(t-\tau)\right) \right| \le \beta_k, \qquad (11)$$

let

$$\left| \left(\frac{\partial z_k}{\partial x}\right)^T \mathcal{A}x \right| \le \frac{\delta_k \beta_k}{\varrho}, \qquad (12)$$

$$\left| \left(\frac{\partial z_k}{\partial x}\right)^T \mathcal{A}_\tau x(t-\tau) \right| \le \frac{(1-\delta_k)\beta_k}{\varrho}, \qquad (13)$$

where $0 \le \delta_k \le 1$, $\left| \frac{\partial w_0^k}{\partial z_k} \right| \le \varrho$. It is obvious that (11) holds if (12) and (13) hold. For (12), we have

$$\left(\frac{\partial z_k}{\partial x}\right)^T \mathcal{A} S_h^{-1} \mathcal{A}^T \left(\frac{\partial z_k}{\partial x}\right) + x^T S_h x \le 2\frac{\delta_k \beta_k}{\varrho}. \qquad (14)$$

Note, $\dot{V}(x_t) < 0$ means $x^T P_{11h} x$ is bounded for any $x(0) \in \mathbb{C}$. Observing the Lyapunov function (7) and the set $\mathbb{M}$, we have $x^T P_{11h} x \le 1$. Introducing a variable P_h satisfying $P_{11h} \le P_h$ and letting $S_h = \delta_k \beta_k P_h$ yield

$$\left(\frac{\partial z_k}{\partial x}\right)^T \mathcal{A} P_h^{-1} \mathcal{A}^T \left(\frac{\partial z_k}{\partial x}\right) \le \left(\frac{\delta_k \beta_k}{\varrho}\right)^2. \qquad (15)$$

Applying Schur complement to (15) and noticing $\left(\frac{\partial z_k}{\partial x}\right)^T \left(\frac{\partial z_k}{\partial x}\right) \le \psi_k \Leftrightarrow \left(\frac{\partial z_k}{\partial x}\right) \left(\frac{\partial z_k}{\partial x}\right)^T \le \psi_k I$, we get

$$\begin{bmatrix} \left(\frac{\delta_k \beta_k}{\varrho \sqrt{\psi_k}}\right)^2 P_h & \mathcal{A}^T \\ \mathcal{A} & I \end{bmatrix} \ge 0. \qquad (16)$$

For (13), if $t > \tau$ and $x(t-\tau) \in \mathbb{C}$, from the analysis above, we have $x(t-\tau)^T P_h x(t-\tau) \le 1$, if $t \le \tau$, $x(t-\tau) = $

$\phi(t)$, we also have $x(t-\tau)^T P_h x(t-\tau) \leq 1$, using similar method as above we get

$$\begin{bmatrix} \left(\frac{(1-\delta_k)\beta_k}{\varrho\sqrt{\psi_k}}\right)^2 P_h & \mathcal{A}_\tau^T \\ \mathcal{A}_\tau & I \end{bmatrix} \geq 0. \qquad (17)$$

Then, let $\bar{M} = M_1^{-T}$, pre-and post-multiply both sides of $\Omega < 0$ with $\text{diag}\{M_1^{-1}, M_1^{-1}, M_1^{-1}, M_1^{-1}\}$ and its transpose respectively and defining $K_i \bar{M} = \bar{K}_i$, $K_{\tau i}\bar{M} = \bar{K}_{\tau i}$, $\bar{M}^T P_{11h}\bar{M} = \bar{P}_{11h}$, $\bar{M}^T P_{12h}\bar{M} = \bar{P}_{12h}$, $\bar{M}^T P_{22h}\bar{M} = \bar{P}_{22h}$, $\bar{M}^T Q_h \bar{M} = \bar{Q}_h$, $\bar{M}^T Z \bar{M} = \bar{Z}$, $\bar{M}^T \hat{P}_h \bar{M} = \hat{P}_h$ we get (5). Pre-and post-multiply both sides of (16) and (17) with $\text{diag}\{M_1^{-1}, M_1^{-1}\}$ and its transpose respectively we get (6). ∎

Remark 1: $\mathbb{M}$ is very complicated and hard to be measured directly because there are double integrations and the time derivative of the initial state $\dot{\phi}(t)$. In order to estimate the the local stabilization region, we propose the following two methods:

(I) For $\int_{-\tau_0}^{0}\int_{\theta}^{0} \dot{\phi}(s)^T Z\dot{\phi}(s)\, d_s d_\theta$, supposing $\phi(t)$ and $\dot{\phi}(t)$ are smooth in $\begin{bmatrix} -\tau & 0 \end{bmatrix}$ and $\dot{\phi}(t)$ can be bounded as $\phi(t_2) \leq \dot{\phi}(t) \leq \phi(t_1)$ for some $t_1, t_2 \in \begin{bmatrix} -\tau & 0 \end{bmatrix}$, thus $\dot{\phi}(t)$ can be expressed as $\dot{\phi}(t) = \sum_{q=1}^{2}\gamma_q(t)\phi(t_q)$ with $0 \leq \gamma_q(t) \leq 1$, $\sum_{q=1}^{2}\gamma_q(t) = 1$ for $t \in \begin{bmatrix} -\tau & 0 \end{bmatrix}$, then we have

$$\int_{-\tau}^{0}\int_{\theta}^{0} \dot{\phi}(s)^T Z\dot{\phi}(s)\, d_s d_\theta = \int_{-\tau}^{0}\int_{\theta}^{0} F_1 d_s d_\theta$$
$$\leq \int_{-\tau}^{0}\int_{\theta}^{0} F_2 d_s d_\theta = \int_{-\tau}^{0}\int_{\theta}^{0}\left(\sum_{q=1}^{2}\gamma_q(s)\phi_{qq}\right) d_s d_\theta,$$

$$F_1 = \sum_{q=1}^{2}\gamma_q^2(s)\phi_{qq} + \gamma_1(s)\gamma_2(s)(\phi_{12}+\phi_{21}),$$
$$F_2 = \sum_{q=1}^{2}\gamma_q^2(s)\phi_{qq} + \gamma_1(s)\gamma_2(s)(\phi_{11}+\phi_{22}),$$
$$\phi_{qq} = \phi(t_q)^T Z\phi(t_q), q = 1,2.$$

Let $\bar{Z} = \phi(\nu)^T Z\phi(\nu)$, we have

$$\tau \int_{-\tau_0}^{0}\int_{\theta}^{0}\left(\sum_{q=1}^{2}\gamma_q(s)\phi_{qq}\right) d_s d_\theta \leq \frac{1}{2}\tau^3 \max_{-\tau \leq \nu \leq 0}(\bar{Z}). \quad (18)$$

For $\mathbb{P}(0)$, supposing $P_{12h_{(0)}}^T = P_{12h_{(0)}} \geq 0$ and let $J = \int_{-\tau}^{0}\phi(s)\,ds$, we have

$$\mathbb{P}(0) \leq \phi(0)^T P_{11h_{(0)}}\phi(0) + J^T P_{22h_{(0)}}J$$
$$+ J^T P_{12h_{(0)}}J + \phi(0)^T P_{12h_{(0)}}\phi(0),$$

thus

$$\mathbb{P}(0) \leq \max_{-\tau \leq \nu \leq 0}(L_1) + \tau^2 \max_{-\tau \leq \nu \leq 0}(L_2) \qquad (19)$$

$$L_1 = \phi(\nu)^T\left(P_{11h_{(0)}} + P_{12h_{(0)}}\right)\phi(\nu),$$
$$L_2 = \phi(\nu)^T\left(P_{22h_{(0)}} + P_{12h_{(0)}}\right)\phi(\nu).$$

For $V_1(\phi(0)) + \int_{-\tau}^{0}\phi(s)^T Q_{h(s)}\phi(s)\,ds$, it has been shown in [33] that $V_1(x_t)$ is path-independent and can be rewritten as $V_1(x_t) = 2\int_{0}^{1}\vartheta x^T \hat{P}_{h(\vartheta x)}x\,d\vartheta$, $\vartheta \in \mathbb{R}$. Let $\hat{P}_i \leq G_1$, $Q_h \leq G_2$, we have $V_1(x_t) \leq \left(2\int_{0}^{1}\vartheta d\vartheta\right)\left(x^T G_1 x\right) = x^T G_1 x$ and the following

$$V_1(x_0) + \int_{-\tau}^{0}\phi(s)^T Q_{h(s)}\phi(s)\,ds$$
$$\leq \max_{-\tau \leq \nu \leq 0}(\bar{G}_1) + \tau \max_{-\tau \leq \nu \leq 0}(\bar{G}_2), \qquad (20)$$
$$\bar{G}_1 = \phi(\nu)^T G_1\phi(\nu), \bar{G}_2 = \phi(\nu)^T G_2\phi(\nu).$$

Combining (18)-(20) and considering the constraints $\frac{1}{2}\tau^3 Z \leq N_i$, $P_{11i} + P_{12i} \leq N_i$, $\tau^2(P_{22i}+P_{12i}) \leq N_i$, $G_1 \leq N_i$, $\tau G_2 \leq N_i$, $5N_i \leq \mathcal{P}_{1i}$, we get an estimation of $\mathbb{M}$

$$\bar{\mathbb{M}} := \left\{\phi : \phi(\nu)^T \mathcal{P}_{1h(0)}\phi(\nu) \leq 1, \ \forall \nu \in [-\tau, 0]\right\}.$$

(II) Supposing $\phi(t)$ is time-invariant that is $\dot{\phi}(t) = 0$ and denoted as ϕ_0, then we have

$$V(\phi_0) \leq \phi_0^T G_1\phi_0 + \phi_0^T P_{11h_{(0)}}\phi_0 + \tau^2\phi_0^T P_{22h_{(0)}}\phi_0$$
$$+\tau\phi_0^T\left(P_{12h_{(0)}} + P_{12h_{(0)}}^T\right)\phi_0 + \tau^2\phi_0^T Q_{h_{(0)}}\phi_0.$$

Considering the constraint $G_1 + P_{11i} + \tau^2 P_{22i} + \tau\text{He}(P_{12i}) + \tau^2 Q_i \leq \mathcal{P}_{2i}$, we have $V(\phi_0) \leq \phi_0^T \mathcal{P}_{2h_{(0)}}\phi_0$, and thus, we get $\hat{\mathbb{M}}$ which is also an estimation of $\mathbb{M}$

$$\hat{\mathbb{M}} := \left\{\phi_0 : \phi_0^T \mathcal{P}_{2h_{(0)}}\phi_0 \leq 1, \phi_0 \in \mathbb{R}^n\right\}. \qquad (21)$$

Remark 2: Since the nonlinear system can be expressed as T-S model in $\mathbb{C}$, it is naturally that $\hat{\mathbb{M}}$ (or $\bar{\mathbb{M}}$) must satisfy $\hat{\mathbb{M}} \subset \mathbb{C}$ (or $\bar{\mathbb{M}} \subset \mathbb{C}$, the following is the same) which can be expressed as LMIs by using Lagrange multiplier method. Note, $\hat{\mathbb{M}} \subset \mathbb{C}$ means, any ϕ_0 satisfying $l_d\phi_0 = \pm\epsilon_d$, we have $\phi_0 \mathcal{P}_{2i}\phi_0 \geq 1$. This problem can be expressed as

$$\min\left\{\phi_0^T \mathcal{P}_{2i}\phi_0\, |l_d\phi_0 = \pm\epsilon_d\right\} \geq 1, \qquad (22)$$

and be solved by using Lagrange multiplier method. Defining the Lagrange function $L(\phi_0) = \phi_0^T \mathcal{P}_{2h}\phi_0 + \alpha(l_d\phi_0 \mp \epsilon_d)$, where α is the Lagrange factor, we have

$$\frac{\partial L(\phi_0)}{\partial\phi_0} = 2\phi_0^T\mathcal{P}_{2i} + \alpha l_d = 0, l_d\phi_0 \mp \epsilon_d = 0. \qquad (23)$$

Solving (23), we get $\alpha^* = \mp 2\epsilon_d\left(l_d\mathcal{P}_{2i}^{-1}l_d^T\right)^{-1}$, $\phi_0^* = \pm\epsilon_d\mathcal{P}_{2i}^{-1}l_d^T\left(l_d\mathcal{P}_{2i}^{-1}l_d^T\right)^{-1}$, $d \in \{1, \cdots, n\}$. Substituting ϕ_0^* into (22) and applying Schur complement we get exactly

$$\begin{bmatrix} \epsilon_d^2 & l_d\bar{M} \\ \bar{M}^T l_d^T & \bar{\mathcal{P}}_{2i} \end{bmatrix} \geq 0, \bar{\mathcal{P}}_{2i} = \bar{M}^T\mathcal{P}_{2i}\bar{M}. \qquad (24)$$

In the following we give another method to stabilize (2).

Theorem 2: For a given scalar $\tau > 0$, and parameters λ_1, λ_2, $\bar{\beta}_k$, the closed-loop T-S fuzzy system (2) is asymptotically stabilized in a local region $\hat{\mathbb{M}}$ (or $\bar{\mathbb{M}}$) by the controller (3), if there exist matrices $\begin{bmatrix} \bar{P}_{11j} & \bar{P}_{12j} \\ * & \bar{P}_{22j} \end{bmatrix} > 0, \bar{Q}_j > 0, \bar{\hat{P}}_i > 0,$ $\bar{Z} > 0, \bar{\mathcal{P}}_{2i} > 0, \bar{M}$ such that the following LMIs hold for $i, j, s = 1, \cdots, r$, $d = 1, \cdots, n$, $k = 1, \cdots r - 1$,

$$\bar{\Omega}_{sij} + \bar{\Omega}_{sji} \leq 0, \tag{25}$$

$$\begin{bmatrix} \bar{P}_{11k} & \bar{P}_{12k} \\ * & \bar{P}_{22k} \end{bmatrix} \geq \begin{bmatrix} \bar{P}_{11r} & \bar{P}_{12r} \\ * & \bar{P}_{22r} \end{bmatrix}, \tag{26}$$

$$\bar{\Phi}_{ij} + \bar{\Phi}_{ji} \geq 0, \bar{\Psi}_{ij} + \bar{\Psi}_{ji} \geq 0, \tag{27}$$

$$\bar{\hat{P}}_i + \bar{P}_{11i} + \tau^2 \bar{P}_{22i} + \tau \left(\bar{P}_{12i} + \bar{P}_{12i}^T \right) + \tau^2 \bar{Q}_i \leq \bar{\mathcal{P}}_{2i}, \tag{28}$$

$$\begin{bmatrix} \epsilon_d^2 & l_d \bar{M} \\ \bar{M}^T l_d^T & \bar{\mathcal{P}}_{2i} \end{bmatrix} \geq 0, \tag{29}$$

$$\bar{\Phi}_{ij} = \begin{bmatrix} \left(\frac{\delta_k \bar{\beta}_k}{\sqrt{\sigma_k}} \right)^2 \bar{\mathcal{P}}_{2i} & * \\ \bar{\mathcal{A}} & I \end{bmatrix},$$

$$\bar{\Psi}_{ij} = \begin{bmatrix} \left(\frac{(1-\delta_k) \bar{\beta}_k}{\sqrt{\sigma_k}} \right)^2 \bar{\mathcal{P}}_{2i} & * \\ \bar{\mathcal{A}}_\tau & I \end{bmatrix},$$

$$\bar{\Omega}_{sij} = \begin{bmatrix} \bar{\Omega}_{11ij} & \bar{\Omega}_{12ij} & \bar{\Omega}_{13ij} & \bar{\Omega}_{14ij} \\ * & \bar{\Omega}_{22ij} & \bar{\Omega}_{23ij} & \bar{\Omega}_{24ij} \\ * & * & \bar{\Omega}_{33ij} & \bar{P}_{12i}^T \\ * & * & * & \bar{\Omega}_{44ij} \end{bmatrix},$$

$$\bar{\Omega}_{11ij} = \text{He} \left(\bar{P}_{12i} + \bar{\mathcal{A}} \right) + \bar{Q}_i - 4\bar{Z} + \sum_{k=1}^{r-1} \bar{\beta}_k \left(\bar{P}_{11k} - \bar{P}_{11r} \right),$$

$$\bar{\Omega}_{13ij} = \bar{P}_{22i} + 6\tau^{-1} \bar{Z} + \sum_{k=1}^{r-1} \bar{\beta}_k \left(\bar{P}_{12k} - \bar{P}_{12r} \right),$$

$$\bar{\Omega}_{33ij} = -12\tau^{-2} \bar{Z} + \sum_{k=1}^{r-1} \bar{\beta}_k \left(\bar{P}_{22k} - \bar{P}_{22r} \right),$$

and the controller gains are designed as $K_i = \bar{K}_i \bar{M}^{-1}$, $K_{\tau i} = \bar{K}_{\tau i} \bar{M}^{-1}$.

Proof: Let

$$\frac{dh_k}{dt} = \left(\frac{\partial h_k}{\partial x} \right)^T \dot{x} = \left(\frac{\partial h_k}{\partial x} \right)^T \left(\mathcal{A}_h x + \mathcal{A}_{\tau h} x \left(t - \tau \right) \right), \tag{30}$$

and suppose $\left(\frac{\partial h_k}{\partial x} \right)^T \left(\frac{\partial h_k}{\partial x} \right) \leq \sigma_k$, the rest is similar to the proof of Theorem 1 to ensure $\left| \frac{dh_k}{dt} \right| \leq \bar{\beta}_k$, thus omitted. ∎

In Theorem 2, $\dot{P}_{11h}$ (the same to $\dot{P}_{12h}$ and $\dot{P}_{22h}$) is presented as $\dot{P}_{11h} \leq \sum_{k=1}^{r-1} \bar{\beta}_k \left(P_{11k} - P_{11r} \right)$, $P_{11k} - P_{11r} \geq 0$. Another simple and direct method is $\dot{P}_{11h} \leq \sum_{i=1}^r \bar{\beta}_i P_{11i}$. The two methods are applicable for different examples. Sometimes, the same results can be obtained by applying both of them, sometimes better results are obtained by applying the first than by the second because $P_{11k} - P_{11r}$ is smaller than P_{11k} and sometimes better results can be obtained by applying the second than by the first because of the constraints

$P_{11k} - P_{11r} \geq 0$.

Remark 3: The contributions of this paper are as follows: 1), Compared with the latest results such as [25] and [26], the LK function designed in this paper is dependent on the membership function. The time derivatives of membership functions is analyzed such that the designed controller satisfies the upper bounds on the time derivatives of the membership functions defined a priori. Thus, the results in this paper are less conservative than the existing ones and the simulations show this point. 2), The local stabilization region is estimated by two methods. One method is applicable for the time-varying initial conditions and the other one is applicable for the time-invariant initial conditions.

Remark 4: The differences between Theorem 1 and Theorem 2 are as follows: 1), Theorem 1 focuses on the fuzzy systems obtained by Sector Nonlinearity method [34] and uses the mod function and floor function $\lfloor \cdot \rfloor$ to deal with the time derivatives of the membership functions. Theorem 2 does not consider the details of the membership function and is applicable to the fuzzy systems obtained by other methods. 2), For some cases, the membership functions are independent of the system states, for example, $h_1 = \sin(t)$, $h_1 = \frac{2}{\pi} \arctan(t)$ or $h_1 = \frac{1}{1+\exp(3-t)}$, the time derivatives can be bounded as $\left| \frac{dh_i}{dt} \right| \leq \bar{\beta}_i$. For this kind of situations, Theorem 2 without the constraints (27)-(29) becomes the conditions for global stabilization. 3), $k = 1, \cdots, p$ in Theorem 1 and $k = 1, \cdots, r - 1$ in Theorem 2. This is because the k in Theorem 1 depends on the number of premise variables p, while the k in Theorem 2 depends on the number of fuzzy rules r.

IV. NUMERICAL EXAMPLES

Example 1: Consider the two-rule fuzzy system that has been studied in [25], [26] where the system and input matrices are as follows:

$$A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$A_{\tau 1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.$$

This two-rule fuzzy system has been studied extensively in the literature in the past several years and the goal is to compute the maximum delay τ under which the fuzzy system can be stabilized by the designed controller. Using different methods to compute the maximum delay τ we get Table 1 which shows that better results can be obtained by using the method proposed in this paper than the ones in the literatures.

In this paper, δ_k is set as $\delta_k = 0.5$, then searching β_k from small to large, of course, δ_k can also be searched to get further less conservative results. λ_1 and λ_2 are searched by using the most common brute-force algorithm and the searching scope is $[-1 \quad 4]$ with step 0.01. For example, the largest delay $\tau = 1.4257$ ($\varepsilon = 1.26$) is obtained in [26], while applying the method in this paper we get $\tau = 1.6421(\beta_1 = 0.01)$, $\tau = 1.6219(\beta_1 = 0.1)$ for Theorem 1 by searching $\lambda_1 = 3.77$, $\lambda_2 = 2.1$, and $\tau = 1.6263(\bar{\beta}_1 = 0.01)$, $\tau = 1.6113(\bar{\beta}_1 = 0.1)$ for Theorem 2 by searching $\lambda_1 = 2.95$, $\lambda_2 = 1.85$.

For Theorem 1, supposing $z_1 = x_1$, $w_0^1 = h_1 = \dfrac{1 - \sin(x_1)}{2}$, $\mathbb{C} = \left\{ x : |x_i| \leq \dfrac{\pi}{2}, i = 1, 2 \right\}$, $\delta_1 = 0.5$, $\left| \dfrac{\partial w_0^1}{\partial z_1} \right| \leq \dfrac{1}{2} = \varrho$, $\left(\dfrac{\partial z_k}{\partial x} \right)^T \left(\dfrac{\partial z_k}{\partial x} \right) \leq 1 = \psi_1$, $\epsilon_1 = \epsilon_2 = \dfrac{\pi}{2}$, $l_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $l_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$. For Theorem 2, we only need know $h_1 = \dfrac{1 - \sin(x_1)}{2}$, and $\left(\dfrac{\partial h_1}{\partial x} \right)^T \left(\dfrac{\partial h_1}{\partial x} \right) \leq \dfrac{1}{4} = \sigma_1$. It is observed from Table 1 that larger delay τ can be obtained by Theorem 1 than Theorem 2 as discussed in Remark 3, but if this example is obtained by Local approximation method, Theorem 1 becomes infeasible.

Comparing with [26], in order to demonstrate what lead to the less conservative results, we consider five cases: **(I)** means $\hat{P}_h = 0$; **(II)** means $\hat{P}_h = 0$ and $\lambda_2 = 0$; **(III)** means $\hat{P}_h = 0$, $\lambda_2 = 0$ and $Q_h = Q$; **(IV)** means $\hat{P}_h = 0$ and $\begin{bmatrix} P_{11h} & P_{12h} \\ * & P_{22h} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$; **(V)** means $\begin{bmatrix} P_{11h} & P_{12h} \\ * & P_{22h} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}$, $\lambda_2 = 0$ and $Q_h = Q$.

For this example, it is concluded from Table 1 that $Q_h = Q$ has no effect on the results and λ_2 can improve the results but the improvement is little. In addition, observing the differences between **(III)** and **(V)**, we can get larger delay even without the line-integral Lyapunov function $V_1(x_t)$. That is to say that the analysis of the time derivative of the membership function is very important.

Example 2: Consider the following nonlinear system

$$\dot{x}(t) = \begin{bmatrix} -\dfrac{1}{RC} & \dfrac{1}{C} \\ -\dfrac{1}{L} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(t - \tau)$$
$$+ \begin{bmatrix} 0 \\ -\dfrac{1}{L}(R_M x_2(t) - V_{in} - V_D) \end{bmatrix} u(t).$$

Let $z_1 = x_2, w_0^1 = h_1(z) = \dfrac{x_2 - \bar{d}}{\bar{D} - \bar{d}}, h_2 = 1 - h_1$, in the compact set $\mathbb{C} = \{ x : |x_i| \leq 8, i = 1, 2 \}$, we get a two-rule T-S model with the following system and input matrices:

$$A_1 = A_2 = \begin{bmatrix} -\dfrac{1}{RC} & \dfrac{1}{C} \\ -\dfrac{1}{L} & 0 \end{bmatrix}, A_{\tau 1} = A_{\tau 2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -\dfrac{1}{L}(R_M \bar{D} - V_{in} - V_D) \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ -\dfrac{1}{L}(R_M \bar{d} - V_{in} - V_D) \end{bmatrix},$$

where $L = 0.9858$, $C = 0.2025$, $V_D = 0.82$, $V_{in} = 30$, $R = 6$, $R_M = 0.27$, $\bar{D} = 8$, $\bar{d} = -8$.

In Remark 1, two sets $\bar{\mathbb{M}}$ and $\hat{\mathbb{M}}$ are obtained to estimate the stabilization region $\mathbb{M}$. We use this example to show the relationship between $\bar{\mathbb{M}}$ and $\hat{\mathbb{M}}$. In order to get the relationship easily, we do not optimize the parameter λ_1, λ_2 and just let

TABLE I
THE MAXIMUM DELAY τ OBTAINED BY DIFFERENT METHODS

	[20]	[24]	[13]	[25]	[26]
τ	1.0947	1.3088	1.3169	1.4224	1.4257
	$\beta_1 = 0.01$ Th. 1	$\beta_1 = 0.1$ Th. 1	$\beta_1 = 0.01$ Th. 2	$\beta_1 = 0.1$ Th. 2	
τ	$\lambda_1 = 3.77, \lambda_2 = 2.1$		$\lambda_1 = 2.95, \lambda_2 = 1.85$		
	1.6421	1.6219	1.6263	1.6113	
(I)	$\lambda_1 = 2.45, \lambda_2 = 0.75$		$\lambda_1 = 1.15, \lambda_2 = 0.11$		
	1.6380	1.5729	1.5841	1.5038	
(II)	$\lambda_1 = 1.25$		$\lambda_1 = 1.15$		
	1.6078	1.5566	1.5703	1.4961	
(III)	$\lambda_1 = 1.25$		$\lambda_1 = 1.15$		
	1.6078	1.5566	1.5703	1.4961	
(IV)	$\lambda_1 = 1.2, \lambda_2 = -0.05$		$\lambda_1 = 1.2, \lambda_2 = -0.05$		
	1.2430		1.2430		
(V)	$\lambda_1 = 1/1.26$		$\lambda_1 = 1/1.26$		
	1.4257		1.4257		

$\beta_1 = 1$, $\lambda_1 = 1$, $\lambda_2 = 0$. The initial conditions ϕ are time-varying in $\bar{\mathbb{M}}$ and time-invariant in $\hat{\mathbb{M}}$, however, ϕ in $\bar{\mathbb{M}}$ has to satisfy some constraints such as $\phi(t_2) \leq \dot{\phi}(t) \leq \phi(t_1)$ for some $t_1, t_2 \in \begin{bmatrix} -\tau & 0 \end{bmatrix}$. These constraints lead to that $\bar{\mathbb{M}}$ is smaller than $\hat{\mathbb{M}}$ for the same delay τ. For example, as $\tau = 1$, applying Theorem 1 we get Figure 1 which shows that $\hat{\mathbb{M}}$ is larger than $\bar{\mathbb{M}}$. In addition, the trajectories of six initial states in the stabilization region are also plotted in Figure 1. They are all stabilized by the corresponding controller.
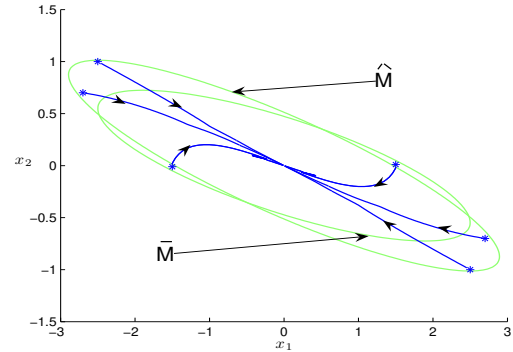

Fig. 1. The estimated stabilization region and six trajectories starting on the boundary.

Figure 2(a) shows two states trajectories starting from $\phi_0 = \begin{bmatrix} 1.5 & 0.1 \end{bmatrix}$ and $\phi(t) = \begin{bmatrix} 1.5e^t & 0.1e^t \end{bmatrix}^T$ for $t \in \begin{bmatrix} -1 & 0 \end{bmatrix}$ respectively. As $t = 0$, they have the same initial value $\phi_0 = \begin{bmatrix} 1.5 & 0.1 \end{bmatrix}$, but as time goes by, the trajectories are different. This is because if $t < 1$, the fuzzy system becomes $\dot{x}(t) = A_h x(t) + A_{\tau h} \phi(t - 1) + B_h u(t)$ for $\bar{\mathbb{M}}$ and $\dot{x}(t) = A_h x(t) + A_{\tau h} \phi_0 + B_h u(t)$ for $\hat{\mathbb{M}}$. Figure 2(b) shows the time derivatives of the membership function always satisfy $\left| \dfrac{\partial w_0^k}{\partial z_k} \left(\dfrac{\partial z_k}{\partial x} \right)^T \dot{x} \right| \leq \beta_1 = 1$.

V. CONCLUSIONS

In this paper, we have studied the problem of local stabilization for continuous-time T-S fuzzy systems with time delay. A new Lyapunov-Krasovskii function which is dependent on the
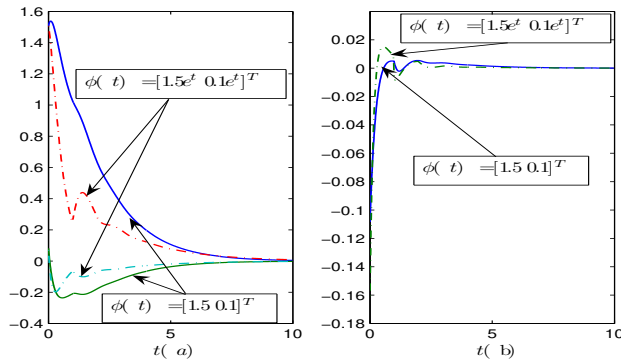
Fig. 2. The trajectories of the system states and $H = \frac{\partial w_0^k}{\partial z_k} \left(\frac{\partial z_k}{\partial x} \right)^T \dot{x}$ for different initial conditions $\phi(t)$ and ϕ_0.

membership function has been designed to deal with the time delay. Two methods have been proposed to analyze the time derivatives of the membership function. One is applicable to the T-S fuzzy systems obtained by Sector Nonlinearity method and the other one is applicable to the systems obtained by other methods.

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