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Localization
&
Exact Holography

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Université Pierre et Marie Curie Paris VI

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Based on

- A.D. João Gomes, Sameer Murthy, *"Quantum Black Holes, Localization, and the Topological String,"* arXiv:1012.0265

- A.D. João Gomes, Sameer Murthy, *"Localization and Exact Holography,"* arXiv:1105.nnnn

- A.D. João Gomes, Sameer Murthy, Ashoke Sen; *"Supersymmetric Index from Black Hole Entropy,"* arXiv:1009.3226

Two Related Motivations

Entropy of black holes remains one of the most important and precise clues about the microstructure of quantum gravity.

Can we compute exact quantum entropy of black holes including all corrections both microscopically and macroscopically?

Holography has emerged as one of the central concepts regarding the degrees of freedom of quantum gravity.

Can we find simple example of AdS/CFT holgraphy where we might be able to 'prove' it exactly?

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Quantum Entropy

Black Hole Entropy   Bekenstein [72]; Hawking[75]

For a BPS black hole with charge vector (q, p), *for large charges*, the leading Bekenestein- Hawking entropy precisely matches the logarithm of the degeneracy of the corresponding quantum microstates

$$\frac{A(q, p)}{4} = \log(d(q, p)) + O(1/Q)$$

Strominger & Vafa [96]

This beautiful approximate agreement raises two important questions:

- What exact formula is this an approximation to?
- Can we systematically compute corrections to both sides of this formula, perturbatively and nonperturbatively in $1/Q$ and may be even exactly for arbitrary finite values of the charges?

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AdS_2/CFT_1 Holography

Quantum Entropy and AdS_2/CFT_1 Sen [08]

- Near horizon geometry of a BPS black hole is $AdS_2 \times S^2$. Quantum entropy can be defined as as partition function of the AdS_2.

$$W(q, p) = \left\langle \exp \left[- i \, q_i \int_0^{2\pi} A^i \, d\theta \right] \right\rangle_{\mathrm{AdS}_2}^{\mathrm{finite}} .$$

Functional integral over all string fields. The Wilson line insertion is necessary so that classical variational problem is well defined.

The black hole is made up of a complicated brane configuration. The worldvolume theory typically has a gap. Focusing on low energy states gives CFT_1 with a degenerate, finite dimensional Hilbert space. Partition function $d(q, p)$ is simply the dimension of this Hilbert space.

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AdS_2/CFT_1 Holography

Functional Integral Boundary Conditions

- For a theory with some vector fields A^i and scalar fields ϕ^a, we have the fall-off conditions

$$
\begin{aligned}
ds_0^2 &= v\left[(r^2 + \mathcal{O}(1))\,d\theta^2 + \frac{dr^2}{r^2 + \mathcal{O}(1)}\right], \\
\phi^a &= u^a + \mathcal{O}(1/r), \qquad A^i = -i\,e^i(r - \mathcal{O}(1))d\theta, \qquad (1)
\end{aligned}
$$

- Magnetic charges are fluxes on the S^2. Constants v, e^i, u^a fixed to attractor values v_*, e_*^i, u_*^a determined purely in terms of the charges, and set the boundary condition for the functional integral.

- Quantum entropy is purely a function of the charges (q, p).

The functional integral is infrared divergent due to infinite volume of the AdS_2. Holographic renormalization to define the finite part.

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AdS_2/CFT_1 Holography

Renormalized functional integral

- Put a cutoff at a large $r = r_0$.

- Lagrangian $\mathcal{L}_{bulk}$ is a local functional, hence the action has the form

$$S_{\text{bulk}} = C_0 r_0 + C_1 + \mathcal{O}(r_0^{-1}) \,,$$

  with C_0, C_1 independent of r_0.

- C_0 can be removed by a boundary counter-term (boundary cosmological constant). C_1 is field dependent to be integrated over.

Quantum Entropy gives a proper generalization of Wald entropy to include not only higher-derivative *local* terms but also the effect of *integrating over massless fields*. This is essential for duality invariance and for a systematic comparison since nonlocal effects can contribute to same order.

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To summarize, our most ambitious goal will be twofold.

1. Compute $d(q, p)$ from bound state dynamics of branes.
2. Compute $W(q, p)$ from the bulk for arbitrary *finite* charges by evaluating the functional integral over string fields in AdS_2. Check if the two agree.

The first problem has now been solved in some cases. We now know the exact spectrum of both half and quarter-BPS dyonic black holes for *all charges* at *all points* in the moduli space for certain $\mathcal{N} = 4$ theories. Dijkgraaf, Verlinde, Verlinde; [96] Gaiotto, Strominger, Yin; David, Sen [05];

David, Jatkar, Sen; Dabholkar, Nampuri; Dabholkar, Gaiotto [06];

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Microscopic Counting

Exact microscopic degeneracy of half-BPS black holes

To be concrete let us consider a particularly simple example.

- Heterotic string compactified on $T^5 \times S^1$. A state with momentum n and winding w along S^1 is half-BPS if in the right-moving ground state with arbitrary left-moving excitations. Partition function of 24 left-moving transverse bosons gives

$$Z(\tau) = \frac{1}{\eta^{24}(\tau)} \,,$$

Dabholkar & Harvey [89]

- The degeneracy depends only on the T-duality invariant $N := n\,w$ and is given by

$$d(N) = \int_C e^{-2\pi i \tau N} Z(\tau) d\tau \,,$$

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Microscopic Counting

Rademacher expansion

d(N) admits an *exact* expansion

$$d(N) = \sum_{c=1}^{\infty} Kl(N; -1; c) \left(\frac{2\pi}{c}\right)^{14} \tilde{I}_{13}\left(\frac{4\pi\sqrt{N}}{c}\right)$$

where

$$\tilde{I}_{13}(z) = \frac{1}{2\pi i} \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{1}{t^{14}} e^{t + \frac{z^2}{4t}} \, dt,$$

is a modifield Bessel function of index 13, and

$$Kl(N; -1; c) = \sum_{\substack{a,d \in (\mathbb{Z}/\mathbb{Z})^* \\ ad = 1 \, mod \, c}} \exp\left(\frac{2\pi i d N}{c}\right) \cdot \exp\left(\frac{-2\pi i a}{c}\right).$$

is called the "Kloosterman sum". This sum simplifies for $c = 1$ being equal to 1, but for other values of c it shows a nontrivial dependence on N.

- Our goal now will be to evaluate the formal expression for W(q,p) by doing the functional integral over string fields in AdS_2.

- This is of course highly nontrivial and may even seem foolishly ambitious.

- Surprisingly, one can go quite far using localization techniques to reduce the functional integral to finite number of *ordinary integrals*.

- With enough supersymmetry, it seems possible to even evaluate these ordinary integrals all the way under certain assumptions.

Bulk of my talk will be about some recent progress in the evaluations of $W(q, p)$ functional integral using localization.

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Localization

- Consider a supermanifold $\mathcal{M}$ with an odd (fermionic) vector field Q be such that $Q^2 = H$ for some compact bosonic vector field H.

- To evaluate an integral of a Q-invariant function h with Q-invariant measure we first deform it

$$I := \int_{\mathcal{M}} d\mu \, h \, e^{-S} \qquad \rightarrow \qquad I(\lambda) := \int_{\mathcal{M}} d\mu \, h \, e^{-S - \lambda QV},$$

where V is a fermionic, H-invariant function.

- It is easy to see $I'(\lambda) = 0$ and thus $I(\lambda)$ is independent of λ. Hence, instead of at $\lambda = 0$, one can evaluate it at $\lambda = \infty$ where semiclassical approximation is exact.

Witten[88, 91], Duistermaat-Heckmann [82], Schwarz & Zaboronsky [95]

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Choice of the Supercharge

- In this limit, the functional integral *localizes* onto the critical points of the functional $S^Q := QV$. This reduces the functional integral over field space to a this localizing submanifold.

- To apply to our problem, we pick Q which squares to $H = 4(L - J)$. Here L generates rotation of the Euclidean AdS_2 which is a disk and J generates a rotation of S^2, so H is compact.

- Given this choice of Q we choose the localizing action functional to be

$$S^Q = QV; \qquad V = (Q\Psi, \Psi)$$

where Ψ denotes schematically all fermions of the theory.

To apply this directly in string theory is not feasible given the state of string field theory. So we will first solve a simpler problem in supergravity.

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Localization

A simpler problem in supergravity

- Consider $\hat{W}(q, p)$ which is the same functional integral but in supergravity coupled to only $n_v + 1$ vector multiplets.
- This is still a complicated functional integral over *spacetime* fields.

We will show using localization that this functional integral reduces to an ordinary integral over $n_v + 1$ real parameters. Huge simplification.

To apply localization inside the functional integral, one requires an *off-shell* formulation. In general, off-shell supergravity is notoriously complicated but for $\mathcal{N} = 2$ vector multiplets an elegant formalism exists that gauges the full superconformal group.

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Multiplets and Off-shell supersymmetry transformations

- *Gravity multiplet*: Vielbein, spin connection, auxiliary fields and fermions

- *Vector multiplet*: vector field A_μ^I, complex scalar X^I and an $SU(2)$ triplet of auxiliary fields Y_{ij}^I and fermions. Here i is $SU(2)$ doublet.

$$\mathbf{X}^I = \left(X^I, \Omega_i^I, A_\mu^I, Y_{ij}^I \right)$$

$$\delta\Omega_i = 2\slashed{D}X\epsilon_i + \frac{1}{2}\varepsilon_{ij}F_{\mu\nu}\gamma^{\mu\nu}\epsilon^j + Y_{ij}\epsilon^j + 2X\eta_i \,,$$

where ϵ, η are the (superconformal) supersymmetry parameters.

de Wit, Lauwers, van Holten, Van Proeyen [1980]

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Off-shell supergravity for Localization

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Importance of being off-shell

- The beauty of off-shell supergravity consists in the fact that the supersymmetry transformations are specified once and for all and do not depend on the choice of the action. This is crucial for localization both at conceptual and computational level.

- In particular, auxiliary fields which are normally eliminated from the physical action, will play an important role and will acquire nontrivial position dependence for the localizing instanton solutions.

With this setup, the bosonic part $(Q\Omega, Q\Omega)$ of the QV action is a sum of perfect squares. Setting each of these terms to zero gives a set of first-order differential equations. Happily, it turns out they can be solved exactly to obtain an analytic solution.

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Off-shell supergravity for Localization

The bosonic part of the localizing action $(Q\Psi, Q\Psi)$

$$\cosh(\eta)\big[K - 2\,\mathrm{sech}(\eta)H\big]^2$$

$$+ \quad 4\cosh(\eta)\big[H_1 + H\tanh(\eta)\big]^2 + 4\cosh(\eta)[H_0^2 + H_2^2 + H_3^2]$$

$$+ \quad 2A\left[f_{01}^- - J - \frac{1}{A}\left(\sin(\psi)J_3 - \sinh(\eta)J_1\right)\right]^2$$

$$+ \quad 2B\left[f_{01}^+ + J - \frac{1}{B}\left(\sin(\psi)J_3 + \sinh(\eta)J_1\right)\right]^2$$

$$+ \quad 2A\left[f_{03}^- + \frac{1}{A}\left(\sin(\psi)J_1 + \sinh(\eta)J_3\right)\right]^2$$

$$+ \quad 2B\left[f_{03}^+ + \frac{1}{B}\left(\sin(\psi)J_1 - \sinh(\eta)J_3\right)\right]^2$$

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Off-shell supergravity for Localization

$$+ \quad 2A \left[f_{02}^{-} + \frac{1}{A} \left(\sin(\psi) J_0 + \sinh(\eta) J_2 \right) \right]^2$$

$$+ \quad 2B \left[f_{02}^{+} - \frac{1}{B} \left(\sin(\psi) J_0 + \sinh(\eta) J_2 \right) \right]^2$$

$$+ \quad \frac{4 \cosh(\eta)}{AB} \left[\sinh(\eta) J_0 - \sin(\psi) J_2 \right]^2$$

$$+ \quad \frac{4 \cosh(\eta) \sinh^2(\eta)}{AB} [J_1^2 + J_3^2] \, ,$$

where

$$H_a^I := e_a^\mu \partial_\mu H^I \, , \quad J_a^I := e_a^\mu \partial_\mu J^I \, ,$$

$$A := \cosh(\eta) + \cos(\psi) \, , \qquad B := \cosh(\eta) - \cos(\psi) \, .$$

It is understood that all squares are summed over the index I.

Localizing instanton Solution

$$X^I = X_*^I + \frac{C^I}{r} \,, \qquad \bar{X}^I = \bar{X}_*^I + \frac{C^I}{r}$$

$$Y_1^{I1} = -Y_2^{I2} = \frac{2C^I}{r^2} \,, \qquad f_{\mu\nu}^I = 0 \,.$$

Solves a major piece of the problem by identifying the off-shell field configurations onto which the functional integral localizes. Thus, a functional integral is reduced to a finite dimensional ordinary integral. This instanton is *universal* and does not depend on the physical action.

Scalar fields move away from the attractor values X_*^I inside the AdS_2 'climbing up' the entropy function potential. Q supersymmetry is still maintained because *auxiliary fields* get nontrivial position dependence.

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Localizing Instanton and its Renormalized action

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We now need to evaluate the physical action on the localizing
instantons after proper renormalization to compute $S_{ren}(C, q, p)$

$$
\begin{aligned}
& (-i(X^I \bar{F}_I - F_I \bar{X}^I)) \cdot (-\frac{1}{2}R) + [i\nabla_\mu F_I \nabla^\mu \bar{X}^I \\
+ \ & \frac{1}{4} i F_{IJ}(F_{ab}^{-I} - \frac{1}{4}\bar{X}^I T_{ab}^{ij} \varepsilon_{ij})(F^{-abJ} - \frac{1}{4}\bar{X}^J T_{ab}^{ij} \varepsilon_{ij}) \\
- \ & \frac{1}{8} i F_I(F_{ab}^{+I} - \frac{1}{4}X^I T_{abij} \varepsilon^{ij}) T_{ab}^{ij} \varepsilon_{ij} - \frac{1}{8} i F_{IJ}Y_{ij}^I Y^{Jij} - \frac{i}{32} F(T_{abij}\varepsilon^{ij})^2 \\
+ \ & \frac{1}{2} i F_{\widehat{A}} \widehat{C} - \frac{1}{8} i F_{\widehat{A}\widehat{A}}(\varepsilon^{ik}\varepsilon^{jl}\widehat{B}_{ij}\widehat{B}_{kl} - 2\widehat{F}_{ab}^- \widehat{F}_{ab}^-) \\
+ \ & \frac{1}{2} i \widehat{F}^{-ab} F_{\widehat{A}I}(F_{ab}^{-I} - \frac{1}{4}\bar{X}^I T_{ab}^{ij} \varepsilon_{ij}) - \frac{1}{4} i \widehat{B}_{ij} F_{\widehat{A}I} Y^{Iij} + \mathrm{h.c.}] \\
- \ & i(X^I \bar{F}_I - F_I \bar{X}^I) \cdot (\nabla^a V_a - \frac{1}{2}V^a V_a - \frac{1}{4}|M_{ij}|^2 + D^a \Phi^i{}_\alpha D_a \Phi^\alpha{}_i) \,.
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Renormalized action

- Substituting our localizing instanton solution in the above action we can extract the finite piece after removing the leading divergent piece linear in r_0 by holographic renormalization.

- After a tedious algebra, one obtains a remarkably simple form for the renormalized action S_{ren} as a function of $\{C^I\}$.

$$S_{ren}(\phi, q, p) = -\pi q_I \phi^I + \mathcal{F}(\phi, p) \tag{2}$$

with $\phi^I := e_*^I + 2iC^I$ and $\mathcal{F}$ given by

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where e_*^I are the attractor values of the electric field.

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- Note that $S_{ren}(\phi, q, p)$ equals precisely the *classical* entropy function $\mathcal{E}(e, q, p)$. In particular, $\exp(S_{ren})$ is the topological string partition function. The physics is however completely different.

- $\mathcal{E}(e, q, p)$ depends on the the attractor values X_* of the scalar fields. $S_{ren}(\phi, q, p)$ depends on the value of the scalar fields at the center of AdS_2. This difference is very important.

- Even though scalar fields are fixed at the boundary, their value at the origin can fluctuate and we can integrate over them for large values.

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We at present lack a useful definition of functional integral over string fields. To apply localization, we proceed in three steps.

Three Steps:

1. Integrate out massive string and Kaluza-Klein modes to obtain a local Wilsonian effective action for the massless supergravity fields.

2. Solve a supergravity problem to evaluate $\hat{W}(q, p)$.

3. Use the results in Step II to evaluate $W(q, p)$ There are $\mathbb{Z}_c$ orbifolds of AdS_2 that have the same boundary conditions and hence contribute to the functional integral. Hence, $W(q, p)$ has the form

$$W(q, p) = \sum_{c=1}^{\infty} W_c(q, p).$$

Sen[09, 10], Pioline & Murthy [10]

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Reducing the problem to supergravity

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Reducing the problem to supergravity

Evaluation $W_c(q, p)$ is related to the problem of evaluation of $\hat{W}(q, p)$ in a simple way, under certain assumptions.

General form of the answer for $c = 1$

$$W_1(n, w) = \int_{\mathcal{M}_Q} e^{S_{ren}(n,w,\phi)} |Z_{inst}(\phi, n, w)|^2 \ Z_{det}(\phi) \ [d\phi]_\mu \ .$$

- $[d\phi]_\mu$ is the induced measure on the localizing manifold.

- $|Z_{inst}|^2$ is contribution of instantons localized at north pole of S^2 and anti-instantons localized at the south pole.

- $Z_{det}(\phi)$ is one loop determinant.

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Reducing the problem to supergravity

Evaluation $W_c(q, p)$ is related to the problem of evaluation of $\hat{W}(q, p)$ in a simple way, under certain assumptions.

General form of the answer for $c = 1$

$$W_1(n, w) = \int_{\mathcal{M}_Q} e^{S_{ren}(n,w,\phi)} |Z_{inst}(\phi, n, w)|^2 \, Z_{det}(\phi) \, [d\phi]_\mu \ .$$

- $[d\phi]_\mu$ is the induced measure on the localizing manifold.

- $|Z_{inst}|^2$ is contribution of instantons localized at north pole of S^2 and anti-instantons localized at the south pole.

- $Z_{det}(\phi)$ is one loop determinant.

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No off-shell formalism with $\mathcal{N} = 4$ with finite number of auxiliary fields. So we will proceed in the $\mathcal{N} = 2$ language.

Assumptions

- Drop two gravitini multiplets. These contain four vector fields but no scalar fields. If the black hole does not couple to these vector fields, it should be reasonable to drop this.

- Drop hyper multiplets. The off-shell supersymmetry transformations of the vector multiplets do not change by adding hypers. So our localizing instantons will continue to exist.

- Drop D-terms. A large class of D-terms are known not contribute to Wald entropy.   de Wit, Katmadas, Van Zalk [10]

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Reducing the problem to supergravity

Prepotential

For our example with this $\mathcal{N} = 2$ restriction, the prepotential is

$$F(X, \hat{A}) = -\frac{1}{2}\frac{X^1}{X^0}\sum_{a,b=2}^{23} C_{ab}X^a X^b - \frac{\hat{A}}{64}\frac{X^1}{X^0},$$

in the Type-IIA frame dual to the heterotic frame. Here C_{ab} is the intersection matrix for the 22 2-cycles of $K3$. The charge configuration (n, w) corresponds in this frame to choosing $p^1 = -w$ and $q_0 = n$.

Renormalized Action

$$S_{ren} = -\pi n\phi^0 - 4\pi\frac{w}{\phi^0} + \frac{\pi}{2}\frac{w}{\phi^0}C_{ab}\phi^a\phi^b.$$

Evaluation of W_1

Induced Measure and one-loop determinants

Using the fact that the scalar manifold is special Kähler, can deduce the measure over the ϕ-space knowing the measure on X-space.

$$\frac{1}{w^2} \prod_{a=0}^{23} d\phi^a.$$

The localizing action is purely quadratic in the fields. Hence the determinant factor is one.

Instantons

Euclidean NS5-branes contribute for quarter-BPS dyons. However, the half-BPS dyons preserve additional supersymmetries that are broken by the NS5-brane which leads to additional fermions zero modes. Hence no brane-instanton contribution.

Form of the Integral

The ϕ-integration then takes the form

$$\int \frac{d\phi^1}{\phi^0} \int \frac{d\phi^0 \phi^0}{w^2} \prod_{a=2}^{23} d\phi^a \exp\left[-\pi n \phi^0 - 4\pi \frac{w}{\phi^0} + \frac{\pi}{2} \frac{w}{\phi^0} C_{ab} \phi^a \phi^b\right].$$

Substituting $t = -4\pi w/\phi^0$ and $a = \phi^a/\phi^0$, we get

$$\int da \int \frac{dt}{t^3} \exp[t + \frac{4\pi^2 N}{t}] \int \prod_{a=2}^{23} d\phi^a \exp\left[-\frac{t}{8} C_{ab} \phi^a \phi^b\right].$$

Residual Duality Symmetry

The residual $a \to a+1$ is an unbroken discrete duality symmetry of the integrand as well as the charge configuration. Hence must be modded out.

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Putting it all together

Final Answer for $W_1(n, w)$

Conformal compensator

The ϕ^0 variable can be thought of as the conformal compensating field. Conformal factor of the metric has wrong sign kinetic term in Euclidean gravity and hence its contour of integration must be analytically continued to make the functional integral well-defined.

The Gaussian integrals can be readily done to obtain

$$\mathcal{C} \int_{-i\infty}^{+\infty} \frac{dt}{t^{14}} \exp[t + \frac{z^2}{4t}] \quad with \quad z = 4\pi\sqrt{N} .$$

Up to an overall constant, this is precisely the integral representation of the first term in the Rademacher expansion — $I_{13}(4\pi\sqrt{N})$!

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Putting it all together

Nonpertrubative Corrections

There is a family of freely acting supersymmetric $\mathbb{Z}_c$ orbifolds with twists on $AdS_2 \times S^2$ on shifts along S^1. The shift can be effected by modifying the gauge field at infinity. Hence the Wilson line gives a phase both for winding and momentum. Since the orbifold action is freely acting, one obtains the same localizing instanton solution but the renormalized action is divided by c because of the reduced volume.

For each c we then obtain

$$\sum_{a,d \in (\mathbb{Z}/\mathbb{Z})^*} e^{2\pi i(\frac{nd}{c} - \frac{wa}{c})} \int da \int \frac{dt}{c^2 t^3} \exp[t + \frac{4\pi^2 N}{c^2 t}] Z_{det}(c) \ .$$

The one-loop determinant has a c dependence.

One-loop determinant

It is easy to see that

$$Z_{det}(c) = \frac{Z_{det}(1)}{c^{12}}$$

which is a consequence of the fact that each scalar field has a zero mode, there are 24 scalar fields. The nonzero modes cancel between bosons and fermions. Factor of $1/\sqrt{c}$ for each zero mode by ultralocality.

For each c we then obtain

$$\mathcal{C} \sum_{a,d \in (\mathbb{Z}/\mathbb{Z})^*} e^{2\pi i(\frac{nd}{c} - \frac{wa}{c})} \frac{1}{c^{14}} \tilde{I}_{13}\left(\frac{4\pi\sqrt{nw}}{c}\right) \, .$$

with the same overall constant as before. We seem to reproduce the full Rademacher expansion (almost!).

Kloosterman Sum and T-duality Invariance.

The Kloosterman sum satisfies

$$Kl(n, -w; c) = Kl(nw, -1; c)$$

if a and d satisfy $ad = 1 \bmod(c)$. Integers a and d naturally enter our story from the Wilson lines but *without* the mod c constraint.

Our orbifold is a symmetric twist on $AdS_2 \times S^2$ and a shift in the momentum-winding lattice. Twisting of right-moving fermions gives ground state energy. The level matching condition becomes

$$\frac{1}{c}(1 - \frac{1}{c}) + \frac{ad}{c^2} = 0 \quad \bmod \ \frac{1}{c},$$

which is precisely $ad = 1 \bmod c$. Plausible explanation, but not clear why other shifts are not allowed, nor if such geometric reasoning is OK.

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Summary and Outlook

Summary

Localization of the functional integral in Sugra and String Theory

- We have shown that full functional integral of supergravity coupled to vector fields on AdS_2 localizes onto the submanifold $\mathcal{M}_Q$ of critical points of the functional S^Q where Q is a specific supersymmetry.

- We have obtained exact analytic expression for a family of nontrivial complex instantons as *exact* solutions which are completely *universal* and independent of the form of the physical action.

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Summary and Outlook

W(n, w) reproduces the full Rademacher expansion

$$W(n, w) = \mathcal{C} \sum_{c=1}^{\infty} Kl(N; -1; c) \left(\frac{2\pi}{c}\right)^{14} \tilde{I}_{13}(\frac{4\pi\sqrt{N}}{c})$$

with

$$\tilde{I}_{13}(z) = \frac{1}{2\pi i} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{1}{t^{14}} e^{t + \frac{z^2}{4t}} dt,$$

and

$$Kl(N; -1; c) = \sum_{\substack{d \in (\mathbb{Z}/\mathbb{Z})^* \\ ad=1 \bmod c}} \exp(\frac{2\pi i dN}{c}) \cdot \exp(\frac{-2\pi i a}{c}).$$

Some of the assumptions need to be better justified. Requires off-shell realization of at least the charge Q with vector, hypers, D-terms or $\mathcal{N} = 4$ field content. Interesting problem in supergravity.

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Summary and Outlook

$N = nw$	$d(N)$	$\log d(N)$	$4\pi\sqrt{N}$	$\log \tilde{I}_{13}(4\pi\sqrt{N})$
1	24	3.17	12.56	3.94
2	324	5.78	17.77	6.23
3	3200	8.07	21.76	8.31
4	25650	10.15	25.13	10.24
17	6599620022400	29.51	51.81	28.87
18	21651325216200	30.70	53.31	30.03
19	69228721526400	31.86	54.77	31.16
20	216108718571250	33.00	56.19	32.28

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Summary and Outlook

Comparison with Earlier Work

It seems possible to compute finite size corrections to the entropy of BPS black holes in a systematic way going well beyond Bekenstein-Hawking.

The leading Bessel function was partially derived in Dabholkar [04] and Dabholkar, Denef, Pioline, Moore [05]. However,

- it relied on the unproven OSV conjecture Ooguri, Strominger, Vafa [04];
- it was at best an asymptotic expansion since there was no systematic way of determining the measure or the range of integration since the OSV conjecture applied to a mixed ensemble.
- it was not clear how to include the nonperturbative corrections to obtain for example the subleading Bessel functions.
- it was not clear how to incorporate holomorphic anomaly systematically.

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Summary and Outlook

Our derivation follows from first principles using standard rules of functional integration and localization (under certain assumptions and caveats) within the framework of holography.

- Functional integral localizes onto a nontrivial localizing instanton solution of the *off-shell theory*. Auxiliary fields play an important role.

- The measure or the range of integration can be determined following usual collective coordinate quantization. Gives the exact Bessel function and not just the asymptotic expansion.

- Nonperturbative corrections from orbifolds give subleading Bessel functions. Their sum is natural in *microcanonical* ensemble. It is justified to keep subleading saddles because of localization.

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From Gravity to Number Theory

- The Rademacher expansion is an *exact* expansion. It is rapidly convergent but at no finite order can one assert integrality of the sum. It is a nontrivial fact that all these terms add up to an integer and that we can see the entire expansion from the bulk.

- This indicates an underlying integral structure to quantum gravity. If we have two very close but different integers, the bulk theory will be able to distinguish the two. This would be never evident from semiclassical Bekenstein-Hawking formula.

- It is intriguing that two AdS_2 bulk theories which may have very different field content but which yield the same integer $W(q, p)$ are expected to be dual since they will be both dual to the same CFT_1. This is rare but possible.

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Summary and Outlook

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- On a philosophical note, our computation indicates that the quantum gravity in the bulk is as fundamental as the boundary field theory, with its own rules of computation. It is an exact dual description rather than a coarse-grained description. It is worth exploring the AdS_2/CFT_1 duality further including correlation functions.

- It is remarkable that a bulk functional integral can reproduce an integer. The fact that the bulk is able to 'see' this quantization may be relevant for information retrieval.

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