

Localization in Wireless Sensor Networks: A Dempster-Shafer Evidence Theoretical Approach

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Abstract

This paper proposes a data fusion technique aimed at achieving highly accurate localization in a wireless sensor network with low computational cost. This is accomplished by fusing multiple types of sensor measurement data including received signal strength and angle of arrival. The proposed method incorporates a powerful data fusion technique, one that has never before been used in low cost localization of a stationary node, known as Dempster-Shafer Evidence Theory. Many useful functions of this theory, including sampling, aggregation, and plausibility, are integrated into the localization method. From there, the algorithm determines whether a set of given measurements belong to a particular county. Motivated by the flexible nature of Dempster-Shafer Theory, a multitude of network setups and combinations of available measurement features are tested to verify the performance of the proposed method. Performance of the proposed approach is evaluated using numerical results obtained from extensive simulations. When compared with the results of existing approaches in similarly constructed scenarios, the proposed localization technique achieves up to 98% accuracy in less than a tenth of the run-time required under presently established algorithms.

Keywords: Wireless Sensor Networks, Localization, Dempster-Shafer Theory

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1. Introduction

Wireless sensor networks have been of great interest in today's world due to their wide variety of practical applications and purposes. Practicality can range from long distance navigational systems and localized contact points for emergency services to industrial detection of carbon monoxide (Yang, Zhou, Lv, Wei, and Song, 2015) and community detection of social clusters (Li, Qin, and Song, 2016). In addition to emergencies and navigation, typical uses of these networks can include tracking, security, and information (Lloret, Tomas, Garcia, and Canovas, 2011).

While the demand for and scope of wireless sensor networks (or WSNs) continue to grow, the need for identifying a node's location within such a network becomes increasingly vital. However, achieving localization with a steady balance of minimal power, low overall cost, and high accuracy (Mao, Fidan, and Anderson, 2007) remains one of the greatest challenges in the area of WSNs.

1.1. Localization Techniques

A brief overview of the most common unimodal localization techniques of both low cost and high cost categories is presented in Fig. 1. In this scenario, cost refers to the amount for significant computational resources required to achieve accurate localization. Some of the most common modern localization techniques are well-known even outside of scientific research areas due to

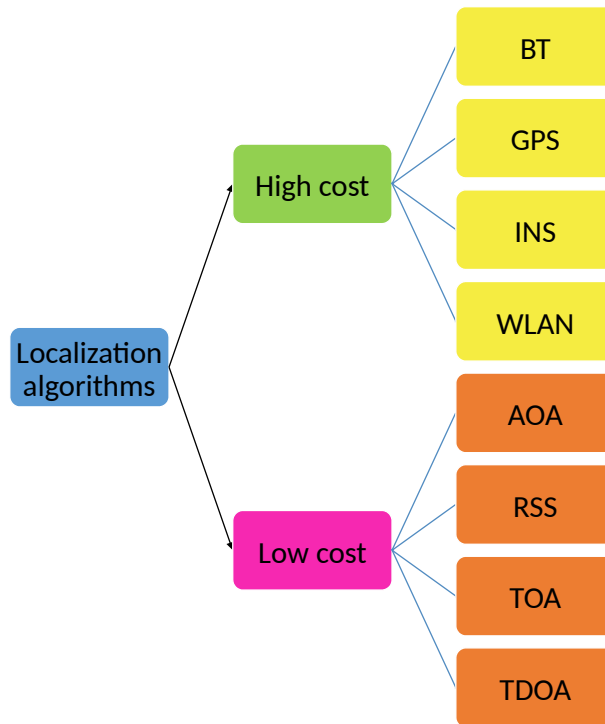


Figure 1: Classification of localization techniques.

their popularity in modern consumer electronics. These include global positioning systems (GPS), wireless local area networks (WLAN), and bluetooth (BT). GPS is perhaps the most widely known method due to its frequent use in navigational assistance through standalone devices and through embedded navigational capabilities in smartphones and tablets. Bluetooth is primarily appealing for short range solutions and thus can be ideal specifically for local positioning within a WSN (Shen, Chen, and Lu, 2008). WLAN, or Wi-Fi, is another short range solution whose significance lies within its high localization accuracy throughout indoor WSN environments (Shen, Chen, and Lu, 2008). Inertial Navigation Systems (INS) are not quite as common as the other three methods but are noteworthy due to their high potential for seamless integration with GPS systems (Bhatt, Aggarwal, Devabhaktuni, and Bhattacharya, 2012). All four of these techniques share the common characteristic of requiring a high computational cost to achieve a desirable level of accuracy. For instance, a GPS-based solution requires sufficient resources in every node to receive a signal from a positioning satellite, resulting in long

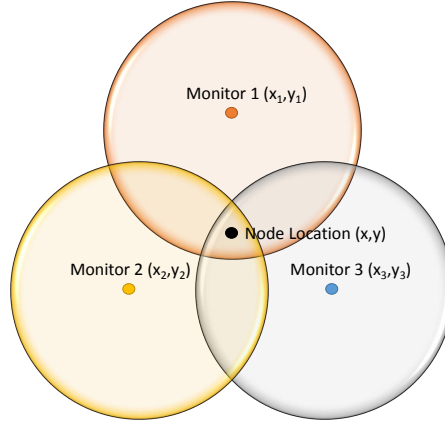


Figure 2: Depiction of RSS Trilateration in a three-monitor WSN Setup.

connection times, external signal outages, and shorter battery life (Bulusu, Heidemann, and Estrin, 2000). Bluetooth and WLAN are also prone to faster battery drain, as any smartphone or laptop user is quick to observe. One potential resolution to this resource issue involves restricting measurement capabilities to a small handful of nodes while reducing all other nodes to a bare minimum level of communication capability. These specialized, higher powered nodes are known as cluster heads, or anchor nodes (Mao, Fidan, and Anderson, 2007). In order to utilize the remaining nodes, however, a vital further step is to apply lower cost methods of a less computational and more mathematical nature in order to achieve the full potential of effective localization.

Many common low cost techniques include received signal strength (RSS), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and hop count. In the context of this research, RSS is defined as the electric field at the receiving node divided by the distance between said node and the transmitting antenna. In outdoor environments, RSS can be considered as a simple mathematical method of distance calculation, free from the restrictions of walls, multipath effects, and humidity (Garcia, Martinez, Tomas, and Lloret, 2007; Garcia, Tomas, Boronat, and Lloret, 2009) that are typically associated with indoor WSN environments. Due to RSS's inverse relationship with squared distance (Xu, Lin, Lang, Zhang, and Wang,

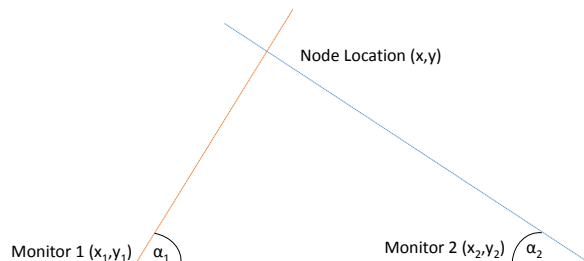


Figure 3: Depiction of AOA-based localization.

2010) in outdoor WSNs, highly accurate locations can be determined with three or more RSS measurements from three unique points. This is accomplished by trilateration (Mazuelas et al, 2009), as can be seen in Fig. 2. Trilateration is particularly beneficial when three or more anchor nodes are available but is ineffective in WSN setups in which only two measurements are available. That is, under two RSS measurements, only bilateration can be achieved, resulting in two possible intersecting points and thus a 50% chance of achieving accurate localization. AOA is the arrival angle of the emitted source signal observed at an anchor node (as shown in Fig. 3) and is a measurement dependent on TDOA. More specifically, AOA is calculated using the differences between arrival times of a transmitted signal (Patwari et al, 2005). TDOA is dependent on time differences between nodes (Cakir, Kaya, and Cakir, 2014) and is another effective means of localization by process of multilateration (Pelant and Stejskal, 2011). TOA, on the other hand, is simply the amount of time required for a signal to move from a transmitting node to a receiving node (Patwari et al, 2005). Like RSS, AOA provides a 50% chance of accurate localization using intersecting angles, but it also requires expensive antenna arrays (Patwari et al, 2005). In addition, TOA faces the drawback of having a dependence on source signal transmission time to measure propagation time between sensors (Shen, Ding, Dasgupta, and Zhao, 2008). TDOA measurements, on the other hand, strengthen noise,

leading to hindered performance (Shen, Ding, Dasgupta, and Zhao, 2008).

In spite of the aforementioned drawbacks, many of these techniques, such as RSS and AOA, function adequately on their own. However, it has been shown that combining multiple, varied types of data can enhance results and understanding, effectively providing the best of all worlds in regards to each data type's advantages (Xing et al, 2009; Tseng, Kuo, Lee, and Huang, 2004). However, the very act of combining measurement types—data fusion—poses a unique challenge.

1.2. Dempster-Shafer Theory and Its Existing Applications

Decision fusion is a form of data fusion in which the desired output is a decision rather than a quantitative value. Dempster-Shafer (DS) Theory is a decision fusion technique that is based on Bayesian combinational probability but undertakes a drastically different foundation with regards to combinational factors (Nguyen and Walker, 1993). While Bayesian statistics rely primarily upon combinations of internal probability factors within a system (Chiodo and Mazzanti, 2006), typically through the use of propositions or random variables, DS Theory is contingent upon external evidence factors (Limbourg, Savic, Petersen, and Kochs, 2007; Lipeng, Juan, and Laibin, 2011). Each factor consists of (i) a range of data in which the desired output value can be found as well as (ii) a probability of confidence that the data range is in fact reliable. DS Theory is meant to model information that, under more traditional Bayesian methods, would be considered incomplete (Nguyen and Walker, 1993). Some localization problems have incorporated DS Theory in the past, such as robot localization (Clerentin, 2000) and indoor localization of a mobile user (Kasebzadeh, 2014). Prior to the validation of this research, however, Dempster-Shafer Theory has never before been applied to low cost localization of a stationary node in a large outdoor WSN.

1.3. Research Motivations

The overall objective of this research is to develop an algorithm for localization in a wireless sensor network that is both highly accurate and of low computational cost. Time-critical localization of WSNs is a topic of increasing concern in today's world, as a variety of civil and military applications are heavily dependent on these two properties. For instance, many college campuses and some urban areas have a network of emergency phones scattered across town so that individuals in distress can reach emergency services (Meyer, 2012). Using effective localization techniques such as that proposed

in this paper, emergency services can dispatch police, fire, or medical services to a caller's location at a significantly improved response time. In addition, the wireless aspect of these networks would ease any infrastructure-based complexities that currently exist with current wired phone stations.

1.4. Overview of Subsequent Sections

The remainder of this paper is organized as follows. Section 2 expands upon many of the previous subsections by providing an extensive theoretical background of DS Theory and WSN localization techniques. Section 3 provides an overview of how the research study was conducted, including generation of the relevant WSN data, the properties and parameters of the sensor network, and the determination of the method's accuracy and performance. Section 4 presents the results of the experiment in a variety of scenarios, a comparison of said results to those of existing localization techniques, and a brief discussion of the impact thereof. Lastly, conclusions are presented in Section 5.

2. Mathematical Background and Formulation

This section establishes three vital concepts that comprise the core of this research study, provided in three subsections. The first is Dempster-Shafer Theory, or more specifically the concepts and properties thereof. The second contains the types of sensor node measurements needed for the localization technique. Finally, the third subsection describes the technique itself, which formulates the relationship between the two concepts to form a comprehensive location-estimating algorithm.

2.1. Dempster-Shafer Theory

From a theoretical context, Dempster-Shafer Theory can be best described as a dynamic generalization of Bayesian probability theory, forming a divergence based on the concept of ignorance (Bloch, 1996). Typical Bayesian approaches assume an ignorance quantity of zero, or equivalently 100% confidence, for any probabilistic value, which is usually appropriate when a probability factor is internal. Because DS Theory is based upon external factors, however, this property can no longer be unconditionally assumed. Hence, an array of evidence factors is used to determine all possible outcomes in a given scenario (e.g. the distance between a sensor node and an anchor node in a WSN). The finite set of all possible mutually exclusive

values is known as the frame of discernment and is denoted by Θ . From there, the union of all subsets is given by the power set 2^Θ .

One of the most significant fundamental concepts of DS Theory is the basic probability assignment (BPA) function, which is essentially the equivalent of the random variable in Bayesian probability theory. It is also known as the belief variable (Auer, Luther, Rebner, and Limbourg, 2010) and is denoted as m . Suppose A_1, \dots, A_n are all possible sets within a frame of discernment, noting that $A_i \in 2^\Theta$. Then

$$m : 2^\Theta \rightarrow [0, 1], \sum_{i=1}^n m(A_i) = 1, m(\emptyset) = 0, \quad (1)$$

where \emptyset denotes the empty set. Each set A_i takes the form of the interval $([x, \bar{x}])$, denoting the lower and upper bounds of an evidence factor, respectively (Auer, Luther, Rebner, and Limbourg, 2010). Each BPA m consists of three vital properties: a lower bound, an upper bound, and a degree of confidence μ that lies between the values of 0 and 1 (University of Duisenberg-Essen, 2012). Thus, ignorance can be defined as simply $1 - \mu$.

2.1.1. Belief and Plausibility

Belief and Plausibility are two of the most vital core functions of an aggregate BPA, as they represent upper and lower bounds in probability, respectively. In a frame of discernment, they are defined as

$$Bel(B) = \sum_{A_i \subset B} m(A_i) \quad (2)$$

$$Pl(B) = 1 - \sum_{A_i \cap B = \emptyset} m(A_i), \quad (3)$$

where $m(A_i)$ denotes the BPA mass function for a set A_i .

2.1.2. Set Representation

A BPA contains three vital properties: a lower bound x , an upper bound \bar{x} , and a degree of confidence μ . Hence, representation of a BPA as a set of these three properties becomes a highly convenient organizational method. In such a case, the resultant BPA m would be defined as

$$m = \{x, \bar{x}, \mu\}. \quad (4)$$

Likewise, because calculation of belief and plausibility requires multiple BPAs, or the CDF of an aggregate BPA, representation of the individual assignments

as a set of sets is also beneficial. Suppose $m_1^u, m_2^u, \dots, m_n^u$ represent all BPAs in Θ such that

$$m = \begin{Bmatrix} m_1^u \\ m_2^u \\ \vdots \\ m_n^u \end{Bmatrix} = \begin{Bmatrix} \{x_1^u, \bar{x}_1^u, \mu_1^u\} \\ \{x_2^u, \bar{x}_2^u, \mu_2^u\} \\ \vdots \\ \{x_n^u, \bar{x}_n^u, \mu_n^u\} \end{Bmatrix} \quad (5)$$

and $\bar{x}_1^u < \bar{x}_2^u < \dots < \bar{x}_n^u$. The latter property is of particular importance, as this sorted order of upper bounds can be used to define belief from a more specific, set-oriented context. Let the superscript u indicate that contents of m are arranged in order of increasing upper bound. Also, recall that μ is the degree of confidence in each BPA and thus can be defined as $\mu_1^u, \mu_2^u, \dots, \mu_n^u$. Then belief can be represented as

$$Bel = \begin{Bmatrix} \{\bar{x}_1^u, \mu_{1,new}^u\} \\ \{\bar{x}_2^u, \mu_{2,new}^u\} \\ \{\bar{x}_3^u, \mu_{3,new}^u\} \\ \vdots \\ \{\bar{x}_n^u, \mu_{n,new}^u\} \end{Bmatrix}, \quad (6)$$

where

$$\mu_{1,new}^u = \mu_1^u \quad (7)$$

and

$$\mu_{n,new}^u = \sum_{i=1}^{n-1} \mu_{i,new}^u. \quad (8)$$

Now suppose m is rearranged and $m_1^u, m_2^u, \dots, m_n^u$ are renamed such that

$$m = \begin{Bmatrix} m_1^l \\ m_2^l \\ \vdots \\ m_n^l \end{Bmatrix} = \begin{Bmatrix} \{x_1^l, \bar{x}_1^l, \mu_1^l\} \\ \{x_2^l, \bar{x}_2^l, \mu_2^l\} \\ \vdots \\ \{x_n^l, \bar{x}_n^l, \mu_n^l\} \end{Bmatrix}, \quad (9)$$

where $x_1^l < x_2^l < \dots < x_n^l$ and the superscript l indicates that contents of m are now arranged in order of increasing lower bound. Hence, all conditions

are available to define plausibility as

$$Pl = \left\{ \begin{array}{c} \{\bar{x}_1^l, \mu_{1,new}^l\} \\ \{\bar{x}_2^l, \mu_{2,new}^l\} \\ \{\bar{x}_3^l, \mu_{3,new}^l\} \\ \vdots \\ \{\bar{x}_n^l, \mu_{n,new}^l\} \end{array} \right\}, \quad (10)$$

where

$$\mu_{1,new}^l = \mu_1^l \quad (11)$$

and

$$\mu_{n,new}^l = \sum_{i=1}^{n-1} \mu_{i,new}^l. \quad (12)$$

In other words, the top and bottom rows represent the least and most believable outcomes, respectively. The latter is the row of greatest importance, where \bar{x}_n^u is the value of the most believable outcome and $\mu_{n,new}^u$ is the belief probability, which in this case is always 100% or 1, since it is the sum of all confidence values. Similarly, the top and bottom rows of the belief set respectively represent the least and most plausible outcomes. Also likewise to belief, the most believable row is of most significance, as \bar{x}_n^l signifies the most plausible outcome and $\mu_{n,new}^l$ is another 100% probability. Due to these powerful methods of outcome prediction, extensive analysis of belief and plausibility serve as the initial foundations and formations of this research.

2.2. Categories of Sensor Node Measurements

The proposed localization technique is dependent on a multitude of sensor measurements, namely RSS, AOA, and standby. Before these values can be mathematically defined, however, the distance between any sensor node and any anchor node must also be evident. Suppose the coordinate $[x_n, y_n]$ refers to the position of a sensor node and $[x_m, y_m]$ is the position of a monitor, or anchor node. Then, the distance d can be defined as

$$d_{actual} = \sqrt{(x_m^2 - x_n^2) + (y_m^2 - y_n^2)}, \quad (13)$$

where all aforementioned variables are measured in meters. For RSS calculation in outdoor WSN environments, multiple formulas exist for exact

calculation but share the common property of having an inverse proportionality with squared distance (Xu, Lin, Lang, Zhang, and Wang, 2010; Patwari et al, 2005). Hence, in the context of this research, RSS is simply defined as the inverse of distance squared, or $RSS = 1/d_{actual}^2$.

Due to the generic nature of this interpretation, no specific unit is needed, as long as the generic unit contains the factor of m^{-1} . AOA, however, is defined in a more specific context as

$$AOA = \begin{cases} \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) & \text{if } y_n > y_m \text{ and } x_n > x_m \\ \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) + 180 & \text{if } y_n \neq y_m \text{ and } x_n < x_m \\ \tan^{-1}\left(\frac{y_n - y_m}{x_n - x_m}\right) + 360 & \text{if } y_n < y_m \text{ and } x_n > x_m \end{cases} \quad (14)$$

with resultant units in degrees. Standby is a unique type of distance measurement in which a sensor node with this feature enabled is detected by a dedicated standby node, which measures the distance thereof by inverse square root RSS calculation. This is enabled primarily in network terrains with smaller area, as each sensor node's signal strength must be within range of the standby node. Let $[x_{sb}, y_{sb}]$ be the location of the standby node. Then the standby distance is defined mathematically as

$$d_{SB} = \sqrt{(x_{sb}^2 - x_n^2) + (y_{sb}^2 - y_n^2)}. \quad (15)$$

2.3. Proposed Representation of Distance Range as BPA

In order to fuse any of the varied measurement types together, each measurement must be converted into a common value. Such a value in this case is best represented as the distance from a sensor node to an anchor node, as determined by inverse square root RSS calculation. Due to this method of distance calculation, the experimental setup will require RSS to be enabled at all times. If a distance or range of distances can be found for all available cluster heads, then a node's location can be estimated by an enhanced process of trilateration, in which enhancement is based on the effects of data fusion. The challenge associated with reaching such a goal becomes the task of estimating such a distance range for each monitor. Thus, the need for effective organization and evaluation of optimal distance values is where Dempster-Shafer Theory becomes a mechanism of great interest.

Because external evidence factors are the primary means by which DS-based estimation can occur, it becomes necessary to obtain distances and measurement sets that are independent of those given as functional inputs.

This distinction will be explained at greater length in Subsection 3.2 by means of test data and full data sets. Because such data is external, the amounts and ranges of measurements must be reduced based on feature-specific criteria. Suppose these range values, r , are defined as

$$r = \{r_{RSS}, r_{AOA}, r_{SB}\}, \quad (16)$$

where $r_{RSS} > 1$, $0 < r_{AOA} < 1$, and $r_{SB} = -1$, the latter value indicating that there is no range for standby. If the inverse of a given feature measurement is less than the range r_i and the range is not -1 , then the lower and upper limits of such ranges are defined as $|0, 1/r_i|$. Otherwise, if the value is greater than or equal to r_i , then the limits are $|\frac{s_i}{r_i s_i + 1}, s_i|$, where s_i is the particular feature measurement and i is the feature type. A range of -1 , on the other hand, matching feature measurements between both data sets. The next step is to find external distance values from external feature measurements that lie within the corresponding range. After filtering of all such distance values, the minimum and maximum of the remaining values become the lower and upper bounds of the resultant BPA. Because all BPAs will aggregate in a later step, the confidence value can simply be 1 for the time being. Thus, based on Equation 4, we have

$$m_i = \{d_{min}, d_{max}, 1\}, \quad (17)$$

where d_{min} denotes the minimum external distance value and d_{max} denotes the maximum.

Once a BPA is formed for each active feature, aggregation can then occur. At certain times, manually adding new BPAs can be an exhaustive process, in which case sampling from a distribution becomes an ideal solution. This process, also known as discretization, involves generating n discrete samples based on a lesser amount of initialized BPAs, which collectively form a cumulative density function (CDF) of BPA structures (Tonon, 2004). Thus, to smoothen the transition between BPAs, sampling each m_i by a consistent function handle, such as inverse normalization, is recommended. Once sampling has been applied, the aggregate BPAs can now be considered as an adimensional, unit neutral entity, as required for mass assignments. Now each BPA closely fits the parameters of Equation 4 and is written as

$$m_i = \{\underline{x}, \bar{x}, \frac{1}{fn}\}, \quad (18)$$

where $\frac{1}{f_n}$ denotes an evenly distributed confidence probability in terms of f active features and n BPAs. The resultant aggregate BPA for n samples thus becomes

$$\lambda = \{\lambda_{RSS}, \lambda_{AOA}, \lambda_{SB}\}, \quad (19)$$

where

$$\lambda_{RSS} = \begin{Bmatrix} m_{1,RSS} \\ m_{2,RSS} \\ \vdots \\ m_{n,RSS} \end{Bmatrix}, \lambda_{AOA} = \begin{Bmatrix} m_{1,AOA} \\ m_{2,AOA} \\ \vdots \\ m_{n,AOA} \end{Bmatrix}, \lambda_{SB} = \begin{Bmatrix} m_{1,SB} \\ m_{2,SB} \\ \vdots \\ m_{n,SB} \end{Bmatrix} \quad (20)$$

This is under the assumption that all three features are enabled. If one or more features are unavailable, then their corresponding λ values are simply omitted. With all aggregation intact, the next step is the actual data prediction process.

2.3.1. Formulation of Proposed Algorithm

Because Dempster-Shafer Theory is a new concept of keen interest in the area of low cost WSN localization, a clear relation between actual distance and believed or plausible distance was not available in any found literature. Thus, some preliminary experimentation was needed to understand how *Bel* and *Pl* functions could potentially correlate measured values with calculated values. The resultant analysis from belief and plausibility calculations showed that the former method contained little to no effect in predicting a given distance while the latter showed encouragingly high correlation. Thus, after extensive exploration, we propose a completed high accuracy algorithm based on plausibility, as presented in Fig. 4.

As a simple example of how the proposed method works, suppose there exists a WSN of five sensor nodes, two sensor nodes, a standby node, and a fusion center, as depicted in Fig. 5 that encompasses a one square meter area (details of these node types are explained further in Subsection 3.1). Here, we wish to locate the sensor node located at $[0.4, 0.6]$ meters, given only a list of possible node locations and a set of signal measurements (RSS, AOA, and standby, as measured by the standby node). For each anchor node, or monitor, the technique (via the fusion center) conducts an external survey of candidate distance values that satisfy feature-based constraints. Out of these values, the minimum and maximum thereof encompass the bounds of a BPA.

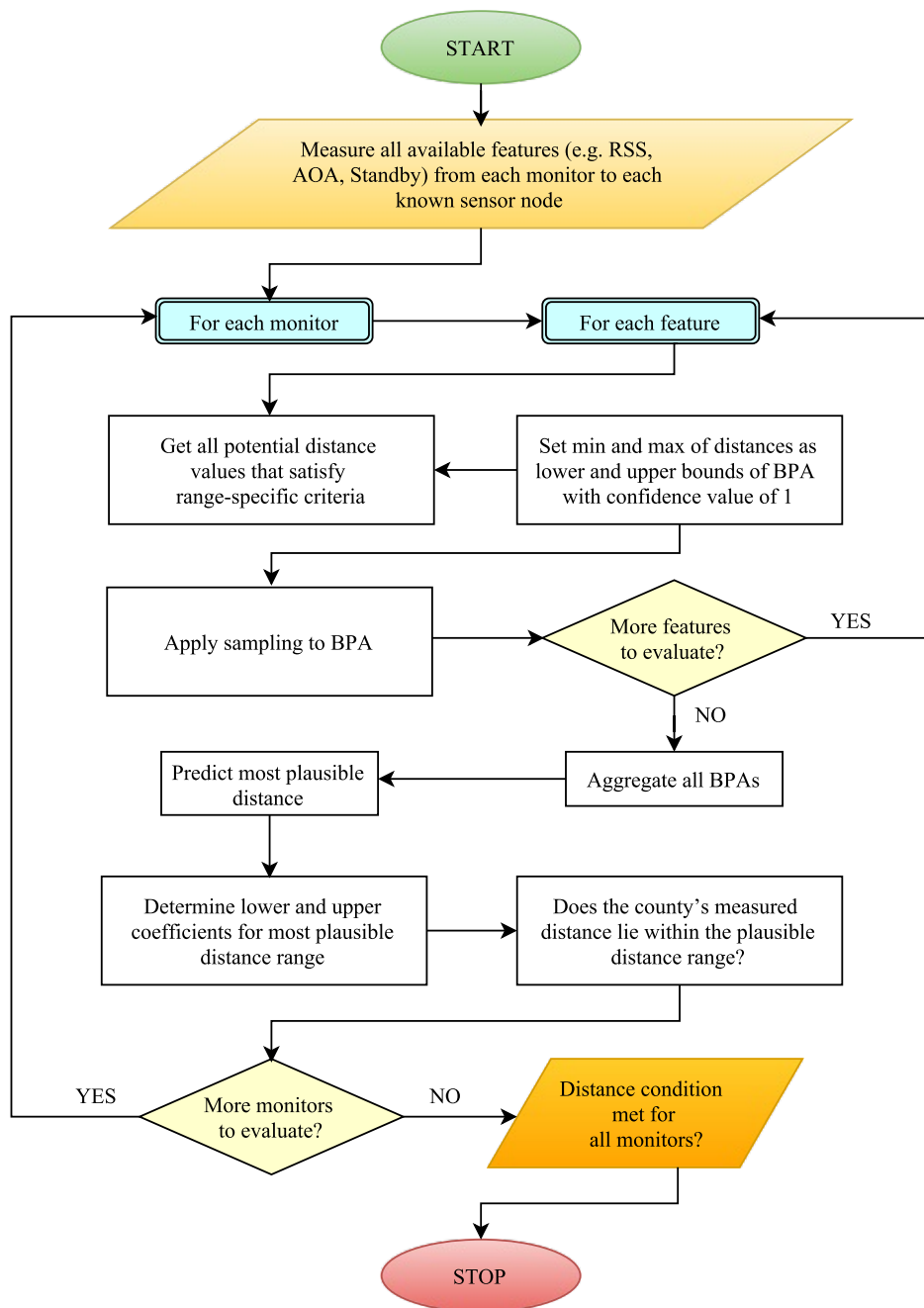


Figure 4: Flowchart of proposed DS localization algorithm.

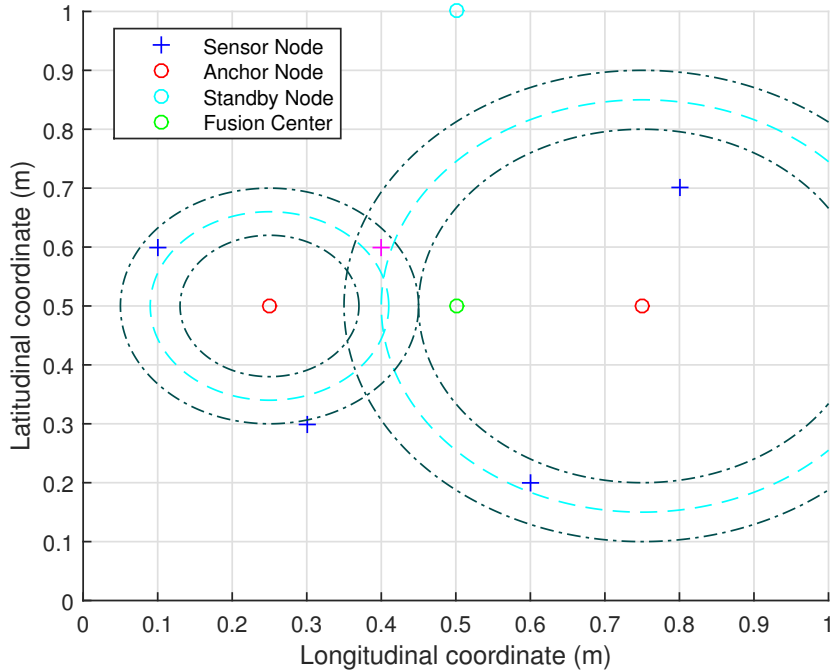


Figure 5: WSN of 5 sensor nodes, 2 anchor nodes, standby node, and fusion center.

After the algorithm repeats this process for every feature type, it aggregates all BPAs. Next, it executes the plausibility function on the distance BPAs and selects the resultant distance value of greatest confidence. This results in a radius of plausibility for each monitor, as indicated by the lightly colored circles of Fig. 5.

The end goal here is to generate a region of feasibility in which the target node could lie. For this, the algorithm incorporates a pair of coefficients, determined by preliminary WSN configuration, which forms inner and outer radii, as indicated by the dark circles in the figure. Finally, we inspect the intersection of inner and outer circles to find that only one node location lies within the region and thus matches the given set of measurements. In essence, this problem can be perceived as a multiple choice exam, and while choosing from only five nodes in this case does not seem too daunting, it is in large scale WSNs with more than a hundred nodes that this technique can truly flourish.

3. Simulation Setup

A variety of WSN simulation scenarios was created and analyzed using the MATLAB scientific programming environment (The Mathworks, Inc., 2013). The resultant software package includes all facets needed for experimental simulation. This includes specific parameters and qualities of the WSN, selectively randomized generation of data, and a complete implementation of the technique proposed in Fig. 4. In addition, two auxiliary algorithms designed to aid the execution of the core technique are also proposed as well as multiple methods for calculating the algorithm's accuracy.

An overview of the default simulation parameters in the experimental setup is as follows:

1. 100 nodes deployed throughout a rectangular region of 1000-by-1000 meters
2. Two or three anchor nodes (both options tested separately in identical networks)
3. Anchor nodes in both two-monitor and three-monitor setups located at [150,100] and [850,100] meters; third anchor node in three-monitor setups located at [500,100] meters
4. Standby node placed at [500,800] meters; location of fusion center is negligible
5. One-to-one mapping of counties and nodes
6. Range criteria of 20, 0.002, and -1 for RSS, AOA, and standby, respectively
7. Setup tested for every combination of features in which RSS is active (as RSS is needed to calculate distance from node to monitor)
8. Discretization consisting of 10 samples as an inverse normalized distribution
9. Differences of -1.5 meters or less for actual distance minus predicted distance not considered in determining a minimum difference between distances
10. Distance range weights determined through iterative training (see Subsection 3.4.1) to be 1.3 and 0.95
11. Experimental results as the average of ten independent experiments in which the WSN is regenerated upon every trial
12. Zero noise factor assumed

Details of the variables and terminologies above are explained in the proceeding subsections.

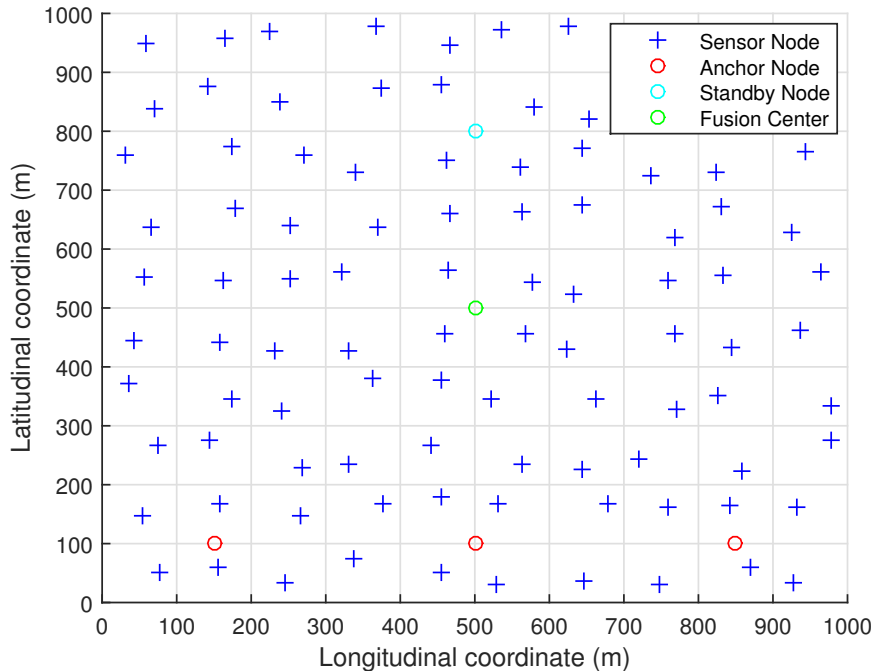


Figure 6: WSN of 100 sensor nodes, 3 anchor nodes, standby node, and fusion center.

3.1. WSN Setup

The sensor network in this simulation consists of four vital components: the sensor nodes, the anchor nodes, the standby node, and the fusion center. The sensor nodes comprise the vast majority of the components in this setup, totaling 100 out of 105 nodes in a three-monitor setup. The anchor nodes, also known as monitors or cluster heads, are placed strategically throughout the WSN to measure the available properties of each and every sensor node, such as RSS, AOA, and operating condition (standby). Such data is then transferred from an anchor node to a fusion center node, which processes all information using the algorithms and methods given in the following subsections. In other words, the fusion center assumes the role of the entire localization software, minus the data generation stage. In addition, a standby node measures the distance of any sensor nodes that are in standby mode. The complete WSN setup is presented in Fig. 6.

3.1.1. Data Value Ranges

In this network setup, the origin of the coordinate system is positioned at the lower left hand corner of the map, as indicated in Fig. 6. County names and monitor names are given as numbers ranging from one to the total number of counties and monitors available, respectively. All monitors are horizontally aligned by having a common latitudinal coordinate of 100 meters as well as longitudinal coordinates from 150 to 850 meters. Ranges of sensor node coordinates are based on the size of the WSN and can be placed anywhere more than 30 meters from any border. Based on the criteria of the latter two data types, distances between sensor nodes and monitors can vary from 40 to 1360 meters. Because RSS is merely the inverse square root of distance in the case of outdoor environments, it can range from $1/1360^2$ to $1/40^2$. AOA can be any positive number of degrees up to 360, and the operating condition, or standby, distance can range from 0 to 920 meters.

3.2. Data Generation

A multitude of sensor nodes, counties to which each node belongs, and various measurements of nodes relative to various monitors have been written to and read from a single comma separated value (CSV) file. This is known as the full data set. Each sensor node encompasses multiple rows of data, in which each row corresponds to a particular monitor observing the node and its measurements. Each category of data, such as county number, distance, or a type of measurement, takes the form of a column. The full data set contains 100 samples of sensor node data for every one of the 100 counties in the WSN, and each county is measured from three monitors, thereby totaling 30,000 rows overall.

A second CSV file known as the test data set contains the county locations and measurements that are to be tested against the full data set. The test data contains only one sensor node per county. The location and measurement values are different from any of the nodes of the corresponding county in the full data. Hence, the test data set contains 300 total rows of data for three monitor WSN setups and 200 rows for two monitor setups.

Each sample of data in each file contains ten columns, determined as follows:

1. Anchor node number
2. X coordinate of anchor node
3. Y coordinate of anchor node

4. County number
5. County x coordinate
6. County y coordinate
7. Distance from county to anchor node
8. RSS value
9. AOA value
10. Standby value

Worth noting is that the first three values are intended to be treated independently of measurement data in accordance with the objectives provided in Subsection 3.5. In addition, the seventh column is optional to the algorithm, as any distance value can be calculated directly from the corresponding RSS value due to a consistent inverse proportionality between RSS and squared distance.

3.3. IPP Toolbox

Aiding in the MATLAB implementation of DS Theory is the acquisition of the open source IPP Toolbox (University of Duisenberg-Essen, 2012), released under the GNU General Public License (GPL). For the first time in WSN localization, this package has been utilized to execute all DS-specific functions in the simulation, such as formation of BPAs, aggregation and sampling of BPAs, and calculation of plausibility.

3.4. Algorithmic Implementation

The algorithm was programmed by following the flowchart provided in Fig. 4 and refining the technique in the form of the pseudocode provided in Algorithm 1. The pseudocode was then converted into MATLAB-specific syntax and thoroughly tested to ensure proper functionality and accurate representation of the proposed algorithm.

An important distinction to note is that of the two data files presented in Subsection 3.2. The localization algorithm uses the full data set only for obtaining distance values for corresponding measurements that fit given range criteria in the distance collection stage. This essentially serves as an external data set that helps to establish the lower and upper bounds of an evidence factor, or BPA. The rest of the algorithm utilizes the internal test data set, particularly for establishing input values for county locations and measurement sets.

```

1: test_data ← read measured data
2: full_data ← read potential data
3: range ← feature-specific range values
4: for all monitors do
5:   for all active features do
6:     distances ← all distances from full data features satisfying range
7:     BPA_per_feat ← min_distance, max_distance, 1
8:     apply sampling to and aggregate BPA
9:   end for
10:  pl_dist ← most plausible distance
11:  min_accept, max_accept ← coefficients of acceptable distance range
12:  meas_dist ← measured distance from county to monitor
13:  decision_per_mon ← measured distance is in acceptable range
14:  difference_per_mon ← measured - plausible distance
15: end for
16: decision ← all decisions per monitor are positive
17: difference ← minimum difference per monitor
18: return decision, difference

```

Algorithm 1: Pseudocode of DS localization algorithm.

3.4.1. Determination of Acceptable Distance Values

As stated in Line 12 of Algorithm 2, a major factor in determining the desired outputs in the next subsection is the formation of acceptable distance values. Unlike artificial neural networks and other conventional machine learning techniques, DS Theory does not require an elaborate training stage. Conversely, however, care must be taken to determine a precise distance range to ensure optimal localization accuracy. Hence, a simple iterative approach is taken to find the most effective weights for the lower and upper bounds of an acceptable distance range. The resultant approach is detailed in Algorithm 3. For the sake of preserving the training-independent property of DS Theory, and due to the highly intensive time requirement when compared to Algorithm 2, this method is not intended to be executed upon every run of the core algorithm. Instead, it is only to be run during the initial formation of the WSN as well as significant changes thereof. In other words, if a WSN undergoes minor changes, such as the addition or removal of one anchor node or the relocation of sensor nodes across short distances, the impact of the

```

1: test_data ← read measured data
2: full_data ← read potential data
3: for all candidate upper bounds do
4:   for all candidate lower bounds do
5:     for all combinations of available features do
6:       generate WSN data
7:       run Alg 3 set number of times given bounds and features
8:       decision_matrix, matching_county ← average outputs of Alg 3
9:       longAcc ← long accuracy
10:      shortAcc ← short accuracy
11:      matchAcc ← match accuracy
12:     end for
13:     avgAcc ←  $\frac{1}{3} \frac{1}{no.feats.} \sum_{features} (longAcc + shortAcc + matchAcc)$ 
14:   end for
15: end for
16: bestAcc ← maximum avgAcc
17: best_lower, best_upper ← bounds that result in best accuracy
18: for all combinations of available features do
19:   generate WSN data
20:   evaluate Alg 3 using best bounds and given features
21: end for

```

Algorithm 2: Pseudocode of distance range training algorithm.

acceptable distance ranges is too low to noticeably alter the accuracy of the core algorithm. The three types of accuracy portrayed in Algorithm 2 as well as the details of Algorithm 3 are provided in Subsection 3.6.

In machine learning techniques, the data used for training is intended to contain many more training samples than validation samples (Krogh and Vedelsby, 1995). Hence, in this approach, WSN data generation and Algorithm 3 execution are run repeatedly and independently three times as much as they are run in validation and training-independent stages. Due to the high computational time and cost required for training, only a limited number of weight combinations are considered. As will be indicated in the experimental results in Subsection 4.1, the resultant lower and upper bound coefficients of the maximum plausibility value are ultimately chosen to be 1.3 and 0.95, respectively.

3.5. *Desired Outputs*

In order to address two distinctly different methods of application, the proposed technique delivers two separate outputs that respectively address the following questions:

1. Given a set of different measurement types and the location of a given county, do the measurements belong to said county?
2. Given a set of different measurement types and the locations of all available counties, which county most closely matches the measurements?

3.6. *Accuracy Calculation*

Three different types of accuracy are established in calculating the effectiveness of the two questions posed in the previous subsection: long accuracy, short accuracy, and matching accuracy. The former two methods correspond to the first question, in which a single county location and a single list of measurements are given as inputs and a binary decision of one or zero is the sole output. To test long accuracy, every combination of county and measurement set is run through the algorithm as the two inputs. From there, each output is a square matrix whose side length is equal to both the number of counties and the number of measurement sets. In such a matrix, each row corresponds to each measurement set and each column corresponds to each county. Finally, long accuracy is calculated by comparing the resultant matrix against an identity matrix of the same dimensions. The resultant formula is as follows:

$$Accuracy = 100 * \frac{cells_{matching}}{cells_{total}}. \quad (21)$$

Short accuracy follows a similar process except that only the matching counties and measurement sets are tested, e.g. county 1 with measurement set 1, county 2 with measurement set 2, etc. The results are recorded in an output vector of length equal to the number of counties, rather than a two-dimensional square matrix of the same side length. The vector is then compared against a vector of ones of equal length, once again using Equation 21.

The third accuracy type is based on the second question, in which a single list of measurements and a full list of all counties are the two inputs and the number corresponding to the most likely county match is the output. To test matching accuracy, every measurement set (in order of their matching

```

1: test_data ← read measured data
2: full_data ← read potential data
3: range ← feature-specific range values
4: for all measurement sets do
5:   for all counties do
6:     decision, difference ← Alg 1 using test_data, full_data, range
7:   end for
8:   index ← minimum difference
9:   matching_county ← county at index
10: end for

```

Algorithm 3: Pseudocode of DS testing algorithm.

county numbers) is tested against the list of counties to form a resultant output vector of size similar to that in short accuracy. The contents of said vector take the form of county numbers. This array is then compared against the ideal vector, which is simply every county number in numerical order, on the basis that the measurement sets are given in the same order. The two matrices are then compared with the same formula as before for calculating accuracy.

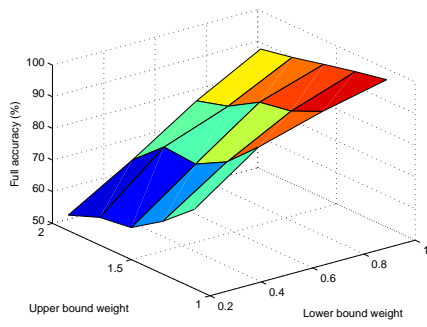
In order to thoroughly evaluate all three types of accuracy, a simple testing algorithm is portrayed in the pseudocode of Algorithm 3, evaluating the core localization algorithm in order to extract all desired distance difference and county match values.

4. Simulation Results and Discussion

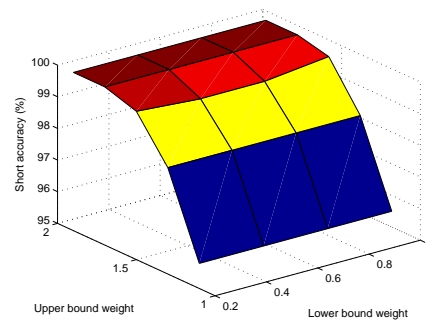
This section provides the results for the MATLAB simulation setup and parameters detailed in the previous section. This includes both the preliminary training-based approach intended only for initial WSN implementations and the core training-independent approach. The latter of the two serves as the primary focus of discussion due to its resultant combination of high accuracy and low computational cost.

4.1. Results Under Distance Range Weight Training

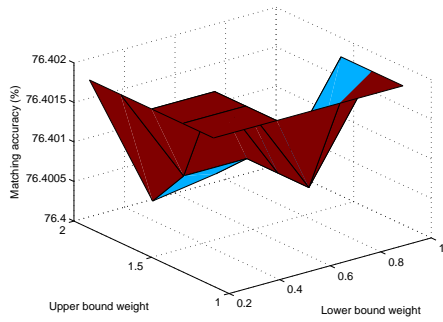
20 possible combinations of desired values, as detailed in Subsubsection 3.4.1, served as the candidate inputs for the training-based portion of the simulation. The results for all three accuracy types as well as the averages



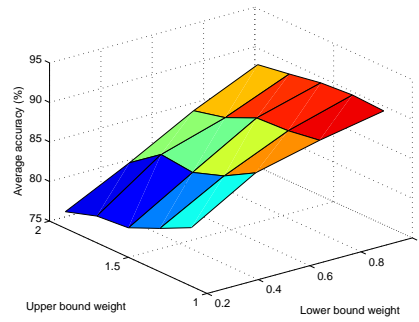
(a) Full Accuracy



(b) Short Accuracy



(c) Matching Accuracy



(d) Average Accuracy

Figure 7: Plots of accuracy versus upper bound weight versus lower bound weight for preliminary weight training.

thereof are presented in Fig. 7. Each result is the average of 30 independent trials, that is, three times the amount of trials run in the preceding subsection. As the surface plots in the figure indicate, full accuracy reaches optimal levels when the lower bound weight is maximized and the upper bound weight is minimized. Short accuracy appears to be almost independent of lower bound weight values and optimizes at the maximum upper bound. Matching accuracy follows the most uniform distribution of accuracy values, varying from 76.000% to 76.002%, optimizing at multiple possible points. Effectively, this form of accuracy can be considered negligible in the attempt to find a single best weight combination. The average accuracy is the primary decision factor of weight selection and optimizes upon a maximum lower bound of 0.95 and an upper bound of 1.3, located in a tightly contained region in which average accuracy narrowly exceeds 90%. Hence, the respective upper and lower weights of 1.3 and 0.95 have been verified to produce the most accurate data and are thus used consistently under the implementation detailed in the following subsection.

4.2. Training-Independent Results

The training-independent portion of the simulation was tested in 24 different scenarios resultant from four independent variables: the set of active features, the number of monitors, the type of accuracy, and the trial-based random generation of the WSN. These results reflect the overall effectiveness of Dempster-Shafer Theory, data fusion, and plausibility, as established in Section 2. This simulation used two and three as the desired numbers of monitors, as these amounts have been established minimums under unimodal RSS and AOA based localization methods. For reliability and redundancy purposes, the simulator regenerated the WSN prior to each of the ten experimental trials for each of the above combinations of scenarios. The accuracy results for all test cases are given in Table 1, and the runtime results are provided in Table 2, where each value for each table is the average run time for all trials.

4.2.1. Accuracy

As Table 1 indicates, the full accuracy test generates consistently high results in the steady area of 97.0-97.5% accuracy for all eight scenarios. Thanks to the powerful core of the localization technique that is DS Theory, the algorithm is proven to be consistently effective for both two-monitor and three-monitor setups under all feasible combinations of active features. Results

Accuracy Monitors	Full (%)		Short (%)		Match (%)		Average (%)	
	2	3	2	3	2	3	2	3
RSS	97.04	97.49	97.20	98.50	56.17	66.64	83.47	87.54
RSS, SB	97.00	97.43	100.0	100.0	94.58	96.07	97.19	97.83
RSS, AOA	97.03	97.47	98.13	98.97	57.81	67.29	84.33	87.92
RSS, AOA, SB	97.01	97.44	100.0	100.0	92.52	94.21	96.51	97.22

Table 1: Simulation results for two-monitor and three-monitor WSNs.

of the short accuracy test climb even higher with a value range of 97-100%. Under the matching accuracy test, results span a wider range of accuracy results from 56.1% to 94.6%. The average of all three accuracy types indicate a somewhat less diverse range of 83.5% to 97.8%. Empirically, county matching can be considered most effective when only RSS and operating condition are enabled. The absence of operating condition results in the only accuracy values lower than 94% as well as the only short accuracy values below 100%. In other words, when a WSN is distributed across a small enough terrain that a standby node is usable, average accuracy of over 95% is easily achieved. In contrast, larger terrains in which standby measurements are unfeasible can only reach average accuracy values of 80-90%.

The full accuracy test appears to be the only evaluation type that results in a higher percentage in the absence of data fusion. While fusing multiple types of data is intended to enhance an accuracy relative to that of a single type of data, this test shows that full accuracy is most effective when only RSS data is available. The other evaluation methods, however, clearly demonstrate that fusion of multiple measurement types do in fact enhance accuracy. Combining this observation with the fact that differences in full accuracies are minimal (all values for each monitor are within 0.06% of one another) can easily infer that data fusion in this context is still an overall success.

Also worth noting is that RSS as a sole feature forms the weakest accuracy and may pale in comparison to traditional RSS-based localization when only RSS is available. One should keep in mind, however, that all other remaining feature combinations, that is, any combination of two or more features, proves that fusion of multiple available types of data in this algorithm is sig-

Runtime (ms)	2Mon	3Mon
Total	787478.00	1375261.00
Per Feature Set	196869.00	343815.20
Per Iteration	19.69	34.38
Per Node	0.197	0.344

Table 2: Runtime results for two-monitor and three-monitor WSNs.

nificantly more effective than any conventional unimodal technique. Thus, RSS should be considered of lower importance, as the scope of this research focuses primarily on the advantages and successes of data fusion.

Another successful outcome is the high accuracy involved in two-monitor WSNs. Most established localization methods require three or more monitors to observe a sensor node’s measurements, but in this algorithm, two-monitor WSNs can localize reliably with a less-than-0.5% decrease in accuracy. From the standpoint of having RSS as the only enabled feature, achieving a high accuracy with three monitors is trivial, as this can be achieved purely through trilateration. Under two monitors, however, only bilateration is available, meaning that if distance values calculated from RSS measurements are exact, there is still only a 50% chance of obtaining the correct location. Thus, the monumental incorporation of DS Theory can achieve highly accurate localization with only two monitors just as easily as with three or more.

4.2.2. Computational Runtime

For each trial, we recorded the total runtime for an entire simulation under all four available feature sets. From there, we averaged the calculated time per feature set, then divided by all $100^2 = 10000$ possible input combinations to find average runtime per iteration. Finally, we divided the latter runtime by all 100 nodes to find the average runtime per node. Table 2 provides the average of each result for both two-monitor and three-monitor WSNs. The program conducted each trial on consistent computer hardware and software, with specifications consisting of a standard Windows version of MATLAB on a quad core 2.60 GHz processor and 8.0 GB of RAM. The significance of these results will be explained in greater detail over the next subsection.

Algorithm	PSO	WSLA	WSRA	DS
Runtime (μ s)	114570	7800	9700	344

Table 3: Comparison of computational runtime among different localization techniques.

4.3. Comparisons to Other Established Algorithms

To gain further perspective into the effectiveness of the proposed technique, we compared our results to those of four previously established localization approaches. These consisted of particle swarm optimization (PSO), maximum-likelihood estimation (MLE), and a weighted search-based localization algorithm (WSLA). Also included is a weighted search-based refinement algorithm combined with the latter technique that is referred by Yao and Jiang as WSLA+WSRA but is reported here as simply WSRA. For the compared methods, the simulated network consisted of 100 sensor nodes with eight anchor nodes while each experiment contained 30 independent trials averaged together.

4.3.1. Accuracy

The results are based on those presented in (Yao and Jiang, 2015) under the assumptions of no positioning error on anchor nodes and no noise factor. Hence, we determined our error values under aforementioned conditions, then subtracted each value from one and converted the result to a percentage. Because the authors reported their error values as a plot rather than in exact numerical form, we had to estimate the error values by eyeing and measuring the plots. Hence, to minimize any potential reporting error, we indicated the accuracy of each algorithm was reported as a whole number percentage. For the DS-based technique, we based our accuracy by the average of full accuracy values for three-monitor setups, rounded to the nearest whole number percentage for consistency.

As Fig. 8 indicates, the proposed technique contains a noticeable edge in accuracy in comparison to the other methods as well as a significant lead over PSO. Although some parameters had to be tweaked between this method and the others, results are undeniably positive. Thanks to the novel capabilities of Dempster-Shafer Theory, the proposed localization technique has a distinct advantage on par with, if not over, multiple established state-of-the-art localization techniques.

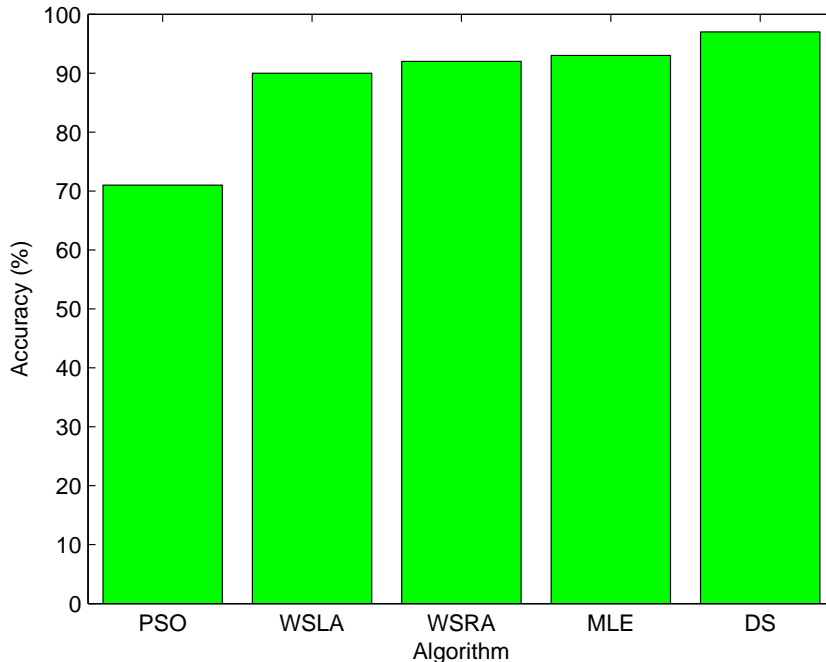


Figure 8: Comparison of accuracy among different localization techniques.

4.3.2. Computational Runtime

In addition, we compared the average runtime per node for a three-monitor WSN setup with three of the four aforementioned techniques, based on data given in (Yao and Jiang, 2015). The table provides the competing data as the average CPU time per node with no noise factor. Hardware specifications and MLE runtime were not reported. All method runtimes are provided in Table 3. Due to the unique and efficient nature of DS Theory, the proposed technique has an overwhelmingly shorter CPU runtime when compared to all reported competing methods, hence solidifying such an algorithm as a novel low cost technique.

5. Conclusions and Future Work

This paper presented an efficient approach to localization in wireless sensor networks. This approach was based heavily upon the innovative statistical framework known as Dempster-Shafer Evidence Theory, a novel data fusion

technique that has never before been used in low cost WSN localization. While DS Theory contains many unique and powerful properties that can aid in data prediction and analysis, the concept of prediction by plausibility is of the most vital levels of importance and interest. This is the result of its consistently high accuracy, ranging up to 98%, under a variety of network setups and feature sets as well as its overwhelmingly low computational cost requirements when compared to other established techniques.

5.1. Future Work

Support vector machines (SVMs) have been of a high level of interest in the field of WSN localization, almost to the same extent as DS Theory, due to its fast localization potential and efficient use of processing resources. Thus, the most likely next step in this research will involve the fusion of DS Theory with SVMs. As for more specific details of future work, training of the upper and lower bound weights was done from a limited iterative approach. Although this brief technique resulted in multiple high levels of accuracy above 90%, accuracy could be even further enhanced through more elaborate optimization methods. Possibilities include biologically inspired methods, such as Genetic Algorithm or Ant Colony Optimization.

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References

- Auer, E., Luther, W., Rebner, G., and Limbourg, P. (2010) A Verified MATLAB Toolbox for the Dempster-Shafer Theory. *Workshop on the Theory of Belief Functions*.
- Bhatt, D., Aggarwal, P., Devabhaktuni, V., and Bhattacharya, P. (2012) Seamless Navigation via Dempster Shafer Theory Augmented by Support Vector Machines. *Proceedings of the 25th International Technical Meeting of The Satellite Division of the Institute of Navigation*, 98-104.

- Bloch, I. (1996) Some aspects of Dempster-Shafer evidence theory for classification of multi-modality medical images taking partial volume effect into account. *Pattern Recognition Letters*, 17, 905-919.
- Bulusu, N., Heidemann, J., and Estrin, D. (2000) GPS-less low-cost outdoor localization for very small devices. *IEEE Personal Communications*, 7(5), 28-34.
- Cakir, O., Kaya, I., and Cakir, O. (2014) Investigation the effect of the time difference of arrival sets on the positioning accuracy for source localization. *Signal Processing and Communications Applications Conference (SIU)*, 2249-2252.
- Chiodo, E. and Mazzanti, G. (2006) Bayesian Reliability Estimation Based on a Weibull Stress-Strength Model for Aged Power System Components Subjected to Voltage Surges. *IEEE Transactions on Dielectrics and Electrical Insulation*, 13(1), 146-159.
- Clerentin, A. (2000) A localization method based on two omnidirectional perception systems cooperation. *IEEE International Conference on Robotics and Automation*, 2, 1219-1224.
- Imprecise Probability Propagation Toolbox (IPP Toolbox)* (2012) Essen, Germany: University of Duisberg-Essen, Department of Information Logistics, (<https://www.uni-due.de/informationslogistik/ipptoolbox.php>). Last accessed March 7, 2016.
- Garcia, M., Martinez, C., Tomas, J., and Lloret, J. (2007) Wireless Sensors Self-Location in an Indoor WLAN Environment. *SensorComm 2007. International Conference on Sensor Technologies and Applications*, 146-151.
- Garcia, M., Tomas, J., Boronat, F., and Lloret, J. (2009) The Development of Two Systems for Indoor Wireless Sensors Self-location. *Ad Hoc and Sensor Wireless Networks*, 8(3-4), 235-258).
- Kasebzadeh, P. Indoor localization via WLAN path-loss models and Dempster-Shafer combining. *2014 International Conference on Localization and GNSS (ICL-GNSS)*, 1-6.
- Krogh, A. and Vedelsby, J. (1995) Neural Network Ensembles, Cross Validation, and Active Learning. In G. Tesauro, D. Touretsky, and T. Leen (eds.)

- Advances in Neural Information Processing Systems (volume 7)* (231-238). Cambridge, MA: MIT Press.
- Li, S., Qin, Z., Song, H. (2016) A Temporal-Spatial Method for Group Detection, Locating and Tracking. *IEEE Access*, 1-10.
- Limbourg, P., Savic, R., Petersen, J., and Kochs, H. (2007) Fault tree analysis in an early design stage using the Dempster-Shafer theory of evidence. In T. Aven and J. E. Vinnem (eds.) *European Conference on Safety and Reliability - ESREL 2007* (713-722). Taylor & Francis, Stavanger.
- Lipeng, G., Juan, J., and Laibin, D. (2011) An Improved Fusion Algorithm of Evidence Theory. *Cross Strait Quad-Regional Radio Science and Wireless Technology Conference*, 1495-1498.
- Lloret, J., Tomas, J., Garcia, M., and Canovas, A. (2009) A Hybrid Stochastic Approach for Self-Location of Wireless Sensors in Indoor Environments. *Sensors*, 9(5), 3695-3712.
- Mao, G., Fidan, B., and Anderson, B. (2007) Wireless sensor network localization techniques. *Computer Networks*, 51(10), 2529-2553.
- MATLAB version 8.1* (2013) Natick, Massachusetts: The MathWorks, Inc.
- Mazuelas, S., Bahillo, A., Lorenzo, R., Fernandez, P., Lago, F., Garcia, E., Blas, J., and Abril, E. (2009) Robust Indoor Positioning Provided by Real-Time RSSI Values in Unmodified WLAN Networks. *IEEE Journal of Selected Topics in Signal Processing*, 3(5), 821-831.
- Meyer, C. (2012) Mass Notification? There's an App for That. *Security*, 2-4.
- Nguyen, H. and Walker, E. (1993) On Decision-Making using Belief Functions. In *Advances in the Dempster-Shafer Theory of Evidence*, 311-330.
- Patwari, N., Ash, J., Kyperountas, S., Hero, A., Moses, R., and Correal, N. (2005) Locating the Nodes: Cooperative Localization in Wireless Sensor Networks. *IEEE Signal Processing Magazine*, 22(4), 54-69.
- Pelant, M., Stejskal, V. (2011) Multilateration System Time Synchronization via Over-Determination of TDOA Measurements. *2011 Tyrrhenian International Workshop on Digital Communications - Enhanced Surveillance of Aircraft and Vehicles (TIWDC/ESAV)*, 12-14.

- Shen, H., Ding, Z., Dasgupta, S., and Zhao, C. (2014) Multiple Source Localization in Wireless Sensor Networks on Time of Arrival Measurement. *IEEE Transactions on Signal Processing*, 62(8), 1938-1949.
- Shen, X., Chen, W., and Lu, M. (2008) Wireless Sensor Networks for Resources Tracking at Building Construction Sites. *Tsinghua Science and Technology*, 13(1), 78-83.
- Tonon, F. (2004). Using random set theory to propagate epistemic uncertainty through a mechanical system. *Reliability Engineering and System Safety*, 85(1-3), 169-181.
- Tseng, Y., Kuo, S., Lee, H., and Huang, C. (2004) Location Tracking in a Wireless Sensor Network by Mobile Agents and Its Data Fusion Strategies. *The Computer Journal*, 47(4), 448-460.
- Xing, G., Tan, R., Liu, B., Wang, J., Jia, X., and Ji, C. (2009) Data fusion improves the coverage of wireless sensor networks. *Proceedings of the 15th annual international conference on Mobile computing and networking*, 157-168.
- Xu, J., Liu, W., Lang, F., Zhang, Y., and Wang, C. (2010) Distance Measurement Model Based on RSSI in WSN. *Wireless Sensor Network*, 2(8), 606-611.
- Yang, J., Zhou, J., Lv, J., Wei, W., and Song, H. (2015) A Real-Time Monitoring System of Industry Carbon Monoxide Based on Wireless Sensor Networks. *Sensors*, 15(11), 29535-29546.
- Yao, Y., Jiang, N. (2015) Distributed Wireless Sensor Network Localization Based on Weighted Search. *Computer Networks: The International Journal of Computer and Telecommunications Networking*, 86, 57-75.