Localization of Intense Electromagnetic Waves in a Relativistically Hot Plasma

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We consider nonlinear interactions between intense short electromagnetic waves (EMWs) and a relativistically hot electron plasma that supports relativistic electron holes (REHs). It is shown that such EMW-REH interactions are governed by a coupled nonlinear system of equations composed of a nonlinear Schrödinger equation describing the dynamics of the EMWs and the Poisson-relativistic Vlasov system describing the dynamics of driven REHs. The present nonlinear system of equations admits both a linearly trapped discrete number of eigenmodes of the EMWs in a quasistationary REH and a modification of the REH by large-amplitude trapped EMWs. Computer simulations of the relativistic Vlasov and Maxwell-Poisson system of equations show complex interactions between REHs loaded with localized EMWs.

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The nonlinear interaction of high-intensity ultrashort electromagnetic waves with plasmas is of primary interest for the fast ignitor concept of the inertial confinement fusion and for the development of high power sources of hard electromagnetic (EM) radiation, as well as for laser-plasma particle and photon accelerators, and compact astrophysical objects containing intense electromagnetic bursts. Recent progress in the development of superstrong electromagnetic pulses with intensities $I \sim 10^{21}$–10^{23} W/cm^2 has also made it possible to create relativistic plasmas in the laboratory by a number of experimental techniques. At the focus of an ultraintense short electromagnetic pulse, the electrons can acquire velocities close to the speed of light, opening the possibility of simulating in the laboratory conditions and phenomena that, usually, belong to the astrophysical realm. In fact, nonlinear interactions between intense short electromagnetic pulses and a background plasma give rise to a number of nonlinear effects [1,2] associated with relativistic electron mass increase in the electromagnetic fields and the plasma density modification due to relativistic radiation ponderomotive force. In the past, several authors [3–8] presented theoretical [3–6] and particle-in-cell simulation [7,8] studies of intense electromagnetic envelope solitons in a cold plasma, where the plasma slow response to the electromagnetic waves (EMWs) is modeled by the electron continuity and relativistic momentum equations, supplemented by Poisson's equation. Assuming beamlike particle distribution functions, relativistic electromagnetic solitons in a warm quasineutral electron-ion plasma have been investigated [9]. Experimental observations [10] show bubblelike structures in proton images of laser-produced plasmas, which are interpreted as remnants of electromagnetic envelope solitons.

In this Letter, we present fully relativistic nonlinear theory and computer simulations for nonlinearly coupled intense localized EMW and relativistic electron hole (REH) structures in a relativistically hot electron plasma by adopting the Maxwell-Poisson-relativistic Vlasov system which accounts for relativistic electron mass increase in the electromagnetic fields and relativistic radiation ponderomotive force [2], in addition to trapped electrons which support the driven REHs. Such a scenario of coupled intense EMWs and REHs is absent in any fluid treatment [3–6] of relativistic electromagnetic solitons in a plasma.

We first present the relevant equations describing the action of intense laser light on the electrons in a relativistically hot collisionless plasma, as well as the electromagnetic wave equation accounting for the relativistic electron mass increase and the electron density modification due to the radiation relativistic ponderomotive force [1] $F = -m_e c^2 \partial\gamma/\partial z$, where m_e is the electron rest mass, c is the speed of light in vacuum, and $\gamma = (1 + p_z^2/m_e^2c^2 + e^2|\mathbf{A}|^2/m_e^2c^4)^{1/2}$ is the relativistic gamma factor. Here, p_z is the z component of the electron momentum, $\mathbf{A}$ is the perpendicular (to $\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the z direction in a Cartesian coordinate) component of the vector potential of the circularly polarized EMWs, and e is the magnitude of the electron charge. The dynamics of nonlinearly coupled EMWs and REHs is governed by

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{1}{\alpha^2}\frac{\partial^2 \mathbf{A}}{\partial z^2} + \int_{-\infty}^{\infty} \frac{f}{\gamma} dp_z \mathbf{A} = 0, \tag{1}$$

$$\frac{\partial f}{\partial t} + \frac{p_z}{\gamma}\frac{\partial f}{\partial z} + \frac{\partial(\phi - \gamma/\alpha^2)}{\partial z}\frac{\partial f}{\partial p_z} = 0, \tag{2}$$

and

$$\frac{\partial^2 \phi}{\partial z^2} = \int_{-\infty}^{\infty} f dp_z - 1, \tag{3}$$

where $\mathbf{A}$ is normalized by $m_e c/e$, ϕ by T_e/e, p_z by $m_e V_{Te}$, and z by r_D. We have denoted $\gamma = (1 + \alpha^2 p_z^2 + |\mathbf{A}|^2)$, $V_{Te} = (T_e/m_e)^2$, $\alpha = V_{te}/c$, $r_D = V_{Te}/\omega_p$, and $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$. In Eq. (1), we have used the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$, and have excluded the longitudinal (z) component $\partial^2\phi/\partial t \partial z = j_z$, where j_z is the parallel current density, by noticing that this component is equivalent to Poisson's equation (3). This can be seen from $\partial^3\phi/\partial t\partial z^2 = \partial j_z/\partial z = -\partial\rho/\partial t$, where the last equality follows from the electron continuity equation which relates the charge and current densities, viz. ρ and j_z, respectively. Integration of the last expression with respect to t yields $\partial^2\phi/\partial z^2 = -\rho \equiv \int_{-\infty}^{\infty} f dp_z - 1$, which is Eq. (3) above.

Far away from the REH, where $\phi = |\mathbf{A}| = 0$, the electrons are assumed to obey a Jüttner-Synge distribution function [11] $\tilde{f}_0(p_z) = a_0\exp[-(\sqrt{1+\alpha^2 p_z^2} - 1)/\alpha^2]$, where the normalization constant is $a_0 = \{\int_{-\infty}^{\infty} \times \exp[-(\sqrt{1+\alpha^2 p_z^2} - 1)/\alpha^2]dp_z\}^{-1} = \alpha\exp(-\alpha^{-2})/2K_1(\alpha^{-2})$. For the Jüttner-Synge distribution function, the last term in the left-hand side of Eq. (1) takes the value

$$\int_{-\infty}^{\infty} \frac{\tilde{f}_0(p_z)}{\sqrt{1+\alpha^2 p_z^2}} = \frac{K_0(\alpha^{-2})}{K_1(\alpha^{-2})} \equiv \Omega_p^2, \tag{4}$$

where K_0 and K_1 are the modified Bessel functions of second kind. The frequency Ω_p represents the normalized (by ω_p) relativistic plasma frequency at equilibrium.

We now investigate the properties of driven REHs which move with the constant speed v_0 relative to the observer (bulk plasma) frame. Accordingly, we use the ansatz $f(p_z, \xi)$ for the relativistic electron distribution function, and assume that ϕ and $|A|^2$ depend on ξ only, where $\xi = z - v_0 t$. Then, Eqs. (2) and (3) take the form

$$\left(-v_0 + \frac{p_z}{\gamma}\right)\frac{\partial f}{\partial\xi} + \frac{\partial(\phi - \gamma/\alpha^2)}{\partial\xi}\frac{\partial f}{\partial p_z} = 0, \tag{5}$$

and

$$\frac{d^2\phi}{d\xi^2} = \int_{-\infty}^{\infty} f dp_z - 1, \tag{6}$$

respectively. The general solution of Eq. (5) is $f = f_0(\mathcal{E})$, where f_0 is some function of one variable and $\mathcal{E} = -v_0 p_z + (1/\alpha^2)(\gamma - 1/\gamma_0) - \phi$ is the energy integral. Here, we have denoted $\gamma_0 = 1/\sqrt{1-\alpha^2 v_0^2}$. We note that trapped electrons have negative energy while untrapped (free) electrons have positive energy.

In the slowly varying envelope (WKB) approximation, viz. $\mathbf{A} = (1/2)A(z,t)(\hat{\mathbf{x}} + i\hat{\mathbf{y}})\exp(-i\omega_0 t + ik_0 z) +$ complex conjugate, Eq. (1) can be written as

$$2i\omega_0\left(\frac{\partial A}{\partial t} + v_g\frac{\partial A}{\partial z}\right) + \frac{1}{\alpha^2}\frac{\partial^2 A}{\partial z^2} - \left(\int_{-\infty}^{\infty}\frac{f}{\gamma}dp_z - \Omega_p^2\right)A = 0, \tag{7}$$

where $\omega_0 = (\Omega_p^2 + k_0^2/\alpha^2)^{1/2}$ is the EMW frequency, $v_g = k_0/\alpha^2\omega_0$ is the group velocity, and $\gamma = \sqrt{1+\alpha^2 p_z^2 + |A|^2}$. Introducing $A = W(\xi)\exp(-i\theta t + iKz)$ into Eq. (7), where W is a real-valued normalized (by $m_e c/e$) function, we obtain $K = \alpha^2\omega_0(v_0 - v_g)$ and

$$\frac{d^2 W}{d\xi^2} - \lambda W - \alpha^2(\Omega^2 - \Omega_p^2)W = 0, \tag{8}$$

where $\lambda = -2\omega_0\alpha^2\theta + \alpha^4\omega_0^2(v_0^2 - v_g^2)$ represents a nonlinear frequency shift, and the gamma factor becomes $\gamma = \sqrt{1+\alpha^2 p_z^2 + W^2}$. Here, $\Omega^2 = \int_{-\infty}^{\infty}(f/\gamma)dp_z$ represents the square of the local electron plasma frequency that accounts for the relativistic electron mass increase.

A condition for the untrapped electron is that far away from the REH, the electron distribution function should smoothly connect to the Jüttner-Synge distribution function. In order to impose this condition for the free electrons, we will use the solution

$$f_{\mathrm{f}} = f_0(\mathcal{E}) = \tilde{f}_0[\tilde{p}(\mathcal{E})], \tag{9}$$

where $\tilde{p}(\mathcal{E})$ is a function of the energy such that $\tilde{p}(\mathcal{E}) \to p$ when $W \to 0$ and $\phi \to 0$. Such a function can be found with the help of the energy integral by setting $-v_0\tilde{p}(\mathcal{E}) + (1/\alpha^2)(\sqrt{1+\alpha^2\tilde{p}^2(\mathcal{E})} - 1/\gamma_0) = \mathcal{E}$. Solving for $\tilde{p}(\mathcal{E})$, we have

$$\tilde{p}_{\pm}(\mathcal{E}) = \gamma_0^2 v_0\left(\alpha^2\mathcal{E} + \frac{1}{\gamma_0}\right) \pm \frac{\gamma_0^2}{\alpha}\sqrt{\left(\alpha^2\mathcal{E} + \frac{1}{\gamma_0}\right)^2 - \frac{1}{\gamma_0^2}}, \tag{10}$$

where $\tilde{p}_+(\mathcal{E})$ and $\tilde{p}_-(\mathcal{E})$ correspond to a modified momentum for free electrons on each side of the trapped electron population in momentum space. Using $\tilde{p}(\mathcal{E}) = \tilde{p}_+(\mathcal{E})$ and $\tilde{p}(\mathcal{E}) = \tilde{p}_-(\mathcal{E})$ in Eq. (9), we obtain the distribution function for free electrons. In the limit of vanishing energy, $\mathcal{E} = 0$, we have $\tilde{p}_{\pm}(0) = \gamma_0 v_0$, and the value of the distribution function is $f_{\mathrm{f}}|_{\mathcal{E}=0} = f_0(0) = \tilde{f}_0(\gamma_0 v_0) = a_0\exp[-(\gamma_0 - 1)/\alpha^2]$, which should be matched with the distribution function for the trapped electrons with $\mathcal{E} = 0$ in order to obtain a continuous distribution function. For the trapped electrons, we choose a relativistic Maxwell-Boltzmann distribution with a negative "temperature," viz. $f_{\mathrm{t}} = a_0\exp[\alpha^{-2}(1-\gamma_0) - \beta\mathcal{E}]$, where β is a trapping parameter, leading to a vortex distribution [12] for $\beta < 0$. Clearly, the separatrix between the free and trapped electron distributions is found where the energy integral $\mathcal{E} = 0$. Solving for p in $\mathcal{E} = 0$, we have

$$p_{\pm} = \gamma_0^2 v_0\left(\alpha^2\phi + \frac{1}{\gamma_0}\right) \pm \frac{\gamma_0^2}{\alpha}\sqrt{\left(\alpha^2\phi + \frac{1}{\gamma_0}\right)^2 - \frac{1+W^2}{\gamma_0^2}}, \tag{11}$$

where p_- and p_+ constitute the limits between the trapped and free electron distributions in momentum space. Using these limits, we can then write

$$f = \begin{cases} a_0\exp[-\frac{1}{\alpha^2}(\sqrt{1+\alpha^2\tilde{p}_+^2(\mathcal{E})} - 1)], & p > p_+, \\ a_0\exp[-\frac{\gamma_0 - 1}{\alpha^2} - \beta\mathcal{E}], & p_- \le p \le p_+, \\ a_0\exp[-\frac{1}{\alpha^2}(\sqrt{1+\alpha^2\tilde{p}_-^2(\mathcal{E})} - 1)], & p < p_-. \end{cases} \tag{12}$$

Integrating the distribution function (12) over the momentum space, we obtain the total electron number density as a function of ϕ and W, which are calculated self-consistently by means of the Poisson and Schrödinger equations, respectively.

Figure 1 exhibits the influence of intense EMWs on REHs, described by the coupled system of Eqs. (6) and (8). In the Schrödinger equation (8), λ represents the eigenvalue, and the square of the local electron plasma frequency, Ω^2, enters as a "potential." We see that for larger electromagnetic fields W, the REH potential ϕ becomes larger and the REH wider, admitting larger eigenvalues λ. This can be explained in that the relativistic ponderomotive force of localized EMWs pushes the electrons away from the center of the REH, leading to an increase of the electrostatic potential and a widening of the REH. We see that the depletion of the electron density in the REH is only minimal, while the local electron plasma frequency Ω is strongly reduced owing to the increased mass of the electrons that are accelerated by the REH potential; the maximum potential $\phi_{\max} \approx 15$ in Fig. 1 corresponds in physical units to a potential $\alpha^2\phi_{\max} \times 0.5 \times 10^6 \approx 1.2 \times 10^6$ V, accelerating the electrons to gamma factors of ≈ 6. The linear trapping of EMWs in REHs is displayed in Fig. 2, where we have assumed a zero electromagnetic field ($W = 0$) in the expression for γ used in Eq. (8) and in the energy integral. The eigenvalue problem admits a discrete set of localized eigenfunctions with positive eigenvalues, and in this case we found two even and one odd eigenfunctions corresponding to three different eigenvalues. The numerical method used to solve the boundary value problem (6), where ϕ was set to zero at the boundaries and for a given W, was a slightly modified Newton's method. Equation (8) has been solved as a linear eigenvalue problem for W, where the amplitude $W_{\max}$ of the EMW was kept fixed to obtain new values on W and λ. Then, the procedure of solving for ϕ and W was repeated until convergence. The second derivatives were approximated with a second-order centered scheme with the function values set to zero at the boundaries.

In order to study the dynamics of interacting solitary structures composed of localized REHs loaded with trapped EMWs, we have numerically solved the time-dependent, relativistic Vlasov equation (2) together with the Schrödinger equation (7). The results are displayed in Figs. 3 and 4. As an initial condition to our simulations, we used solutions to the quasistationary equations described above, where the left REH initially has the speed $v_0 = 0.7$ and is loaded with EMWs with $W_{\max} = 1.5$, while the right REH has the speed $v_0 = -0.3$, and is loaded with EMWs with $W_{\max} = 2.5$. Further, we used $k_0 = v_g = 0$ in the initial condition for A and in the solution of Eq. (7). In Fig. 3 we display the phase space distribution of the electrons and the electromagnetic field amplitude at different times. We see that the REHs loaded with trapped EMWs collide, merge, and then split into two REHs, while there are two strongly peaked EMW envelopes at $z \approx 30$ and $z \approx 70$ remaining after the splitting of the REH. A population of electrons has also been accelerated to large energies, seen at $z = 100$ in the lower left panel of Fig. 3. The time development of the EMW amplitudes, REH potential, the squared local plasma frequency, and the electron number density is shown in Fig. 4. We observe collision and splitting of the REHs and creation of the two localized EM envelopes at $z \approx 70$, clearly visible in the left two panels at $t > 150$. The EM solitary waves are created by the combined action of relativistic electron mass increase and relativistic ponderomotive force of localized EMWs, which have been further intensified due to nonlinear interactions where collapsing REH has deposited its EM energy into the plasma. The interacting REHs also excite large-amplitude electrostatic waves, seen as oscillations in the REH potential for $t > 100$. In our numerical

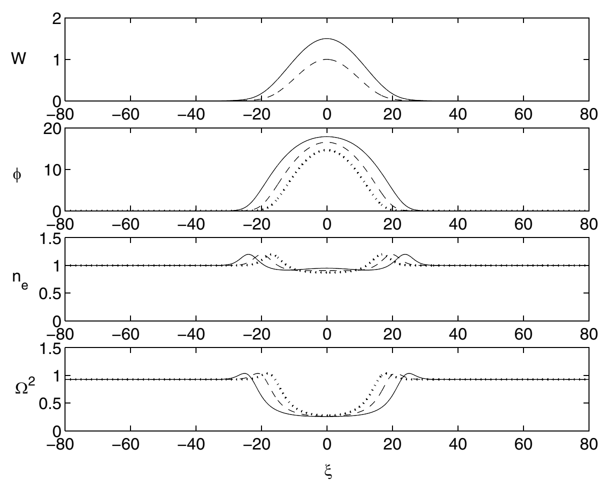

FIG. 1. Large-amplitude trapped EMW envelope (upper panel), the potential (second panel), the electron number density (third panel), and the square of the local electron plasma frequency (lower panel) for large-amplitude EMWs with a maximum amplitude of $W_{\max} = 1.5$ (solid lines) and $W_{\max} = 1.0$ (dashed lines), and as a comparison a REH with small-amplitude EMWs which have $W_{\max} \ll 1$ (dotted lines). The nonlinear frequency shift for the $W_{\max} = 1.5$ case is $\lambda = 0.099$ and for the $W_{\max} = 1.0$ case it is $\lambda = 0.095$, to be compared with the small-amplitude case which has $\lambda = 0.088$. Parameters are: $v_0 = 0.7$, $\alpha = 0.4$, and $\beta = -0.5$. The selected values of β are related to the maximum REH potential according to a specific relation similar to one in Ref. [12], which will be presented in a separate paper.

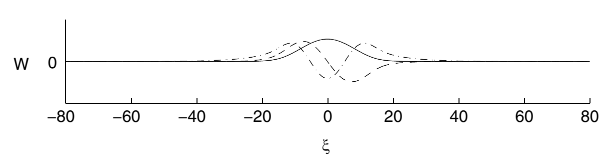

FIG. 2. Small-amplitude trapped EMWs (in arbitrary units) in a REH. Three eigenstates of trapped EMWs exist, corresponding to the eigenvalues $\lambda_1 = 0.088$ (solid line), $\lambda_2 = 0.053$ (dashed line), and $\lambda_2 = 0.013$ (dash-dotted line). The parameters used are the same as for the dotted lines in Fig. 1.

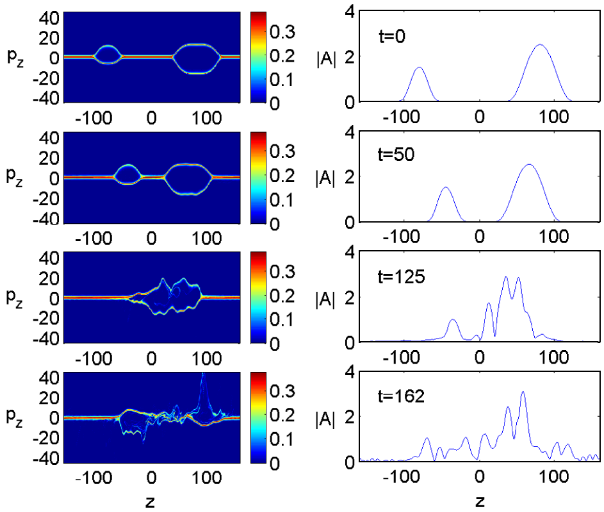

FIG. 3 (color online). Phase space plots of the electron distribution function (left panels) and the modulus of the electromagnetic field (right panels) for $t = 0$, $t = 50$, $t = 125$, and $t = 162$.

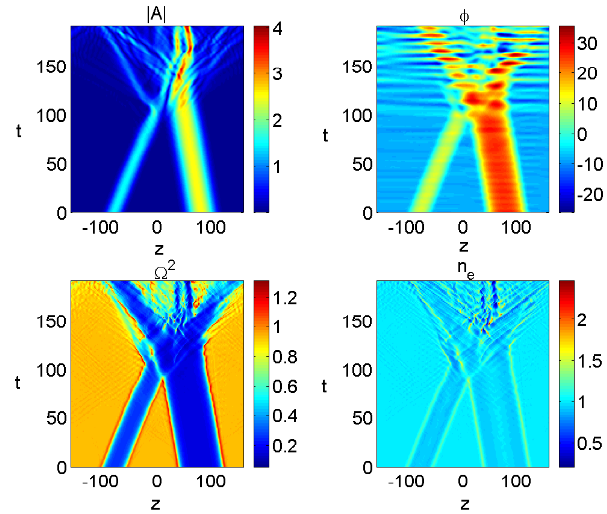

FIG. 4 (color online). The electromagnetic field (upper left panel), potential (upper right panel), squared local plasma frequency (lower left panel), and electron density (lower right panel) for two colliding REHs.

simulations, we used a compact Padé scheme [13] to approximate the p_x derivatives and a pseudospectral method to approximate the x derivative and to solve Poisson's equation, while we used the standard fourth-order Runge-Kutta scheme for the time-stepping.

To summarize, we have presented the first theoretical and simulation studies of intense electromagnetic wave interactions with REHs in a relativistically hot plasma. Our plasma slow response to intense EMWs is unique in that it accounts for nonisothermal relativistic electron distribution including trapped electrons. The results of our analytical theory and numerical analysis reveal that localized EMWs, which are trapped in the REH, push electrons away from the center of the REH, leading to an increase of the electrostatic REH potential and a widening of the REH. Physically, this happens due to the relativistic electron mass increase in the intense EM fields and the relativistic ponderomotive force which pushes the electrons away from the REH. We have also carried out simulations of the dynamics of two interacting REHs loaded with trapped EMWs. We find that due to complex nonlinear interactions EMWs are further intensified, and that the radiation pressure of localized light expels electrons locally, creating electron density cavities which trap intensified light outside the REHs. The interaction has thus given rise to pure EMW solitary waves, while some EMW energy remains trapped in the REHs. In the interaction, energy is also released into a strong acceleration of electrons. Hence, our results add a new dimension to nonlinear interactions between arbitrary large amplitudes, intense electromagnetic waves, and radiation driven intense REHs, which are uncovered by the fluid or nonrelativistic plasma slow response [3–7,9] involved in the description of relativistic electromagnetic solitons in plasmas. In conclusion, we stress that intense localized light driven huge electric fields (several tens of MV/cm) associated with the REH can accelerate electrons to extremely high energies. In fact, our results should be useful in understanding nonlinear collective effects appearing in the present generation inertial confinement fusion schemes, in laboratory astrophysics using intense short laser pulses, as well as in compact astrophysical objects (like radio galaxies, quasars or radio pulsars, supernova remnants, in the vicinity of black holes, etc.) containing high-intensity short electromagnetic bursts.

[1] J. T. Mendonça, *Theory of Photon Acceleration*, (IOP, Bristol, 2001).
[2] P. K. Shukla *et al.*, Phys. Rep. **138**, 1 (1986); R. Bingham *et al.*, Plasma Phys. Controlled Fusion **46**, R1 (2004).
[3] V. A. Kozlov *et al.*, JETP **49**, 75 (1979).
[4] P. K. Kaw *et al.*, Phys. Rev. Lett. **68**, 3172 (1992).
[5] T. Zh. Esirkepov *et al.*, JETP Lett. **68**, 36 (1998).
[6] D. Farina and S. V. Bulanov, Phys. Rev. Lett. **86**, 5289 (2001).
[7] S. V. Bulanov *et al.*, Phys. Rev. Lett. **82**, 3440 (1999).
[8] N. M. Naumova *et al.*, Phys. Rev. Lett. **87**, 185004 (2001).
[9] M. Lontano *et al.*, Phys. Plasmas **10**, 639 (2003).
[10] M. Borghesi *et al.*, Phys. Rev. Lett. **88**, 135002 (2002).
[11] S. R. de Groot *et al.*, *Relativistic Kinetic Theory, Principles and Applications* (North-Holland, Amsterdam, 1980).
[12] S. Bujarbarua and H. Schamel, J. Plasma Phys. **25**, 515 (1981); H. Schamel, Phys. Plasmas **7**, 4831 (2000).
[13] S. K. Lele, J. Comput. Phys. **103**, 16 (1992).