Localization of Superradiance near a Photonic Band Gap

Sajeev John and Tran Quang
Department of Physics, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A7
(Received 22 June 1994)

We describe collective spontaneous emission of N two-level atoms placed within a photonic band-gap material. When the atomic resonance frequency lies at the band edge, superradiant emission remains localized in the vicinity of the atoms. This leads to a steady state with spontaneously broken symmetry in which the atomic system acquires a macroscopic polarization. The superradiant decay rate is proportional to $N^{2/3}$ and N^2 for isotropic and anisotropic 3D band gaps, respectively. The corresponding peak intensity of superradiance is proportional to $N^{5/3}$ and N^3, respectively.

PACS numbers: 71.55.Jv, 32.80.–t, 42.50.Fx

Photon localization, in three-dimensional dielectric media, opens a new frontier for fundamental phenomena in classical and quantum electrodynamics. Following the initial prediction [1] of this effect, attention has focused on systematic methods for the experimental realization of strong localization of light and its consequences in laser physics.

While studies in strongly disordered dielectrics have revealed signatures of incipient localization [2,3], the proposal of creating a complete photonic band gap [4,5] offers the most systematic route to this goal. In recent years, several dielectric structures have been predicted [6–8] and observed [8] to exhibit a photonic band gap (PBG), a range of frequencies for which no propagating electromagnetic modes are allowed. The existence of PBG materials gives rise to a number of interesting phenomena including the suppression of spontaneous emission [4], the formation of strongly localized states of light [5], and photon-atom bound states [9]. Spontaneous emission in a PBG displays distinct features from those in free space such as oscillatory behavior, fractional steady-state atomic population on the excited state, and subnatural linewidth [10,11]. These are all direct consequences of localization.

Although photonic band gaps are analogous to electronic band gaps in semiconductors, there are many intriguing aspects of photons which are not shared by electronic systems. Among these are laser action and superradiance. These are related to the bosonic nature of light through which many photons can occupy the same mode. The recent observation of laser action in strongly scattering media [12] motivates studies in this new direction. In this paper we derive theoretically the nature of collective spontaneous emission of N two-level atoms whose resonance frequency lies at the edge of an isotropic or anisotropic 3D photonic band gap. It is shown that the collective decay rate is proportional to $N^{2/3}$ and N^2 for isotropic and anisotropic 3D band gaps, respectively. The corresponding peak intensity is proportional to $N^{5/3}$ and N^3, respectively. That is, the collection of atoms near a 3D band edge can radiate faster ($\sim N^2$) and more intensely ($\sim N^3$) than Dicke superradiance in free space. We show that a fraction of the superradiant emission remains localized in the vicinity of the atoms leading to a steady state in which the atomic system acquires a macroscopic polarization and retains a nonzero atomic population in the excited state. This novel form of spontaneous symmetry breaking is the analog of lasing without a cavity mode. The collective emission near the photonic band edge is accompanied by self-induced oscillations, a simple illustration of the "ringing" regime in superradiance. In addition to being a fundamental phenomenon, localization of superradiance may play an important role in low threshold microlasers based on photonic band gap engineering. It suggests that a light emitting diode operating near a photonic band edge will exhibit very high modulation speed and coherence properties without recourse to external mirrors or even a true cavity mode.

We consider a Dicke model [13–16] of N identical two-level atoms coupled to the radiation field in a three-dimensional periodic dielectric. The atoms have excited state $|2\rangle$, ground state $|1\rangle$, and resonant transition frequency ω_{21}. The Hamiltonian of the system in the interaction picture takes the form

$$H = \sum_{\lambda} \hbar \Delta_{\lambda} a_{\lambda}^{\dagger} a_{\lambda} + i\hbar \sum_{\lambda} g_{\lambda} (a_{\lambda}^{\dagger} J_{12} - J_{21} a_{\lambda}), \quad (1)$$

where $J_{ij} = \sum_{k=1}^{N} |i\rangle_{kk}\langle j|$ $(i, j = 1, 2)$ are the collective atomic operators, a_{λ} and $a_{\lambda}^{\dagger}$ are the radiation field annihilation and creation operators, $\Delta_{\lambda} = \omega_{\lambda} - \omega_{21}$ is a detuning of the radiation mode frequency ω_{λ} from the atomic resonant frequency ω_{21}, and $g_{\lambda} = (\omega_{21} d_{21}/\hbar)(\hbar/2\epsilon_0 \omega_{\lambda} V)^{1/2} \mathbf{e}_{\lambda} \cdot \mathbf{u}_d$ is the atomic field coupling constant. Here d_{21} and $\mathbf{u}_d$ are the absolute value and the unit vector of the atomic dipole moment, V is the sample volume, $\mathbf{e}_{\lambda} \equiv \mathbf{e}_{\mathbf{k},\sigma}$ are the two transverse (polarization) unit vectors, and ϵ_0 is the Coulomb constant.

Assume that the radiation field is initially in the vacuum state. The equations of motion for $\langle J_{12}(t)\rangle$ and $\langle J_3(t)\rangle = \langle J_{22}(t)\rangle - \langle J_{11}(t)\rangle$ are

$$\frac{d}{dt}\langle J_{12}(t)\rangle = \int_0^t G(t - t')\langle J_3(t) J_{12}(t')\rangle \, dt', \quad (2a)$$

$$\frac{d}{dt}\langle J_3(t)\rangle = -2\int_0^t G(t-t')\langle J_{21}(t)J_{12}(t')\rangle\,dt' + \text{c.c}. \tag{2b}$$

Here $G(t-t') = \sum_\lambda g_\lambda^2 e^{-i\Delta_\lambda(t-t')}$ is the delay Green's function, and $\langle A\rangle$ indicates the expectation value of the system operator A. The Green's function $G(t-t')$ depends strongly on the dispersion relation and density of states of the medium. For the purpose of discussion we consider two simple models of a PBG for electromagnetic waves in a three-dimensional periodic dielectric. In model I, we assume the dispersion is isotropic with respect to the wave vector $\mathbf{k}$. The simplest model dispersion relation which exhibits an isotropic PBG while retaining the correct behavior in the limit of very low and very high frequencies is

$$\omega_{\mathbf{k}}/c = \text{sgn}(k-k_0)\sqrt{(k-k_0)^2+\gamma^2} + \sqrt{k_0^2+\gamma^2}. \tag{3}$$

Here $k \equiv |\mathbf{k}|$ and k_0 and γ are parameters related to the dielectric microstructure. The two-valued nature of the square-root function is made explicit by the presence of the function $\text{sgn}(k-k_0) = +1$ for $k > k_0$ and -1 for $k < k_0$. The square-root function has branch point singularities at $k = k_0 \pm i\gamma$. The presence of the sign function indicates that the branch cut should be placed along the line connecting these two branch points. Physically, this corresponds to placing an isotropic photonic band gap of width $\Delta\omega/c = 2\gamma$ centered about the frequency $\omega_0/c = \sqrt{k_0^2+\gamma^2}$. Also $\omega_0/c \to k(k_0/\sqrt{k_0^2+\gamma^2})$ as $k \to 0$ and $\omega_k/c \simeq k + (\sqrt{k_0^2+\gamma^2} - k_0)$ for $k \gg k_0$. Near the band edge $\omega_c/c = \sqrt{k_0^2+\gamma^2} + \gamma$, the photon density of states is singular. For $k \simeq k_0$, we may simplify the dispersion relation by the effective mass approximation $\omega_{\mathbf{k}} \simeq \omega_c + A(k-k_0)^2$, where $A = 1/(2\gamma)$.

The singular density of states is an artifact of the isotropic model. In the anisotropic model II, which we describe later, the density of states in fact vanishes at ω_c. While both models exhibit localized superradiance and spontaneous symmetry breaking, the collective time scale factors for superradiant emission are qualitatively different for the two cases.

The delay Green's function $G(t-t')$ can be written for the isotropic PBG of model I as

$$G(t-t') = \frac{\omega_{21}^2 d_{21}^2}{6\pi^2\epsilon_0\hbar}\int_0^\Lambda \frac{k^2}{\omega_k} e^{-i(\omega_k-\omega_{21})(t-t')}\,dk. \tag{4}$$

Here we converted the mode sum over the transverse plane wave into an integral and performed the angular integral. $\Lambda = mc/\hbar$ is the cutoff in the photon wave vector. Photons of energy higher than the electron rest mass mc^2 probe the relativistic structure of the electron wave packet [17]. Using the effective mass isotropic dispersion relation, integration of Eq. (4) yields $G(t-t') = \beta^{3/2}e^{-i\pi/4}/\sqrt{\pi(t-t')}$, where $\beta^{3/2} = \omega_{21}^{7/2}d_{21}^2/6\pi\epsilon_0\hbar c^3$. For simplicity we assume $\omega_{21} = \omega_c$, i.e., the atomic resonance frequency lies at the band edge frequency ω_c. A discussion of superradiance at general frequencies using the full dispersion relation (3) will be presented elsewhere.

To discuss the possibility of spontaneous symmetry breaking during the process of superradiant emission, we introduce a very small external perturbation which endows the atomic system with an infinitesimal polarization. This is analogous to the addition of a small magnetic field h in describing the thermodynamic phases of a collection of N magnetic moments. A ferromagnetic phase transition at zero field is described by taking the limit $h \to 0$ only after the thermodynamic limit $N \to \infty$. Accordingly, we find that an infinitesimal initial polarization of the atomic dipoles gives rise to a macroscopic polarization in the steady state limit $t \to \infty$. Assume that initially the atomic system is in the state [16]

$$|\psi_N\rangle = \prod_{k=1}^N (\sqrt{r}|1\rangle + \sqrt{1-r}|2\rangle)_k, \tag{5}$$

where $r \ll 1$, i.e., atoms are mostly populated in the excited state $|2\rangle$ and the atomic coherence is infinitesimal. Such a state can be created by interaction of atoms with an external pulse [16]. Qualitatively similar results to the ones we present occur for various values of the initial atomic inversion per atom $\langle J_3(0)\rangle/N$ and for an infinitesimal initial polarization $\langle J_{12}(0)\rangle/N$. The role of different initial conditions and quantum fluctuations on superradiance in a PBG will be discussed in detail elsewhere. The system can be considered semiclassical [15,16] and equations of motion for $x(t) \equiv \langle J_{12}(t)\rangle/N$ and $y(t) \equiv \langle J_3(t)\rangle/N$ may be obtained from (2) by factorizing the quantum expectation value of the operator products

$$\frac{dx}{dt} = Ny(t)\int_0^t G(t-t')x(t')\,dt', \tag{6a}$$

$$\frac{dy}{dt} = -2Nx^*(t)\int_0^t G(t-t')x(t')\,dt' + \text{c.c}. \tag{6b}$$

It is easy to verify, using the isotropic, effective mass solution to the Green's function (4), that x and y are functions of the dimensionless, scaled, time variable $\beta N^{2/3}t$. The factor $\beta N^{2/3}$ is analogous to a bandwidth parameter in solid state physics. As a result of the band edge mediated interaction between atoms, the effective Rabi splitting is enhanced and the spectrum is broadened by a factor of $N^{2/3}$. In the Markovian approximation [15], it is assumed that the dynamical evolution of the system at time t is determined entirely by the state of the system at time t. Neglecting memory effects, Eq. (6) yields

$$\langle J_3(t)\rangle = -N\tanh\{B[(t/\tau)^{3/2}-1]\}, \tag{7}$$

where $B = \text{arctanh}(1-2r/N)$ and $\tau = 3^{2/3}\pi^{1/3}B^{2/3}/2\beta N^{2/3}$. It is clear from Eq. (7) that the collective decay rate at the band edge of an isotropic PBG is proportional to $N^{2/3}$ and the delay time τ of superradiance is proportional to $N^{-2/3}$. As a result, the superradiant intensity, which is proportional to $-d\langle J_3(t)\rangle/dt$, scales as $N^{5/3}$. One can see from Eq. (7) that $\lim_{t\to\infty}\langle J_3(t)\rangle = -N$. In

the Markovian approximation, all atoms decay to the ground state and there is no localization of superradiance. While the Markovian approximation does give the correct collective time scale factors, it fails to properly account for memory and feedback effects. These effects are particularly important in a photonic band gap where photons can bind to atoms [9].

To recapture this localization effect, we solve the system of Eqs. (6a) and (6b) exactly using numerical methods [18]. In Fig. 1 we plot the atomic population inversion $\langle J_3(t)\rangle/N$ (solid curve) and atomic dipole moment $D(t)/N = |\langle J_{12}(t)\rangle|/N$ (dashed curve) as a function of $\beta N^{2/3}t$. In Fig. 2 we plot the phase $\mu(t)$ of the atomic polarization $\langle J_{12}(t)\rangle$ for the same initial condition (5). Clearly, the collective spontaneous emission at the edge of a PBG displays striking distinctions from the free space case: (i) In the steady-state limit the population inversion $\langle J_3(t)\rangle/N$ is not equal to -1. This follows from the fact that the single atomic population inversion in the excited state $|2\rangle$ remains nonzero [11]. This signifies localization of superradiant emission in the vicinity of the atoms. (ii) The atomic polarization evolves from its infinitesimal initial value to a steady-state macroscopic value. This is distinct from the free space superradiance where the atomic steady-state polarization is equal to zero. This spontaneous symmetry breaking in the atomic polarization field is analogous to lasing without a cavity mode. It suggests the possibility of observing macroscopic quantum coherent superpositions of states. (iii) The evolution of $\langle J_3(t)\rangle$, $D(t)$, and $\mu(t)$ displays collective self-induced oscillation instead of a simple decay as it is in free space. These oscillations are analogous to the collective Rabi oscillations of N Rydberg's atoms in a resonant high-Q cavity [19]. In addition to amplitude oscillations, the phase of the macroscopic polarization rotates in the steady-state limit with a frequency proportional to the magnitude of vacuum Rabi splitting. (iv) The collective time scale factor for the isotropic PBG is proportional to $N^{2/3}$ rather than N as it is in free space. That is, the collective decay rate of superradiance is proportional to $N^{2/3}$ and the peak superradiance intensity, which is proportional to $-d\langle J_3(t)\rangle/dt$, is proportional to $N^{5/3}$ rather than N^2.

Our numerical results reveal qualitatively similar behavior for different initial conditions. In particular, macroscopic polarization emerges for *any* initial state for which $\langle J_3(0)\rangle/N > 0$, and the steady-state limit $\langle J_{12}(\infty)\rangle/N$ is independent of the initial (infinitesimal) value $\langle J_{12}(0)\rangle/N$. The delay time required for superradiant emission, however, decreased noticeably as $\langle J_{12}(0)\rangle/N$ was varied from 10^{-4} to 10^{-3}. The magnitude of the macroscopic steady-state polarization $\langle J_{12}(\infty)\rangle/N$ decreases monotonically from 0.42 to 0.15 as the initial inversion $\langle J_3(0)\rangle/N$ was decreased form 0.95 to 0.3. In the absence of population inversion [$\langle J_3(0)\rangle/N < 0$], we find that macroscopic polarization (in the long time limit) occurs only if the initial state itself has a macroscopic polarization.

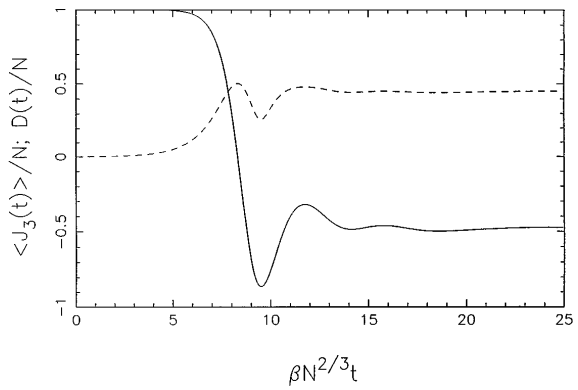

FIG. 1. Atomic inversion $\langle J_3(t)\rangle/N$ (solid curve) and amplitude of the atomic polarization $D(t)/N = |\langle J_{12}(t)\rangle|/N$ (dashed curve) in an isotropic PBG as a function of the scale time $\beta N^{2/3}t$ for initial condition (10) with $r = 10^{-6}$.

The collective time scale factor of $N^{2/3}$ was found above using the isotropic PBG (model I). This exponent of N, however, depends sensitively on the dimension of the phase space occupied by band-edge photons of vanishing group velocity and the resulting band-edge singularity in the overall photon density of states. In an isotropic band edge, we have overestimated this phase space using the entire sphere $|\mathbf{k}| = k_0$. For a real dielectric crystal in three dimensions with an allowed point-group symmetry, the band edge is associated with a point $\mathbf{k} = \mathbf{k}_0$ (or a finite collection of symmetry related points) rather than the entire sphere $|\mathbf{k}| = |\mathbf{k}_0|$. In model II, we choose the "effective mass" dispersion relation to be of the form

$$\omega_{\mathbf{k}} \cong \omega_c + A(\mathbf{k} - \mathbf{k}_0)^2. \tag{8}$$

Using the anisotropic dispersion relation (8), the Green's function in Eq. (2) and its integral for the case of $\omega_c t \gg 1$ become

$$G(t - t') \cong -\frac{1}{\sqrt{2}}\,\beta_3^{1/2} e^{i\pi/4}/(t - t')^{3/2}, \tag{9a}$$

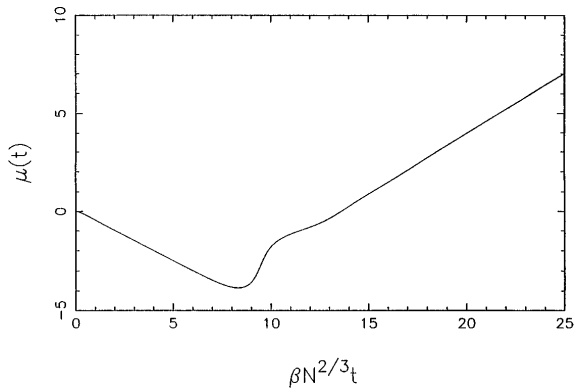

FIG. 2. Phase angle of the atomic polarization as a function of $\beta N^{2/3}t$ for the same parameter as in Fig. 1.

$$\int_0^t G(t - t')\,dt' = \sqrt{2}\,\beta_3^{1/2} e^{i\pi/4}/\sqrt{t} - i\sqrt{2\pi}\,\omega_c^{1/2}\beta_3^{1/2}, \tag{9b}$$

where $\beta_3^{1/2} = \omega_{21}^2 d_{21}^2/8\sqrt{2}\,\hbar\epsilon_0\pi^{3/2}A^{3/2}\omega_c$. It is straightforward to verify, using Eqs. (6a) and (6b) and the Green's function (9a), that x and y are now functions of a new dimensionless time variable $\beta_3 N^2 t$. In the Markovian approximation, $\langle J_3(t)\rangle$ is again determined from Eqs. (6a), (6b), and (9b),

$$\langle J_3(t)\rangle = -N \tanh\{B[(t/\tau_3)^{1/2} - 1]\}, \tag{10}$$

where $\tau_3 = B^2/4\beta_3 N^2$. It is clear from Eq. (10) that in an anisotropic 3D PBG, the collective decay rate is proportional to N^2. As a result, the superradiance intensity, which is proportional to $-d\langle J_3(t)\rangle/dt$, is proportional to N^3.

Model II exhibits both localization of superradiance and spontaneous symmetry breaking. To see this, we solved Eqs. (6a) and (6b) numerically [18] using Eqs. (9). In Fig. 3 we plot $\langle J_3(t)\rangle/N$ (solid curve) and $D(t)/N = |\langle J_{21}(t)\rangle|/N$ (dashed curve) as a function of $\beta_3 N^2 t$. As before, superradiance is localized in the vicinity of atoms and the system tends to the steady state with macroscopic atomic polarization and rotating phase.

The strong dependence of the collective processes on the structure of the photonic band gap can be associated with the density of states near the band edge of a PBG. As discussed in Refs. [9,11], the isotropic dispersion relation leads to a photonic density of states $\rho(\omega)$ at a band edge $\omega \geq \omega_c$ which behaves as $(\omega - \omega_c)^{-1/2}$. For the anisotropic dispersion (8) this becomes $\rho(\omega) \sim (\omega - \omega_c)^{1/2}$.

In conclusion, we have demonstrated that localization and macroscopic coherence in superradiant emission occur near a photonic band edge even in the absence of a dielectric defect mode or other cavity mode. This suggests the possibility that ordinary light emission in a perfectly periodic dielectric may exhibit coherence properties with infinitesimal threshold. For the case of a physical three-dimensional anisotropic gap, the superradiant emission occurs much faster and with higher peak intensity than conventional superradiance. These results are based on a simple model of point superradiance in which N atoms are confined to a region smaller than the wavelength of light. The spontaneous atomic polarization in the steady state is analogous to the emergence of a "superfluid" order parameter for photons. If the volume of superradiance is made larger than a cubic wavelength, alternate forms of spontaneous symmetry breaking may arise from the spatial dependence of the effective atomic resonance dipole-dipole interaction (RDDI). In general, RDDI causes a breaking of the permutation symmetry of the N-atom wave function (5). This may diminish the magnitude of the macroscopic steady-state polarization, and lead to a "Bose glass" under certain circumstances.

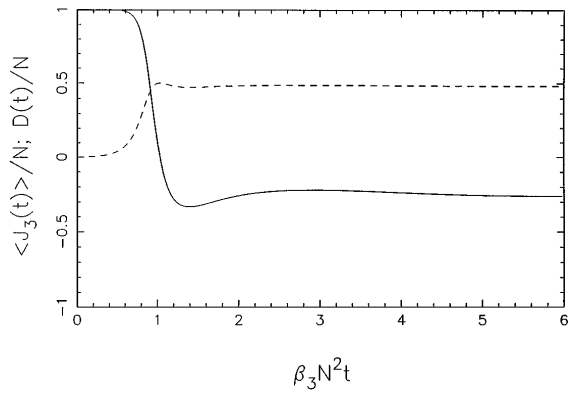

FIG. 3. Atomic inversion $\langle J_3(t)\rangle/N$ (solid curve) and amplitude of the atomic polarization $D(t)/N = |\langle J_{12}(t)\rangle|/N$ (dashed curve) in an anisotropic 3D PBG as a function of the scale time $\beta_3 N^2 t$ for $\xi_N = (2\pi\omega_c/\beta_3 N^2)^{1/2} = 10$ and $r = 10^{-6}$.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and Ontario Laser and Lightwave Research Center.

[1] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[2] N. Garcia and A. Z. Genack, Phys. Rev. Lett. **66**, 1850 (1991).
[3] A. Z. Genack and N. Garcia, Phys. Rev. Lett. **66**, 2064 (1991).
[4] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
[5] S. John, Phys. Rev. Lett. **58**, 2486 (1987).
[6] K. M. Ho, C. T. Chan, and C. M. Soukoulis, Phys. Rev. Lett. **65**, 3152 (1990).
[7] Z. Zhang and S. Satpathy, Phys. Rev. Lett. **65**, 2650 (1990).
[8] E. Yablonovitch, T. J. Gmitter, and K. M. Leung, Phys. Rev. Lett. **67**, 2295 (1991).
[9] S. John and J. Wang, Phys. Rev. Lett. **64**, 2418 (1990); Phys. Rev. B **43**, 12 772 (1991).
[10] R. F. Nabiev, P. Yeh, and J. J. Sanchez-Mondragon, Phys. Rev. A **47**, 3380 (1993).
[11] S. John and T. Quang, Phys. Rev. A **50**, 1764 (1994).
[12] N. M. Lawandy, R. M. Balachandran, A. S. L. Gomes, and E. Sauvain, Nature (London) **368**, 436 (1994).
[13] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[14] G. S. Agarwal, *Quantum Optics*, Springer Tracts in Modern Physics Vol. 70 (Springer-Verlag, Berlin, 1974).
[15] M. Gross and S. Haroche, Phys. Rev. **93**, 301 (1982); S. Haroche and J. M. Raimond, Adv. At. Mol. Phys. **30**, 347 (1985).
[16] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975), Chap. 8.
[17] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
[18] L. M. Delves and J. L. Mohamed, *Computational Methods for Integral Equations* (Cambridge University, Cambridge, 1985).
[19] Y. Kaluzny, P. Goy, M. Gross, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **51**, 1175 (1983).