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LOCALLY CLOSED SETS AND LC-CONTINUOUS FUNCTIONS

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ABSTRACT. In this paper we introduce and study three different notions of generalized continuity, namely LC-irresoluteness, LC-continuity and sub-LC-continuity. All three notions are defined by using the concept of a locally closed set. A subset S of a topological space X is locally closed if it is the intersection of an open and a closed set. We discuss some properties of these functions and show that a function between topological spaces is continuous if and only if it is sub-LC-continuous and nearly continuous in the sense of Ptak. Several examples are provided to illustrate the behavior of these new classes of functions.

KEY WORDS AND PHRASES. Locally closed set, LC-continuous, nearly continuous.
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1. INTROUDCTION.

In 1921 Kuratowski and Sierpinski [1] considered the difference of two closed subsets of an n-dimensional euclidean space. Implicit in their work is the notion of a locally closed subset of a topological space (X,T). Following Bourbaki [2], we say that a subset of (X,T) is locally closed in X if it is the intersection of an open subset of X and a closed subset of X. Stone [3] has used the term FG for a locally closed subset.

The following results indicate that locally closed subsets are of some interest in the setting of local compactness, Cech-Stone compactifications, or Cech complete spaces. From Engelking [4] we have:

1. If (X,T) is Hausdorff and C is a locally compact subspace of X, then C is locally closed [4, p. 140, Ex. A].
2. If (X,T) is locally compact and Hausdorff and $C \subseteq X$, then C is locally compact if and only if C is locally closed [4, p. 140, Ex. B].
3. If X is completely regular, then X is locally closed in βX if and only if X is locally compact [4, follows from Theorem 6, page 137].

4. If X is completely regular and Cech complete and C is locally closed in X, then C is Cech complete [4, follows from Theorem 3, page 144].

Stone [3] has studied the absolutely FG spaces - the spaces that in every embedding are locally closed. He has shown that, for Hausdorff spaces, the hereditarily absolute FG spaces coincide with the hereditarily locally compact spaces.

The results of Borges [5] show that locally closed sets play an important role in the context of simple extensions. For example, if T(A) denotes the simple extension of T by A, then if (X,T) is regular and $A \subseteq X$, we have that (X, T(A)) is regular if and only if A is locally closed in X [5, Theorem 3.2].

In 1922 Blumberg [6] introduced the concept of a real valued function on euclidean space being densely approached at a point of its domain. This notion was generalized in 1958 to general topological spaces by Ptak [7] who used the term nearly continuous. The concepts of nearly continuous and nearly open functions are important in functional analysis especially in the context of open mapping and closed graph theorems. We refer the reader to the work of Ptak [7], Pettis [8], Noll [9] and Wilhelm [10], [11], for example.

In this paper we consider a new class of generalized continuous functions which are called LC-continuous functions. Such a function is defined by requiring the inverse image of each open set in the codomain to be locally closed in the domain. The significance of this notion is that LC-continuity is the continuity dual of nearly continuity, that is a function is continuous if and only if it is nearly continuous and LC-continuous. This theorem enables us to obtain interesting variations of results from functional analysis. We quote two to illustrate. If G is a Baire topological group and H is a separable (or Hausdorff and Lindelof) topological group, then a homomorphism $f: G \to H$ is continuous if and only if it is sub-LC-continuous, Husain [13, p. 222]. If X is a Baire topological vector space, Y is a topological vector space and $f: X \to Y$ is linear, then f is continuous if and only if it is sub-LC-continuous, Husain [13, p. 224].

In section 2 we consider the properties of locally closed subsets, while section 3 introduces the classes of LC-irresolute, LC-continuous and sub-LC-continuous functions. Section 4 is concerned with some of the properties of these functions, and relevant examples are provided in section 5. We note that throughout this paper no separation properties are assumed unless explicitly stated.

2. LOCALLY CLOSED SETS.

Let S be a subset of a topological space (X,T). We denote the closure of S, the interior of S, and the boundary of S with respect to T by T cl S, T int S, and T bd S respectively, usually suppressing the T when there is no possibility of confusion. The relative topology on S with respect to T is denoted by T/S. We will denote the set of all reals by R and the set of all positive integers by N. Unless otherwise mentioned R carries its usual topology.

DEFINITION [2]. A subset S of a space (X,T) is called locally closed if $S = U \cap F$ where $U \in T$ and F is closed in (X,T).

We denote the collection of all locally closed subsets of (X,T) by LC(X,T).

REMARKS. (i) A subset S of (X,T) is locally closed if and only if X-S is the union of an open set and a closed set.
(ii) Every open [resp. closed] subset of (X,T) is locally closed. (iii) For any space (X,T), LC(X,T) is closed under finite intersections. In particular, any interval in R is locally closed. (iv) The complement of a locally closed subset need not be locally closed. Hence the finite union of locally closed subsets need not be locally closed (see e.g. Corollary 2). (v) A subset S of a space (X,T) is said to be nearly open if $S \subseteq \text{int}(\text{cl } S)$. Nearly open sets are known also as preopen sets [14]. Ganster and Reilly [12] have shown that a subset S of (X,T) is open if and only if it is nearly open and locally closed. In particular, a dense subset is open if and only if it is locally closed. (vi) Spaces in which every singleton is locally closed are called T_D spaces [15].

The following result is essentially a restatement of I.3.3, Proposition 5, of [2].

PROPOSITION 1 [2]. For a subset S of a space (X,T) the following are equivalent.

(i) S is locally closed.
(ii) $S = U \cap \text{cl } S$ for some open set U.
(iii) $\text{cl } S - S$ is closed.
(iv) $S \cup (X - \text{cl } S)$ is open.
(v) $S \subseteq \text{int}(S \cup (X - \text{cl } S))$.

Recall that (X,T) is called submaximal if every dense subset is open. Using (iv) of Proposition 1 we immediately get

COROLLARY 1. A space (X,T) is submaximal if and only if every subset of (X,T) is locally closed.

The next result indicates where to look in order to find locally closed sets besides open sets and closed sets.

PROPOSITION 2. (i) Let (X,T) be a T_1 space and let S be a discrete subset of (X,T). Then S is locally closed. (ii) Let (X,T) be dense-in-itself and S be a discrete subset. Then X-S is locally closed if and only if S is closed.

PROOF. Let S be a discrete subset of the T_1 space (X,T), i.e. for each $x \in S$ there is an open set U_x such that $U_x \cap S = \{x\}$. If $U = \bigcup\{U_x \mid x \in S\}$ then it is easily verified that $S = U \cap \text{cl } S$. This proves (i). In order to prove (ii), observe that in a dense-in-itself space any discrete subset has empty interior.

COROLLARY 2. If $S = \{1/n \mid n \in N\}$ then S is locally closed in R whereas R - S is not.

Note that in Proposition 2 (ii) the assumption that (X,T) is dense-in-itself cannot be dropped. Consider a space (X,T) whose set D of isolated points is a proper dense subset. Then clearly D is a nonclosed discrete subset whereas X - D is closed hence locally closed.

Our next four results exhibit some of the basic properties of locally closed sets.

PROPOSITION 3. Let (X,T) be a space and let $Z \in LC(X,T)$. If $A \in Z$ and $A \in LC(Z,T/Z)$ then $A \in LC(X,T)$.

PROPOSITION 4. Let A and B be locally closed subsets of a space (X,T). If A and B are separated, i.e. if $A \cap \text{cl } B = \text{cl } A \cap B = \phi$, then $A \cup B \in LC(X,T)$.

PROOF. Suppose there are open sets U and V such that $A = U \cap \text{cl } A$ and $B = V \cap \text{cl } B$. Since A and B are separated we may assume that $U \cap \text{cl } B = V \cap \text{cl } A = \phi$. Consequently $A \cup B = (U \cup V) \cap \text{cl } (A \cup B)$ showing that $A \cup B \in LC(X,T)$.

THEOREM 1. Let $\{Z_i \mid i \in I\}$ be either an open cover or a locally finite closed cover of a space (X,T) and let $A \subseteq X$. If $A \cap Z_i \in LC(Z_i, T/Z_i)$ for each $i \in I$ then $A \in LC(X,T)$.

PROOF. First suppose that $\{Z_i \mid i \in I\}$ is an open cover of (X,T). For each $i \in I$, since $A \cap Z_i \in LC(Z_i,T/Z_i)$ we may assume that $A \cap Z_i = V_i \cap \text{cl}(A \cap Z_i)$ where $V_i \in T$ and $V_i \subseteq Z_i$. Now $V_i \cap \text{cl } A = V_i \cap Z_i \cap \text{cl } A \subseteq V_i \cap \text{cl}(A \cap Z_i) = A \cap Z_i$. Hence if $V = \cup \{V_i \mid i \in I\}$ we have $V \cap \text{cl } A = A$.

Now suppose that $\{Z_i \mid i \in I\}$ is a locally finite closed cover of (X,T). For each $i \in I$, since $A \cap Z_i \in LC(Z_i, T/Z_i)$ we have $A \cap Z_i = V_i \cap \text{cl } (A \cap Z_i)$ where $V_i \in T$. Let $x \in A$. Since $\{Z_i \mid i \in I\}$ is a locally finite closed cover, hence a point-finite and closure-preserving cover, there is a finite subset $I_x \subseteq I$ such that $x \in Z_i$ if $I \in I_x$ and $x \notin \bigcup\{Z_i \mid i \in I - I_x\}$. Moreover, there is an open set U_x containing x such that $U_x \subseteq \bigcap\{V_i \mid i \in I_x\}$ and $U_x \cap (\cup \{Z_i \mid i \in I - I_x\}) = \phi$. If $U = \bigcup \{U_x \mid x \in A\}$ then clearly $A \subset U \cap \text{cl } A$. Let $y \in U \cap \text{cl } A$. Then $y \in U_x$ for some $x \in A$. Since $y \in \text{cl } A = \cup \{\text{cl}(A \cap Z_i) \mid i \in I\}$ we have $y \in \text{cl}(A \cap Z_j)$ for some $j \in I$. Hence $j \in I_x$ and $U_x \subseteq V_j$. Thus $y \in V_j \bigcap \text{cl}(A \cap Z_j) = A \cap Z_j \subseteq A$. It follows that $A = U \cap \text{cl } A$.

PROPOSITION 5. For each $i \in I$, let (X_i, T_i) be a space and let $S_i \in LC(X_i,T_i)$. If $S_i = X_i$ except for finitely many $i \in I$, then ΠS_i is a locally closed subset of the product space ΠX_i.

In general one cannot expect that the set-theoretic complement of a locally closed set is locally closed. The following result characterizes those spaces in which a locally closed subset has necessarily a locally closed complement. Recall that a subset S of a space (X,T) is said to be semi-open if $S \subseteq \text{cl (int } S)$.

THEOREM 2. For a T_1 space (X,T) the following are equivalent:

(i) $S \in LC(X,T)$ if and only if $X-S \in LC(X,T)$.

(ii) $LC(X,T)$ is closed under finite unions.

(iii) The boundary of each open set is a discrete subset.

(iv) The boundary of each semi-open set is a discrete subset.

(v) Every semi-open set is locally closed.

PROOF. (i) <=>(ii) is obvious.

(ii) ==> (iii): Let U be open and let $x \in \text{bd}U = \text{cl } U \cap (X-U)$. By assumption, if $S = U \cup \{x\}$ then $S \in LC(X,T)$. Let $S = V \cap \text{cl } S$ for some open set V. One easily verifies that $V \cap \text{bd } U = \{x\}$.

(iii) ==> (iv): Let S be semi-open in (X,T) and let $U = \text{int } S$. Then $\text{bd } S \subseteq \text{bd } U$ and hence bd S is a discrete subset.

(iv) ==> (v): Let S be semi-open in (X,T). For each $x \in S \cap \text{bd } S$ there is an open set U_x such that $U_x \cap \text{bd } S = \{x\}$. If $U = \text{int } S \cup [\cup \{U_x \mid x \in S \cap \text{bd } S\}]$ it is easily verified that $S = U \cap \text{cl } S$.

(v) ==> (i): We will show that any union of an open set and a closed set is locally closed. Let $A = U \cup F$ where U is open and F is closed. We may assume that $U \cap F = \phi$. If $S = U \cup (\text{cl } U \cap F)$ then S is semi-open and hence $S = V \cap \text{cl } S = V \cap \text{cl } S = V \cap \text{cl } U$ for some open set V. If $W = V \cup (X - \text{cl } U)$ then clearly $A = W \cap \text{cl } A$. Thus $A \in LC(X,T)$.

3. LC-CONTINUOUS FUNCTIONS.

In this section we define three distinct notions of LC-continuity and study some of their immediate consequences.

DEFINITION. A function $f: X \to Y$ between spaces (X,T) and (Y,σ) is called

(i) LC-irresolute if $f^{-1}(M) \in LC(X,T)$ for each $M \in LC(Y,\sigma)$.

(ii) LC-continuous if $f^{-1}(V) \in LC(X,T)$ for each $V \in \sigma$.

(iii) sub-LC-continuous if there is a subbase (or, equivalently, a base) B for (Y,σ) such that $f^{-1}(V) \in LC(X,T)$ for each $V \in B$.

Let us note that these concepts have also obvious local forms. A function $f:(X,T) \to (Y,\sigma)$ is called LC-irresolute [resp. LC-continuous] at a point $x \in X$ if for each $M \in LC(Y,\sigma)$ [resp. $M \in \sigma$] satisfying $f(x) \in M$ there is an open neighbourhood U of x such that $U \cap \text{cl } f^{-1}(M) \subseteq f^{-1}(M)$. $f: (X,T) \to (Y,\sigma)$ is said to be sub-LC-continuous at $x \in X$ if there is an open subbase B for the neighbourhood filter of f(x) such that $f^{-1}(V) \in LC(X,T)$ whenever $V \in B$. It is easily verified that $f:X \to Y$ is LC-irresolute [resp. LC-continuous, resp. sub-LC-continuous] if and only if it is LC irresolute [resp. LC-continuous, resp. sub-LC-continuous] at each point of X.

From the previous definition it follows immediately that we have the following implications: continuous ==> LC-irresolute ==> LC-continuous ==> sub-LC-continuous. However, none of these implications can be reversed. Example 1 provides a function which is LC-irresolute but not continuous. In Example 2 we have constructed an LC-continuous function which is not LC-irresolute. Example 3 and Example 4 provide functions which are sub-LC-continuous but fail to be LC-continuous.

Our next two results are immediate consequences of Corollary 1 and Theorem 2 respectively.

PROPOSITION 6. A space (X,T) is submaximal if and only if every function having X as its domain is LC-continuous.

PROPOSITION 7. Let (X,T) be a space in which bd U is a discrete subset for each open set U. Then for any space (Y,σ) and any LC-continuous function $f: X \to Y$, f is LC-irresolute.

The importance of LC-continuous functions shows up in their relationship to nearly continuous functions. Recall that a function $f: (X,T) \to (Y,\sigma)$ is said to be nearly continuous [7] if the inverse image of each open set is nearly open. The following result is an improvement of the decomposition theorem in [12].

THEOREM 3. A function $f: X \to Y$ between spaces (X,T) and (Y,σ) is continuous if and only if f is nearly continuous and sub-LC-continuous.

PROOF. Let f be nearly continuous and sub-LC-continuous. Let B be a base for (Y,σ) such that $f^{-1}(V) \in LC(X,T)$ whenever $V \in B$. Now let $W \in \sigma$ and $f(x) \in W$. There is a $V \in B$ such that $f(x) \in V \subseteq W$. Since $f^{-1}(V)$ is nearly open and locally closed it is open [12], hence x int $f^{-1}(W)$. This proves the continuity of f. The converse is obvious.

REMARK. Example 1 and Example 5 illustrate that near continuity and LC-continuity are independent of each other.

4. SOME PROPERTIES OF LC-CONTINUOUS FUNCTIONS.

It is obvious that if we consider the restriction of a function to an arbitrary subspace then LC-irresoluteness, LC-continuity or sub-LC-continuity are preserved. Example 3 illustrates that a function can be continuous on the elements of a cover of the domain but need not be LC-continuous. As an analogue to the case of continuous functions we have, however, the following result which is an immediate conseqence of Theorem 1.

PROPOSITION 8. Let $\{Z_i \mid i \in I\}$ be either an open or a locally finite closed cover of the space (X,T). Let $f: (X,T) \to (Y,\sigma)$ be such that $f|Z_i : Z_i \to Y$ is LC-irresolute [resp. LC-continuous, resp. sub-LC-continuous] for each $i \in I$. Then f is LC-irresolute [resp. LC-continuous, resp. sub-LC-continuous].

Concerning compositions of functions, the composition of two LC-irresolute functions is clearly LC-irresolute. It is also easy to verify that whenever the composition of a continuous function and an LC-continuous function is defined, it is LC-continuous. In contrast to this we have the following two results.

PROPOSITION 9. The composition of an LC-continuous function and an LC-irresolute function need not be sub-LC-continuous.

PROOF. Let $A = \{1/n \mid n \in N\}$. But Corollary 2, A is locally closed in R and R-A is not. Let $f: R \to R$ be as in Example 2, i.e. $f(x) = x$ if $x \in A$ and $f(x) = 0$ if $x \in R-A$. Then f is LC-continuous. Define $g: R \to R$ be setting $g(x) = 0$ if $x \leq 0$ and $g(x) = 1$ if $x > 0$. Then g is clearly LC-irresolute. If $h = gof$ then $h(x) = 0$ if $x \in R-A$ and $h(x) = 1$ if $x \in A$. Since the only possible preimages of sets under h are ϕ, R, A and R-A, h is not even sub-LC-continuous.

PROPOSITION 10. The composition of a sub-LC-continuous function and a continuous function need not be sub-LC-continuous.

PROOF. Take a sub-LC-continuous function $f:(X,T) \to (Y,\sigma)$ which is not LC-continuous (e.g. the function in Example 3). Hence there is a set $V \in \sigma$ such that $f^{-1}(V) \notin LC(X,T)$. Now $\sigma^* = \{\phi,V,Y\}$ is a topology on Y and the identity function $id:(Y,\sigma) \to (Y,\sigma^*)$ is continuous. The composition idof, however, fails to be sub-LC-continuous.

To every function $f: X \to Y$ one can assign the graph function $g_f: X \to X \times Y$ defined by $g_f(x) = (x,f(x))$.

PROPOSITION 11. Let $f: X \to Y$ be a function between spaces (X,T) and (Y,σ).

(i) If f is sub-LC-continuous then g_f is sub-LC-continuous.

(ii) If f is LC-irresolute then g_f need not be LC-continuous.

PROOF. Let B be a subbase for (Y,σ) such that $f^{-1}(V) \in LC(X,T)$ whenever $V \in B$. Then $\{U \times V \mid U \in T, V \in B\}$ is a subbase for the product topology on [illegible]. Since $g_f^{-1}(U \times V) = U \cap f^{-1}(V)$, g_f is sub-LC-continuous. This proves (i). To prove (ii) let $f: R \to R$ as in Example 1, i.e. $f(x) = 0$ if $x \leq 0$ and $f(x) = 1$ if $x > 0$. If $B = \{(0,1)\} \cup \{(1/n,1) \mid n \in N\}$ then B is closed in $R \times R$. If $V = R \times R - B$ then

$g_f^{-1}(V) = R - \{1/n \mid n \in N\}$ is not locally closed by Corollary 2, hence g_f fails to be LC-continuous.

Proposition 11 shows that the diagonal function of a family of LC-irresolute, thus LC-continuous, functions will not be LC-continuous in general. Our last result shows that sub-LC-continuous functions behave much better in this respect. Its proof is an immediate consequence of Proposition 5 and thus is omitted.

PROPOSITION 12. (i) For each $i \in I$ let $f_i: X_i \to Y_i$ be sub-LC-continuous. If $f = \Pi f_i$ then $f: \Pi X_i \to \Pi Y_i$ is sub-LC-continuous. (ii) For each $i \in I$ let $f_i: X \to Y_i$ be sub-LC-continuous. If $f = \Delta f_i: X \to \Pi Y_i$, i.e. $(f(x))_i = f_i(x)$ for each $i \in I$, then f is sub-LC-continuous.

5. EXAMPLES.

In this section we provide some examples in order to illustrate the various notions of generalized continuity which were introduced and discussed in the previous sections. We point out that in most cases we consider real-valued functions on the real line so that very natural spaces are involved in producing our counter examples.

EXAMPLE 1. Define a function $f: R \to R$ by setting $f(x) = x$ if $x < 0$ and $f(x) = 1$ if $x > 0$. For any subset $V \subseteq R$ we have $f^{-1}(V) = V \cap (-\infty,0)$ if $1 \notin V$ and $f^{-1}(V) = V \cup (0,\infty)$ if $1 \in V$. One easily checks that f is LC-irresolute. Obviously f is not continuous.

EXAMPLE 2. Let $A = \{1/n \mid n \in N\} \subseteq R$. By Corollary 2, A is locally closed in R and R-A is not. Define $f: R \to R$ by setting $f(x) = x$ if $x \in A$ and $f(x) = 0$ if $x \in R-A$. Since $f^{-1}(\{0\}) = R-A$, f fails to be LC-irresolute. If $V \subseteq R$ is open and $0 \notin V$ then $f^{-1}(V) \subset A$ and is locally closed by Proposition 2. If $0 \in V$ then $f^{-1}(V)$ is a cofinite, hence an open subset of R. This shows that f is LC-continuous.

EXAMPLE 3. Define $f: R \to R$ by setting $f(x) = x$ if $x \neq 0$ and $f(0) = 1$. For any subset $V \subset R$ we have $f^{-1}(V) = V - \{0\}$ if $1 \notin V$ and $f^{-1}(V) = V \cup \{0\}$ if $1 \in V$. Hence, if V is an open interval then $f^{-1}(V)$ is locally closed. Thus f is sub-LC-continuous. Now let $V = R - (\{0\} \cup \{1/n \mid n \in N,\ n > 2\})$. Then V is open and dense. Since $1 \in V$ we have $(\mathrm{cl}\, f^{-1}(V)) - f^{-1}(V) = \{x \in R \mid x \neq 1/n$ for each $n > 2\}$ which is not closed, so $f^{-1}(V)$ is not locally closed by Proposition 1. Hence f is not LC-continuous.

EXAMPLE 4. Let Y be the Sorgenfrey line and $f: R \to Y$ be the identity function. Clearly f is sub-LC-continuous. If $B = \{-1/n \mid n \in N\}$ then Y-B is open in Y but not locally closed in R. Thus f is not LC-continuous.

EXAMPLE 5. There is a bijective, open and nearly continuous function $f: R \to Y$, Y a metrizable space, such that f is LC-continuous at no point.

Let D_1 be the set of all rationals in R and let $D_2 = R - D_1$. If $Y = D_1 \oplus D_2$ then the identity function $f: R \to Y$, $f(x) = x$ for each $x \in R$, is the desired function. This example is due to Berner [16].

EXAMPLE 6. There is a Hausdorff space (X,T) and a bijective LC-irresolute function $f: X \to Y$, Y a discrete space, such that f is continuous at no point.

We use the so-called Bourbaki construction. Let X be the set of reals and T_e be the euclidean topology on X. Let α be a maximal filter consisting of dense subsets of (X,T_e) and let T be the topology on X having $T_e \cup \alpha$ as a subbase. It is well known that (X,T) is Hausdorff submaximal and dense-in-itself. If Y is the set of reals carrying the discrete topology then the identity function f: X → Y is LC-irresolute since (X,T) is submaximal. However, f is continuous at no point since (X,T) is dense-in-itself.

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