MultiCraft

International Journal of Engineering, Science and Technology
Vol. 3, No. 2, 2011, pp. 157-162

INTERNATIONAL JOURNAL OF ENGINEERING, SCIENCE AND TECHNOLOGY
www.ijest-ng.com

Location of collinear equilibrium points in the generalised photogravitational elliptic restricted three body problem

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Abstract

We have discussed the location of collinear equilibrium points in the generalised photogravitational restricted three body problem. The problem is generalised in the sense that both primaries are oblate spheroid. They are source of radiation as well. We have found the solution for the location of collinear point L_1. We found that location of collinear point L_1 is affected by eccentricity, oblateness and radiation factor terms. The same method may be applied for location of collinear points $L_2$and L_3.

Keywords: Collinear points, generalised photogravitational, ERTBP.

1. Introduction

Radzievskii (1950) formulated the photogravitational restricted three body problem. This arises from the classical problem when one of the masses is an intense emitter of radiation. Arnold (1961) studied the stability of positions of equilibrium of a Hamiltanian system in the general elliptic case. Chaudhary (1966) studied the periodic orbits of the third kind and stability of the generating solution in the elliptical restricted three body problem. Bhatanagar (1969) examined periodic orbits of collision in the elliptic restricted three body problem. Sharma and Subbarao (1975) studied the restricted three body problem when the primaries are oblate spheroids. Sharma (1982) investigated the linear stability of triangular libration points when the more massive primary is a source of radiation and oblate spheroid as well. He also examined the linear stability of libration points of the photogravitational restricted three body problem when the smaller primary is an oblate spheroid. Beauge (1996) gave a note on a global expansion of the disturbing function in the planar elliptic restricted three body problem. Khasan (1996) studied librational solutions to the photogravitational restricted three body problem by considering both primaries as radiating. He also examined the stability of collinear and triangular points. Khasan (1996a) also studied three dimensional periodic solutions to the photogravitational Hill problem. He investigated restricted photogravitational elliptic three body problem. Sahoo and Ishwar (2000) studied the stability of collinear equilibrium points in the generalised photogravitational elliptic three body problem. Kumar and Ishwar (2004) studied the equations of motion in the generalised photogravitational elliptic restricted three body problem. Kumar and Ishwar (2009) investigated the solutions of generalised photogravitational elliptic restricted three body problem.

Hence, we thought to establish location of collinear equilibrium points in the generalised photogravitational elliptic restricted three body problem. In restricted three body problem, when the primaries move on ellipse is called ERTB (elliptic restricted three body problem). The problem is generalised in the sense that both primaries are considered as oblate spheroid. They are source of radiation as well.

2. Location of Collinear Points

The equilibrium points are the solutions of the equations:

This paper is presented in 5[th] National Conference on Applicable Mathematics in Wave Mechanics and Vibrations (WMVC-2010) held at Kakatiya University, Warangal, India,13-15 March,2010.

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial \zeta} = 0$$

where

$$f = \frac{1}{(1-e^2)^{1/2}}\left[\frac{\xi^2+\eta^2}{2} + \frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)q_1A_1}{2r_1^3} + \frac{\mu q_2 A_2}{2r_2^3}\right\}\right] \tag{1}$$

is the force function in the equation of motion of our problem.

and

$$r_1^2 = (\xi-\xi_1)^2 + \eta^2 + \zeta^2$$

$$r_2^2 = (\xi-\xi_2)^2 + \eta^2 + \zeta^2$$

μ = mass parameter

q_1 = radiation parameter of m_1

q_2 = radiation parameter of m_2

A_1 = oblateness parameter of m_1

A_2 = oblateness parameter of m_2

n = average angular velocity

e = eccentricity of the orbit.

The equation $\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial \zeta}$ =0 gives

$$\xi - \frac{1}{n^2}\left\{\frac{(1-\mu)q_1(\xi-\xi_1)}{r_1^3} + \frac{\mu q_2(\xi-\xi_2)}{r_2^3} + \frac{3(1-\mu)q_1A_1(\xi-\xi_1)}{2r_1^5} + \frac{3\mu q_2A_2(\xi-\xi_2)}{2r_2^5}\right\} = 0 \tag{2}$$

$$\eta\left\{1 - \frac{1}{n^2}\left(\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)q_1A_1}{2r_1^5} + \frac{3\mu q_2A_2}{2r_2^5}\right)\right\} = 0 \tag{3}$$

$$-\frac{\zeta}{n^2}\left(\frac{(1-\mu)q_1}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)q_1A_1}{2r_2^3} + \frac{3\mu q_2A_2}{2r_2^2}\right) = 0 \tag{4}$$

For collinear equilibrium points lying on the line joining the primaries i.e. on ξ -axis, we have from equation (2)

$$\xi - \frac{1}{n^2}\left(\frac{(1-\mu)q_1(\xi-\xi_1)}{\left[(\xi-\xi_1)^2\right]^{3/2}} + \frac{\mu q_2(\xi-\xi_2)}{\left[(\xi-\xi_2)^2\right]^{3/2}} + \frac{3(1-\mu)q_1A_1(\xi-\xi_1)}{2\left[(\xi-\xi_1)^2\right]^{5/2}} + \frac{3\mu q_2A_2(\xi-\xi_2)}{2\left[(\xi-\xi_2)^2\right]^{5/2}}\right) = 0$$

or,

$$\begin{aligned} &2(\xi-\xi_1)^2(\xi-\xi_2)^2\left\{n^2\xi(\xi-\xi_1)^2(\xi-\xi_2)^2 - (1-\mu)q_1(\xi-\xi_2)^2 - \mu q_2(\xi-\xi_1)^2\right\} \\ &-3(1-\mu)q_1A_1(\xi-\xi_2)^4 - 3\mu q_2A_2(\xi-\xi_1)^4 = 0 \end{aligned} \tag{5}$$

This is ninth degree equation, so we shall get nine roots of ξ . Three equilibrium points lie on the X-axis. One root of ξ is greater than ξ_2 , other root lies between ξ_1 and ξ_2 and the third root is less than ξ_1 :

Case I

.

In this case $\xi > \xi_2$ and we consider

$$\xi - \xi_2 = \rho$$

Therefore

$$\xi - \xi_1 = 1 + \rho$$

Hence

$$\xi_2 - \xi_1 = 1$$

Since c is the centre of mass

$$(1-\mu)\,\xi_1 + \mu\,\xi_2 = 0$$

which gives

$$\xi_1 = -\mu$$

and

$$\xi = 1 + \rho + \xi_1 = 1 + \rho - \mu$$

Substituting these values in the equation (5), we get

$$2n^2\rho^9 + 2n^2(5-\mu)\rho^8 + 2n^2(10-4\mu)\rho^7 + \left[2n^2(10-6\mu) - 2q_1(1-\mu) - 2\mu q_2\right]\rho^6 +$$
$$\left[2n^2(5-4\mu) - 4q_1(1-\mu) - 8\mu q_2\right]\rho^5 + \left[2n^2(1-\mu) - 2q_1(1-\mu) - 12\mu q_2 - 3(1-\mu)q_1A_1 - 3\mu q_2A_2\right]\rho^4$$
$$-\left[8\mu q_2 + 12\mu q_2A_2\right]\rho^3 - \left[2\mu q_2 + 18\mu q_2A_2\right]\rho^2 - 12\mu q_2A_2\rho - 3\mu q_2A_2 = 0 \tag{6}$$

For verification we put $n=1$, $q_1=1$, $q_2=1$, $A_1=0$, $A_2=0$ we get the same result as in classical case. Let γ_1 be the solution of the classical case i.e. when $e=0$, $A_1=A_2=0$, $q_1=q_2=1$, and due to the presence of these terms the location will be slightly changed

Let the new value of ρ be defined by $\rho = \gamma_1 + \delta$, where δ is very-very small and let $q_1 = 1-\beta_1$, $q_2 = 1-\beta_2$ where β_1, β_2 < < 1. Substituting the value of ρ in the equation (6), we get

$$\rho = \gamma_1 + \delta = \gamma_1 + \left[U_1X_1 + (U_1Y_1 + V_1X_1)e^2 + (U_1Y_1 + V_1X_1)A_1 + (U_1Y_1 + V_1X_1)A_2\right] +$$
$$(X_1 + Y_2e^2 + y_1A_1 + Y_1A_2)\left[-2(1-\mu)\{(\gamma_1^6 + 2\gamma_1^5 + \gamma_1^4) + (6\gamma_1^5 + 10\gamma_1^4 + 4\gamma_1^3)U_1X_1\}\right]\beta_1 +$$
$$(6\gamma_1^5 + 20\gamma_1^4 + 24\gamma_1^3 + 12\gamma_1^2 + 2\gamma_1)U_1X_1\}]\beta_2 +$$
$$(X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)[3(1-\mu)\{(\gamma_1^4 + 4\gamma_1^3)U_1X_1\}]A_1 +$$
$$(X_1 + Y_1e^2 + Y_1A_1 + Y_1A_2)[3\mu\{\gamma_1^4 + 4\gamma_1^3 + 6\gamma_1^2 + 4\gamma_1 + 1)$$
$$+ (4\gamma_1^3 + 12\gamma_1^2 + 12\gamma_1 + 4)U_1X_1\}]A_2 \tag{7}$$

where γ_1 is the value of ρ, the distance between L_1 and smaller primary in the classical case. We have found the value of U_1, V_1, X_1, Y_1 as in Sahoo and Ishwar (2000),

$$U_1 = \frac{1}{a}\left[-2\gamma_1^9 - 2(5-\mu)\gamma_1^8 - 2(10-4\mu)\gamma_1^7 - 2(10-6\mu)\gamma_1^6 - 2(5-4\mu)\gamma_1^5 - 2(1-\mu)\gamma_1^4\right]$$
$$+ 2\gamma_1^6 + 4(1+\mu)\gamma_1^5 + 2(1+5\mu)\gamma_1^4 + 8\mu\gamma_1^3 + 2\mu\gamma_1^2$$

$$V_1 = \frac{-3}{a}\left[\gamma_1^9 + (5-\mu)\gamma_1^8 + (10-4\mu)\gamma_1^7 + (10-6\mu)\gamma_1^6 + (5-\mu)\gamma_1^5 + (1-\mu)\gamma_1^4\right]$$

$$X_1 = \left[\frac{1}{a}\left\{18\gamma_1^8 + 16(5-\mu)\gamma_1^7 + 28(5-2\mu)\gamma_1^6 + 24(5-3\mu)\gamma_1^5 + 10(5-4\mu)\gamma_1^4 + 8(1-\mu)\gamma_1^3\right\}\right.$$
$$\left. - \left\{12\gamma_1^5 + 20(1+\mu)\gamma_1^4 + 8(1+5\mu)\gamma_1^3 + 24\mu\gamma_1^2 + 4\mu\gamma_1\right\}\right]^{-1}$$

$$Y_1 = \frac{\left[-\frac{1}{a}\left\{27\gamma_1^8 + 24(5-\mu)\gamma_1^7 + 42(5-2\mu)\gamma_1^6 + 36(5-3\mu)\gamma_1^5 + 15(5-4\mu)\gamma_1^4 + 12(1-\mu)\gamma_1^3\right\}\right]}{\left[\frac{1}{a}\left\{18\gamma_1^8 + 16(5-\mu)\gamma_1^7 + 28(5-2\mu)\gamma_1^6 + 24(5-3\mu)\gamma_1^5 + 10(5-4\mu)\gamma_1^4 + 8(1-\mu)\gamma_1^3\right\} - \left\{12\gamma_1^5 + 20(1+\mu)\gamma_1^4 + 8(1+5\mu)\gamma_1^3 + 24\mu\gamma_1^2 + 4\mu\gamma_1\right\}\right]^2}$$

a =semi-major axis of ellipse.

Hence, we located collinear point L_1.

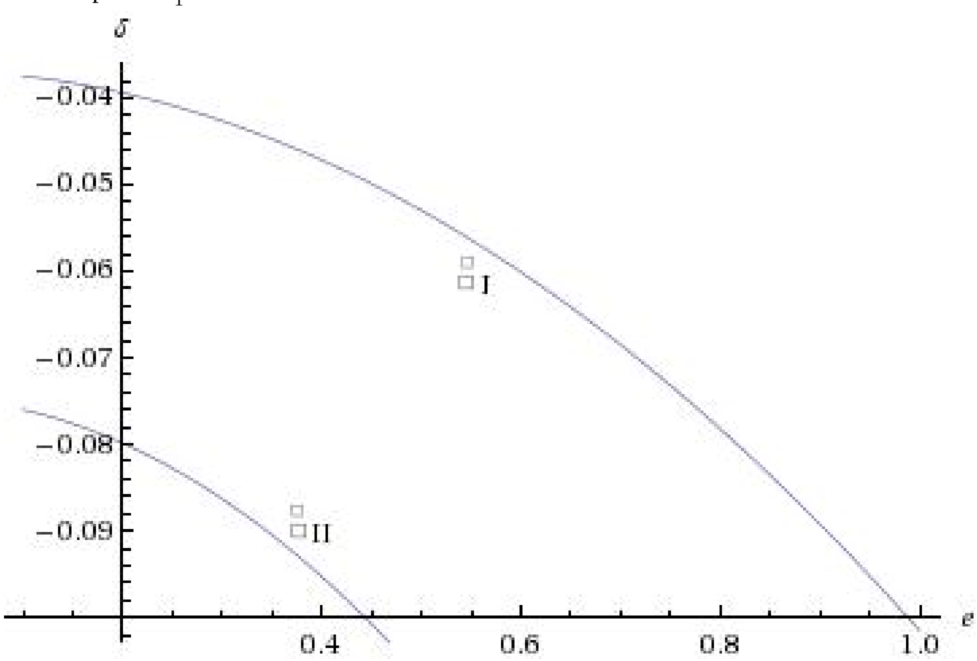
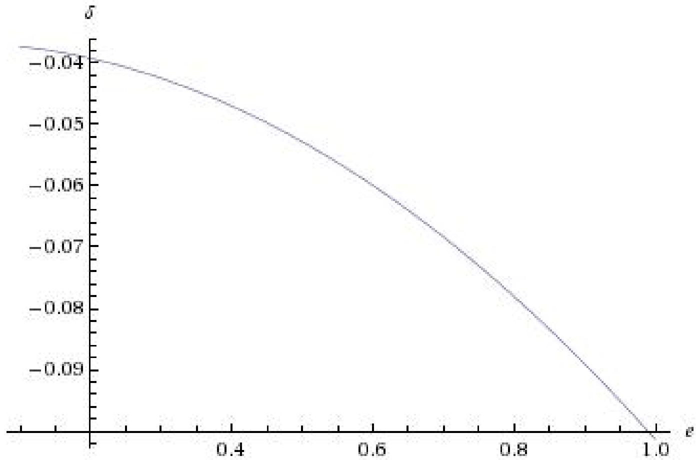
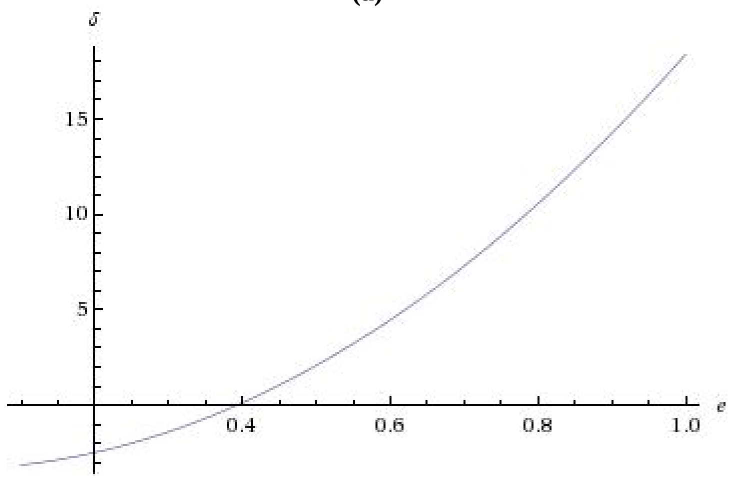


Fig.-1: Effect of A_1, A_2, q_1 *and* q_2 on the position of L_1



(a)



(b)

Fig.-2:Effect of mass parameter μ on the position of L_1

For numerical calculation we have used a=.0001, $\beta_1 = \beta_2 = 0.0001$, and plotted the above graphs for deviation δ Vs eccentricity . In figure 1 we consider $A_1 = 0.0025$ and $\mu = 0.00025$, where curve I represents the effect of oblateness coefficients $A_2 = 0.0025$ and curve II for $A_2 = 0.0050$. The effect of mass parameter μ is depicted in figure 2 with (a) : $\mu = 0.00025$ and (b): $\mu = 0.0025$. One can see that the deviation δ in (a) is decreasing while in (b) this δ is increasing with eccentricity e . We have also computed the effect of β_1, β_2 on the position of L_1 but this effect is very insignificant, and the graphs are similar to the above figures even if the values of A_1, A_2 are interchanged.

3. Conclusion

Thus, we conclude that collinear point L_1 is affected by oblateness, eccentricity and radiation factor terms. Also when these terms are neglected, we get the same terms as in the classical elliptical restricted three body problem. The same method may be applied for location of collinear points L_2 and L_3. Numerically we have obtained that the position of L_1 is deviated from classical case for various values of parameters.

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Received September 2010
Accepted June 2011
Final acceptance in revised form June 2011