J. Mod. Power Syst. Clean Energy (2019) 7(6):1709–1720
https://doi.org/10.1007/s40565-019-0532-7

Locational marginal price share: a new structural market power index

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Abstract Market power is known as the ability of units and generation companies (GenCos) to change electricity price profitably. As cleared in the definition, locational marginal price (LMP) is the most important key in market power evaluation. Therefore, the main objective of the paper is to analyze market power of units and GenCos based on their abilities to change electricity price. At the first step, using Karush-Kuhn-Tucker (KKT) conditions of Lagrangian method, LMP is decomposed into four main components. These components indicate the share of each unit at the LMP of each bus. These values are calculated by the proposed analytical method, and cannot be obtained using simulation methods. At the second step, "unit-based LMP_S" index, which indicates the contribution factor of each unit at LMP of each bus, is proposed as a new structural market power index. This index is also used as an effective tool to determine the most profitable coalition between two units. Using that, the market operator can predict highly potential collusions. Moreover, "GenCos-based LMP_S" index is proposed. Using this effective tool, the contribution of each GenCo, which owns multiple units in various buses, at the LMP of each bus is discovered. The proposed market power indices are calculated on the IEEE 24-bus test system and compared with some conventional structural market power indices. Incremental profits of units due to change of unit's strategies verify the accuracy of proposed method.

Keywords Lagrange relaxation method, Locational marginal price (LMP) share, Market power, Coalition

CrossCheck date: 8 March 2019

Received: 11 October 2018 / Accepted: 8 March 2019 / Published online: 7 June 2019

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1 Introduction

Market power is the ability of a generation company (GenCo) to increase the electricity price profitably [1, 2]. Therefore, the main issue in market power studies is analyzing the electricity price to determine the effective factors to change it. The price sensitivity to the effective factors can be detected by simulation-based or analytical methods. Simulation-based methods model the problem with more details. On the other hands, the analytical methods, despite considering simplifying assumptions, reveal structural relationships of market factors. These structural analyses make an excellent understanding of the market. The main motivation of the paper is to use analytical methods to analyze market power of units and GenCos based on their ability to change electricity price.

A comprehensive overview of market power definitions and indices is investigated in [3]. Moreover, in [4], some research works about market power exercising and detection techniques are reviewed. The regulatory should monitor the electricity market to evaluate the ability of market players for exercising market power [5]. For example, the level of competitiveness in the Dutch electricity market is studied over 2006–2011 in [6]. The regulatory evaluates the market power using some indices. The market power indices may be quantity-based [7] such as Herfindahl-

Hirschman index (HHI), pivotal supplier index (PSI) and residual supply index (RSI), and price-based such as output gap and Lerner index (LI) [8]. GenCos owning a great share of the market or locating at the strategic bus of the network are able to exercise market power through strategic behavior [9]. This action can be applied in two ways: capacity withholding (i.e., decreasing the power lower than the available capacity) and financial withholding (i.e., raising the bid price higher than the marginal cost). In [10], capacity withholding in a day-ahead market is analyzed using bi-level optimization. Reference [11] proposes several indices to evaluate capacity withholding in transmission-constrained electricity markets. In this reference, three electricity market types including oligopoly, perfectly competitive, and perfectly competitive with the nodal prices are considered and analyzed. In [12], the effect of forward contract and demand elasticity on the market power is investigated. Reference [13] proposes a new method to evaluate capacity withholding based on the supply function equilibrium and Cournot model. In [14], some new indices are introduced to calculate actual market power exercised by GenCos in the electricity market considering transmission constraints. In this reference, both physical withholding and financial withholding are analyzed.

From another perspective, market power analyses are categorized into three types: behavioral analyses, structural analyses and competition models. Behavioral analyses use actual information of the market after market clearing process. The LI is the most popular behavioral market power index [15, 16]. The authors of [17] propose a virtual electricity market model to assess the market power. They investigate the behaviour of GenCos from the viewpoint of the regulatory body, and assume that the electricity market includes renewable energy resources. Simulation models are commonly based on the game theory approaches [18]. The competition models simulate the interaction of competing generation companies to identify how market power might be formed in power markets.

Structural analyses discover the potential of market power. The most important structural market power indices are market share, HHI [19], PSI [20] and RSI [21]. The effect of demand shifting to mitigate market power is theoretically discussed in [9] using a bi-level optimization model. In [22], transmission-constrained network flow (TCNF) index is proposed as a functional market power measure. TCNF integrates three market power indices including residual-supply-based, network-flow-based and minimal-generation-based indices. In [23], a conjectured supply function equilibrium approach is applied to consider transmission constraints in market power analyses. The authors in [24] propose a structural market power evaluation based on locational marginal price (LMP) decomposition. They calculate the weighted coefficients of the generation units to discover the structural market power, thus providing a good analytical groundwork for market power evaluation based on the LMP share (LMP_S). However, [24] does not provide a market power index to quantify the value of market power of units based on the contribution on the LMP.

Form the literature review, it can be concluded that:

1) Although many literatures have considered electricity price at their market power analysis, there is no market power index which is directly defined based on the LMP.
2) Some market power indices ignore the transmission constraints which lead to unreliable results in evaluating the market power.
3) Most of the previous works that have applied simulation-based methods need high computational effort to detect market power. Therefore, they are not suitable to utilize as a subroutine in repetitive and time-consuming applications or for any large networks.

Therefore, it is necessary to propose new analytical LMP-based market power indices in transmission-constrained electricity market.

The main goal of this paper is to propose new structural market power indices for units and GenCos, based on their LMP_S at the market equilibrium point. This goal is achieved in two steps.

At the first step, by developing the previous model in [24], new extended LMP decomposition is proposed at Lemma 1. In our approach, LMP at bus n (LMP_n) is decomposed into four main components. These components indicate the LMP_S of all units at each bus and are the basis for definition of the new market power indices.

At the second step we propose:

1) "Unit-based LMP_S" indices which are categorized into self and cross LMP_S, where ① Self LMP_S indicates the contribution factor of each unit at the LMP of its located bus. Therefore, this is introduced as a powerful unit's market power index. ② Cross LMP_S determines the contribution factor of each unit at LMP of other units. Using this, new tool for evaluating coalition profitability between units are proposed.
2) "GenCos-based LMP_S" index which is defined as the total contribution of all GenCo's units at electricity price of GenCo's located buses.

We can say that one of the most important contributions of this paper is to analytically reveal the market power of units and GenCos based on the market power definition, i.e. the ability to change electricity price profitably.

Finally, the proposed market power indices are calculated on the IEEE 24-bus test system. The simulation results are compared with other structural market power indices. The incremental profits of units due to change of the unit's strategies verifies the results obtained by proposed indices.

The rest of this paper is organized as follows. Market structure is presented in Section 2. Section 3 includes the developed LMP decomposition. New structural market power indices are proposed in Section 4. The simulation results on the IEEE 24-bus test system are presented in Section 5. Finally, the paper is summarized and concluded in Section 6.

2 Market structure

This paper focuses on the closed auction with nondiscriminatory pricing rules, which is the most commonly accepted structure for the spot electricity markets around the world. The auction problem under DC power flow constraints can be expressed as [24–29]:

$$\begin{cases} \min\limits_{P_i} \sum\limits_{i=1}^{N} \left(a_i P_i + \dfrac{b_i}{2} P_i^2 \right) \\ \text{s.t. } \sum\limits_{i=1}^{N} P_i \\ \qquad \underline{\alpha}_l \le \sum\limits_{i=1}^{N} \gamma_{l,i} P_i \le \bar{\alpha}_l \qquad l = 1, 2, \ldots, L \\ \qquad P_{i,\min} \le P_i \le P_{i,\max} \qquad i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where a_i and b_i are intercept and slope of the bid function of unit i; N and L are the number of units and transmission lines; P_d is the total load; P_i, $P_{i,\min}$ and $P_{i,\max}$ are the generations of unit i, lower and upper generation limits, respectively; $\gamma_{l,i}$ is the line flow distribution factor of line l due to the generation of unit i; $\underline{\alpha}_l$ and $\bar{\alpha}_l$ are the lower and upper limits derived from the maximum line flow l. The transmission lines constraints are expressed as restrictions on the weighted sum of the production of units [24]. The LMPs and the dispatch of the generators are calculated by solving the market problem (1). The Lagrangian method is employed to solve the optimization problem (1) [24, 30].

3 New LMP decomposition

According to the definition of market power, there is a direct relationship between the market power and LMPs. Therefore, this paper developed the previous LMP decomposition in [24] to analyze the ability of all GenCos to exercise market power. This decomposition is a powerful analytical tool for defining new structural market power indices based on LMP_S of units. LMPs and dispatch of the generators are resulted from solving the auction problem (1) at the specified load level of the network, which is indicated as "market equilibrium point". Therefore, the fully dispatched units, marginal units and units with minimum generation are determined at the market equilibrium point. These units are numbered from high-cost to low-cost in Lemma 1. Therefore, at the market equilibrium point, the first $K_{\min}$ units, which are the high-cost units, are limited to their minimum generations, and the last $K_{\max}$ units reach to their maximum generations. Here $K_{\min}$ and $K_{\max}$ are the numbers of low and high cost units, respectively.

Against the previous work [24] which decomposed LMP into three segments, Lemma 1 expresses the new decomposition of LMP_n into four main components to achieve all units' contribution. It is deduced from solving the Lagrange equation under the Karush-Kuhn-Tucker (KKT) conditions at the market equilibrium point [24].

Lemma 1 *For the specified network topology and fixed vector load* $\boldsymbol{P}$ *and based on the DC load flow, the* LMP_n *is obtained as follows*:

$$\begin{aligned} LMP_n = A_{0,n} + \sum_{i=K_{\min}+1}^{N-K_{\max}} A_{i,n} a_i \\ + \sum_{i=N-K_{\max}+1}^{N} A'_{i,n} P_{i,\max} + \sum_{i=1}^{K_{\min}} A''_{i,n} P_{i,\min} \end{aligned} \tag{2}$$

where $A_{0,n}$, $A_{i,n}$, $A'_{i,n}$ and $A''_{i,n}$ *are the constant coefficient and the coefficients of* a_i, $P_{i,\max}$ and $P_{i,\min}$, *respectively*.

Proof The Lemma 1 is proved in two steps. At the first step, the KKT conditions at the market equilibrium point are analyzed. At the second step, by using the results of the KKT conditions, the Lemma 1 is proved.

1) *Step 1*: KKT conditions.

Same as the proof of Lemma 1 in [24], LMP_n is obtained. Moreover, a linear relationship between the vectors $\boldsymbol{\Gamma}_{\max}$, $\boldsymbol{a}$, $\boldsymbol{P}_{\min}$ and $\boldsymbol{P}_{\max}$ in (3) is obtained from the KKT conditions:

$$\begin{aligned} \boldsymbol{\Gamma}_{\max} = \boldsymbol{\beta}^{-1} \times \boldsymbol{C} - \boldsymbol{\beta}^{-1} \times \boldsymbol{\alpha} \times \boldsymbol{a} - \boldsymbol{\beta}^{-1} \times \boldsymbol{D} \times \boldsymbol{P}_{\min} \\ - \boldsymbol{\beta}^{-1} \times \boldsymbol{E} \times \boldsymbol{P}_{\max} \end{aligned} \tag{3}$$

where $\boldsymbol{\Gamma}_{\max}$ is the Lagrangian multiplier vector of the congested lines flow constraints; $\boldsymbol{P}_{\min}$ is the vector of minimum generation of high cost units; $\boldsymbol{P}_{\max}$ is the vector of maximum generation of low cost units; $\boldsymbol{a}$ is the vector of the intercept of the bid functions; $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{E}$ are the middle vector variables; $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{D}$ are defined in [24] and the elements of $\boldsymbol{C}$ and $\boldsymbol{E}$ are defined in (4).

$$C(l) = \bar{\alpha}_l - \sum_{j=K_{\min}+1}^{N-K_{\max}} \gamma_{l,j} \frac{P_d}{C_1 b_j}$$
$$E(l,j) = \gamma_{l,j} - \frac{\sum_{i=K_{\min}+1}^{N-K_{\max}} \frac{\gamma_{l,i}}{b_i}}{C_1} \tag{4}$$

2) *Step 2*: LMP decomposition.

Same as the proof of Lemma 1 in [24], the LMP_n is obtained in the vector form in (5).

$$LMP_n = C_2 + \boldsymbol{C}_3 \times \boldsymbol{P}_{\min} + \boldsymbol{C}_4 \times \boldsymbol{P}_{\max} + \boldsymbol{A} \times \boldsymbol{a} + \boldsymbol{B}_n \times \boldsymbol{\Gamma}_{\max} \tag{5}$$

where

$$\begin{cases} C_2 = \dfrac{P_d}{C_1} \\ \boldsymbol{C}_3 = \dfrac{-\ \text{ones}(1, K_{\min})}{C_1} \\ \boldsymbol{C}_4 = \dfrac{-\ \text{ones}(1, K_{\max})}{C_1} \end{cases} \tag{6}$$

On the other hand, based on (3), $\boldsymbol{\Gamma}_{\max}$ has a linear relationship with $\boldsymbol{a}$, $\boldsymbol{P}_{\min}$ and $\boldsymbol{P}_{\max}$. Therefore, LMP_n can be recalculated by substituting (3) into (5).

$$\begin{aligned} LMP_n &= A_{0,n} + \boldsymbol{A}_n \times \boldsymbol{a} + \boldsymbol{A}_n^{'} \times \boldsymbol{P}_{\max} + \boldsymbol{A}_n^{''} \times \boldsymbol{P}_{\min} \\ &= A_{0,n} + \sum_{i=K_{\min}+1}^{N-K_{\max}} A_{i,n} a_i \\ &\quad + \sum_{i=N-K_{\max}+1}^{N} A_{i,n}^{'} P_{i,\max} + \sum_{i=1}^{K_{\min}} A_{i,n}^{''} P_{i,\min} \end{aligned} \tag{7}$$

where

$$\begin{cases} A_{0,n} = C_2 + \boldsymbol{B}_n \times \boldsymbol{\beta}^{-1} \times \boldsymbol{C} \\ \boldsymbol{A}_n = \boldsymbol{A} - \boldsymbol{B}_n \times \boldsymbol{\beta}^{-1} \times \boldsymbol{\alpha} \\ \boldsymbol{A}_n^{'} = \boldsymbol{C}_4 - \boldsymbol{B}_n \times \boldsymbol{\beta}^{-1} \times \boldsymbol{E} \\ \boldsymbol{A}_n^{''} = \boldsymbol{C}_3 - \boldsymbol{B}_n \times \boldsymbol{\beta}^{-1} \times \boldsymbol{D} \end{cases} \tag{8}$$

Thus, the Lemma 1 is proved.

4 LMP_S: new structural market power index

According to (2) in Lemma 1, the electricity price can be decomposed into its constitutive components. Also, the influence of different units on the electricity price at each bus can be evaluated. The values $A_{i,n}a_i$, $A_{i,n}^{'}P_{i,\min}$ and $A_{i,n}^{''}P_{i,\min}$ in (2) represent the contribution of marginal, high cost and low cost units in the electricity price at bus n (LMP_n), respectively. Therefore, these values can be employed for structural market power evaluation at different buses. Accordingly, this paper proposes new structural market power indices, by using the concept of LMP_S of each unit at each bus. These market power indices are categorized into "unit-based LMP_S" and "GenCos-based LMP_S" indices.

4.1 Unit-based LMP_S indices

According to the new LMP decomposition in Lemma 1, $LMP\_S_{i,n}$ can be proposed as a new structural market power index as follow:

$$LMP\_S_{i,n} = \begin{cases} \dfrac{\left|A_{i,n}a_i\right|}{LMP_n} & i \text{ belongs to marginal units} \\ \dfrac{\left|A_{i,n}^{'}P_{i,\min}\right|}{LMP_n} & i \text{ belongs to high-cost units} \\ \dfrac{\left|A_{i,n}^{''}P_{i,\max}\right|}{LMP_n} & i \text{ belongs to low-cost units} \end{cases} \tag{9}$$

where $LMP\_S_{i,n}$ indicates the contribution factor of unit i at the electricity price of bus n. Since the coefficients $A_{i,n}$, $A'_{i,n}$ and $A''_{i,n}$ can be positive or negative values, $LMP\_S_{i,n}$ is defined in (9) by the absolute function. If, supposedly, unit i is at bus k, without considering the collusion, LMP_k is certainly the most important price for unit i. In other words, unit i benefits from the increase of the LMP_k. Therefore, LMP_S_i is defined as (10), if unit i is located at bus k:

$$LMP\_S_i = LMP\_S_{i,k} \tag{10}$$

Using this definition, LMP_S_i indicates the contribution factor of unit i at its located bus. Therefore, the high value of LMP_S_i means the high market power value of unit i.

4.2 Evaluation of multi-agent coalition formation

According to the location of unit j, LMP_j indicates the LMP where unit j is located. Therefore, using (9), $LMP\_S_{i,j}$ indicates the contribution factor of unit i at the electricity price of unit j.

This index can evaluate the coalition profitability between two units. To this, for each unit i we evaluate $LMP\_S_{i,j}$ from two viewpoints:

1) Determining unit j which has the maximum value of $LMP\_S_{i,g}$ ($g = 1,\ 2,\ \ldots,\ N$). This means that unit i applies its maximum effect on LMP of unit j. Hence, units i can form a coalition by unit j. In this situation unit i changes its bid to increase the LMP of unit j.
2) Determining unit k which has the maximum value of $LMP\_S_{g,i}$ ($g = 1,\ 2,\ \ldots,\ N$). This means that LMP_i is mostly affected by unit k. So, units i can form a

coalition by unit k. In this situation unit k changes its bid to increase the LMP of unit i.

Therefore, the most profitable coalition from viewpoint unit i occurs if $j = k$. It must be noted that $LMP\_S_{i,j}$ is not necessarily equal to $LMP\_S_{j,i}$. So, if unit j is the best choice to form a coalition with unit i, this does not mean that unit i is also the best choice to form a coalition with unit j. Certainly, if unit j be the best choice to form a coalition with unit i and vice versa, these units have a good situation to form coalition.

By sorting the values of $LMP\_S_{i,g}$ ($g = 1, 2, \ldots, N$) and $LMP\_S_{g,i}$ ($g = 1, 2, \ldots, N$) from largest to smallest, the best units' candidates to form coalition by unit i are determined.

4.3 GenCos-based LMP_S indices

A GenCo is a company that may own multiple units at different buses in a network. As $LMP\_S_{i,n}$ ($i = 1, 2, \ldots, K$; $n = 1, 2, \ldots, N$) demonstrates the contribution factor of each unit at LMP of each bus, it can be used to define a powerful GenCos-based structural market power index. Note that one of the most important contributions of this paper is to reveal and use this structural relationship in the GenCos market power index definition. Suppose U_j represents the set of all units owned by GenCo j (GC_j). The contribution factor of GC_j at the electricity price of bus n is defined as (11):

$$LMP\_S_{GC_j,n} = \sum_{i \in U_j} LMP\_S_{i,n} \quad j = 1, 2, \cdots, J \tag{11}$$

Since $LMP\_S_{GC_j,n}$ is formed from the total share of all units owned by GC_j at LMP_n, the high value of $LMP\_S_{GC_j,n}$ indicates the high market power value of GenCo j to determine LMP_n. If N_j is the set of all buses where U_j is located, without considering the collusion, LMP_n ($n \in N_j$) certainly are the most important prices for GC_j. In other words, GenCo j benefits from the increase of LMP_n ($n \in N_j$). Therefore, $LMP\_S_{GC_j}$ is defined as:

$$LMP\_S_{GC_j} = \sum_{n \in N_j} LMP\_S_{GC_j,n} = \sum_{n \in N_j} \sum_{i \in U_j} LMP\_S_{i,n} \tag{12}$$

This structural LMP_S-based index indicates the total market power of GC_j in the power market.

The general framework of this research is presented in Fig. 1. This figure illustrates the process of calculating the proposed indices.

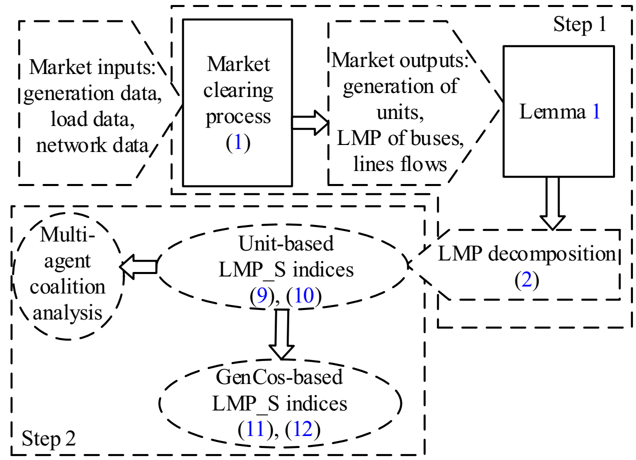

Fig. 1 Framework of research

5 Case study

In this paper, the IEEE 24-bus modified reliability test system (MRTS) with 32-generation units owned by 9 GenCos and 17 load points is used to evaluate the proposed structural market power indices. The proposed method is simulated using MATLAB software. The topology of the test system, the location of the GenCos and the load point are shown in Fig. 2. Details of the transmission system can be found in [24, 31–33].

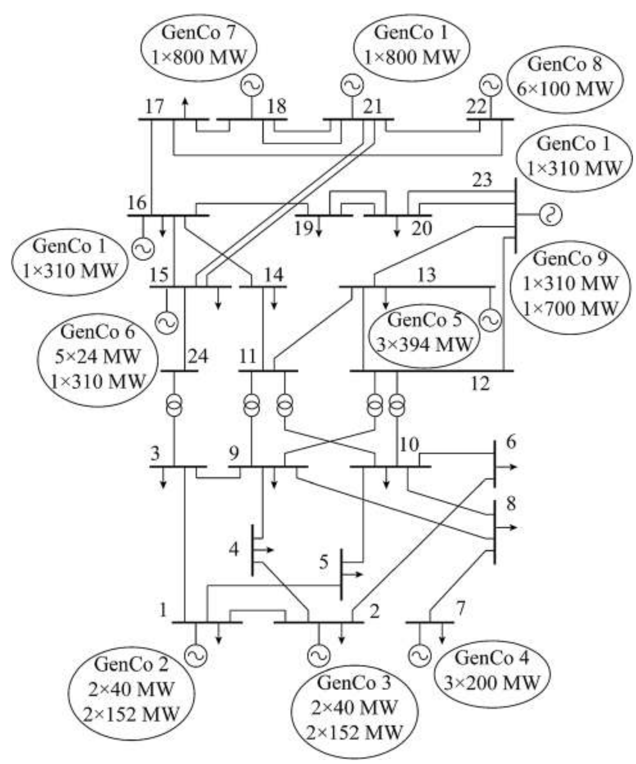

Fig. 2 MRTS topology

Table 1 GenCos' data for MRTS

GenCo number	Unit number	Bus	$P_{i,\max}$ (MW)	a_i (\$/MWh)	b_i (\$/MW^{2}h)
1	27	23	310	9.537	0.00559
1	31	21	800	5.230	0.00007
1	32	16	310	9.537	0.00559
2	19, 20	1	40	24.842	0.36500
2	21, 22	1	152	10.239	0.03840
3	23, 24	2	40	24.842	0.36500
3	25, 26	2	152	10.239	0.03840
4	13–15	7	200	17.974	0.02748
5	16–18	13	394	18.470	0.01011
6	7–11	15	24	21.227	0.37937
6	12	15	310	9.537	0.00559
7	30	18	800	5.230	0.00007
8	1–6	22	100	1.000	0
9	28	23	310	9.537	0.00559
9	29	23	700	9.587	0.00315

GenCos' data are presented in Table 1 [33, 34]. In this study, the minimum generation of units is considered zero.

The market is dispatched at the maximum load level (5700 MW) using the DC load flow. At the market equilibrium point, 5 expensive units are limited to their minimum generations, 14 low-cost units are bound by their generation caps and 13 marginal units are remained. Furthermore, transmission lines 10 (from bus 10 to bus 6), 23 (from bus 14 to 16) and 28 (from bus 16 to 17) are congested.

The LMPs and the decomposed components of Lemma 1 at the generation buses are presented in Table 2 and Fig. 3.

The underlined numbers in Table 2 indicate the located bus of each unit. It is expected that low cost units contribute at the LMP of each bus by the negative sign. It means that by reducing the generation of low-cost units the LMP increases. According to Fig. 3, the participation of units 21, 22 and 25, 26 at the LMPs of buses 1 and 2 is clear. However, because of congestion, there are some unexpected values in Table 2. For example, LMPs of buses 1 and 2 decrease by reducing the generation of low-cost unit 29.

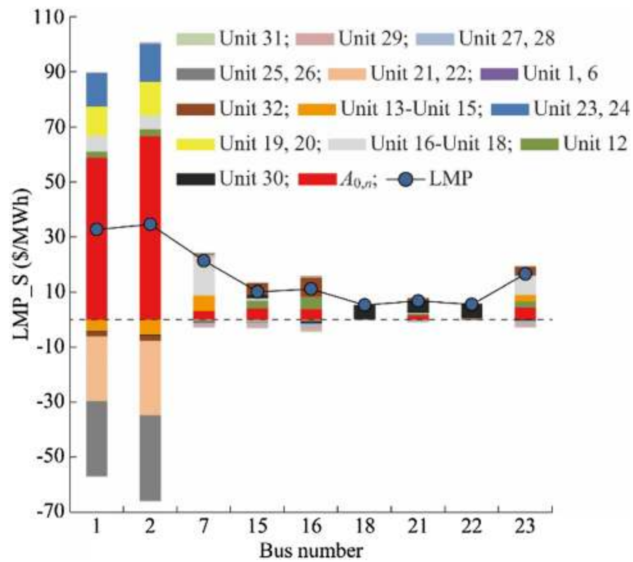

Fig. 3 LMPs and their constructive components

5.1 Evaluation of unit-based market power indices

According to (9), the values of new market power index $LMP\_S_{i,n}$ for each unit are shown in Table 3.

In order to explain the efficiency of the proposed indices, using the indices values of Table 3, the share index of all marginal units on the LMP of units 29-32 at buses 23, 18, 21 and 16, are plotted in Fig. 4. According to this figure, unit 30 has the maximum share on the LMP of units 30 and 31. The LMP of units 29, 31 and 32 are highly affected by units 12 and 32.

Table 2 LMPs and their constructive components

N	LMP	$A_{0,n}$	$A_{i,n}a_i$							$A'_{i,n}P_{i,\max}$					
			12	13–15	16–18	19–20	23–24	30	32	1–6	21–22	25–26	27–28	29	31
1	32.666	58.758	2.295	− 1.401	1.900	5.285	6.051	− 0.330	− 1.575	− 0.007	− 11.805	− 13.516	0.085	0.193	− 0.335
2	34.535	66.636	2.578	− 1.818	1.618	6.051	6.931	− 0.418	− 1.859	− 0.008	− 13.516	− 15.481	0.152	0.344	− 0.376
7	21.304	3.149	− 0.172	1.843	4.593	− 0.146	− 0.189	0.130	− 0.320	0.000	0.325	0.422	− 0.386	− 0.872	0.024
15	10.113	4.076	2.800	− 0.066	0.202	0.092	0.103	1.149	4.285	− 0.010	− 0.204	− 0.230	− 0.386	− 0.871	− 0.420
16	11.099	3.839	4.285	− 0.123	0.054	− 0.063	− 0.074	− 0.727	6.794	− 0.013	0.140	0.166	− 0.593	− 1.338	− 0.624
18	5.273	0.114	0.026	0.001	− 0.002	0.000	0.000	5.226	− 0.017	− 0.007	0.001	0.001	0.002	0.003	− 0.042
21	6.790	1.356	0.895	− 0.020	0.062	0.028	0.032	3.948	1.332	− 0.008	− 0.064	− 0.071	− 0.120	− 0.271	− 0.161
22	5.535	0.329	0.176	− 0.003	0.009	0.005	0.005	5.005	0.216	− 0.007	− 0.010	− 0.012	− 0.019	− 0.044	− 0.063
23	16.482	4.416	2.124	0.815	2.298	− 0.019	− 0.033	− 0.365	3.261	− 0.006	0.042	0.075	− 0.487	− 1.100	− 0.309

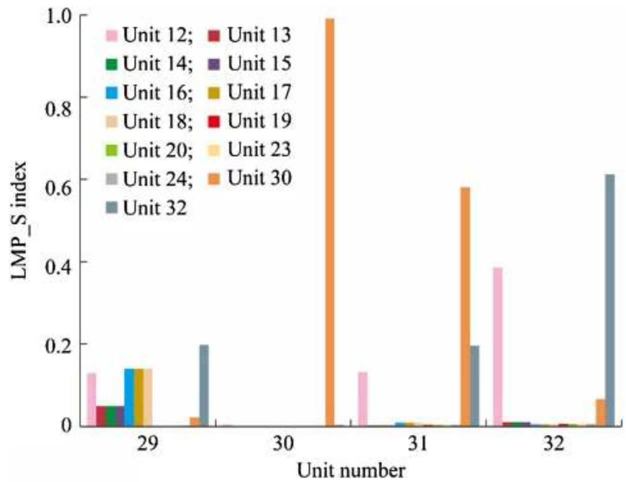

Fig. 4 LMP_S of units 29–32

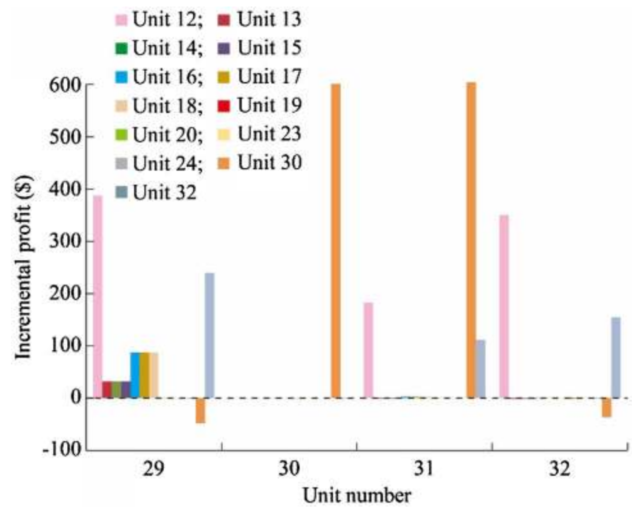

Fig. 5 Incremental profit of units 29–32

To evaluate the potential of market power indicated by proposed index, the bid of each marginal unit is increased by 1 $/MWh and the market is dispatched again. This process is separately performed for all marginal units. Then the incremental profits of units 29-32 are calculated and plotted in Fig. 5.

From the comparison of Fig. 4 with Fig. 5, it is concluded that the proposed index can effectively predict the market power of the units. For example, Fig. 5 shows that the profit of unit 31 is highly affected by the increase of the bid of unit 30. This incremental profit was predictable by proposed index in Fig. 4.

The underlined numbers in Table 3 indicate the located bus of each unit. These values demonstrate the proposed LMP_S_i market power index for each unit as introduced in (7). The following results can be concluded from Table 3.

1) The bordered cell in each row (bus) shows the highest market power value at that bus, based on the $LMP\_S_{i,n}$ market power index. It should be noted that the highest market power value at each bus does not necessarily belong to the units located at that bus. For example, the highest LMP_S value at bus 1 belongs to units 25 and 26, which are located at bus 2.
2) The bolded value in each column (unit) indicates that the unit has the highest market power value at that row (bus) among all buses. For instance, units 13-15 at bus 7 have the highest market power at their bus. However, this is not a common result. For example, unit 12 has the best market power value at bus 16 with $LMP\_S_{12,16}$ of 0.3861, although it is located at bus 15.
3) A unit prefers to have the highest market power value at its bus (underlined and bolded value) with the maximum LMP_S value at that bus (bordered cell). In this study, units 25, 26, 30 and 32 are the best units to apply market power, since their $LMP\_S_{i,n}$ values are underlined, bolded and bordered simultaneously.
4) The collusion can be formed if the bordered and underlined $LMP\_S_{i,n}$ values in each row are not placed coincident in one unit. For instance, if the bidding strategy of unit 30 at bus 18 increases by 1 $/MWh and the network is dispatched again, then the revenue of unit 1 at bus 22 increases by $ 95.7, reaching $ 649, i.e. 17.3% variation. Therefore, the proposed market

Table 3 Values of $LMP\_S_{i,n}$ index

N	Located unit	$LMP\_S_{i,n}$ index of marginal units							$LMP\_S_{i,n}$ index of low-cost units					
		12	13–15	16–18	19, 20	23, 24	30	32	1–6	21, 22	25, 26	27, 28	29	31
1	19–22	0.0703	0.0429	0.0582	0.1618	0.1852	0.0101	0.0482	0.0002	0.3614	0.4138	0.0026	0.0059	0.0103
2	23–26	0.0746	0.0526	0.0469	**0.1752**	**0.2007**	0.0121	0.0538	0.0002	**0.3914**	**0.4483**	0.0044	0.0100	0.0109
7	13–15	0.0081	**0.0865**	**0.2156**	0.0068	0.0089	0.0061	0.0150	0.0000	0.0153	0.0198	0.0181	0.0409	0.0011
15	12	0.2768	0.0065	0.0200	0.0091	0.0102	0.1136	0.4238	0.0010	0.0202	0.0227	0.0382	0.0862	0.0415
16	32	**0.3861**	0.0111	0.0049	0.0057	0.0067	0.0655	**0.6122**	0.0011	0.0126	0.0149	**0.0534**	**0.1206**	**0.0563**
18	30	0.0050	0.0002	0.0003	0.0001	0.0001	**0.9910**	0.0031	**0.0013**	0.0001	0.0002	0.0003	0.0006	0.0080
21	31	0.1319	0.0029	0.0091	0.0042	0.0047	0.5814	0.1961	0.0012	0.0094	0.0105	0.0177	0.0399	0.0237
22	1–6	0.0319	0.0005	0.0017	0.0008	0.0009	0.9042	0.0391	0.0012	0.0019	0.0021	0.0035	0.0079	0.0113
23	27–29	0.1288	0.0494	0.1394	0.0011	0.0020	0.0221	0.1979	0.0004	0.0025	0.0045	0.0296	0.0668	0.0188

power index could be a good basis for the structural analysis of the potential of a collusion formation in the market.

In order to verify the efficiency of the proposed index, the values of structural market power indices including capacity market share (CMS), generation market share (GMS), transmission-constrained minimal generator index (TCMGI) [16], and LMP_S are compared in Table 4. The units are ordered from low cost to high cost. In this table, "L", "M" and "H" denote low cost, marginal and high cost units, respectively. The bolded value in each column shows the maximum value of market power index.

The proposed LMP_S index demonstrates that the marginal units 30 and 32 have the highest market power value. Note that GMS and TCMGI indices do not show this result. Table 4 shows that our index can discover and evaluate the market power of marginal units while the other indices cannot. In other word, LMP_S index quantifies the ability of units to change the electricity price. This property is not directly founded in other structural market power indices.

To evaluate the results of Table 4, the bidding strategy of each unit increases by 1 \$/MWh and the market is re-dispatched. This process is repeated for all units. Figure 6 shows the incremental value of the unit's profit at each case. This figure verifies the performance accuracy of LMP_S index to detect a market power value based on the market power definition, i.e. the ability to change the electricity price profitably.

5.2 Analysis of potential of multi-agent coalition formation

The units that have the greatest impact on the price of each other have the greatest motivation to form a coalition.

Table 4 Comparison of structural market power indices

Unit number	Capacity (MW)	P_i (MW)	CMS	GMS	TCMGI	LMP_S_i
1–6 (L)	100	100	0.0147	0.018	0	0.0012
21–22 (L)	152	152	0.0223	0.027	71.20	0.3614
25–26 (L)	152	152	0.0223	0.027	**81.06**	0.4483
27–28 (L)	310	310	0.0455	0.054	0	0.0296
29 (L)	700	700	0.1028	0.123	59.71	0.0668
31 (L)	800	800	**0.1175**	**0.140**	0	0.0237
12 (M)	310	103.1	0.0455	0.018	0	0.2768
13–15 (M)	200	121.2	0.0294	0.021	0	0.0865
19–20 (M)	40	21.4	0.0059	0.004	0	0.1618
23–24 (M)	40	26.6	0.0059	0.005	0	0.2007
30 (M)	800	613.8	**0.1175**	0.108	0	**0.9910**
32 (M)	310	279.4	0.0455	0.049	0	0.6122
7–11 (H)	24	0	0.0035	0	0	0

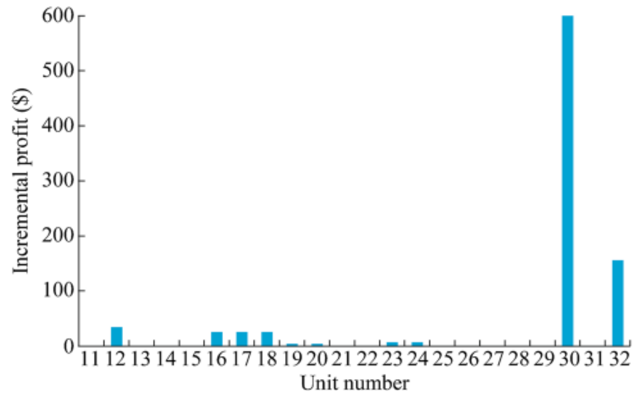

Fig. 6 Incremental of profits of units

The summation of $LMP\_S_{g,i}$ ($g = 1, 2, \ldots, N$, $g \neq i$), can be named as total coalition profitability of unit i (TCP_i). The high value of TCP_i indicates that unit i has high ability to change the LMP of other units. Therefore, we sort the generation units based on their TCP_i index in descending order and the results are plotted in Fig. 7. *LMP_S* values are scaled on the left vertical axis. Obviously, the summation of the values of bars for each unit equals to its TCP_i.

As indicated in this figure, although the units 25 or 26 do not have high market power ($LMP\_S_{25} = LMP\_S_{26} = 0.4483$), these units have a good position to form a coalition ($TCP_{25} = TCP_{26} = 3.12$). On the other hand, unit 30 has a good market power by $LMP\_S_{32} = 0.991$ but does not have good situation in view point of collusion formation by $TCP_{32} = 1.23$. Furthermore, it can be said that unit 32 which has the second rank of market power and the third rank of TCP has a proper location from both collusion and market power points of view.

As mentioned in Section 4.2, $LMP\_S_{i,j}$ reveals this potential of multi-agent coalition formation. For example, Fig. 8 determines the best units' candidates to form coalition with units 12 and 32.

The values of Fig. 8 are obtained from Table 3. As indicated from the blue lines, unit 32 is the best unit to form coalition in view point of unit 12. In other words, with the bid change of unit 12, the LMP of units 32 has the most change (0.4238). Moreover, LMP of unit 12 takes the maximum impact from the bid change of unit 32.

The red lines indicate that unit 32 takes the maximum impact from its bid change (0.6122). Therefore, this unit has a good market power and may not be willing to form coalition. However, this unit is highly impacted from unit 12 (0.4238) and can accept the coalition offer from unit 12.

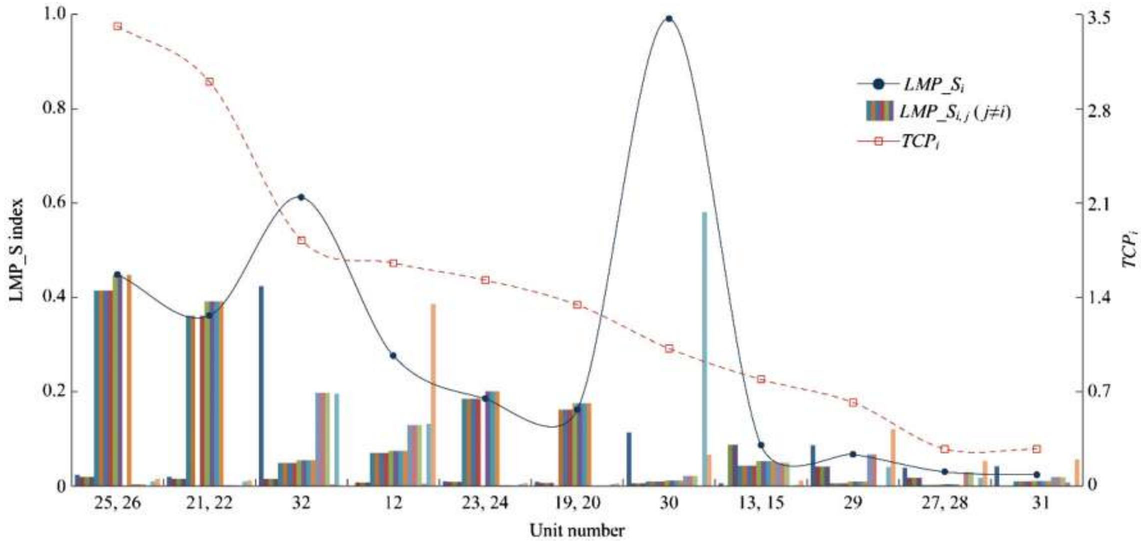

Fig. 7 Comparison between TCP and LMP_S of units

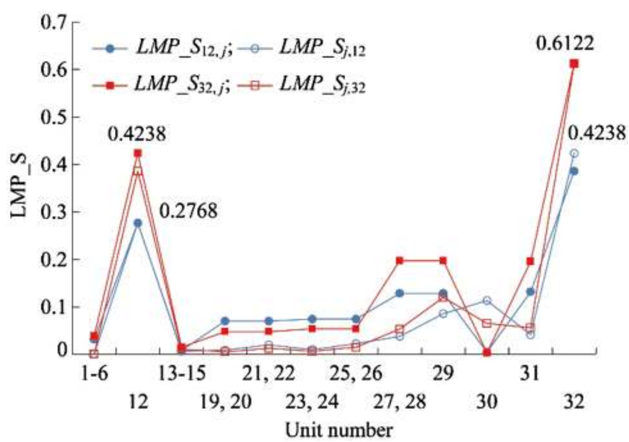

Fig. 8 Units' candidates to form coalition by units 12 and 32

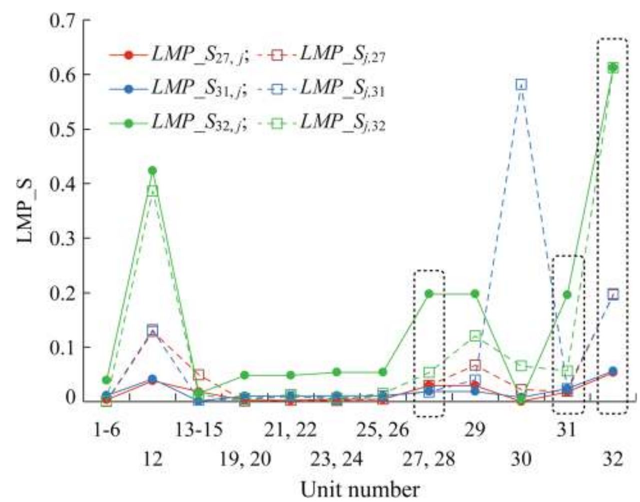

Fig. 9 Values of LMP_S of GenCo 1 at LMP of other units

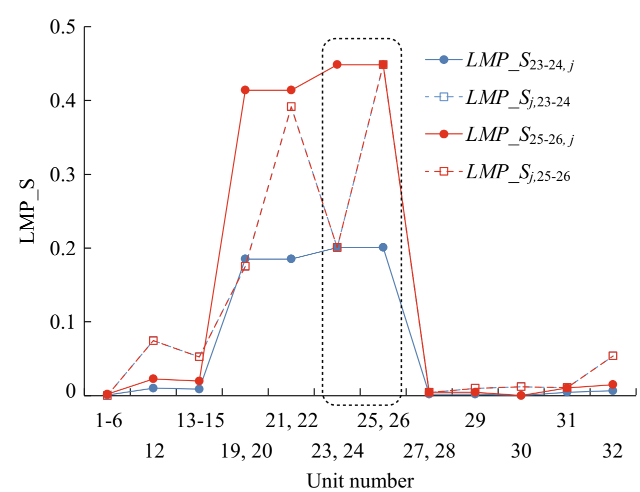

Fig. 10 Values of LMP_S of GenCo 3 at LMP of other units

5.3 GenCos-based market power evaluation

The previous section evaluates the coalition between multi units. This analysis can be used to evaluate the coalition between the units of GenCos with multiple units. Figures 9 and 10 show the values of LMP_S for units of GenCos 1 and 3, respectively. The dashed boxes in these figures indicate the units of GenCo.

As indicated from blue lines in Fig. 9, unit 30 is the best choice for coalition by unit 31, but this unit is not owned by GenCo 1. The green and red lines indicate that unit 12 is a good choice which is not owned by GenCo 1. Therefore, GenCo 1 does not have good situation in view point of the coalition between its units.

From Fig. 10, it can be concluded that the units of GenCo 3 have a good situation for coalition. By using the GenCos-based market indices, this qualitative analysis is expressed by quantitative value. Section 4.3 proposed a

Table 5 Values of $LMP\_S_{GCj,n}$ index

N	LMP	$LMP\_S_{GC_j,n}$ index for each GenCo							
		1	2	3	4	6	7	8	9
1	32.666	0.061	1.046	1.198	0.129	0.070	0.010	0.001	0.009
2	34.535	0.069	**1.133**	**1.298**	0.158	0.075	0.012	0.001	0.014
7	21.304	0.034	0.044	0.057	**0.260**	0.008	0.006	0.000	0.059
15	10.113	0.503	0.059	0.066	0.020	0.277	0.114	0.006	0.124
16	11.099	**0.722**	0.037	0.043	0.033	**0.386**	0.066	0.007	**0.174**
18	5.273	0.012	0.000	0.001	0.001	0.005	**0.991**	**0.008**	0.001
21	6.790	0.237	0.027	0.030	0.009	0.132	0.581	0.007	0.058
22	5.535	0.054	0.006	0.006	0.001	0.032	0.904	0.008	0.012
23	16.482	0.246	0.007	0.013	0.148	0.129	0.022	0.002	0.096
Net	–	1.206	1.046	1.298	0.260	0.277	0.991	0.008	0.096

new market power index $LMP\_S_{GC_j,n}$ in (11) to evaluate the structural market power of GenCos. Table 5 shows the values of the $LMP\_S_{GC_j,n}$ index for each GenCo. The last row of the table indicates the $LMP\_S_{GC_j,n}$ index as defined in (12). The underlined numbers in each column (GenCo) show the located buses of GenCo's units. The bolded value in each column indicates the bus where GenCo has the highest market power value among all buses. The bordered cell in each row (bus) shows the highest market power value at that bus. The GenCos 1, 3, 4 and 7 have the desirable situations, since their $LMP\_S_{GC_j,n}$ values are underlined, bolded and bordered simultaneously. If the bordered and underlined $LMP\_S_{GC_j,n}$ values in each bus are not placed coincident in one GenCo, the collusion can be formed. This condition is observed at buses 1, 15, 21 and 22.

The values of GenCo's market power index CMS, GMS and the proposed LMP_S index are gathered in Table 6, to compare the proposed index by other structural market power index.

The underlined-bolded number in each column shows the maximum value of the market power index. The bolded number indicates the second one. As shown in Table 6, the GenCo 3, which does not have high market share in view point of CMS and GMS, has the highest LMP_S index $LMP\_S_{GC_j}$. It must be noted that this quantitative analysis confirms the qualitative result obtained from Fig. 10.

Table 6 Comparison of GenCos-based structural market power indices (case MRTS)

GenCo number	Capacity (MW)	P_i (MW)	CMS	GMS	$LMP\_S_{GC,j}$
1	1420	1389.4	**0.209**	**0.244**	**1.2055**
2	384	346.9	0.056	0.061	1.0464
3	384	357.1	0.056	0.063	**1.2979**
4	600	363.6	0.088	0.064	0.2596
6	430	103.1	0.063	0.018	0.2768
7	800	613.8	0.117	0.108	0.9910
8	600	600.0	0.088	0.105	0.0075
9	1010	1010.0	**0.148**	**0.177**	0.0963

6 Conclusion

This paper presents new structural market power indices for units and GenCos, based on their LMP_S. To calculate the proposed indices, initially, LMP is decomposed into four main components. These components indicated LMP_S of units at each bus. Then new structural market power indices, using the concept of LMP_S of each unit are proposed. These market power indices are categorized into "unit-based LMP_S" and "GenCos-based LMP_S" indices. Unit-based LMP_S index indicates the contribution factor of each unit at the electricity price of each bus. Moreover, this index determined the best units' candidates to form coalition by each unit. GenCos-based LMP_S index is formed from the total share of all units owned by a GenCo at electricity price of GenCo's located buses. The proposed market power indices are calculated for the IEEE 24-bus test system. The following conclusions can be drawn from the simulation results.

1) Because of congestion, there are some unexpected behaviors at the market, for instance, the LMPs of some buses decrease by reducing the generation of some low-cost units, unexpectedly. The largest production share does not mean the highest market power. The proposed LMP_S index shows that the units with lower capacities can effectively change LMPs profitably.
2) The collusion can be formed if the highest LMP_S in each bus belongs to the unit at another bus. The proposed market power index is a good basis for the structural analysis of market power and the potential of a collusion formation in market.

3) Some units do not have high market power but may have a good position to form a coalition. On the other hand, a unit with a good market power may not have a good situation in view point of collusion formation.
4) Using the proposed GenCos-based market indices, the situation of each GenCo in view point of the coalition between its units can be expressed quantitatively.
5) LMP_S index has a good performance accuracy to detect the market power based on the market power definition, i.e. the ability to change the electricity price profitably.

The proposed LMP_S market power indices provide a good analytical tool to evaluate the structural market power. In the future works, this method could be extended in double-sided markets to discover the effect of price elastic loads in market power mitigation. Moreover, the proposed method could be developed to calculate the incremental profits of units analytically. Using that it is possible to define new market power indices based on the incremental profit of units.

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