Locus equations are an acoustic expression of articulator synergy

Khalil Iskarous,[a] Carol A. Fowler, and D. H. Whalen
Haskins Laboratories, 300 George Street, New Haven, Connecticut 06511

(Received 16 December 2009; revised 20 July 2010; accepted 22 July 2010)

The study investigated the articulatory basis of locus equations, regression lines relating F2 at the start of a Consonant-Vowel (CV) transition to F2 at the middle of the vowel, with C fixed and V varying. Several studies have shown that consonants of different places of articulation have locus equation slopes that descend from labial to velar to alveolar, and intercept magnitudes that increase in the opposite order. Using formulas from the theory of bivariate regression that express regression slopes and intercepts in terms of standard deviations and averages of the variables, it is shown that the slope directly encodes a well-established measure of coarticulation resistance. It is also shown that intercepts are directly related to the degree to which the tongue body assists the formation of the constriction for the consonant. Moreover, it is shown that the linearity of locus equations and the linear relation between locus equation slopes and intercepts originates in linearity in articulation between the horizontal position of the tongue dorsum in the consonant and to that in the vowel. It is concluded that slopes and intercepts of acoustic locus equations are measures of articulator synergy. © *2010 Acoustical Society of America.* [DOI: 10.1121/1.3479538]

PACS number(s): 43.70.Fq, 43.70.Bk, 43.70.Mn [DAB] Pages: 2021–2032

I. INTRODUCTION

The acoustic measurement of place of articulation for consonants and the acoustic measurement of the amount of coarticulation are two long-standing problems in phonetics. Previous work on consonant place of articulation, using articulatory-acoustic models, within the acoustic theory of speech production (Stevens, 1998), has found certain acoustic correlates of place, but the variability found in empirically-based studies has not provided an easy match to the theoretical predictions. Indeed, in the field of automatic speech recognition, which relies on high quality estimation of acoustic features, it has recently been argued that it is necessary to develop more powerful statistical techniques for investigating speech dynamics to overcome the difficulties of estimating place and amount of coarticulation from acoustics (Deng, 2006). One method that has been proposed for solving both problems, and that is robust to various sources of variability, is locus equations (Nearey and Shammass, 1987; Sussman *et al.*, 1991; Fowler, 1994; Sussman *et al.*, 1998; Brancazio and Fowler, 1998). Lindblom (1963) showed that, for CV syllables, where C is an oral stop, if F2 at the beginning of the vowel transition is plotted against F2 in the middle of the vowel, for C fixed and V variable, a highly linear relation emerges. This linear relation is parameterized by a slope and an intercept. The slope indicates the amount of change of F2 at the beginning of the vowel transition for a unit change in F2 at the vowel midpoint, and the intercept indicates F2 at the beginning of the vowel transition for a zero F2 at vowel midpoint. That is, the intercept is at the intersection of the regression line with the dependent variable axis. It has been observed in many experiments that slopes descend in the series labial > velar > alveolar, and the magnitudes of the intercepts increase in the opposite order. Later it was also shown that there is a linear relation between the intercepts and the slopes, termed Second Order Locus Equations (SOLE) (Chennoukh *et al.*, 1997), when the intercepts are taken as the dependent variable and the slopes as the independent variable. This is surprising, since in a linear relation, the intercept is, in general, independent of the slope. The robustness of the pattern of slopes in English and other languages (Sussman *et al.*, 1993) has led to their use as a marker of both place of articulation and coarticulation in a variety of studies of typical and atypical populations (Sussman *et al.*, 1998; Robb and Blomgren, 1997; Baillargeon *et al.*, 2002; Gibson and Ohde, 2007). However the interpretation of slopes and intercepts of locus equations still remains problematic. For instance, given the statistical meaning of intercept as the value for consonant F2 when vowel F2 is zero, it seems strange that such a number, which represents a seemingly non-observable quantity, is capable of distinguishing different consonants. Also, the articulatory phenomena that lead to a linear relation in acoustics are still unknown, and it is not known why the slopes and intercepts pattern the way they do. The reasons for the linear relation between intercepts and slopes and why locus equation measures can provide information about both place of articulation and amount of coarticulation are also not known. The aim of the present study is to further research on coarticulation by showing that these issues are intimately related to each other, and that addressing them allows for more empirically informed use of locus equations to study variability in both coarticulation and the linguistic use of place of articulation in contrasting consonants in the world's languages.

There is agreement that locus equations are not an accident of speech production (Sussman *et al.*, 1998), but their articulatory origin is unclear (Fowler, 1994; Sussman *et al.*,

---

[a] Author to whom correspondence should be addressed. Electronic mail: iskarous@haskins.yale.edu

1998; Löfqvist, 1999). One hypothesis is that locus equations indicate degree of coarticulation between a consonant and a following vowel (Krull, 1987, 1989; Fowler, 1994). Two studies using computer simulations of speech production have indeed shown that manipulation of production parameters that increases degree of coarticulation will raise the slopes of locus equations (Chennoukh *et al.*, 1997; Lindblom and Sussman, 2004). However, two empirical studies of speech production have failed to definitively link locus equation slopes to coarticulation degree across segments of English (Löfqvist, 1999; Tabain, 2000). Another hypothesis is that consonant coarticulation resistance, the degree to which different consonants resist the influence of a contiguous vowel (Bladon and Al-Bamerni, 1976; Recasens, 1985; Recasens and Espinosa, 2009) is the source of locus equations (Fowler, 1994; Fowler and Brancazio, 2000; Tabain, 2000). Fowler and Brancazio (2000) showed that there is a correlation between the measures of coarticulation resistance by Recasens (1985) and the locus equation slopes measured by Sussman *et al.* (1991). However, the link between locus equations and coarticulation resistance has remained indirect, with no articulatory study definitively showing a direct link between them. Another articulatory hypothesis about the origin of locus equations is encapsulated in Sussman *et al.* (1998)'s Orderly Output Constraint, according to which there is a constraint on the speech production mechanism that forces different amounts of coarticulation between different vowels and each consonant that would yield, through the nonlinear articulatory-acoustic transform, a linear locus equation relation in the acoustics that could serve as a relational invariant for stops. To date there has not been an empirical articulatory study that demonstrates a nonuniformity in coarticulation that would yield the linearity of locus equations and specific patterning of their statistical parameters.

Our first step is to show that a statistical exploration of the slopes and intercepts of locus equations yields insight into which of the articulatory origins of locus equations is most likely to be correct (Sec. II). Furthermore the statistical relations invoked yield a relation between slopes and intercepts that is useful for appreciating why SOLE relations exist. In the following, we pursue the hypothesis that F2 locus equation relations are based on a relation involving the horizontal position of the back of the tongue, because this is the articulatory factor most directly linked to F2 (Joos, 1948; Delattre, 1951).

II. COARTICULATION RESISTANCE AND LOCUS EQUATIONS

Recasens and Espinosa (2009, p. 2288) defined the coarticulation resistance of a consonant or a vowel as "… a measure of its degree of articulatory variability as a function of phonetic context." Recasens (1985) measured coarticulation resistance for each of the Catalan consonants he investigated by calculating the standard deviation of F2 at the release of the consonant, where variability is computed over vowels following that consonant. This measure reflects the degree of resistance, because a low standard deviation would indicate a high degree of constraint by the consonant on the articulatory properties determining F2 at consonant release, forcing F2 to vary little. In contrast, a high standard deviation indicates little such constraint, allowing large vowel variability of F2 at the vowel to cause large variability at the consonant release. Recasens and Espinosa (2009) used a slightly different method for measuring coarticulation resistance using articulatory measurements. In this method, the centroid of an articulatory measure of the same consonant in different vowel environments is made, then the Euclidean distance between the measure in each context and the centroid is calculated. Coarticulation resistance is then determined as the average of all such distances, as the vowel varies. But as Recasens and Espinosa (2009) mention, the standard deviation measure and the average deviation from the centroid yield essentially the same result. The main difference is that the centroid measure is useful for comparing the vertical and horizontal components of resistance. Fowler and Brancazio (2000) showed that the more highly resistant a consonant is, the lower its locus equation slope. However the reason for this correlation remained unclear. In this section, we show that coarticulation resistance and locus equation slope and intercept are directly related.

In the theory of bivariate regression, there is a basic relation that expresses the slope of a regression line relating two random variables to the standard deviations of those variables and the correlation coefficient between the variables: $b=\rho(\sigma_y/\sigma_x)$, where b is the regression slope, ρ is the correlation coefficient, σ_y is the standard deviation of the dependent variable, and σ_x is the standard deviation of the independent variable (Yule, 1897). That is, the slope of a regression line is the ratio of the standard deviation of the dependent variable to the standard deviation of the independent variable, weighted by the correlation coefficient. When applied to the case at hand, the formula for locus equations is $Locus\ Equation\ Slope=\rho(\sigma_{F2_C}/\sigma_{F2_V})$, where σ_{F2_C} is the standard deviation of F2 at consonant release and σ_{F2_V} is the standard deviation of F2 at the vowel. But σ_{F2_C} is exactly Recasens (1985)'s measure of coarticulation resistance. Dividing σ_{F2_C} by σ_{F2_V} amounts to normalizing the variability measure of coarticulation resistance measured at the consonant by the variability at the vowel midpoint. Therefore, if the correlation coefficient does not vary appreciably for the different consonants, then the slope of the regression line for each consonant is *directly* related to the normalized coarticulation resistance for the consonant. Specifically, high coarticulation resistance, measured as a low standard deviation of F2 at the consonant normalized by variability of F2 across vowels, directly leads to a low locus equation slope. And low coarticulation resistance would, by the same logic, directly lead to a locus equation slope close to 1, since the variability of F2 measured at consonant release would be nearly the variability measured at the midpoint of the vowel. Therefore, the independently motivated and tested measurement of coarticulation resistance as a standard deviation of F2 at the consonant release, when normalized, is equivalent to the slope of a locus equation. To the knowledge of the authors, this has not been pointed out either in the locus equation literature or in the coarticulation resistance literature.

There is also a formula for the intercept of a regression line in terms of the slope, given the averages of the dependent and independent variables (Hayduk, 1987, p. 15):

$$\text{Intercept} = \text{Average of Dependent Variable} - (\text{Slope}^{*}\text{Average of Independent Variable}). \tag{1}$$

Even though, in general, the slope is independent of the intercept, they become linearly dependent when the averages of the dependent and independent variables are given. The assumption behind this equation is that the dependent variable is a linear function of the independent variable, and that they are both drawn from the same type of probability distribution. The equation falls out from the fact that the expected value of the dependent variable is a function of the expected value of the independent variable, due to the linear relation. For locus equations, the formula for the intercept would be:

$$\text{LE Intercept} = \text{Average of F2 at Cons. Rel.} - (\text{LE Slope}^{*}\text{Avg. of F2 at V Mid}). \tag{2}$$

That is, the locus equation intercept is high if the average of F2 at the consonant release is high and is low if the average of F2 at the vowel or the slope is high. The average F2 at the mid vowel would vary for different consonants, if there is C-to-V coarticulation, because this would influence the average value of F2 at the vowel. Average F2 at consonant release is likely to be influenced by the frontness of the tongue at consonant release and lip rounding, because these are the two articulatory factors most likely to affect F2 (Fant, 1960).[1] The locus equation intercept is therefore a complex measure, affected by several different articulatory phenomena: coarticulation resistance, C-to-V carryover coarticulation, and the average position of the tongue back and lips at the consonant release.

III. EXPERIMENT

The first step is to show that the linearity, slopes, and intercepts of locus equations arise in articulation (III.B.1). Then the relation between slopes and intercepts of locus equations encoded in SOLE are also shown to be evident in articulation (III.B.2). Finally, the statistical arguments linking coarticulation resistance with slopes and intercepts discussed in the previous section are tested using the data sets examined.

Several studies have associated the frontness of the body of the tongue in the vocal tract to F2 (Joos, 1948) and (Delattre, 1951). Specifically, when the tongue body is fronted, F2 is generally high, and when it is backed, F2 is generally low. Therefore in order to investigate the articulatory origins of locus equations (LE), the horizontal position of the tongue body at the release of a consonant, across consonant places of articulation, and in the middle of a vowel (the positions in a CV syllable where LE measurements are made) were measured.

TABLE I. Words used from the Wisconsin XRMB database. Each word is followed by the particular CV included in the analysis.

Labial stop	Alveolar stop	Velar stop	Alv. fricative
Put /pʊ/	Too /tu/	School /ku/	Sued /su/
Both /bo/	Dormer /do/	Could /kʊ/	Sewed /so/
Popular /pɑ/	Does /dʌ/	Coat /ko/	Sod /sɑ/
But /bʌ/	Dark /dɑ/	Country /kʌ/	Sawed /sɔ/
Special /pɛ/	Day /de/	Conversation /kɑ/	Said /sɛ/
Back /bæ/	Damage /dæ/	Cash /kæ/	Sad /sæ/
Been /bɪ/	Didn't /dɪ/	Making /ki/	Sid /sɪ/
People /pi/	Ingredients /di/	Came /ke/	Seed /si/

A. Methods

Two sets of data were analyzed, one using Electromagnetic Midsagittal Articulography and the other using X-ray Microbeam.

1. Electromagnetic midsagittal articulography Data (EMMA)

Data from one female speaker of American English were analyzed. The utterances were disyllables /əCV/. C varied over /b/, /d/, /g/, /v/, /ð/, /z/ and V varied over /i/, /ei/, /ɛ/, /ʌ/, /u/, and /a/ in a carrier sentence "Enough–Bub." There were 15 repetitions of each utterance, for a total of 540 utterances. EMMA coils tracked the motion of the lower and upper lips, jaw, and 4 points on the tongue. The most anterior pellet was on the tip complex, approximately a cm from the tip itself, and the posterior pellet was placed as far back as the speaker would allow, approximately on the tongue body. The other two pellets were placed equidistantly from the other two. More details on the data collection procedures can be found in Fowler (2005). The data were acoustically segmented to obtain the consonant release and midpoint of the second vowel. For each CV syllable, the position of the articulators at the same two points in time were then extracted. F2 was automatically extracted using Linear Predictive Coding formant estimation on the first 20 ms of the consonant transition and a 20 ms window centered at the middle of the vowel. Both 20 ms windows were pre-emphasized and Hamming windowed. The formants were estimated by peak-picking.

2. X-ray microbeam data (XRMB)

Eight Obstruent-Vowel syllables, each of one place, were extracted from productions by 38 speakers from the Wisconsin X-ray Microbeam database (Westbury, 1994). The consonants were labial stops, alveolar stops, velar stops, and alveolar fricatives, but they varied in voicing, and the vowel sets used for each of the stops varied. Almost all of the syllables used were stressed. Table I presents the words used. Half of the labial obstruents were voiceless, one of the alveolar obstruents was voiceless, and all the velar obstruents were voiceless. The advantage of using the Wisconsin XRMB database is that it is possible to analyze data from many speakers, which is rare in speech production research. It was hypothesized that if the results for the EMMA speaker are supported by the data set from XRMB, then the results are quite robust, despite the large expected variability due to the number of speakers and the difference in vowels and consonants included in each category. Another advantage of using the database is that the tokens used are real words from real sentences, therefore, if the results based on these data agree with studies based on isolated non-meaningful syllables, it would affirm the robustness of the locus equation phenomenon. However, the disadvantage from the experimental design perspective is that the utterances are not contextually-controlled lists. Regarding the fact that the vowels used with each of the consonants were not the same, it is important to note that the slopes and intercepts of locus equations have already been shown to be relatively robust to the variation in the vowel set used with each consonant, as can be seen in the fact that various studies have used different vowels, but the pattern of slopes and intercepts among the consonants is relatively stable despite variability in the vowels (Sussman *et al.*, 1991; Modarresi *et al.*, 2004). Another feature of locus equation data that makes the use of less than ideally controlled data possible is that this type of data is robust to changes in prosodic factors. Specifically, Lindblom *et al.* (2007) found that there are consistent difference in locus equations due to stress, Modarresi *et al.* (2004) found that there are differences due to CV vs. VC differences, and Bakran and Mildner (1995) found that there are consistent differences due to speech style, but these studies also showed that these differences, however consistent, still lead to the same basic differences between the consonants introduced by slope and intercept. Therefore there is some indication in the literature that it is possible to combine data collected in different prosodic conditions.

The X-ray microbeam data are comparable to the EMMA data. The jaw, upper and lower lip were tracked, along with four comparably placed pellets on the tongue. The same procedures used to segment and analyze the EMMA data were used to acoustically segment the XRMB CV's data and to extract the articulatory data and F2 at consonant release and the vowel midpoint.

B. Results

1. First-order locus equations

Figure 1 presents the fitted regression lines relating F2 at consonant release to that at mid vowel (left panels) along with the relations between the horizontal position of the tongue back (TBx) at consonant release and TBx at mid vowel (right panels) for the EMMA study. The reason that the articulatory data are negative is that the data are measured in a coordinate system in which the origin is an occlusal zero at the front of the vocal tract. As can be seen on the left, the relations between F2 at the onset of F2 and the midpoint of F2 vowel are linear across the consonants, as would be predicted based on previous work. But it can be seen on the right that the relations between TBx at the same two moments are also linear. The slopes, intercepts, and explained variation r^2 for each of the relations are given in Table II. The last line of the table gives the correlation between the F2 slopes and the TBx slopes, as well as the correlation between F2 intercepts and TBx intercepts. The explained variability r^2 for both F2 and TBx relations are all above 0.88. The mean r^2 for F2 is 0.92 and the mean r^2 for TBx is 0.95, indicating that both the standard locus equations and the linear relations derived from TBx are highly linear. Although the articulatory and acoustic parameters are not equal, the general patterns are the same and agree with the pattern seen in previous studies. Specifically, (1) labials have the largest slope, velars have intermediate slopes, and alveolars have the smallest slope values; (2) alveolars have the largest magnitude intercepts, followed by velars, and labials. The high correlation between the parameters derived from the acoustics and the parameters derived from the articulation in the last row of Table II indicates that the basic patterns are the same for F2 and TBx.

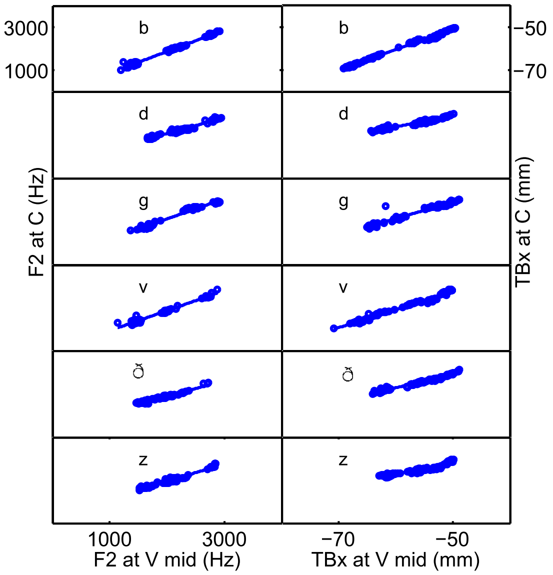

FIG. 1. (Color online) Linear functions relating the horizontal position of the tongue back (TBx) at the consonant release (dependent variable) to TBx at the middle of the vowel (right) and F2 at the consonant release to F2 at the middle of the vowel (left) for each of the consonants examines. Each line fits 90 tokens from six vowels for the EMMA subject.

Table III shows the linear statistics for the same relations across the subjects from the XRMB study. These data were analyzed in two ways: (1) Regression coefficients were calculated within each subject and then averaged (upper numbers in each cell of Table III); (2) Regression coefficients were calculated for data pooled across all subjects (lower number in Table III). Both of these types of analyses have been performed in previous research on locus equations. The slopes from both the articulatory and acoustic data are in the expected order (Labial > Velar > Alveolar), for both the subject-averaged and pooled data. The absolute values of the intercepts for the acoustic and articulatory data also have the same pattern (Alveolar > Velar > Labial). The explained variability for both the acoustic and articulatory equations is less than that in studies that used citation form speech. However Duez (1992) showed that for locus equations calculated from hypoarticulated speech, the explained variability descends by about 20%. Therefore the results here are in agreement with these studies.

Two hypotheses were statistically tested using the slopes and intercepts for the consonants, with variability across subjects. First is the hypothesis that the segments, stops and

TABLE II. Linear parameters for EMMA data.

	F2			TBx		
Segment	Slope	Intercept	r^2	Slope	Intercept	r^2
b	1.004	−141.86	0.9886	0.954	−3.225	0.993
d	0.738	583.883	0.9645	0.491	−26.74	0.948
g	0.934	251.699	0.9778	0.767	−12.571	0.960
v	0.931	1.203	0.9737	0.842	−10.226	0.980
ð	0.689	525.369	0.9502	0.615	−20.220	0.966
z	0.799	354.96	0.9575	0.503	−26.938	0.881
Correlations:		Slope, 0.873			Intercept, −0.892	

fricatives of the same place, are distinguished from each other based on the slopes and intercepts. Second is the hypothesis that there is a significant difference between the patterns shown in the acoustically and articulatorily derived regressions. If the latter is the case, then there is little support for the claim that the linearity of locus equations is already present in the production system. A mixed general linear model test was run with two independent variables Modality (2 levels: Acoustic and Articulatory) and Segment (4 levels: p/b, t/d, k/g, s). The two dependent variables were slope and intercept. A significant effect for segment would be expected if the slopes and intercepts distinguish the segments. If the Modality variable has a significant effect on the slopes or intercepts, it would mean that the articulatory and acoustic patterns for slopes and intercepts are different. To address the fact that data within each participant are correlated, Participant was used as a random factor. Significance values and 95% confidence intervals for the parameters were computed using Markov Chain Monte Carlo sampling (Baayen, 2007). Since the intercepts in the two domains have different units (mm vs. Hz), z-scores were computed within subject for this variable.

The results are shown in Table IV. For slope, Segment had a significant effect on all pairs, except for /s/ vs. /d-t/ (not shown) and /b-p/ vs. /g-k/. These data therefore show that the alveolars with different manners of articulation are not distinguished by slope, indicating that the slopes could be a property of place of articulation, regardless of manner, at least for alveolars (but see Fowler, 1994). The failure of slopes to distinguish labial from velar place of articulation is not unexpected. For instance Sussman *et al.* (1991) presents a histogram of slopes across subjects, which demonstrates that there is considerable overlap for the slopes of these two places across subjects. Intercepts were significantly different across segments, except for /d-t/ vs. /s/. Modality had no significant effect on slopes, but did have a significant effect on intercepts. However the effect size is 0.33 standard deviations. According to Cohen (1992)'s guidelines this is a small to non-existent effect and shows considerable overlap among the distributions. Therefore the hypothesis that place is distinguished using slopes and intercepts is supported, except for the labial-velar distinction. And the hypothesis that there is no difference in linear relations as measured in articulation and acoustics is supported, despite a very small effect on intercepts.

2. Second-order locus equations

As discussed in Sec. I, previous studies have shown a linear relation between intercepts and slopes of locus equations (Chennoukh *et al.*, 1997; Sussman *et al.*, 1998). This linear relation has been termed the Second Order Locus Equation (SOLE), and has been taken to characterize an even more abstract orderly relation in the acoustic output. To determine the SOLE parameters for the data examined here, a regression with the slopes as the independent variable and the intercepts as the dependent variable was performed. For the XRMB data, the average slopes across the subjects were used. Standardized regression was used, because slopes and intercepts are in different units. For the EMMA data, the slope of the Slope-Intercept regression for the articulatory data was 0.997 and for the acoustic data was −0.930. These numbers are also the correlation coefficients, due to standardization. For the XRMB data, the slope for the articula-

TABLE III. Linear parameters for XRMB data. The upper number in each cell contains the average across all subjects, for regressions done within each subject. The lower number in each cell contains the statistics for the regressions performed on the data pooled across subjects.

	F2				TBx			
	p/b	t/d	k/g	s	p/b	t/d	k/g	s
Slope	0.985	0.403	0.902	0.592	0.957	0.301	0.691	0.2719
	0.969	0.535	0.916	0.717	0.98	0.628	0.905	0.701
Int.	−55.52	1079.2	263.68	702.39	−3.24	−35.62	−15.68	−37.221
	−32.33	848.21	244.71	487.15	−1.89	−18.01	−4.01	−14.43
r^2	0.859	0.584	0.866	0.822	0.84	0.457	0.759	0.6434
	0.858	0.635	0.853	0.844	0.829	0.706	0.89	0.69

TABLE IV. Mixed effects GLM results for each place of articulation vs. Labial: effect sizes (Dif), 95% confidence intervals (LB, UB), and MCMC derived p-values.

	Slope				Intercept			
	Dif	LB	UB	p	Dif	LB	UB	p
Art-Ac	−0.027	−0.115	0.062	=0.445	−0.331	−0.555	−0.080	=0.006
d-b	−0.583	−0.657	−0.506	<0.001	1.99	1.67	2.15	<0.001
g-b	−0.074	−0.0797	−0.162	>0.05	0.514	0.303	0.797	<0.001
s-b	−0.394	−0.471	−0.3118	<0.001	1.479	1.169	1.676	<0.001

tory data was 0.998 and for the acoustic data was −0.989. There is therefore an almost one-to-one relation between intercepts and slopes, as is evident in both the acoustic and articulatory data, except that, in the acoustic data, the relation is described by a negative slope. The discrepancy in sign is due to the signs of the coordinates used, with formants always being positive, whereas TBx is negative. This interacts with the fact that posterior motion of the tongue (raising the magnitude of TBx) reduces F2.

3. Coarticulation resistance measurement and slopes

The upper part of Table V, shows for F2 and TBx, the ratio of standard deviations for each of the consonants for the EMMA data. As argued in Sec. II, this is equivalent to the normalized coarticulation resistance. The lower part of the table shows the ratios for the XRMB data. The ratios are in the same order as the locus equations and TBx slopes: Labial > Velar > Alveolar. This may seem like a trivial result, because it was argued in Sec. II that there is an intrinsic relation between coarticulation resistance and slopes, by statistical definition. However, the formula for the regression slope $b=\rho(\sigma_y/\sigma_x)$, also includes the correlation coefficient ρ, which in practice can be quite different for different consonants, and for the XRMB subjects, there was considerable variability in the correlation coefficient. Therefore it could have been the case that variability of the correlation coefficient is a major determiner of the magnitude of the slope. The data in Table V, on the other hand, show that the patterns of the slopes are indeed present in the pattern of coarticulation resistance, and they are equally present in the articulation and in the acoustics.

4. Analysis of intercepts

In Sec. II, it was shown that the regression intercept can be expressed in terms of the average of the dependent variable and the average of the slope-scaled independent variable, using the terms in Eq. (2). In terms of the F2 and TBx regressions, the average dependent variable is the average F2 or TBx at the consonant across all the vowels, the slope is the correlation-scaled normalized coarticulation resistance, and the average of F2 or TBx at the vowel is an indicator of the amount of consonant-to-vowel coarticulation. To examine how each of these factors influences the intercept for the data examined, each of the factors was plotted as a function of consonant in Figs. 2 and 3. The left columns of the figures show the intercept and the slope-scaled vowel average and the right columns show the unscaled averages of the variables. As can be seen from the right column of both figures, both averages fluctuate as a function of consonant. Moreover, the pattern of variation of both averages (C and V) is approximately the same, except that the average at V has a lower magnitude of variability.

Figure 4 shows the trajectories of TBx for each of the consonants from the EMMA data, all aligned to begin at the point of the beginning of the F2 transition in the acoustics. The black time series are for back vowels and the gray time series are for front vowels. In these plots, forward in the vocal tract is up. It can be seen that at the /d/ release, the tongue back is fronted, whether the following vowel is front or back, hence the average TBx at C for /d/ is forward to that of the other consonants. That is, at the moment of the release of F2 for the alveolars, the tongue back is more advanced in the vocal tract than for the consonants of other places of articulation, regardless of the frontness of the following

TABLE V. Normalized coarticulation resistance.

	EMMA					
	b	d	g	v	ð	z
$\sigma_{F2_C}/\sigma_{F2_V}$	1.010	0.751	0.945	0.944	0.707	0.817
$\sigma_{TBx_C}/\sigma_{TBx_V}$	0.957	0.505	0.783	0.851	0.625	0.536
	XRMB					
	b/p	d/t	g/k			s
$\sigma_{F2_C}/\sigma_{F2_V}$	1.042	0.543	0.927			0.6517
$\sigma_{TBx_C}/\sigma_{TBx_V}$	1.122	0.457	0.799			0.334

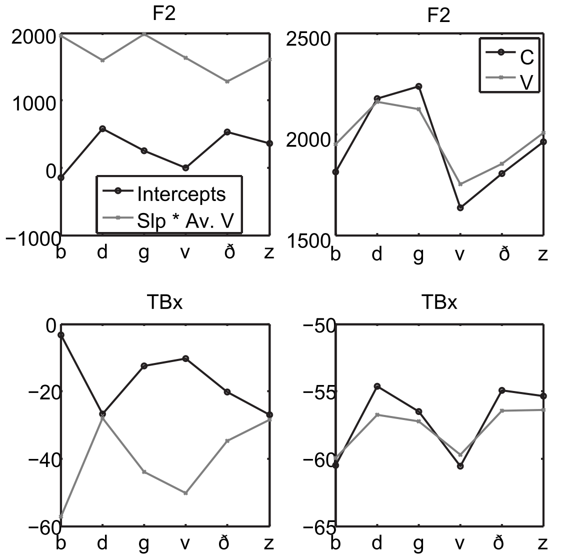

FIG. 2. Analysis of Intercepts into their constituents: slope and average of F2 or TBx at C and V according to Eq. (2) for EMMA data. Frontness increases vertically.

vowel. The advancement of the tongue back is present to a much lesser extent for /g/, and hence the lower average TBx at C. For the labials the tongue back's horizontal position is totally dependent on the following vowel. It can also be seen that there are some small but consistent differences due to manner of articulation. Specifically, the tongue back is more posterior for /z/ than /d/, which is not surprising due to greater constraints on the tongue back for sibilants than stops.

The slope is one of the factors determining the intercept, *given* the averages of the dependent and independent variable. Since the slope is the coefficient of the V average, a curve that is roughly a reflection of the intercept is obtained as can be seen in the left column of Figs. 2 and 3. The intercept is therefore determined through an interaction of three factors, which all vary among the consonants. This interaction is important, since the three factors seem to interact for some consonants to obtain the intercept pattern that is roughly the reverse of the slope pattern. For instance, comparison of F2 averages for /d/ and /g/ EMMA data show that they have approximately the same pattern for the averages at C and V; however the slope difference between the two consonants results in the intercept for /d/ being higher in magnitude than the /g/.

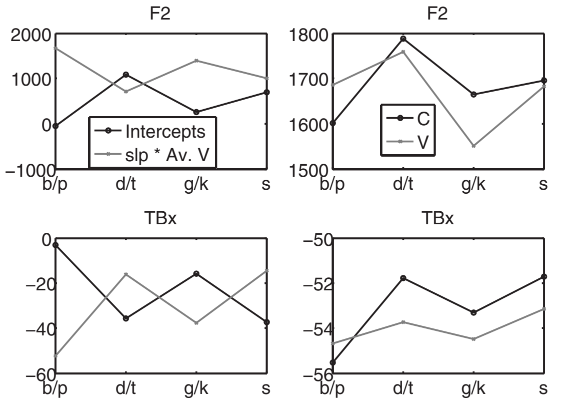

FIG. 3. Analysis of Intercepts into their constituents: slope and average of F2 or TBx at C and V according to Eq. (2) for XRMB data. b/p refers to the labial stop, d/t refers to the alveolar stop, and g/k refers to the velar stop. Frontness increases vertically.

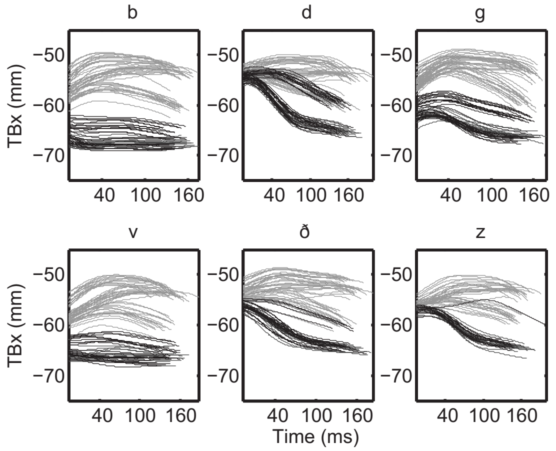

FIG. 4. Trajectories of TBx for consonants from EMMA data: For all consonants, the first articulatory sample in the time series corresponds to the acoustic frame in which the formant transition for the consonant begins. Trajectories in black are for the back vowel context, and trajectories in gray are for front vowel context. Frontness increases vertically.

5. TBx linear relation at closure

Figure 5 compares regression statistics between where C is measured at the middle of the consonant closure (black) vs. at the beginning of the formant transition (gray) for the EMMA data. The mean duration of mid closure to formant transition is 55 ms. The patterns for the slope, standard deviation ratio and intercept are the same for the two sets of relations, but the magnitude of the slope and standard deviation ratio are greater by about 0.2 for the regression using the consonant measure at the beginning of the transition, and the

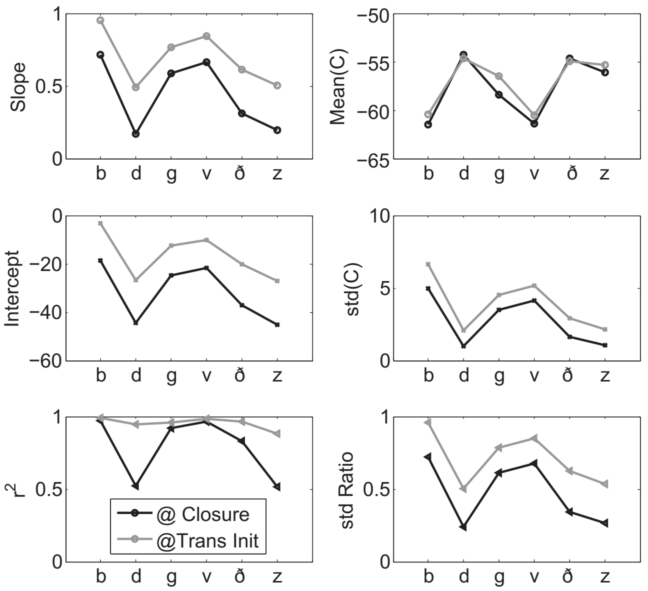

FIG. 5. Comparison of statistics of CV: TBx relation at the initial point of the formant transition and at the point of tightest closure in the consonant for EMMA data. Frontness increases vertically.

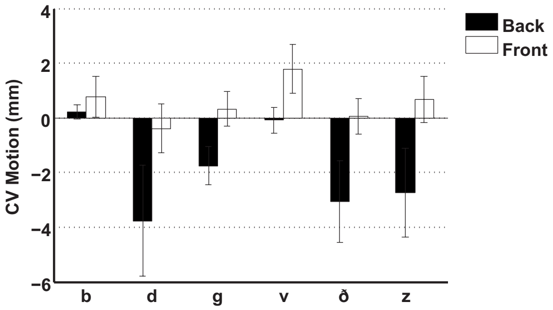

FIG. 6. Spatial effects in coarticulation: Means and standard deviations of change in TBx between initiation of C formant transition and middle of V for each consonant in EMMA data, separated into back vowels (black) and front vowels (white).

magnitude of the intercept is larger for the regression using the consonant measure at the middle of the closure. The explained variability r^2 for the labials and velar are approximately the same for both sets of relations, however, the explained variability for the alveolars is markedly lower. The latter effect suggests that the relation between TBx at the vowel and TBx at the consonant closure for alveolars is simply random. However, examination of the standard deviation of TBx at the consonant (mid panel of the right column of Fig. 5) shows that the standard deviation of TBx at the mid closure is extremely small, suggesting that it is indeed true that TBx at the alveolar consonant closure is not predictable from TBx at the vowel, but the former is extremely limited in variability and is certainly not random. Remarkably, the average TBx at the consonant hardly varies from the middle of the closure to the time at which the formant transition starts. However the standard deviation increases in that duration. This indicates that as time progresses, the vowels have greater and greater influence on TBx. Therefore the standard deviation increase indicates an increase in the influence of the vowel on the consonant. In summary, the basic *pattern* of predictability of C TBx from V TBx is approximately the same, but the *magnitudes* of the regression statistics do differ through an increase in standard deviation of TBx from the middle of the closure to the initiation of the transition.

6. Coarticulation degree and nonuniformity of coarticulation

We tested Sussman *et al.* (1998)'s claim that the linearity of locus equations is the result of nonuniformity of coarticulation across vowels for each consonant. This nonuniformity is required by the Orderly Output Constraint on speech production which is imposed to ensure linearity at the acoustic level (Fig. 14 of Sussman *et al.*, 1998). In particular, we measured spatial characteristics of coarticulation across the front and back vowels within each consonant. The measure is the amount of motion of TBx from the time of initiation of the F2 transition to the middle of the vowel in mm. Figure 6 shows the extent of motion of TBx separated into the front (white) and back (black) vowels. The tongue back exhibits somewhat less motion for the labials than for alveolars or velars. The most marked nonuniformity is between front and back vowels within velars and alveolars, where we see the back vowels exhibiting a great deal more motion than front vowels, i.e., coarticulation is indeed nonuniform, as predicted by Sussman *et al.* (1998).

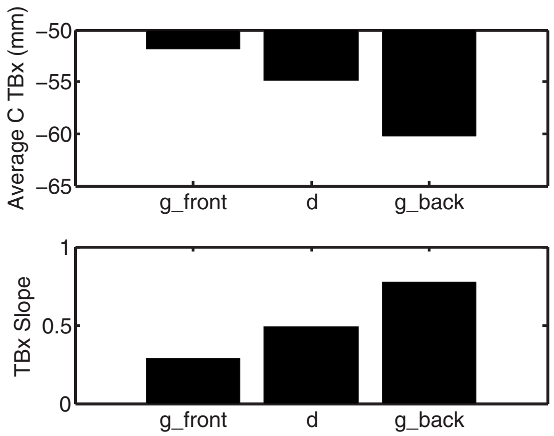

FIG. 7. EMMA velar data split by front and back vowels. Upper panel: Average TBx position at the beginning of F2 onset (c). Lower panel: TBx-based slopes.

7. Front and back velars

It is well-known that in English, and many other languages, there is variability in the location of the position of the constriction location of the velar, depending on surrounding vowels (Dembowski *et al.*, 1998). In terms of locus equations, the slope for the velar in the front vowel context is very low, close to or lower than for /d/, whereas in the back vowel context, the slope is very high, close to or higher than for /b/. As for intercepts, velars in the front vowel context have a very high intercept, close to or higher than for /d/, whereas in the back vowel context, they have an intercept that is lower than /d/. These generalizations tend to be robust despite a great deal of subject variability in the exact placement of slopes and intercepts for a given idiolect, which can be seen in examining the data in several studies (Sussman *et al.*, 1991; Brancazio and Fowler, 1998; Sussman *et al.*, 1993; Sussman, 1999; Guion, 1998; Modarresi *et al.*, 2004). As was done in these studies, the EMMA velars were split by whether they were in front (/i/ and /ei/) or back (/u/ and /a/). Figure 7 shows the average location of TBx at the onset of F2 for the stops and slopes calculated from the TBx values at the F2 onset and the midpoint of the vowel, with the velar split into the front and back contexts. The pattern for the slopes measured articulatorily is basically the same as that shown in the literature, measured acoustically. The average location of TBx at the onset of F2 is expected for the velars, since that location is exactly an indication that the back and front /g/ constriction locations are different form each other. What is more interesting is that the average location of TBx during /d/ is intermediate between those for the front and back /g/, as are the slopes. These results will be discussed in Sec. IV D.

IV. GENERAL DISCUSSION

A. TBx and F2

We have shown here that the linearity, slopes, and intercepts of locus equations are already present in the articulatory system in the relation between TBx at the consonant release and that at the vowel midpoint. The slopes and intercepts, and their second order relations (SOLE) are not exactly the same in the articulatory and acoustic data, of course, but the patterns across the consonants are approximately the same, as is evident in Fig. 1 and Tables I–IV. The statistical analysis in Table III shows that the linear relations are robust in both articulation and acoustics for a large number of speakers, and that, for slope, there is no significant difference between the articulatory and acoustic patterns. For intercepts, there was a significant difference between the articulatory and acoustic patterns, but the magnitude of the effect is 0.33 standard deviations, which is a small difference according to Cohen (1992)'s guidelines.

It has been previously argued that even though articulation has to play a role in realizing locus equations, the articulatory pattern cannot be linear (Sussman *et al.*, 1998), because the articulatory to acoustic transform is nonlinear. According to this view, the articulatory pattern needs to be nonlinear to compensate for the articulatory-to-acoustic nonlinearity, resulting in a linear pattern at the acoustic level. This view is not supported by the data provided here, which shows that the linearity is indeed already present in the articulatory system. How can this be, given that the articulatory-to-acoustic transform is nonlinear? Even though the F2 sensitivity function relating the placement of a constriction to the value of F2 is highly nonlinear, it is locally linear over considerable portions of the vocal tract. Specifically, the maximal advancement of the tongue back from the consonant to the vowel is over a portion of less than 1 cm across subjects, as can be seen in Fig. 6. Over such a distance, the articulatory-to-acoustic transform is locally linear.[2] Indeed, the portion of the vocal tract involved is likely to be somewhere between $\frac{1}{3}$ to $\frac{2}{3}$ of the length of the vocal tract, a region that would extend for about 5 cm for an adult male and 4 cm for an adult female, for which the sensitivity function is linear, as can be seen by inspection in Fant (1960)'s nomograms (his Fig. 1.4–11.a). Anterior-posterior motion of the tongue in that region, as reflected in TBx change between C and V, would cause a proportional change in F2, whether the speaker is a male or a female. Therefore the notion that acoustic locus equation linearity, slopes, and intercepts reflects articulation is not as unexpected as it may seem. The Orderly Output Constraint was posited by Sussman *et al.* (1998) to ensure that there is a nonlinear pattern at the articulatory system level to compensate for the nonlinearity of the articulatory-to-acoustic transform, ensuring linearity at the acoustic level. The present study shows, in contrast, that there is no need for such a constraint, because the linearity is already present at the articulatory level.

B. Coarticulation resistance and articulatory synergy

The statistical formulas for slopes in terms of the correlation coefficient and standard deviations show a direct link between slopes and normalized coarticulation resistance. The data in Table V confirm that the pattern of ratios of standard deviations, the measure of normalized coarticulation resistance, is the same as the pattern of slopes, indicating that the correlation coefficient, which is highly dependent on the data set, does play a role in determining the magnitude of the slope, but that the pattern of slopes is already present in the coarticulation resistance measure. Therefore the magnitude of the slope is a direct reflection of the degree to which each consonant resists coarticulation with the vowel. The demonstration that F2 locus equation linearity is already present in a TBx linear relation implicates constraints on the tongue body as the cause for the different coarticulation resistance values, and hence slopes, for the different consonants. The negative relation between TBx and F2 evident in the correlation coefficients relating the two domains (Table II) is due to the fact that an anterior tongue position leads to a shorter front cavity and a higher F2.

Coarticulation resistance, which is implicated here as an explanation for the slope pattern, is itself, however, a derivative concept in speech production. It derives from "the mechanico-inertial properties of the articulators and their involvement in the formation of a closure or constriction and with manner of articulation demands," according to Recasens and Espinosa (2009) (p. 2288). To understand why the consonants examined have varying degrees of coarticulation resistance and regression slopes, it is necessary to further delve into the articulatory source of intercepts.

The statistical formulas (2) for intercepts in terms of the average TBx or F2 at the consonant relative to the slope-scaled average of TBx or F2 at the vowel contain the link between slopes and intercepts encoded in the SOLE relations. That is, this statistical expansion of the intercept demonstrates that it is related to the slope, when the averages of the dependent and independent variables are fixed. Figures 2 and 3 show the extent to which this expansion accounts for the patterns in both the acoustic and articulatory data. These figures also show that approximately the same basic pattern is evident in the average F2 and TBx at C and V. But why should that be?

The answers to the two questions posed in this section are to be found in the same articulatory source. Examination of Fig. 4 illustrates the basic asymmetry between alveolar consonants and the rest. For alveolar consonants the tongue is advanced for the consonant even at the first evidence of a formant transition into the vowel. This advancement occurs because the tongue tip and blade are very difficult to manipulate on their own. There is no muscular sling, that is, that descends from the alveolar ridge to lift it up. For it to advance to the alveolar ridge, the tongue back must be pushed forward in order to advance the tip. Therefore even though the contrastive aspect of alveolars involves activity in the alveolar region, the advancement of the tongue back to help the tongue tip is essential to accomplishing the linguistically relevant goal. Tongue body advancement to assist alveolar closure is an aspect of the articulatory *synergy* for accomplishing the alveolar task (Saltzman and Munhall, 1989; Manuel and Stevens, 1995). The most direct effect of this tongue advancement is that the average TBx at the consonant for the alveolars has to be significantly forward. This is not true at all for labials, because the tongue is not necessary to help lip closure, and is somewhat true for velars that do necessitate some tongue body advancement to assist the dorsum in making a velar closure. However the advancement of the tongue for velars is less extreme than for the alveolars, simply because the velar place of articulation is already in a back position in the vocal tract. But as is known in the literature, the velar slope and intercept values are an average for those of the back and front velars. Section IV D will discuss the velar data in detail. The expected pattern for the average TBx at C, based on these basic articulatory synergies for articulation, can be seen for the EMMA data in Fig. 2 and for the XRMB data in Fig. 3: alveolar > velar > labial, that is, the pattern of the intercepts. It is also the pattern for the average TBx at the vowel. If the tongue body is maximally advanced for the alveolar consonant, it is expected that the following vowel will be more fronted than usual, raising the average value for TBx at the vowel. This therefore explains why the TBx average pattern is the same at C and V.

Therefore the synergy for consonantal closure involves maximal tongue body advancement for alveolars, intermediate advancement for velars, and no advancement for labials. This also appears to be the origin of the pattern of the normalized coarticulation resistance measures. If the tongue needs to be highly fronted for alveolars, that enforces a very strong constraint on the tongue body prohibiting the flanking vowel from using it, i.e., articulatory synergy is the origin of coarticulation resistance. The tongue body for velars offers less resistance, and the labials even less, because the synergy for labial closure formation does not require the tongue. The involvement of the tongue in the synergies for the various consonants therefore explains how resistant they are to coarticulation, leading to the particular slopes that each has. Since the form of the synergy is in terms of tongue advancement, the pattern of intercepts and averages is also explained. The greater the advancement of the tongue back to assist the formation of the consonant closure, the higher the magnitude of the intercept, the higher the coarticulation resistance, the lower the slope (which inversely measures the resistance) and the more advanced the tongue back is during the following vowel. The various correlations therefore come from one source: articulatory synergy for consonant closure. Recasens and Espinosa (2009) refer to two physical reasons for an increase in coarticulation resistance: 1) mechanico-inertial properties, that is, constant physical properties of an articulator that make it hard to move; 2) involvement of an articulator in formation of a closure, that is, soft constraints on an articulator that vary as the segments vary. Synergistic use of the tongue body to help the tongue tip achieve its closure is a physical cause of coarticulation resistance of the second type.

The data in Fig. 5 illustrate that the linearity for most consonants does not arise only when the consonant is measured at the initiation of the formant transition. The linear relation for predicting consonant TBx from vowel TBx is present even when the measurement of the consonant is made at mid closure. Alveolars are an exception in that the explained variability of the relation is low for the relation computed with the consonant measured at its closure; however this is due to lack of predictability of C TBx from V TBx, which is what a low slope refers to. It does not mean that the relation between the two is random. That is, independence of TBx at the consonant from TBx at the vowel for alveolars is expressed as a low slope for the consonant measured at the initiation of the transition and is expressed as a low r^2 when the consonant is measured at mid closure. There is therefore no support to the idea that a nonlinear articulatory relation is transformed to a linear relation between TBx at the initiation of the F2 transition and F2 at the vowel. The low predictability for alveolars, whether it is expressed as a low slope or low r^2, higher predictability for the velar, and highest predictability for the labial are apparent in articulatory measurements made during the closure.

C. Coarticulation nonuniformity and articulatory synergy

The results on nonuniformity of coarticulation presented in Fig. 6 do not provide support for an Orderly Output Constraint, but do provide support for linking locus equations to tongue body synergy. The spatial asymmetry evident in Fig. 6 is due to the greater motion for back vowels than for front vowels for alveolars and the velar. For the alveolars, the tongue dorsum advances to help the tongue tip. Therefore for a following back vowel, it has to move backward a great deal to achieve the required position for the vowel, from the position required by the synergy for alveolars. And for the velar, as is well known for American English, the tongue dorsum contacts the hard palate more anteriorly before a front vowel than a back vowel reducing the distance the dorsum has to move preceding a front vowel. Sussman *et al.* (1998) are therefore correct in positing that there is nonuniformity in coarticulation across vowels; however that nonuniformity does not seem to be imposed by an extra constraint on speech production requiring it to impose a pattern of nonuniformity to achieve linearity of locus equations at the acoustic level. The nonuniformity is predictable from facts about the synergistic use of the tongue body and the position of the tongue body for front vowels vs. back vowels.

D. Synergy and the velar

Sussman (1999) proposed that the pattern of slopes and intercepts for the front and back velars along with the alveolar /d/ are the result of a special skill in articulation learned just to realize that pattern, and also proposed a special brain circuit for detecting the pattern. We argue that the data in Fig. 7 show that there is no necessity for a specially learned articulatory maneuver to realize the pattern. The pattern of slopes simply follows the pattern of the location of the tongue back during each of the consonants. The front velar is articulated with a highly fronted tongue body, most probably achieved with a part of the tongue that is below the hard palate. It is especially important here that the position of the tongue back for the front /g/ is more anterior than for /d/. As with the tip, there is no way for this part of the tongue to move without the help or *synergy* of other organs. The tongue back has to move frontward and upwards to assist the palatal portion in rising to make a contact. This results in a delay in the start of the vowel, which is effectively registered as higher coarticulation resistance, lower coarticulation degree and lower slope. Therefore no special articulatory mechanisms are required to ensure that the /d/ slope and intercept are exactly between that of the front and back velar. The slopes, coarticulation resistance, and coarticulation degree fall out of the positioning of the tongue required by the synergy to achieve the consonants. In English, there are no palatal consonants, but we predict that in languages that do have them, their slopes would be very low and their intercepts very high, as we see in the front velar in English.

E. How to use locus equations?

Locus equations are a common method for assessing coarticulation. It has been used to study speech development and various atypical populations (Robb and Blomgren, 1997; Baillargeon *et al.*, 2002; Gibson and Ohde, 2007). The results of this study provide a rationale for this use. Specifically, the results show that locus equations are a measure of differences among consonants in the synergistic use of the tongue body. This motor aspect of speech production, how articulators cooperate to achieve linguistic tasks (Saltzman and Munhall, 1989), is a fundamental aspect of coarticulation, and locus equations are an acoustic signature of the synergetic use of a particular organ, the tongue body. Two articulatory phenomena, "coarticulation resistance" and "coarticulation degree," have been correlated with locus equations, and the present results provide no evidence against the use of locus equations to measure these phenomena. However, there is evidence that articulator synergy is a more *primitive* articulatory phenomenon that can explain both the pattern of coarticulation resistance and coarticulatory degree observed among the consonants examined. It remains to be seen whether articulator synergy can also account for the high coarticulatory resistance seen in dark /l/, pharyngealized dentoalveolars, and alveolar trills.

Use of locus equations to measure tongue body synergy could be made to compare the pattern of slopes and intercepts for speakers of different languages or for a typical and atypical population. If the locus equation slope is low for a consonant, the interpretation is that the tongue back is crucial for the achievement of that consonant. This explains why less prominent or hypoarticulated speech leads to higher slopes. If synergistic use of the tongue back to help the tip or dorsum is reduced in less prominent speech or fast speech, then slopes would automatically be higher.

The present study generated no evidence that supports the use of locus equations as invariant or relationally invariant cues for stop location. There is of course a relation between the place of primary constriction of a consonant and whether the tongue back assists in the achievement of that constriction, but this relation is not direct enough to allow for the use of locus equations as a direct measure of place.

V. CONCLUSION

Speech production measures provide evidence for an articulatory basis for locus equations. The function relating the tongue body in the consonant to the vowel is linear, and the pattern of statistics describing the fit is the same as for locus equations. Moreover it was shown that the SOLE relation between slopes and intercepts also emerges from the TBx relations. Statistical arguments, demonstrated through EMMA and XRMB data, show that the slope and intercepts can be seen to be intimately linked to the concept of coarticulation resistance. However, the more basic phenomenon of articulator synergy explains the pattern of resistance observed among the consonants. It was also shown that nonuniformity in coarticulation across the vowels, and the pattern of coarticulation degree, are explainable through the same facts of motor synergy. This study therefore supports using the pattern of locus equation slopes and intercepts to measure the synergistic use of the body of the tongue in accomplishing consonant goals.

ACKNOWLEDGMENTS

This work was supported by NIH-NIDCD grant DC-002717 to Haskins Laboratories. We thank David Berry and two anonymous reviewers for many helpful suggestions. We also thank Anders Löfqvist, Larry Brancazio, and Tine Mooshammer, for helpful discussions and thoughtful comments.

[1] Lip aperture has a major effect on F2, especially due to back vowels, but lip aperture also affects F1 and F3. And it is already known that locus equation relations do not exist for F1 and F3 (Sussman *et al.*, 1998), therefore only horizontal position of the tongue is examined in the experiments.

[2] F3 is not locally linear in that region, which may indeed be why that there is no locus equation for F3. However this does not explain why there is no locus equation for F1.

Baayen, H. (**2007**). *Analyzing Linguistic Data: A Practical Introduction to Statistics* (Cambridge University Press, Cambridge, England) pp. 241–284.

Baillargeon, M., McLeod, A., Metz, D. E., Schiavetti, N., and Whitehead, R. (**2002**). "Preservation of second formant transitions during simultaneous communication: A locus equation perspective," J. Speech Lang. Hear. Res. **35**, 51–62.

Bakran, J., and Mildner, V. (**1995**). "Effect of speech rate and coarticulation strategies on the locus equation determination," in *Proceedings of the XIV ICPHS*, edited by K. Elenius and P. Branderud, Vol. **1**, pp. 26–29 (KTH and Stockholm University, Stockholm).

Bladon, A., and Al-Bamerni, A. (**1976**). "Coarticulation resistance in English /l/," J. Phonetics **4**, 137–150.

Brancazio, L., and Fowler, C. (**1998**). "On the relevance of locus equations for production and perception of stop consonants," Percept. Psychophys. **60**, 24–50.

Chennoukh, S., Carré, R., and Lindblom, B. (**1997**). "Locus equations in the light of articulatory modeling," J. Acoust. Soc. Am. **102**, 2380–2389.

Cohen, J. (**1992**). "A power primer," Psychological Bulletin **112**, 155–159.

Delattre, P. (**1951**). "The physiological interpretation of sound spectrograms," Publications of the Modern Language Association of America **66**, 864–875.

Dembowski, J., Lindstrom, M., and Westbury, J. (**1998**). "Articulator point variability in the production of stop consonants," in *Neuromotor Speech Disorders*, edited by M. P. Cannito, K. M. Yorkston, and D. R. Beukelman (Paul H. Brookes, Baltimore), pp. 27–46.

Deng, L. (**2006**). *Dynamic Speech Models* (Morgan and Claypool, San Rafael, CA) pp. 11–12.

Duez, D. (**1992**). "Second-formant locus patterns: An investigation of spontaneous French speech," Speech Commun. **11**, 417–427.

Fant, G. (**1960**). *Acoustic Theory of Speech Production* (Mouton, The Hague).

Fowler, C. (**1994**). "Invariants, specifiers, cues: An investigation of locus equations as information for place of articulation," Percept. Psychophys. **55**, 597–610.

Fowler, C. (**2005**). "Parsing coarticulated speech in perception: Effects of coarticulation resistance," J. Phonetics **33**, 199–213.

Fowler, C., and Brancazio, L. (**2000**). "Coarticulation resistance of American English consonants and its effects on transconsonantal vowel-to-vowel coarticulation," Lang Speech **43**, 1–41.

Gibson, T., and Ohde, R. (**2007**). "F2 locus equations: Phonetic descriptors of coarticulation in 17- to 22-month-old children," J. Speech Lang. Hear. Res. **50**, 97–108.

Guion, S. G. (**1998**). "The role of perception in the sound change of velar palatalization," Phonetica **55**, 18–52.

Hayduk, L. (**1987**). *Structural Equation Modeling with LISREL: Essentials and Advances* (Johns Hopkins University Press, Baltimore).

Joos, M. (**1948**). *Acoustic Phonetics*, Language Monographs, No. 2 (Suppl. 24) (University of Chicago Press, Chicago) pp. 58–59.

Krull, D. (**1987**). "Second formant locus patterns as a measure of consonant-vowel coarticulation," PERILUS **5**, 43–61.

Krull, D. (**1989**). "Consonant-vowel coarticulation in spontaneous speech and reference words," PERILUS **10**, 101–105.

Lindblom, B. (**1963**). "Spectrographic study of vowel reduction," J. Acoust. Soc. Am. **35**, 1773–1781.

Lindblom, B., Agwuele, A., Sussman, H., and Cortes, E. E. (**2007**). "The effect of emphatic stress on consonant vowel coarticulation," J. Acoust. Soc. Am. **121**, 3802–3813.

Lindblom, B., and Sussman, H. M. (**2004**). "Articulatory and acoustic bases of locus equations," in Proceedings of FONETIK, pp. 8–16.

Löfqvist, A. (**1999**). "Interarticulator phasing, locus equations and degree of coarticulation," J. Acoust. Soc. Am. **106**, 2022–2030.

Manuel, S. Y., and Stevens, K. N. (**1995**). "Formant transitions: Teasing apart consonant and vowel contributions," in *Proceedings of the XIII ICPHS*, edited by K. Elenius and P. Branderud (KTH and Stockholm University, Stockholm), pp. 436–439.

Modarresi, G., Sussman, H., Lindblom, B., and Burlingame, E. (**2004**). "Stop place coding: An acoustic study of CV, VC, and CV sequences," Phonetica **61**, 2–21.

Nearey, T., and Shammass, S. (**1987**). "Formant transitions are partly distinctive invariant properties in the identification of voiced stops," Can. Acoust. **15**, 17–24.

Recasens, D. (**1985**). "Coarticulatory patterns and degrees of coarticulatory resistance in Catalan CV sequences," Lang Speech **28**, 97–114.

Recasens, D., and Espinosa, A. (**2009**). "An articulatory investigation of lingual coarticulatory resistance and aggressiveness for consonants and vowels in Catalan," J. Acoust. Soc. Am. **125**, 2288–2298.

Robb, M., and Blomgren, M. (**1997**). "Analysis of F2 transitions in the speech of stutterers and nonstutterers," J. Fluency Disord. **22**, 1–16.

Saltzman, E., and Munhall, K. (**1989**). "A dynamical approach to gestural patterning in speech production," Ecological Psychol. **1**, 333–382.

Stevens, K. (**1998**). *Acoustic Phonetics* (MIT, Cambridge, MA) pp. 323–487.

Sussman, H. (**1999**). "A neural mapping hypothesis to explain why velar stops have an allophonic split," Brain Lang **70**, 294–304.

Sussman, H. M., Fruchter, D., Hilbert, J., and Sirosh, J. (**1998**). "Linear correlates in the speech signal: The orderly output constraint," Behav. Brain Sci. **21**, 241–299.

Sussman, H. M., Hoemeke, K. A., and Ahmed, F. S. (**1993**). "A cross-linguistic investigation of locus equations as a phonetic descriptor for place of articulation," J. Acoust. Soc. Am. **94**, 1256–1268.

Sussman, H. M., McCaffrey, H. A., and Mathews, S. A. (**1991**). "An investigation of locus equations as a source of relational invariance for stop place categorization," J. Acoust. Soc. Am. **90**, 1309–1325.

Tabain, M. (**2000**). "Coarticulation in CV syllables: A comparison of locus equation and EPG data," J. Phonetics **28**, 137–159.

Westbury, J. (**1994**). *Xray Microbeam Speech Production Database Users Handbook* (University of Wisconsin, Madison, WI).

Yule, G. U. (**1897**). "On the theory of correlation," J. R. Stat. Soc. **60**, 812–854.