

# Logical Development Of Vogel's Approximation Method (LD-VAM): An Approach To Find Basic Feasible Solution Of Transportation Problem

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**Abstract:** The study in this paper is to discuss the limitation of Vogel's Approximation Method (VAM) and developed an improved algorithm after resolving this limitation for solving transportation problem. Vogel's Approximation Method (VAM) is the more efficient algorithm to solve the transportation problem but it has some limitations when highest penalty cost appear in two or more row or column. For that case VAM does not give any logical solution. In this paper we stand a logical approach for this problem and developed an algorithm named by "Logical Development of Vogel's Approximation Method (LD-VAM)" where feasible solution from this method are very close to optimal solution more than VAM.

**Index Terms:** Transportation Problem (TP), LD-VAM, VAM, LPP, Penalty, Transportation Cost.

## 1 INTRODUCTION

The Transportation Problem is the special class of Linear Programming Problem in the field of Applied Mathematics and also in Operation Research. There are some existing algorithms to solve the Transportation Problem such as North West Corner Rule (NWC), Least Cost Method (LCM), and Vogel's Approximation Method (VAM) etc. where Vogel's Approximation Method (VAM) is known as more efficient algorithm to find the basic feasible solution. But VAM has some limitations which are discussed in (2.2) and we fixed this problem and developed a new algorithm in (2.3). To solving Transportation Problem first a fall we have to formulate original problem as a Linear Programming Problem (LPP) for  $m$  sources and  $n$  destinations.

Each  $i^{\text{th}}$  source has a capacity of supply amount that is  $s_i$  and  $d_j$  be the demand amount of each  $j^{\text{th}}$  destination and  $c_{ij}$  be the cost of  $i^{\text{th}}$  source to  $j^{\text{th}}$  destination. The ultimate goal of solving Transportation Problem is to find the amount of product  $x_{ij}$  which will be transfer from  $i^{\text{th}}$  source to  $j^{\text{th}}$  destination so that total transportation cost will be minimized. The Linear Programming Problem (LPP) representing the Transportation problem for  $m$  sources and  $n$  destinations are generally given as:

$$\text{Minimize: } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to:

$$\sum_{j=1}^n x_{ij} \leq s_i, \quad \text{for } i=1,2,3,\dots,m$$

$$\sum_{i=1}^m x_{ij} \geq d_j, \quad \text{for } j=1,2,3,\dots,n$$

$$x_{ij} \geq 0, \quad \text{for all } i, j$$

To applying transportation algorithm for solving TP we have to make a Mathematical model for this LPP.

## 2 METHODOLOGY:

Initially we mention that there are some existing methods for solving transportation problem such as North West Corner Rule (NWC), Least Cost Method (LCM), and Vogel's Approximation Method (VAM) etc. In this section we discuss about Vogel's Approximation Method (VAM) and our proposed method Logical Development of Vogel's Approximation Method (LD-VAM).

### 2.1 EXISTING ALGORITHM FOR VOGEL'S APPROXIMATION METHOD (VAM):

The Vogel's Approximation Method (VAM) is an iterative procedure for computing a basic feasible solution of a transportation problem. This method is better than other two methods i.e. North West Corner Rule (NWC) and Least cost Method (LCM), because the basic feasible solution obtained

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by this method is nearer to the optimal solution. The existing algorithm for Vogel's Approximation Method (VAM) is given below:

**Step-1:** Identify the boxes having minimum and next to minimum transportation cost in each row and write the difference (Penalty) along the side of the table against the corresponding row.

**Step-2:** Identify the boxes having minimum and next to minimum transportation cost in each column and write the difference (Penalty) along the side of the table against the corresponding column. If minimum cost appear in two or more times in a row or column then select these same cost as a minimum and next to minimum cost and penalty will be zero.

**Step-3:**

- Identify the row and column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least cost in the selected row or column. Adjust the supply and demand and cross out the satisfied row or column. If a row and column are satisfied simultaneously, only one of them is crossed out and remaining row or column is assigned a zero supply or demand.
- If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

**Step-4:**

- If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.
- If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.
- If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.
- Otherwise, go to Step-1.

## 2.2 FINDING LIMITATIONS OF VOGEL'S APPROXIMATION METHOD (VAM):

In VAM approach, Penalties are determined by the difference of two minimum costs of each row and each column. The highest penalty indicates that among the two minimum cost, one is the lowest and another is too high. In VAM algorithm select such row or column which contains largest penalty to ensure the least cost in current iteration and avoiding the chance of taking larger cost in next iteration. If highest penalty cost appear in two or more row or column, the VAM algorithm selects any one of them (or select most top row or extreme left column) [(2.) Step-3(b)]. But it is not necessarily true that largest penalty always ensure the lowest cost because difference of two pairs of numbers can be equal when one of the pair is smaller than another pair. As an example, difference of 10 and 5 and difference of 7 and 2 are equal but lowest numbers exist in second pair. For that reason we can say that in VAM algorithm, if the lower pair does not appear in the top most or extreme left position then lowest cost will not be selected in the current iteration so that total transportation cost may not be minimized. For that case VAM may not be given

the lowest feasible solution.

## 2.3 PROPOSED ALGORITHM FOR LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):

From the above discussion of the limitations of Vogel's Approximation Method (VAM) we fixed this problem and proposed an improved algorithm which is named by "Logical Development of Vogel's Approximation Method (LD-VAM)". In this algorithm, we resolved the above discussed problem of VAM *i.e.*, when largest penalty appear in two or more rows or columns then select that row or column among them which contains the least cost and give the maximum possible allocation. The algorithm of LD-VAM is given below: Set  $s_i$  be supply amount of the  $i^{th}$  source and  $d_j$  be the amount of demand of  $j^{th}$  destination and  $c_{ij}$  be the unit transportation cost of  $i^{th}$  source to  $j^{th}$  destination.

**Step-1:** Check: *if*  $s_i < 0$  *and*  $d_j < 0$  *then* Stop.

**Step-2:** *if*  $\sum_i s_i > \sum_j d_j$  *or* *if*  $\sum_i s_i < \sum_j d_j$  *then*

balance the transportation problem adding dummy demand or supply.

**Step-3:**

- Identify the smallest and next to smallest cost of each row and column and calculate the difference between them which is called by penalty. Set  $p_i$  be the row penalty and  $p_j$  be the column penalty.

$$p_i = |c_{ih} - c_{ik}| \quad \text{and} \quad p_j = |c_{hj} - c_{kj}|$$

- If smallest cost appear in two or more times in a row or column then select these same cost as a smallest and next to smallest cost and penalty will be zero.

$$p_i = |c_{ih} - c_{ik}| = 0 \quad \text{and} \quad p_j = |c_{hj} - c_{kj}| = 0$$

**Step-4:** Select  $\max(p_i, p_j)$ . Select lowest cost of that row or column which has largest penalty and allocate maximum possible amount  $x_{ij}$  *i.e.*,  $\min(s_i, d_j)$ . If the lowest cost appears in two or more cells in that row or column then choose the extreme left or most top lowest cost cell.

**Step-5:** If tie occurs in the largest penalties in some rows or columns then select that row or column which contains least cost among them.

**Step-6:** Adjust the supply and demand and cross out the satisfied row or column. If row and column are satisfied simultaneously then crossed out one of them and set zero supply or demand in remaining row or column.

**Step-7:**

- If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.

- b. If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.
- c. If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.
- d. Otherwise go to Step-3.

**3 NUMERICAL ILLUSTRATION:**

Consider some special types of Transportation Problems where highest penalty appear in two or more rows or columns and solve these using Vogel's Approximation Method (VAM) and the proposed method Logical Development of Vogel's Approximation Method (LD-VAM) and compare these results with optimal solution for measuring accuracy.

**3.1 EXAMPLE-1:**

Consider a Mathematical Model of a Transportation Problem in below:

**Table-1.1:**

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
<b>Demand</b>	<b>60</b>	<b>40</b>	<b>100</b>	<b>50</b>	<b>90</b>	

Now we find the feasible solution of this problem using LD-VAM and VAM respectively in below:

**3.1.1 SOLUTION OF EXAMPLE-1 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Iteration-1:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	3
S2	7	9	8	10	4	80	3
S3	9	3 40	7	10	6	30	3
S4	11	4	8	3	9	90	1
<b>Demand</b>	<b>60</b>		<b>100</b>	<b>50</b>	<b>90</b>		
<b>Column Penalty</b>	2	1	1	2	2		

In Iteration-1, the largest penalty appears in three times in S1, S2, S3 rows, but lowest cost appear in (S3, D2) cell. As per LD-VAM algorithm allocate 40 amount commodity in (S3, D2) cell.

**Iteration-2:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	4
S2	7	9	8	10	4	80	3
S3	9	3 40	7	10	6	30	1
S4	11	4	8	3 50	9	40	5
<b>Demand</b>	<b>60</b>		<b>100</b>		<b>90</b>		
<b>Column Penalty</b>	2		1	2	2		

**Iteration-3:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	1
S2	7	9	8	10	4 80		3
S3	9	3 40	7	10	6	30	1
S4	11	4	8	3 50	9	40	1
<b>Demand</b>	<b>60</b>		<b>100</b>		<b>10</b>		
<b>Column Penalty</b>	2		1		2		

**Iteration-4:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	1
S2	7	9	8	10	4 80		
S3	9	3 40	7	10	6 10	20	1
S4	11	4	8	3 50	9	40	1
<b>Demand</b>	<b>60</b>		<b>100</b>				
<b>Column Penalty</b>	1		1		3		

**Iteration-5:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	1
S2	7	9	8	10	4 80		
S3	9	3 40	7	10	6 10	20	2
S4	11	4	8 40	3 50	9		3
<b>Demand</b>	<b>60</b>		<b>60</b>				
<b>Column Penalty</b>	1		1				

**Iteration-6:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13	100	1
S2	7	9	8	10	4	80	
S3	9	3	7	10	6		2
S4	11	4	8	3	9		
Demand	60		40				
Column Penalty	1		2				

**Iteration-7:**

Source	Destination					Supply	Row Penalty
	D1	D2	D3	D4	D5		
S1	10	8	9	5	13		
S2	7	9	8	10	4	80	
S3	9	3	7	10	6		
S4	11	4	8	3	9		
Demand	0						
Column Penalty							

In Iteration-6, only one row has remains with positive supply and demand amount then as per **LD-VAM** algorithm allocates these by Least Cost Method. The final feasible solution table is given below:

**Table-1.2:**

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	10	8	9	5	13	100
S2	7	9	8	10	4	80
S3	9	3	7	10	6	70
S4	11	4	8	3	9	90
Demand	60	40	100	50	90	

**Total Transportation Cost (Using LD-VAM):**

$$(10 \times 60) + (9 \times 40) + (4 \times 80) + (3 \times 40) + (7 \times 20) + (6 \times 10) + (8 \times 40) + (3 \times 50) = 2070$$

**3.1.2 SOLUTION OF EXAMPLE-1 USING VOGEL'S APPROXIMATION METHOD (VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-1.3:**

Source	Destination					Supply	Row Penalty					
	D1	D2	D3	D4	D5							
S1	10	8	9	5	13	100	3	1	1	1	1	1
S2	7	9	8	10	4	80	3	3	3			
S3	9	3	7	10	6	70	3	3	1	1	2	2
S4	11	4	8	3	9	90	1	4	1	1	3	
Demand	60	40	100	50	90							
Column Penalty	2	1	1	2	2							
	2	1	1		2							
	2		1		2							
	1		1		3							
	1		1									
	1		2									

**Total Transportation Cost (Using VAM):**

$$(10 \times 50) + (5 \times 50) + (4 \times 80) + (9 \times 10) + (7 \times 50) + (6 \times 10) + (4 \times 40) + (8 \times 50) = 2130$$

**Observation:** We observed that VAM provides feasible solution for Example-1 is 2130 and LD-VAM provides 2070 which is lower than VAM.

**3.2 Example-2:**

Consider a Mathematical Model of a Transportation Problem in below:

**Table-2.1:**

Source	Destination				Supply
	D1	D2	D3	D4	
S1	7	5	9	11	30
S2	4	3	8	6	25
S3	3	8	10	5	20
S4	2	6	7	3	15
Demand	30	30	20	10	

Now we find the feasible solution of this problem using **LD-VAM** and **VAM** respectively in below.

**3.2.1 SOLUTION OF EXAMPLE-2 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-2.1:**

Source	Destination				Supply	
	D1	D2	D3	D4		
S1	5	5	20	11	30	
S2	4	25	8	6	25	
S3	20	3	8	10	20	
S4	5	2	6	7	10	15
Demand	30	30	20	10		

**Total Transportation Cost (Using LD-VAM):**

$$(7 \times 5) + (5 \times 5) + (9 \times 20) + (3 \times 25) + (3 \times 20) + (2 \times 5) + (3 \times 10) = 415$$

**3.2.2 SOLUTION OF EXAMPLE-2 USING VOGEL'S APPROXIMATION METHOD (VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-2.2:**

Source	Destination				Supply		
	D1	D2	D3	D4			
S1	7	30	5	9	11	30	
S2	25	4	0	3	8	6	25
S3	5	3	8	5	10	10	20
S4	2	6	15	7	3	15	
Demand	30	30	20	10			

**Total Transportation Cost (Using VAM):**

$$(5 \times 30) + (4 \times 25) + (3 \times 0) + (3 \times 5) + (10 \times 5) + (5 \times 10) + (7 \times 15) = 470$$

**Observation:** We observed that VAM provides feasible

solution for Example-2 is 470 and LD-VAM provides 415 which is lower than VAM.

**3.3 EXAMPLE-3:**

Consider a Mathematical Model of a Transportation Problem in below:

**Table-3.1:**

Source	Destination						Supply
	D1	D2	D3	D4	D5	D6	
S1	12	4	13	18	9	2	120
S2	9	16	10	7	15	11	80
S3	4	9	10	8	9	7	50
S4	9	3	12	6	4	5	90
S5	7	11	5	18	2	7	100
S6	16	8	4	5	1	10	60
Demand	75	85	140	40	95	65	

**3.3.1 SOLUTION OF EXAMPLE-3 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-3.2:**

Source	Destination						Supply
	D1	D2	D3	D4	D5	D6	
S1	12	4	13	18	9	2	120
S2	9	16	10	7	15	11	80
S3	4	9	10	8	9	7	50
S4	9	3	12	6	4	5	90
S5	7	11	5	18	2	7	100
S6	16	8	4	5	1	10	60
Demand	75	85	140	40	95	65	

**Total transportation Cost:**

$$(4 \times 55) + (2 \times 65) + (9 \times 25) + (10 \times 40) + (7 \times 15) + (4 \times 50) + (3 \times 30) + (6 \times 25) + (4 \times 35) + (5 \times 100) + (1 \times 60) = 2220$$

**3.4 EXAMPLE-4:**

Consider a Mathematical Model of a Transportation Problem in below:

**Table-4.1:**

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6	1	9	3	70
S2	11	5	2	8	55
S3	10	12	4	7	90
Demand	85	35	50	45	

**3.4.1 SOLUTION OF EXAMPLE-4 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-4.2:**

Source	Destination				Supply
	D1	D2	D3	D4	
S1	6 35	1 35	9	3 35	70
S2	11 5	5	2 50	8	55
S3	10 80	12	4	7 10	90
Demand	85	35	50	45	

Total transportation Cost:

$$(1 \times 35) + (3 \times 35) + (11 \times 5) + (2 \times 50) + (10 \times 80) + (7 \times 10) = 1165$$

**3.5 EXAMPLE-5:**

Consider a Mathematical Model of a Transportation Problem in below:

**Table-5.1:**

Source	Destinations							Supply
	D1	D2	D3	D4	D5	D6	D7	
S1	12	7	3	8	10	6	6	60
S2	6	9	7	12	8	12	4	80
S3	10	12	8	4	9	9	3	70
S4	8	5	11	6	7	9	3	100
S5	7	6	8	11	9	5	6	90
Demand	20	30	40	70	60	80	100	

**3.5.1 SOLUTION OF EXAMPLE-4 USING LOGICAL DEVELOPMENT OF VOGEL'S APPROXIMATION METHOD (LD-VAM):**

Costs are indicating in right-top corner and allocations are indicating in bottom-left corner.

**Table-5.2:**

Source	Destinations							Supply
	D1	D2	D3	D4	D5	D6	D7	
S1	12	7 20	3 40	8 0	10	6	6	60
S2	6 20	9 0	7	12	8 60	12	4 0	80
S3	10	12	8	4 70	9	9	3	70
S4	8	5	11	6	7	9	3 100	100
S5	7	6 10	8	11	9	5 80	6	90
Demand	20	30	40	70	60	80	100	

**Total transportation Cost:**

$$(7 \times 20) + (3 \times 40) + (8 \times 0) + (6 \times 20) + (9 \times 0) + (8 \times 60) + (4 \times 0) + (4 \times 70) + (3 \times 100) + (6 \times 10) + (5 \times 80) = 1900$$

**4 RESULT ANALYSIS:**

In above two examples and some other examples, we observed that feasible solutions using Logical Development of Vogel's Approximation Method (LD-VAM) are lower than Vogel's Approximation Method (VAM) where some of these are exactly equal to the optimal solution and some are very close to optimal solution. The comparison table of these solutions is given below:

**Table-6:**

Transportation Problem	Problem Size	Methods		
		LD-VAM	VAM	Optimal Solution
Example-1	4x5	2070	2130	2070
Example-2	4x4	415	470	410
Example-3	6x6	2220	2310	2170
Example-4	3x4	1165	1220	1160
Example-5	5x7	1900	1930	1900

**5 CONCLUSION:**

In this paper we find a limitation of Vogel's Approximation Method (VAM) and developed an improved algorithm by resolving this limitation named "Logical Development of Vogel's Approximation Method (LD-VAM)" for solving Transportation Problem. From the above examples and other transportation problems, LD-VAM provides the lower feasible solution than VAM which are very close to optimal solution and sometimes equal to optimal solution.

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