Doxastic influence
Stability and flux
Dynamics in the community
Plausibility influence
Further issues

Logical dynamics of belief change in the community

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Outline

1 Doxastic influence

2 Stability and flux

3 Dynamics in the community

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Doxastic influence

Consider influence regarding a single proposition *p*. If I do not believe *p* and some significant number or proportion of my friends do believe it. How should I respond?

(1) ignore their opinions and remain doxastically unperturbed.
(2) If I am influenced to change my beliefs there are at least two ways: I may *revise* so that I too believe *p* (*Rp*) or (more cautiously) *contract*, removing ¬*p* (*Cp*).

Notations: [*Rp*]*B*φ; [*Cp*]¬*B*¬φ. (Assuming success conditions)

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Doxastic influence

We will draw a distinction between two kinds of influence:

(a) influence that leads to revision (strong influence): S*p*.

(b) influence that leads to contraction (weak influence): W*p*.

We define a general operation of social influence regarding *p* (*Ip*) as the program

> *if* S*p* *then* R*p* *else if* W*p* *then* C¬*p;*
> *if* S¬*p* *then* R¬*p* *else if* W¬*p* *then* C*p*

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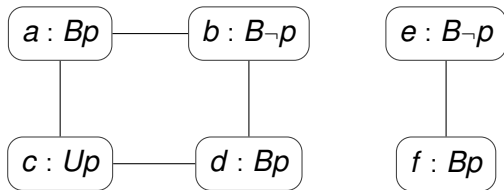**Dynamics in the community**
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Distribution of doxastic states

Three possible doxastic states: belief (Bp), disbelief ($B\neg p$) and no belief (Up) defined as $Up = (\neg Bp \wedge \neg B\neg p)$.

Framework: 'logic in the community,' ([Seligman et al., 2011]):

- friendship is taken to be a symmetric and irreflexive relation.
- A set of agents related by friendship will be called a *social network*.
- A subset of agents that are connected by friendship, in the sense that for any two agents, there is a chain of friends that connect them, is said to be a *community*.
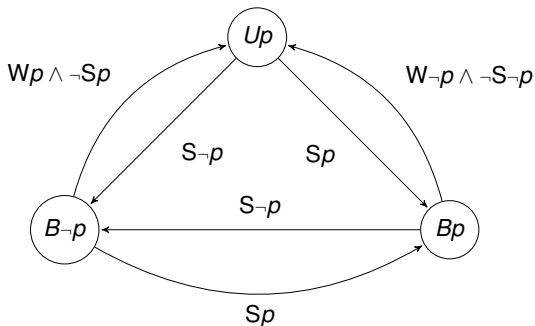
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Distribution of doxastic states: Example

A network of six agents, two communities. Links between nodes indicate friendship. Agent *a* believes *p* (written *a* : *Bp*) and has friends *b* and *c* ; agent *b* disbelieves *p* and has friends *a* and *d*; and so on.

FBp means that all my friends believe *p*, which in the example above is a true description of agents *b* and *c* but not of agents *a* and *d*. The dual operator ⟨*F*⟩ means 'some of my friends'.

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Finite state of automaton

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Threshhold influence

- If $x\%$ of my friends believe p, then I am strongly influenced to believe p.
    - Assume $x = 100$: I am **strongly influenced** to believe p iff **ALL** (at least one) of my friends believe p .
- If at least one of my friends believes p and $x\%$ of my friends disbelieve $\neg p$ (i.e., Bp or Ip), then I am weakly influenced to contract my belief in $\neg p$.
    - Assume $x = 0$: I am **weakly influenced** to contract by belief in $\neg p$ iff **NONE** of my friends believe $\neg p$.

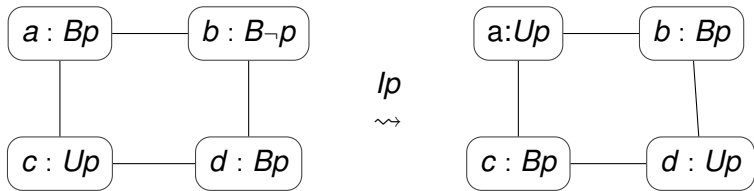Strong and weak influence captured with the following axioms:

$$S\varphi \leftrightarrow (FB\varphi \wedge \langle F \rangle B\varphi)$$
$$W\varphi \leftrightarrow (F\neg B\neg\varphi \wedge \langle F \rangle B\varphi)$$

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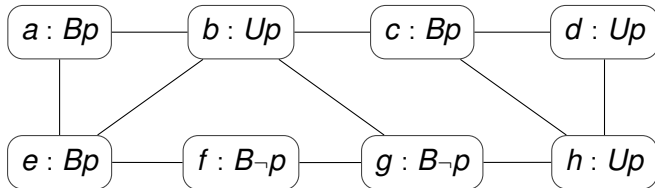
Threshhold influence: Example

Example 1:

Observation: In the configuration on the right, *a* and *d* are both strongly influenced to believe *p*, and so a further application of *Ip* would cause them to revise their beliefs, returning them to their previous doxastic states.

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Stability and flux

A community is *stable* if the operator *lp* has no effect on the doxastic states of any agent in the community. Unanimity within the community is sufficient for stability but not necessary, as is shown below:
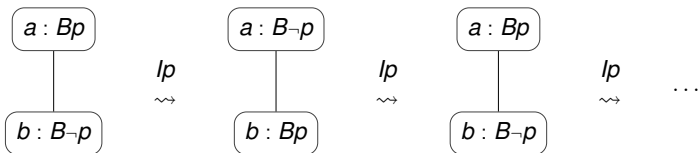
Example 2:

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Stability and flux: one more example

Configurations that never become stable will be said to be *in flux*:

Example 3:

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Characterizing stability

$$\neg(B\neg p \wedge Wp) \;\wedge\; \neg(Up \wedge Sp) \;\wedge\; \neg(Up \wedge S\neg p) \;\wedge\; \neg(Bp \wedge W\neg p)$$

Under the assumption of threshold influence, it is equivalent to

$$\neg(B\neg p \wedge F\neg B\neg p \wedge \langle F\rangle Bp) \;\wedge$$
$$\neg(\neg Bp \wedge \neg B\neg p \wedge FBp \wedge \langle F\rangle Bp) \;\wedge$$
$$\neg((\neg Bp \wedge \neg B\neg p \wedge FB\neg p \wedge \langle F\rangle B\neg p) \;\wedge$$
$$\neg(Bp \wedge F\neg Bp \wedge \langle F\rangle B\neg p)$$

A community is *stable* when every agent in the community satisfies this condition.

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Characterizing flux

[Zhen and Seligman, 2011] contains a theorem that shows how to characterizing flux for the preference dynamics.

Conjecture: A community (of at least two agents) is in flux iff every agent in the community satisfies the condition

$$(FBp \land FFB\neg p) \lor (FB\neg p \land FFBp)$$

In particular, if there is any agent in the community in state *Up*, then the community will eventually become stable – if not stable already.

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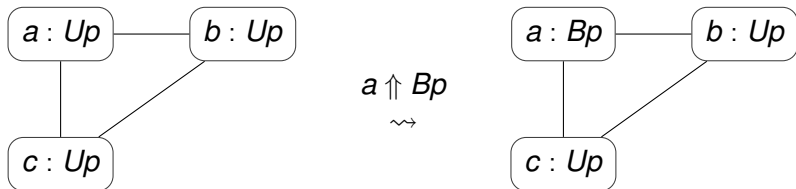Plausibility influence
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Dynamics I: Private belief change in a community

Agents may change their minds for many reasons other than
the influence of their friends' opinions. This raises the question
of if and how such changes are propagated to other members
of the community.

- A very coherent community may resist all changes,
  ensuring that any agent who changes her mind unilaterally
  will soon be brought back into conformity.
- A less coherent community may be highly affected by the
  change, going into flux or even following the agent who
  changed her mind into a new stable configuration.

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Isolated private belief change

Example 4:

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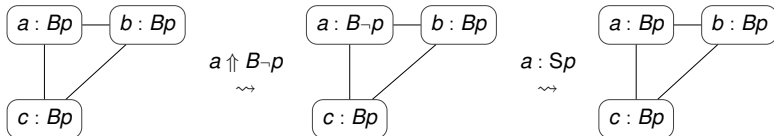**Dynamics in the community**
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More active resistence

Unanimous belief within a community can be strong enough to resist private belief changes even further:
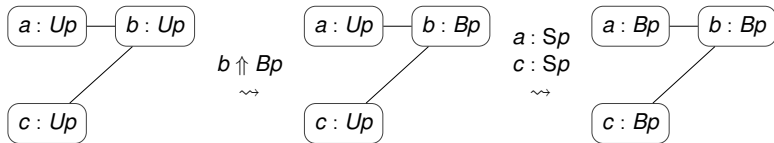
Example 5:

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Dynamics in the community
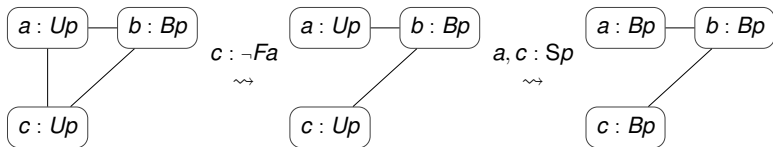**Plausibility influence**
Further issues

location is critical

For a community of undecided agents to be influenced by a private belief change, the location of the agent who comes to believe *p* is critical.

Example 6:

Doxastic influence
Stability and flux
Dynamics in the community
Plausibility influence
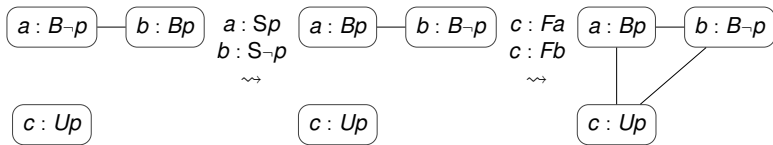**Further issues**

Dynamics II: Gaining and losing friends

Example 7:

We start with a stable distribution of opinions among three mutual friends, with only one believer. One friendship is broken, putting the mutual friend into a position of greater influence.

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Dynamics in the community
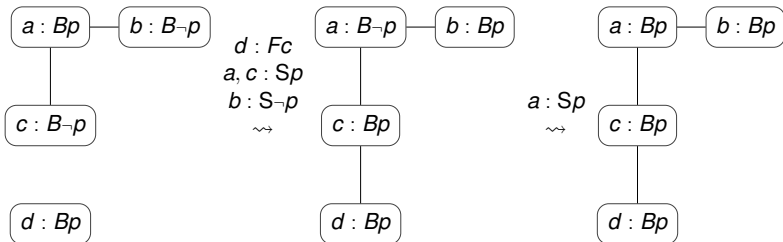Plausibility influence
Further issues

New friend joining in

Example 8:

The oscillating pair of friends at the top is calmed when an indifferent agent joins their circle. In one more step, they will all become undecided.

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A different newcomer

Example 9:

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Stability and flux
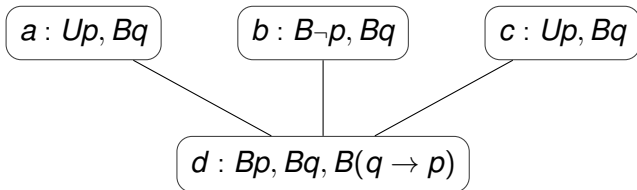Dynamics in the community
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Plausibility influence

We have ignored the interdependence of a person's beliefs:

Example 10:

How is *d* to change her beliefs?

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Four plausibility states

Given a fixed domain W of possible outcomes, consider each agent's judgements regarding the relative plausibility of elements of W. For u and v in W, we write $u \leq_a v$ to mean that a judges v to be at least as plausible as u.
There are now four relevant possible states:

- agent a may find v strictly more plausible than u ($u \leq_a v$ and $v \not\leq_a u$)
- vice versa,
- may regard them as equally plausible ($u \leq_a v$ and $v \leq_a u$)
- have no view at all ($u \not\leq_a v$ and $v \not\leq_a u$).

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Plausibility influence: the simplest case

If they all take v to be at least as plausible as u, so does she, and if they all take v not to be at least as plausible as u, nor does she. We will call this *plausibility influence*($\mathcal{I}$). The action of plausibility influence is characterised by:
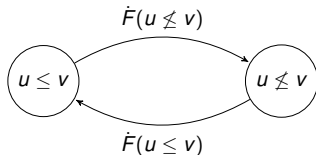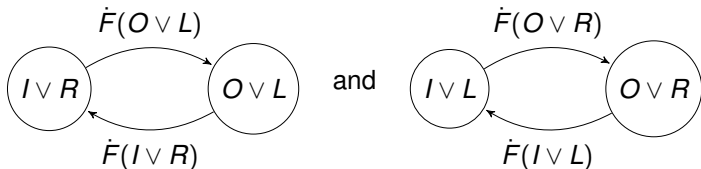

(where $\dot{F}\varphi$ is an abbreviation for ($F\varphi \wedge \langle F\rangle\varphi$), i.e. the version of the universal quantifier that takes it to have existential import.)

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Two more automata

Label *R* (for 'right') *v* is strictly more plausible than *u*, *L* (for 'left') *u* is strictly more plausible than *v*, *I* (for 'impartial') *u* and *v* are equally plausible, and *O* (for 'no opinion').
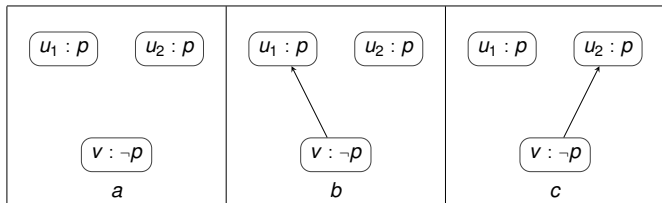
In these terms $u \leq v$ is $(I \vee R)$, $u \not\leq v$ is $(O \vee L)$, $v \leq u$ is $(I \vee L)$, and $v \not\leq u$ is $(O \vee R)$. The dynamics of the two parts of the comparison can be represented as the two automata:
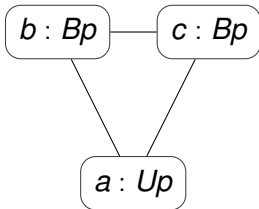
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Example

There are three possible outcomes, u_1, u_2 and v with p true at each of u_1 and u_2 but not at v. agent b regards u_1 as strictly more plausible than v, agent c regards u_2 as strictly more plausible than v. agent a makes no judgements at all.

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Agents *b* and *c* agree that u_1 and u_2 are the only maximally plausible outcomes, and so believe that *p*. Their friend *a* also allows that outcome *v* is maximally plausible and so is undecided about *p*. Thus we have the following configuration:

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After plausibility influence, agent b drops her judgement that u_1 is more plausible than v because it is not supported by either friend. Agent c drops her judgement that u_2 is more plausible than v, making all three agents converge to a's initial view.
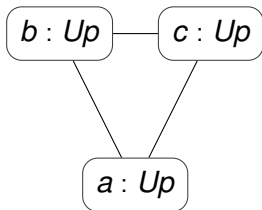
We can interpret this as capturing, to some extent, the influence of reasons rather than mere beliefs.

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Other interesting issues

- Ranked friends
- Reliability
- Belief aggregation
- A right dynamic logic language

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Thank you for your attention!
Email: fenrong@tsinghua.edu.cn

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📄 Seligman, J., Liu, F., and Girard, P. (2011).
Logic in the community.
In Banerjee, M. and Seth, A., editors, *Logic and Its Applications - 4th Indian Conference, ICLA 2011, Delhi, India, January 5-11, 2011. Proceedings*, volume 6521 of *Lecture Notes in Computer Science*, pages 178–188.

📄 Zhen, L. and Seligman, J. (2011).
A logical model of the dynamics of peer pressure.
Electronic Notes in Theoretical Computer Science, 278:275–288.