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John Corcoran, *Logically Equivalent False Universal Propositions with Different Counterexample Sets*.
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This largely expository and pedagogical article discusses the phenomenon indicated by the title and other related phenomena. Two propositions are *logically equivalent* if each is a logical consequence of the other. Our examples use standard first-order logic with the class of numbers [non-negative integers] as universe of discourse. A number *x* is a *counterexample* for a universal proposition "A*x* P(*x*)" iff it is not the case that P(*x*).

In some familiar cases logically equivalent false propositions have the same counterexamples. "Every number that is not even is prime" and "Every number that is not prime is even" are both *counterexemplified* by the non-prime odd numbers.

Moving along a spectrum, we find cases which share some but not all counterexamples. "Every number divides every other number" is counterexemplified by every number except one, whereas its equivalent "Every number is divided by every other number" is counterexemplified by every number except zero.

On the other end of the spectrum there are cases having no counterexamples in common: "Every even number precedes every odd number" is counterexemplified only by even numbers, whereas its equivalent "Every odd number is preceded by every even number" is counterexemplified only by odd numbers. One easy result is that given any non-empty finite set of numbers every false universal proposition is logically equivalent to another counterexemplified exclusively by numbers in the given set.

If *n* is a numeral, then "P(*n*)" and "Q(*n*)" are corresponding instances of universal propositions "A*x* P(*x*)" and "A*x* Q(*x*)". As seen above, it is not necessary for corresponding instances of logically equivalent universal propositions to be logically equivalent. The above phenomenon is quite common; indeed every non-tautological universal proposition is in the same logical form as a false universal proposition that is logically equivalent to another false universal having a different set of counterexamples.