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Long-range earthquake forecasting based on a single predictor

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Summary. For a long-term predictor from which a joint distribution of earthquake occurrence time and magnitude has been obtained, and also a record of past successes, false alarms and failures, Bayesian statistical methods yield predictive information of the kind needed as a basis for decision-making on precautionary measures. The information is presented in terms of risk refinement, intensity probability and success probability. After the event the relative likelihood that a prediction was a success or failure can be estimated. Comparisons can also be made of the performance of different forecasting models. The application of these methods is illustrated by an example based on the proposed swarm-magnitude predictor.

Introduction

The study of earthquake precursors shows promise of supporting a major advance in the estimation of future earthquake occurrences through the detailed forecasting of specific earthquakes. Such forecasts might be expressed adequately for some purposes in terms of ranges of epicentre location, occurrence time and magnitude for each earthquake. A more elaborate presentation is needed if the value of forecasting is to be realized fully as a basis for precautionary measures. Bayesian statistical methods are well suited to the problem of drawing together essentially mixed precursory data into a unified and comprehensive expression of the forecasting information. One facet of this problem – the calculation of risk enhancement for a particular event – has recently been discussed by Vere-Jones (1978). The full information comprises a pattern of risk which extends over the whole region under surveillance and varies continuously in space and time. In addition there is the need to assess results given by any chosen forecasting model and to compare the performances of different models.

The simplest forecasting model which yields the necessary information is one involving a single quantifiable predictor which identifies the earthquake location and is associated with a known joint distribution of occurrence time and magnitude. In the occurrence of long-range precursory phenomena there is much evidence that the earthquake magnitude increases with the logarithm of the precursor time (Rikitake 1975). A possible predictor is the size of the area affected by the precursory phenomenon, since it appears that this area increases with the earthquake magnitude; but in many circumstances the area would be difficult to delineate and so far this proposal has been little developed. Recently a long-range precursory hypothesis has been put forward in which a characteristic sequence of minor earthquake activity is initiated by an earthquake swarm (Sekiya 1976; Evison 1977a, b), and in which the magnitude of the largest earthquake in the swarm is linearly related to the logarithm of the precursor time (Evison 1977c). This swarm magnitude is a predictor of the desired kind.

The following analysis relates to an unspecified predictor P. It is supposed that the validity of this predictor has been established, so that relationships which have been ascertained from past events are applicable to the future. Through the Bayesian approach the past record of successes, false alarms and failures is combined with the estimation errors for occurrence time and magnitude in order to present the forecast. The results will be illustrated by reference to the swarm-magnitude predictor as observed in New Zealand, where there are adequate past data on this predictor, although its validity for future events has yet to be tested and for the present purpose will be assumed.

Observational basis

The required data on the precursory relationship consist of n past observations of the precursory phenomenon and the corresponding earthquake (i.e. successes), r observations of the precursor without a corresponding earthquake (i.e. false alarms), and r' observations of an earthquake without a corresponding precursor (i.e. failures). The $n + r$ precursory events on the one hand and the $n + r'$ earthquakes on the other make up two well-defined populations, which are preferably the largest that homogeneity of data allows, and which are bounded in respect of region, time, magnitude and values of the predictor. The desired expression of forecasts in terms of the regional risk pattern requires information also about the ordinary incidence of earthquakes belonging to the specified population.

For a forecast to be useful as a basis for precautionary measures the size of the earthquake can be better expressed by mapping the intensity rather than merely specifying the magnitude. Information is therefore also required on the distribution of intensity corresponding to any earthquake of specified location and magnitude.

Probabilities of success, false alarm and failure

From the n past successes and r past false alarms, Bayesian methods can be used to estimate the probability of a false alarm, so that this can be allowed for in future predictions. The false alarm probability can be regarded as an unknown parameter θ from a binomial distribution. A prior distribution for θ, which expresses the initial lack of information about θ in the absence of any data, is modified on the basis of r observed false alarms from $n + r$ trials to give a posterior distribution whose expected value is used as an estimate of the false alarm probability.

The Beta family of distributions is a suitably rich and mathematically convenient family from which to choose the prior distribution. If the prior distribution is Beta with parameters u and v then the posterior distribution is also Beta with parameters $r + u$ and $n + v$ (DeGroot 1970, p. 160). The prior distribution then has expected value $u/(u + v)$ and the posterior distribution $(r + u)/(r + u + n + v)$. Large values of u and v would indicate a strongly held, and small values a weakly held, prior belief about the value of θ. Since the actual value of θ

is unknown it seems appropriate to adopt the uniform distribution, i.e. that for which all possible values of θ are equally likely. Accordingly the values $u = 1$ and $v = 1$ have been chosen. We shall denote the event of a successful prediction by S and a false alarm (the complementary event) by $\bar{S}$. Thus $Pr(\bar{S}) = (r+1)/(n+r+2)$. This is an initial false alarm probability for general use in future predictions; it may be modified by further information specific to a given prediction. The failure rate $Pr(F)$, i.e. the probability that an earthquake fails to have a precursor, can be estimated similarly with r' replacing r in the above.

As time passes and additional information becomes available the probability of success can be updated by the use of Bayes' rule. Let G denote all relevant information available at time t_0 after the precursor and suppose $G = G_1 \cap G_2 \cap \ldots G_k$, where the k pieces of information $G_1 \ldots, G_k$ are conditionally independent given S and conditionally independent given $\bar{S}$; i.e. for any subset $\{\alpha_1, \alpha_2, \ldots \alpha_I\}$ of $\{1, 2, \ldots, k\}$

$$Pr\left(\bigcap_{i=1}^{I} G_{\alpha_i} \mid S\right) = \prod_{i=1}^{I} Pr(G_{\alpha_i} \mid S)$$

with a corresponding condition for $\bar{S}$. Let

$$G^j = G_1 \cap G_2 \cap \ldots \cap G_j, \qquad j = 1, \ldots, k.$$

Then $G^k = G$, and G^0 is the event of no additional information. Suppose that $Pr(G_j \mid S)$ and $Pr(G_j \mid \bar{S})$ have been estimated for $j = 1, \ldots, k$. Then Bayes' rule can be applied successively using each new piece of information in turn to update the probability of success; i.e.

$$Pr(S \mid G^j) = \frac{Pr(G_j \mid S) Pr(S \mid G^{j-1})}{Pr(G_j \mid S) Pr(S \mid G^{j-1}) + Pr(G_j \mid \bar{S}) Pr(\bar{S} \mid G^{j-1})} \qquad j = 1, 2, \ldots, k.$$

If the pieces of information are not conditionally independent given S (or $\bar{S}$), for instance if several are symptoms of some phenomenon which may be unrelated to the mainshock, the above procedure is invalid and could lead to a gross error in the estimation.

Joint distribution of magnitude and precursor time

Let P, T and M denote, respectively, the predictive variable, the precursor time and the earthquake magnitude, or some suitably chosen transformations of these (e.g. the logarithm of the precursor time for T). Then as random variables, P, T and M are assumed to be related as follows:

$$T = a_0 + a_1 p + e_1 \tag{1}$$

$$M = b_0 + b_1 t + b_2 p + e_2 \tag{2}$$

where t, p denote observed values of T, P respectively and e_1, e_2 are normal random variables with zero means and unknown variances σ_1^2, σ_2^2 respectively. Let $\hat{a}_i$, $\hat{b}_j$ denote the least-squares estimates of a_i, b_j (where $i = 0, 1; j = 0, 1, 2$) computed from n past observations of P, T and M, and define

$$\hat{T}(p) = \hat{a}_0 + \hat{a}_1 p \tag{3}$$

$$\hat{M}(t, p) = b_0 + \hat{b}_1 t + \hat{b}_2 p. \tag{4}$$

According to the theory of linear regression, the random variable $(T - \hat{T}(p))/\sigma_T(p)$ has a t-distribution with $n - 2$ degrees of freedom and $(M - \hat{M}(t, p))/\sigma_M(t, p)$ has a t-distribution with $n - 3$ degrees of freedom, where $\sigma_T(p)$ and $\sigma_M(t, p)$ are estimates of the standard deviation for prediction of the dependent variable from the regression equations (3) and (4), and depend also on the standard error of estimate and the variance–covariance matrix of the coefficients (e.g. Smillie 1966, p. 49). Let g_k denote the probability density function of the t-distribution with k degrees of freedom. Then the conditional density function $f_{T|P}$ of T given P is given by

$$f_{T|P}(t|p) = \frac{1}{\sigma_T(p)} g_{n-2} \left[\frac{t - \hat{T}(p)}{\sigma_T(p)}\right] \tag{5}$$

and the conditional density function $f_{M|T,P}$ of M given T and P is given by

$$f_{M|T,P}(m|t, p) = \frac{1}{\sigma_M(t, p)} g_{n-3} \left[\frac{m - \hat{M}(t, p)}{\sigma_M(t, p)}\right]. \tag{6}$$

The joint density function $f_{M,T|P}$ of M and T given P is then given by

$$f_{M,T|P}(m, t|p) = f_{M|T,P}(m|t, p) f_{T|P}(t|p). \tag{7}$$

The joint density function may be integrated to obtain the probability of the earthquake occurring in any fixed time interval and magnitude range, assuming that the prediction will be successful. Let S denote the event of a successful prediction and G all information available at some time t_0 after the precursor. At time t_0 the probability $q(t_0; (m_1, m_2), (t_1, t_2))$ of the earthquake occurring in the magnitude range (m_1, m_2) and the future time interval (t_1, t_2) is given by

$$q(t_0; (m_1, m_2), (t_1, t_2)) = \frac{Pr(S|G) Pr(M \in (m_1, m_2), T \in (t_1, t_2)|p)}{Pr(T > t_0|p)}$$

$$= \frac{Pr(S|G) \int_{t_1}^{t_2} \int_{m_1}^{m_2} f_{M,T|P}(m, t|p)\, dm\, dt}{\int_{t_0}^{\infty} f_{T|P}(t|p)\, dt} \tag{8}$$

The estimation of $Pr(S|G)$ has already been discussed.

A simple case is given by $G = G_1$, where G_1 is the event that the earthquake has not occurred up to time t_0. Thus

$$Pr(G_1|S) = Pr(T > t_0) = \int_{t_0}^{\infty} f_{T|P}(t|p)\, dt$$

and $Pr(G_1|\bar{S}) = 1$. Other kinds of information, such as the occurrence or non-occurrence of other precursors, also have potential for use as G_js as our knowledge of precursory phenomena increases.

Use of the precursory area in modifying the distributions

Although the size of the precursory area is not usually well enough known to be used as a predictor it is typically much larger than the source region of the earthquake and it may be considered to put an upper limit on the size of the source region. In view of the well-known relationship between the magnitude of a shallow earthquake and the size of its source region, it is desirable to incorporate such information into the forecast, since it implies an upper limit on the predicted magnitude. One way of doing this, chosen because of its consistency with the later section on intensity probability charts, is described below.

Formulae are assumed to be available indicating the magnitude range of an earthquake, at any specified location, which should produce a given felt intensity at any other specified location.

Let A denote the surface projection of the precursory region. For the predicted earthquake the source region (which includes the aftershock epicentres) is expected to lie within A. The edge of the source region for large shallow earthquakes commonly occurs about midway between isoseismals MM IX and MM X for the mainshock. For any point a in A let $m_c(a)$ (the critical magnitude) be the largest magnitude for which the theoretical MM IX – MM X isoseismal lies entirely within A. The cut-off magnitude is the supremum of all such critical magnitudes, i.e.

$$m_A = \sup_{a \in A} m_c(a).$$

The assumption is that the predicted earthquake cannot exceed the cut-off magnitude. The conditional distribution of M given T and P is modified accordingly. Let $f'_{M|T,P}(m|t,p)$ denote the modified distribution given that $M \leqslant m_A$. Then

$$f'_{M|T,P}(m|t,p) = \frac{f_{M|T,P}(m|t,p)}{\int_{-\infty}^{m_A} f_{M|T,P}(m|t,p)\,dm} \qquad \text{for } m \leqslant m_A, \text{ and}$$

$$= 0 \qquad \text{for } m > m_A.$$

A consequent modification is made to the joint distribution of M and T given P.

$$f'_{M,T|P}(m,t|p) = f'_{M|T,P}(m|t,p) f_{T|P}(t|p).$$

The above procedure has the effect of preventing unrealistically high probabilities being associated with very large magnitudes and to some extent provides a check against the dangers of projecting the regression relationships beyond the range of the data used to generate them.

Risk refinement factor

The object of earthquake prediction from the present viewpoint is to refine the pattern of risk over a region by improving its resolution in time and space. The risk refinement factor is defined here as the ratio of risk under the prediction model to the corresponding risk estimated by means previously available, for example from a knowledge of historical seismicity. A value greater than unity, representing a risk enhancement, occurs for a certain interval of time near the location of a predicted earthquake. At other times and places the factor is less than unity, representing a risk reduction. The enhancement aspect has been discussed by Vere-Jones (1978). The reduction aspect is also of much theoretical and practical interest; for example, conditions of risk reduction might justify some temporary relaxation of precautionary measures.

Let A be the area within which the predicted earthquake is to occur. Under the assumption that earthquakes occur randomly in time and space in some neighbourhood of A, the number of earthquakes in the magnitude range (m_1, m_2) occurring in A in the time range (t_1, t_2) is a Poisson random variable with parameter $\lambda(A; (m_1, m_2), (t_1, t_2))$ where λ is the average rate estimated from historical seismicity. Although there is strong evidence that earthquake occurrence deviates markedly from Poisson behaviour even when short-term clustering is eliminated, and to this extent the estimation of λ is unsatisfactory, the Poisson approach still appears to give the most credible expression of future earthquake risk in the absence of a specific predictive model. Let $p(x; \lambda)$ denote the probability that a Poisson random variable with parameter λ takes the value x. The probability of an earthquake in the magnitude range (m_1, m_2) occurring in A in the time interval (t_1, t_2) is $1 - p(0; \lambda(A; (m_1, m_2), (t_1, t_2)))$. Under the prediction model the probability at time t_1 of the mainshock occurring within the stated time and magnitude limits is $q(t_1; (m_1, m_2), (t_1, t_2))$ (see equation (8)). It is also possible that an earthquake unrelated to the precursor could occur within the stated limits. Assuming that failures occur randomly, the probability of such a coincidental failure is $1 - p(0; Pr(F) \cdot \lambda(A; (m_1, m_2), (t_1, t_2)))$ where $Pr(F)$ is the failure rate.

The risk refinement factor for an earthquake in the region A and magnitude range (m_1, m_2) at time t_1 is defined by

$$R(A, (m_1, m_2), t_1)$$

$$= \lim_{t_2 \to t_1} \frac{q(t_1; (m_1, m_2), (t_1, t_2)) + 1 - p(0; Pr(F) \cdot \lambda(A; (m_1, m_2), (t_1, t_2)))}{1 - p(0; \lambda(A; (m_1, m_2), (t_1, t_2)))}$$

$$= \lim_{t_2 \to t_1} \frac{q(t_1; (m_1, m_2), (t_1, t_2))}{1 - p(0; \lambda(A; (m_1, m_2), (t_1, t_2)))} + Pr(F).$$

In practice R can be approximated quite accurately by computing the ratio on the right side for a short time-interval (say 1 day).

The risk refinement factor can also be computed for a region B in which no precursor has occurred. In this case the only risk under the prediction model is the risk of failure. Thus

$$R(B, (m_1, m_2), t_1) = Pr(F).$$

This is a risk reduction.

Intensity probability charts

Probability charts are a useful means of displaying the risk of a given intensity being experienced during a fixed time period in the light of current predictions. Such charts could be the primary presentation of a forecast as a basis for precautionary measures.

Let B denote any fixed location and A the surface projection of the precursory region. For any point a in A the critical magnitude $m_c(a)$ is defined as before. Let i denote a given fixed intensity and let $(m_1(a), m_2(a))$ denote the magnitude range of an earthquake with epicentre at the point a, given that the felt intensity is i at b. Let $A_\gamma(m)$ denote the set of all possible epicentres for a mainshock of magnitude m, such that the entire source region would lie within A; and let $A_\delta(m)$ denote the subset of epicentres in $A_\gamma(m)$ for which the felt intensity at b would be i. Thus

$$A_\gamma(m) = \{ a \in A : m < m_c(a) \}$$

$$A_\delta(m) = \{ a \in A : m_1(a) < m < \min(m_2(a), m_c(a)) \}.$$

Assuming that the epicentre is equally likely to be at any point in $A_\gamma(m)$, the probability of intensity i being felt at b given that the mainshock has magnitude m is approximately

$$r(m) = \frac{\text{area } A_\delta(m)}{\text{area } A_\gamma(m)}, \qquad \text{if area } A_\gamma(m) \neq 0$$

$$= 0, \qquad \text{otherwise.}$$

At time t_0 after the precursor, the probability of intensity i being felt at b in the future time interval (t_1, t_2) due to the mainshock is

$$s(t_0; b, i, (t_1, t_2)) = \frac{Pr(S|G) \int_{t_1}^{t_2} \int_{-\infty}^{\infty} r(m) f'_{M,T|P}(m, t|p) \, dm \, dt}{\int_{t_0}^{\infty} f_{T|P}(t|p) \, dt}$$

By adding $s(t_0; b, i, (t_1, t_2))$ to the probability of intensity i being felt due to the occurrence of failures, for a grid of points b covering the area of interest, and by interpolating for values in between, a contour plot of the probability of intensity i being felt in the time interval (t_1, t_2) can be produced.

Distinguishing successes from failures

Since predictions are stated in terms of a probability distribution for the time and magnitude of the mainshock, an earthquake which occurs in the precursory region cannot with certainty be called a success or failure of the method. Nevertheless for each such earthquake a decision must be made one way or the other based on the observed time and magnitude. The suggested criterion is to accept an earthquake as a success if the likelihood of a success exceeds the likelihood of a failure according to all the information available at the time of the occurrence.

Suppose that an earthquake occurring in the precursory area has a measured magnitude of $m \pm \Delta m$ and a measured precursor time of $t \pm \Delta t$. Then the relevant likelihood ratio is

$$LR = \frac{q(t_1; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t))}{1 - p(0; Pr(F) \cdot \lambda(A; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t)))}.$$

If $LR > 1$ the earthquake is accepted as a success; if $LR \leqslant 1$ it is called a failure. Although the critical value of unity is arbitrary, the 50–50 balance seems appropriate here since a wrong decision either way has a harmful impact on future predictions using the model.

Under the likelihood ratio criterion an unfulfilled prediction may finally be declared a false alarm when no future earthquake could qualify as a success by the criterion, i.e. when there is no longer any magnitude for which the likelihood ratio exceeds unity. In practice a false alarm might make itself apparent much earlier, for example if information came to light which showed that the supposed precursor had been wrongly identified.

Comparative performance of forecasting models

The record of successes, failures and false alarms in actual forecasting (as distinct from the record for the past data which were used to generate the model) gives some indication of the reliability of a forecasting method. However, it is inadequate for probabilistic models of the kind proposed here in that it takes no account of the uncertainty which is an integral part of such models.

A suggested measure of the performance of a model is the likelihood of the whole sequence of seismic events (restricted to events exceeding some fixed magnitude m_0) commencing at the instigation of forecasting using the model. Two competing methods of forecasting can be compared by looking at the ratio of the likelihoods of the observed record of earthquakes under the two models. This likelihood ratio can be evaluated by splitting up the record of earthquakes into 'elementary events' for which the likelihoods are easily found. As an example, consider the problem of comparing the performance of the model described in this paper with a Poisson model based on the average rate of occurrence for past events. Let $L_1(E)$ and $L_2(E)$ denote the likelihoods of an event E under the respective models. The region under observation would first be divided into sub-regions which are homogeneous for forecasting under both models. The 'elementary events' would then be of the following four kinds.

E_1: the non-occurrence of a predicted earthquake in a precursory region A during a time interval (t_1, t_2). In the notation of the previous sections the likelihoods are

$$L_1(E_1) = p(0; Pr(F) \cdot \lambda(A; (m_0, \infty), (t_1, t_2)))\,(1 - q(t_1; (m_0, \infty), (t_1, t_2)))$$

and

$$L_2(E_1) = p(0; \lambda(A; (m_0, \infty), (t_1, t_2)).$$

E_2: the occurrence of an earthquake in a precursory region A at measured time $t \pm \Delta t$ with magnitude $m \pm \Delta m$.

$$L_1(E_2) = q(t - \Delta t; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t))$$
$$+ 1 - p(0; Pr(F) \cdot \lambda(A; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t)))$$

$$L_1(E_2) = 1 - p(0; \lambda(A; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t))).$$

E_3: the non-occurrence of failures in a homogeneous region B during a time interval (t_1, t_2).

$$L_1(E_3) = p(0; Pr(F) \cdot \lambda(B; (m_0, \infty), (t_1, t_2)))$$

$$L_2(E_3) = p(0; \lambda(B; (m_0, \infty), (t_1, t_2))).$$

E_4: the occurrence of a failure in a homogeneous region B at time $t \pm \Delta t$ with magnitude $m \pm \Delta m$.

$$L_1(E_4) = 1 - p(0; Pr(F) \cdot \lambda(B; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t)))$$

$$L_2(E_4) = 1 - p(0; \lambda(B; (m - \Delta m, m + \Delta m), (t - \Delta t, t + \Delta t))).$$

If E denotes the total earthquake record of events exceeding magnitude m_0 and if

$$E = \bigcap_{j=1}^{J} E_j$$

where the E_j are J elementary events, then the likelihood ratios for the whole sequence can be expressed as the product of likelihood ratios for the elementary events, i.e.

$$\frac{L_1(E)}{L_2(E)} = \prod_{j=1}^{J} \frac{L_1(E_j)}{L_2(E_j)}.$$

If an effective method for forecasting is compared to the Poisson model this ratio should become large after the occurrence of a few earthquakes.

Application to proposed swarm-magnitude predictor

The precursory swarm hypothesis, which has been developed mainly from New Zealand data, involves a long-range predictor P, as has been mentioned above. P is taken here as the

Table 1. Regression data.

Location	Date (UT)	P	T	M	Weight
Inangahua	23.5.68	5.6	3.360	7.1	1.0
Milford Sd	4.5.76	5.1	3.477	7.0	1.0
Puysegur Pt	25.9.68	4.4	2.860	6.5	0.5
Gisborne	4.3.66	4.4	2.746	6.2	1.0
Seddon	23.4.66	3.8	2.748	6.0	1.0
Martins Bay	20.9.74	4.1	2.915	5.9	1.0
Doubtful Sd	2.4.68	4.0	2.773	5.8	1.0
Hastings	21.2.73	4.3	2.614	5.7	0.5
Taradale	26.5.68	4.1	2.627	5.2	0.5
Bainham	18.1.74	3.5	2.441	5.0	1.0
Goose Bay	1.5.70	3.7	2.262	4.9	1.0

average magnitude of the three largest earthquakes in the swarm; in view of the uncertainty in magnitude determinations this seems preferable to taking simply the single largest magnitude, as was done in early investigations (Evison 1977c). T is taken as the logarithm to base 10 of the precursor time, which is the time in days between the swarm onset and the mainshock. M is the mainshock magnitude.

Regressions connecting P, T and M have been obtained for New Zealand from values given in Table 1. The source of basic data is the *New Zealand Seismological Report,* which is published annually by the Seismological Observatory, Geophysics Division, Department of Scientific and Industrial Research, Wellington. Excluded from consideration is the New Zealand region of Quaternary volcanism, in which swarms are comparatively frequent but major earthquakes are rare. Also excluded are major earthquakes which have few or no aftershocks; it appears that precursory swarms and aftershock sequences require similar ambient conditions, including fairly shallow focal depths. The period included in Table 1 is 1966–77. A few events are given half weight in the regressions because the relevant precursory swarms do not quite satisfy the recognition criteria which have been developed for New Zealand conditions. All the statistical quantities required for the present purpose can be obtained from the data in Table 1; it may be of interest that the regression coefficients are:

$$\hat{a}_0 = 0.64, \quad \hat{a}_1 = 0.51, \quad \hat{b}_0 = 0.92, \quad \hat{b}_1 = 1.38, \quad \hat{b}_2 = 0.28.$$

There were a number of false alarms and failures during the period. Table 2 shows how the cumulative score varies with the M threshold. Fairly rapid deterioration of the score is to be expected below some level of M, depending on the capability of the seismograph network to locate the corresponding swarm earthquakes. On the basis of Table 2 the

Table 2. Cumulative score for past events.

Mainshock magnitudes	Successes	Failures	False alarms
$\geqslant 6.2$	4	0	0
$\geqslant 6.1$	4	1	0
$\geqslant 6.0$	5	1	0
$\geqslant 5.9$	6	2	1
$\geqslant 5.8$	7	2	1
$\geqslant 5.7$	8	2	2

condition $M \geqslant 6.0$ has been adopted in what follows, and it is assumed that the false alarm and failure rates are constant for such magnitudes. Thus

$$n = 5, \quad r = 0, \quad r' = 1$$

so that, assuming a uniform prior distribution,

$$Pr(S) = 6/7, \quad Pr(F) = 1/4.$$

The theoretical results obtained above will be illustrated by a sample forecast based on the swarm which preceded the Inangahua event (Table 1). A detailed account of this event has been published (Evison 1978). The precursory data have of course been used in the regressions, and the event has been included as a success in the score for past events. It is of interest, however, to examine the forecast that would be made if a similar precursor were to occur again.

For this precursor the area A (7500 km^2) was situated as shown in Fig. 1(a), and $P = 5.6$. The normal rate λ of earthquake occurrence in A can be estimated by assuming that A is typical of the main seismic region of New Zealand, in which there have occurred 14 shallow earthquakes of magnitude $M \geqslant 7.0$ during the past 130 yr, in a total area of some 2×10^5 km^2.

The usual frequency–magnitude law is

$$\rho(m) = \rho(m_0) \exp[-b(m - m_0)]$$

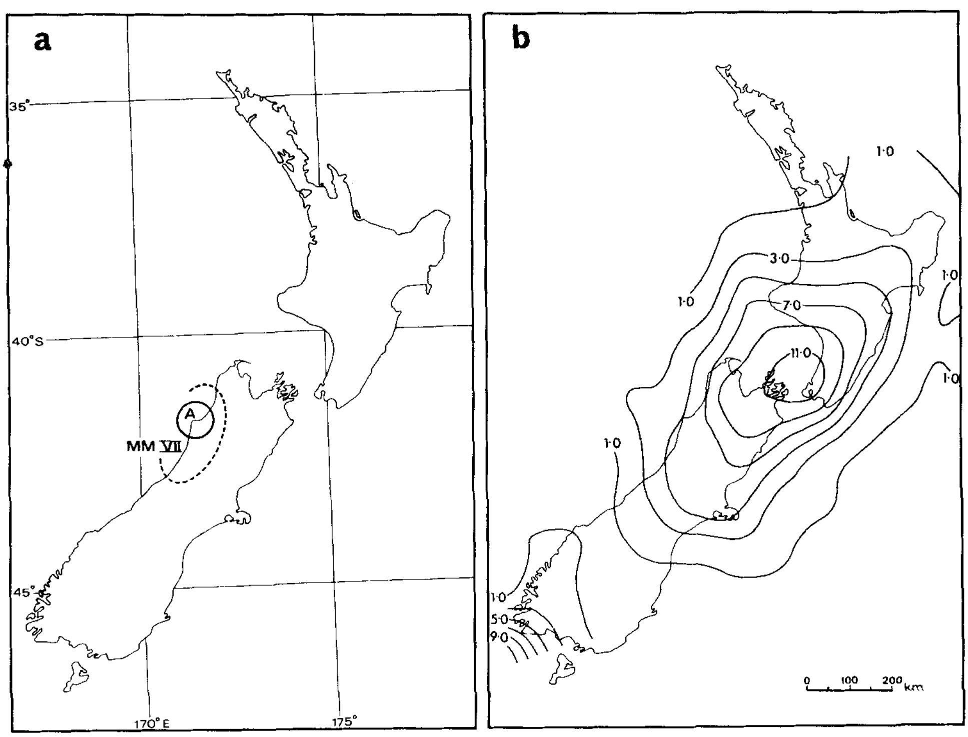

Figure 1. Map of New Zealand showing: (a) Approximate area A of precursor, and isoseismal MM VII, for Inangahua earthquake, 1968 May 23, $M_L = 7.1$. (b) Percentage probability of intensity MM VII occurring during any two-year period, estimated from the historical record (after Smith 1976). Contour interval 2 per cent.

where $\rho(m)$ denotes the average rate of occurrence of events of magnitude greater than m in the precursory area A, and b is a numerical coefficient. Then

$$\lambda\,[A\,;(m_1, m_2), (t_1, t_2)] = (t_2 - t_1)\,[\rho(m_1) - \rho(m_2)].$$

From results given by Eiby (1971) we obtain $b = 2.11$, and the values given above lead to $\rho(7.0) = 0.004$ event/yr.

The presentation of forecasting information in terms of intensity probabilities can be illustrated for New Zealand as a whole. For any location the probability estimated from

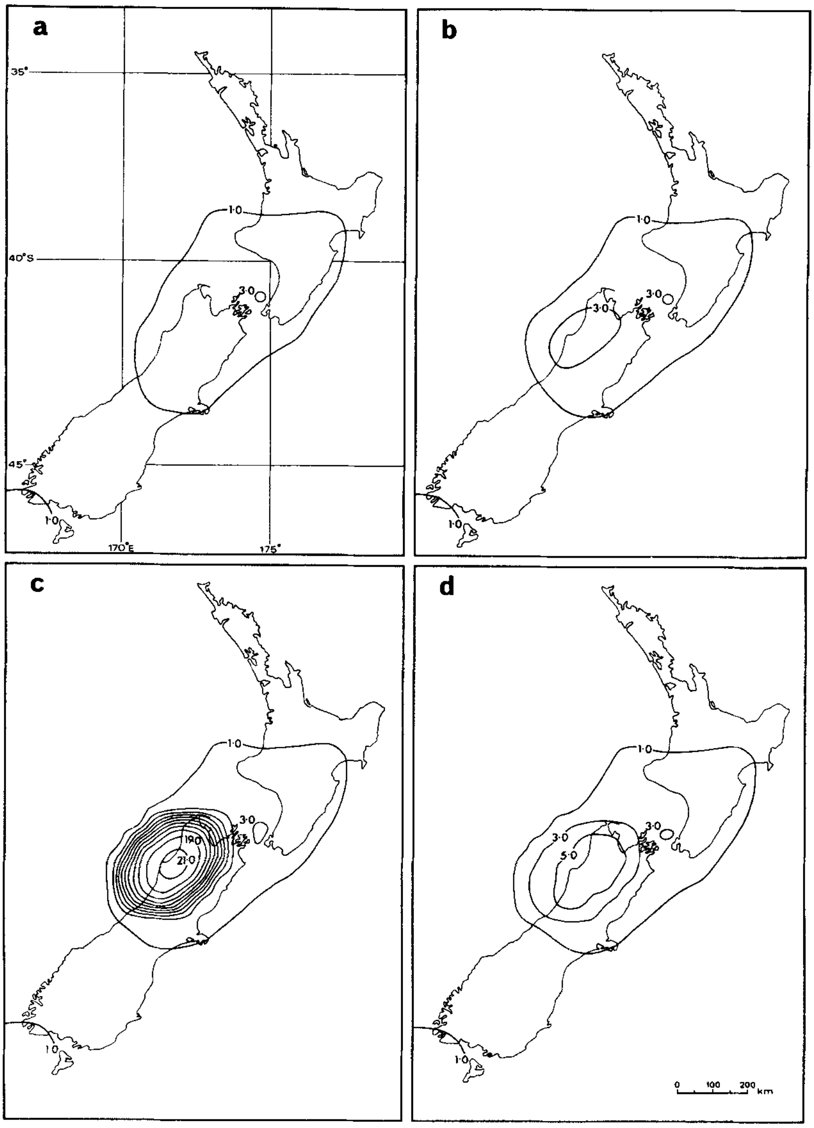

Figure 2. Map of New Zealand showing forecast percentage probabilities of intensity MM VII occurring during specified two-year periods. Contour interval 2 per cent. (a) Any two-year period, given that there are no earthquakes currently forecast. The risk depends on the historical record (Fig. 1(b)) and the forecasting failure rate; this risk is incorporated in (b), (c) and (d). (b) Two-year period commencing 500 day after the onset of the Inangahua precursory swarm. (c) Two-year period commencing 2000 day after the swarm onset. (d) Two-year period commencing 10 000 day after the swarm onset.

historical data is modified by current forecasts and by the probability of failure to forecast. Probabilities can be estimated for any required intensity and time interval; the illustrations given here are for intensity MM VII and for specified two-year intervals. The historical probability, shown in Fig. 1(b), has been derived from the results of Smith (1976), with the usual Poisson assumptions.

The probability of intensity MM VII occurring in New Zealand during any two-year period due to a failure under the precursory swarm hypothesis (assumed valid) is shown in Fig. 2(a). This is computed by dividing the average return time (Smith 1976) by the failure rate, again with the usual Poisson assumptions. As in the other charts in Fig. 2 the probability is expressed in percentages with 1 per cent as the lowest value shown and

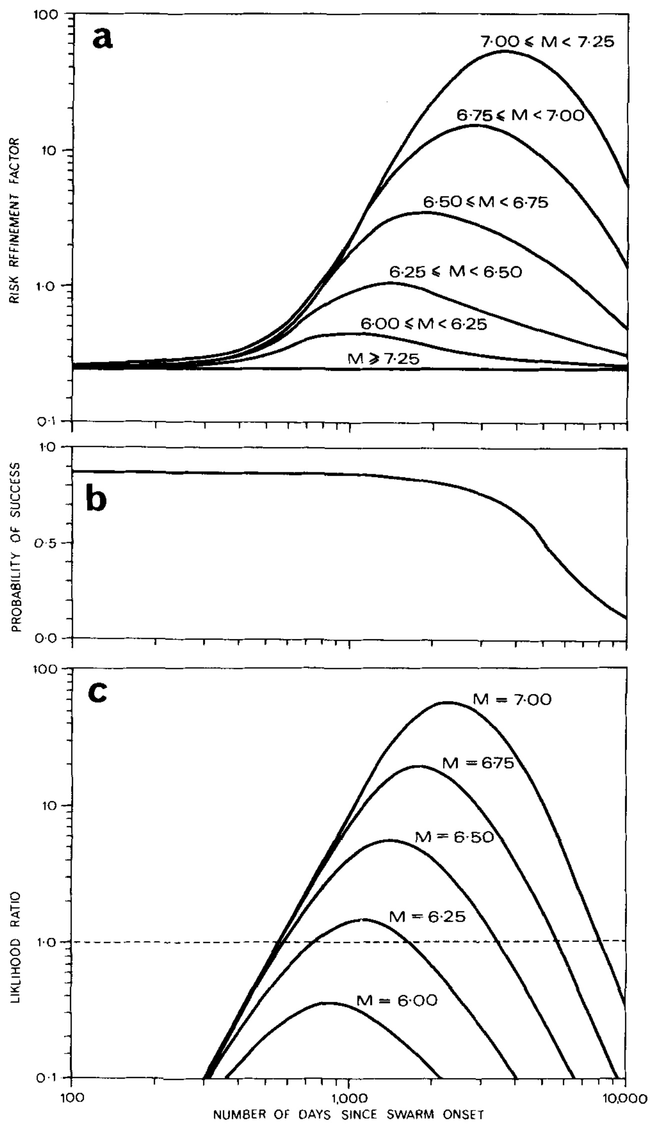

Figure 3. Forecast information exemplified for the Inangahua event. (a) Variation of risk refinement factor in area *A* of Fig. 1(a). (b) Variation of probability that forecast will be successful. (c) Variation of likelihood that the earthquake fulfilled the forecast, relative to likelihood that it did not.

contours at 2 per cent intervals. The effect of the Inangahua forecast is exemplified for selected two-year periods in Fig. 2(b), (c) and (d), it being assumed in each case that no other forecast is current and that the earthquake has not occurred before the start of the respective period. In computing these patterns the formulae used to transform magnitude into intensity are those of Smith (1978). The Inangahua earthquake actually occurred at $t = 2289$ days with magnitude 7.1; the actual isoseismal for intensity MM VII is included in Fig. 1 for comparison with Fig. 2(c).

An impression of the effect achieved by forecasting in improving the resolution of earthquake risk can be gained from a comparison of Fig. 1(b) with Fig. 2. Risk reduction is a prominent feature, as will be appreciated from Fig. 2(a), which is directly comparable with Fig. 1(b) for any two-year period in which the forecasting method is in use but no particular forecast happens to be current. The probability is reduced everywhere by a factor of 4. Risk reduction also occurs almost everywhere during the early and late stages of the currency of a forecast, as Fig. 2(b) and (d) illustrate. On the other hand the manner in which a forecast concentrates the risk in certain regions of space and time is exemplified in Fig. 2(c).

The forecast is presented in Fig. 3(a) in terms of risk refinement in area A as a function of precursor time and mainshock magnitude; the estimated cut-off magnitude is 7.25. The probability that the forecast will be successful in the light of all available information, $Pr(S|G)$, is plotted against time in Fig. 3(b). Here G is simply taken as the information that the earthquake has not yet occurred. If an earthquake does occur the relative likelihood that it marks a successful forecast rather than a failure is shown in Fig. 3(c) as a function of time and magnitude. For the actual Inangahua earthquake the risk refinement factor given by the forecast was about 30 (Fig. 3(a)). At the time of occurrence the probability of a successful forecast was about 0.8 (Fig. 3(b)). Finally, the relative likelihood that the actual earthquake would have represented a successful forecast rather than a failure was greater than 50 (Fig. 3(c)).

These results are presented for illustrative purposes only; they in no way constitute a test of the precursory swarm hypothesis.

Conclusions

A theoretical framework has been developed for the presentation of predictive information in a country or large region in such a way as to provide a sound basis for the taking of precautionary measures. The main concern has been with the use of a single predictor but there is provision for taking account of additional information of which the validity has been independently established. The practical application of risk refinement, intensity probabilities, and the other types of predictive information discussed will require careful study.

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