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Long Range Forces between Hadrons

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Forces between hadrons at large separation are calculated assuming linearly rising quark-quark potential. It is shown that the potential thus calculated are proportional to R^{-3}, and much stronger than those observed in nature. Causes of such disagreement are discussed.

§ 1. Introduction

The nature of the force between quarks that confine them in a hadron has been investigated in some detail. It proved to be very likely that its intensity is independent of the distance r between quarks, i.e., the potential is linear in r except for a possible deviation at very short distances.

The force of this type, sometimes referred to as a string,*) produces a linear Regge trajectory of a hadron. Experimentally the linearity of the trajectories is observed to hold to fairly high angular momenta. Accordingly the linearity of the quark-quark potential is regarded to hold at least for some range of r.

A linear potential like this obviously does not exist between hadrons, each of which is a color-singlet system. However, a long range potential between constituents often produces a "van der Waals potential" between composite particles. Sawada[1] has pointed out that a nucleon-nucleon potential of the form $1/R^6$ follows from a Coulombic interaction between constituents.[2] In the present paper we start with the linear quark-quark potential, calculate long range hadron-hadron potential and show how strong its intensity is.

The origin and dynamics of the quark-quark interaction have not been understood as yet. Here we take the static potential picture seriously and assume an interaction energy which is a function of the relative distance and colors of quarks. It turns out that this leads to a hadron-hadron potential proportional to $1/R^3$ and too strong to be accepted. On the other hand, a proper string**) model such as given by the lattice gauge theory[3] does not produce such a long range potential.

*) The term "string" used in this paper does not necessarily imply a real string but just substitutes for a potential proportional to r or any agent that produces such a potential unless otherwise stated. Actually all the results in this paper were derived from Eq. (1) without making use of the notion of a string.

**) Here this term indicates a string more realistic than that mentioned before.

This point will be discussed in § 6.

§ 2. Quark-quark potential

For the quark-quark interaction, we take the static approximation and consider an adiabatic potential in which quark positions are c-numbers. The linearity of the potential in r and the fact that a colored object does not exist in nature can be understood by introducing an octet of "strings" with colors as their sources.[4] To any colored object a string is attached which can terminate only on another color. For an isolated colored system, the string extends to infinity, so that the energy of the system is linearly divergent and accordingly the colored object cannot exist by itself. In a color-neutral system, a string emitted from a color is reabsorbed by the complementary color in the same system, and the energy of the system remains finite. Since we take the invariance of our theory under $SU(3)$ for granted, the energy of the system must be of the form

$$V=-\frac{c}{2}\sum_{\alpha=1}^{8}\lambda_\alpha^{(i)}\lambda_\alpha^{(j)}r_{ij} \quad \text{if the system is color singlet,}$$

$$=+\infty \qquad \text{otherwise,} \tag{1}$$

where r_{ij} is the distance between quark i and j, and $\lambda_\alpha^{(i)}$ is the color spin matrix of quark i. λ should be replaced by $\lambda^c \equiv -\lambda^T$ for an antiquark. Incidentally, the potential of this form is that obtained by the dipole field theory.[5]

The constant c can be determined from the hadron spectroscopy. Equation (1) gives the potential $V=(16/3)cr$ for a color singlet $q\bar{q}$ system (meson), and to fit the slope of the Regge trajectory c must be taken as

$$c=0.023\ \text{GeV}^2. \tag{2}$$

This value is consistent also with the charmonium binding.

We assume the interaction (1) and calculate the van der Waals potential between hadrons.

§ 3. The van der Waals potential between two mesons

Consider two-meson state ψ_1, as given in Fig. 1(a). Quark 1 and antiquark 2 form a color singlet and so do quark 3 and antiquark 4. In another state ψ_2, Fig. 1(b), two mesons are polarized, not only in the spacial distribution of quarks but also in their color states. The mesons (12) and also (34) are in color octet

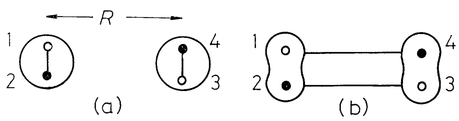

Fig. 1

states, but the total system still remains color singlet.

There will possibly be a transition between ψ_1 and ψ_2, that is, a rearrangement of strings will take place. This causes the decay of the stretched state ψ_2 into two mesons ψ_1. A nonvanishing value of the transition amplitude

$$T_{21}=\langle\psi_2|V|\psi_1\rangle \tag{3}$$

gives rise to a "van der Waals" potential

$$V_M=\sum_{\psi_2}\frac{|\langle\psi_2|V|\psi_1\rangle|^2}{E_1-E_2} \tag{4}$$

between mesons.

Consider two mesons separated by R, much greater than the meson radius. For sufficiently large R, each meson is in the ground state and ψ_1 is given by

$$\psi_1=\frac{1}{3}(\bar{\xi}^1\xi^2)(\bar{\xi}^3\xi^4)Y_{00}(\theta,\phi)Y_{00}(\theta',\phi')f(r)f(r'), \tag{5}$$

where ξ is the color variable and $r\theta\phi$ and $r'\theta'\phi'$ are polar coordinates of $\overleftarrow{(12)}/2$ and $\overleftarrow{(34)}/2$, respectively. $f(r)$ is the radial wave function of the ground state.

When mesons come closer, polarization takes place, i.e., each meson becomes partly color octet and the two combine in color singlet. For spatial wave function it is sufficient to take the following form:

$$\psi_2=\frac{1}{4\sqrt{2}}\sum_{\rho=1}^{8}(\bar{\xi}^1\lambda_\rho\xi^2)(\bar{\xi}^3\lambda_\rho\xi^4).$$

$$\frac{1}{2}(Y_{1-1}(\theta,\phi)Y_{11}(\theta',\phi')+Y_{11}(\theta,\phi)Y_{1-1}(\theta',\phi'))g(r)g(r'). \tag{6}$$

S- and D-states, for example, have no effect on lowering the expectation value of the energy of the system. The radial wave function $g(r)$ as well as the mixing parameter $\varepsilon(R)$ in the wave function,

$$\psi=N(\psi_1+\varepsilon(R)\psi_2),$$

is to be determined so as to minimize the expectation value of the energy for a given value of R. With the interaction (1) the transition amplitude is calculated to be

$$\langle\psi_2|V|\psi_1\rangle=-\frac{32}{9}c\frac{r_0^2}{R}, \tag{7}$$

where

$$r_0=\int_0^\infty r^3f(r)g(r)dr.$$

To attain the minimum of the energy, r_0^2 has to be maximized by varying $g(r)$ under the normalization condition. The solution is

$$g(r) \propto rf(r)$$

and

$$\max r_0^2 = \frac{(\int r^4 f^2 dr)^2}{\int r^4 f^2 dr} \equiv r_m^2 , \tag{8}$$

r_m^2 being the mean square radius of the meson in the ground state.

The expectation value of the energy becomes, for $R \gg r_m$,

$$\langle V \rangle = -\frac{64}{9}\varepsilon(R)\frac{cr_m^2}{R} + 12\varepsilon(R)^2 cR .$$

The last term is the energy of the stretched string in ψ_2. Kinetic energy of the mesons are independent of R, and not needed for the calculation of the potential energy. The minimum of $\langle V \rangle$ with respect to $\varepsilon(R)$ as a variable gives the van der Waals potential between mesons:

$$V_M(R) = \frac{-2^8}{3^5} c \frac{r_m^4}{R^3} \approx -1.05 c \frac{r_m^4}{R^3} . \tag{9}$$

§ 4. Baryon-baryon force

We take the independent particle model for baryons. The wave function of the ground state is expressed as

$$\psi_1 = \frac{1}{6} \xi_\alpha^1 \xi_\beta^2 \xi_\gamma^3 \xi_a^4 \xi_b^5 \xi_c^6 \varepsilon_{\alpha\beta\gamma} \varepsilon_{abc} f(1) \cdots f(6) \tag{10}$$

(Summations with respect to α, β, γ and a, b, c are implied.), where $\varepsilon_{\alpha\beta\gamma}$ is the completely antisymmetric tensor and $f(1)$ stands for $Y_{00}(\theta_1, \phi_1) f(r_1)$, etc. Completely symmetric function of spin and isospin variables is omitted.

The polarized baryon is a color octet P-state. Decuplets do not appear, since they do not combine in a singlet. There are two kinds of polarized state corresponding to two types of color octet wave functions. We take the following form for ψ_2:*)

$$\psi_2 = N[\varepsilon_{\alpha\beta\gamma}\varepsilon_{abc} G^{\pm}(1, 23) G^{\mp}(4, 56) + \text{permutation}],$$

$$G^{\pm}(1, 23) = \varepsilon_1(R) g_{1\pm}(1) f(2) f(3) + \varepsilon_2(R) f(1) \left(g_{2\pm}(2) f(3) + f(2) g_{2\pm}(3)\right),$$

$$g_{i\pm}(1) = Y_{1\pm 1}(\theta_1, \phi_1) g_i(r_1), \tag{11}$$

*) In Eq. (11) color variables ξ are omitted. This form of wave function contains color singlet as well as octets, but it does not affect the result.

where $\varepsilon_1(R)$ and $\varepsilon_2(R)$ are mixing parameters to be varied. The radial functions $g_1(r)$ and $g_2(r)$ are also varied and the extremum is attained when

$$g_1(r) = g_2(r) \propto rf(r) .$$

The mean square radius of the baryon is defined by

$$r_b^2 = \int r^4 f^2(r)\, dr .$$

Similarly to the case of meson-meson, the van der Waals potential between baryons is given by

$$V_B(R) = -\frac{2^4}{3^3} c \frac{r_b^4}{R^3} \approx -0.59 c \frac{r_b^4}{R^3} . \tag{12}$$

Inserting the experimental value of the mean square radius of the proton, $r_p = 0.8$ fm, for r_b and Eq. (2) for c, we have for the nucleon-nucleon potential

$$V_N(R) = -54 \text{ MeV} \left(\frac{r_p}{R}\right)^3 = -28 \text{ MeV} \left(\frac{1}{R \text{ in fm}}\right)^3 . \tag{13}$$

This is comparable in magnitude with the one-pion-exchange potential at nuclear distance, and is larger than the Coulomb interaction for $R < 4.5$ fm. This potential is negligible for molecular binding and is overcome by the gravitational potential only at $R \gtrsim 1$ km. Although it has little effect in celestial problems, our force is about a million times as strong as the gravitational force at $R = 1$m and certainly contradicts the Cavendish experiment.

§ 5. Perturbative QCD

At short distance the inter-quark potential will be dominated by $1/r$ term coming from normal gluon exchange. Let us consider a hypothetical case in which the linear potential is negligible compared with the $1/r$ term, yet $R \gg r_h$, and calculate the van der Waals potential.*) To do this, replace $-c$ in Eq. (1) by $\alpha_s = g^2/4\pi$ and r_{ij} by $1/r_{ij}$. Then Δr_{ij} is replaced by $-\Delta r_{ij}/R^2$, so that T_{21} is simply multiplied by $-\alpha_s/cR^2$. The potential is

$$V_M^{\text{QCD}}(R) = -\frac{2^{10}}{3^4} \frac{\alpha_s^2}{\Delta E} \frac{r_0^4}{R^6} ,$$

where ΔE is the difference $E_2 - E_1$ including kinetic energies but is independent of R. r_0 is given in Eq. (8).

For baryon-baryon interaction also a $1/R^6$ potential is obtained, but the ex-

*) Appelquist and Fischler in Ref. 2) calculated the van der Waals potential for a special case in which four quarks are placed along one line.

pression is more complicated due to the two kinds of polarized states.

In practice, however, the results obtained in this section cannot be compared with experiments, since we know that even at the distance of hadronic size the linear potential is as important as or more important than the $1/r$ term.

§ 6. Discussion

The quark-quark potential of the form (1) leads to a long range potential between hadrons. It is too strong to be compared with experiments. It may be exorbitant to extend the linear potential to macroscopic distances and compare with the gravitation. However, even in nuclear physics a long range Wigner force of considerable size will be at variance with observations.

A possible way out of this trouble seems to assume that the quark-quark potential does not grow linearly to infinity but eventually turns over at some point. We do not know if this assumption is consistent with the linearity of the Regge trajectory, nor we have a principle to determine the cutoff length for the linear potential.

In the wave function we included only ψ_1 and ψ_2, the ground and polarized two hadron states. Actually a stretched string has a tendency to be cut into pieces, i.e., the excited state ψ_2 couples more strongly to multimeson states. The inclusion of states with mesons in addition to the original hadrons in the trial wave function certainly modifies the results: it lowers the energy of the system. However, it is not clear if the potential energy $V(R)=\langle V\rangle_R-\langle V\rangle_{R=\infty}$ is deepened or shallowed by the additional states. It is not inconceivable that meson cloud around hadrons screens the long range potential.

Under the assumption of the instantaneous potential (1), where a string is nothing but a chemical bond representing a linear potential, the rearrangement of strings takes place instantly. However, if we take the string model more seriously, the situation is somewhat different. For instance, consider the lattice gauge theory.[3] Any state is locally gauge invariant, so a hadron is everywhere color neutral. From a quark a string extends to an antiquark (or to a diquark). It requires a factor $1/g^4$ to deform a string through an area a^2, where a is the lattice constant. The transition $\psi_1 \to \psi_2$ in Fig. 1 costs us a factor $(1/g^4)^{Rr_h/a^2}$ which decreases exponentially as $R\to\infty$. No long range potential between hadrons appears in this kind of theory.

In the potential model as described in §§ 3～5, the transition amplitude behaves as R^{-n}, where n is a positive integer, which is the cause of the long range hadron-hadron potential. This feature is common to all theories in which interquark potential is mediated by a (dressed) gluon with a propagator behaving as $1/k^m$ ($m=2$ or 4) in the infrared region.

In conclusion, we may say that a simple superposition of two-body potential does not work in the hadron-hadron problem, and that we need some mechanism for screening the long range hadron-hadron potential, an example of which has been described above. It is also conceivable that the inclusion of mesons or quark-antiquark pairs in the intermediate state in Eq. (4) gives rise to such an effect.

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