Long Run Expectations, Learning and the U.S. Housing Market

Daniel Tortorice, Economics Department, Brandeis University

Working Paper Series

2015 | 85

Long Run Expectations, Learning and the U.S. Housing Market*

Daniel L. Tortorice†

May 20, 2015

Abstract

This paper examines key facts about the U.S. housing market. The price to rent ratio is highly volatile and significantly autocorrelated. Returns on housing are positively autocorrelated. The price to rent ratio is negatively correlated with future returns on housing and future rent growth. Finally, housing returns exhibit significant time varying volatility. I show that a benchmark rational expectations general equilibrium asset pricing model is inconsistent with these facts. I modify the model in two ways to improve its fit with the data. First, I allow for pricing frictions so prices adjust slowly to their fundamental value. Second, I assume the agent does not know if housing fundamentals, captured by rental flows, are stationary or non-stationary and has changing beliefs depending on how well each model fits the current data. I find that these modifications allow the model to increase the volatility of the price to rent ratio and to match the autocorrelation of housing returns. The price to rent ratio then negatively forecasts returns and rent growth. Finally the model generates time varying volatility consistent with the data.

JEL Codes: D83, D84, G12, R21, E03

Keywords: Learning, Expectations, Housing Demand, Asset Pricing

---

*I thank my colleagues at the Brandeis University Economics department and International Business School for helpful comments and Suggestions. All mistakes are mine.

†Department of Economics and International Business School, Brandeis Univeristy. MS 021. 415 South St. Waltham, MA 02452. tortoric@brandeis.edu. (781) 736 5208.

1 Introduction

Given the recent boom and bust in housing markets there is renewed interest in understanding the determinates of U.S. house prices. This paper examines the equilibrium housing price in a general equilibrium asset pricing framework. First I outline several facts about housing prices and rents which are at odds with the standard rational expectations framework. First, there is evidence of excess volatility in house prices. The standard deviation of the price to rent ratio is 15% and the standard deviation of housing returns is 6%, while the standard deviation of the underlying housing rents is only 2.3%. Second, housing returns are significantly positively autocorrelated. Finally, the price to rent ratio is negatively correlated with future returns and rent growth and housing returns show evidence of time varying volatility.

I begin with a standard consumption-based asset pricing model and use a log-linear approximation to the Euler equation as in Campbell (1993) and Restoy and Weil (2011) to solve for the equilibrium house price. The equilibrium house price then depends on future expectations of fundamentals (housing preferences) and consumption. I show that this model is unable to explain the facts outlined in the previous paragraph. Therefore, I adapt the standard model in two ways. First, I allow for prices to adjust slowly to their fundamental value. Second, I assume the agent does not know the true model for housing preference shocks. Specifically, they are unsure if preference shocks are permanent or temporary. They use a Bayesian learning model as in Cogley and Sargent (2005) to learn if the preference process is stationary or non-stationary. Their beliefs change over time depending on how well each model fits the data. While the true process is stationary, the agent does not know this. He puts excessive weight on the non-stationary process, overreacting to temporary changes in market preferences.

Two features of the housing preference processes makes this learning significant. The first is the well know fact that unit root and near unit root processes are very difficult to tell apart in small time series samples. See, for example, Cochrane (1988); Stock (1991). As a result, the agent will almost always put some weight on the non-stationary process even if the true process is stationary. Additionally, after a random sequence of shocks which moves housing preferences away from their long run trend the agent will put additional weight on the non-stationary model. Second, analogous to the analysis of the permanent income hypothesis (see for example Deaton (1992)) if the individuals believes the true process is a unit root process then shocks are permanent. As a results, they will react strongly to news about fundamentals and return volatility will increase.

The sticky price assumption and the learning mechanism allow the model to better match the data. Learning amplifies volatility over the rational expectations benchmark. I find that the learning model generates increased volatility in both housing returns and the HP-filtered house price. Sticky prices allow the model to match the autocorrelation of housing returns. Learning about the true nature of the preference process creates a negative correlation between the price to rent ratio and future returns and rent growth. Additionally, the learning model generates time varying volatility consistent with the data. The learning model generates excessive kurtosis of returns. Data from the learning model is consistent with the positive autocorrelation of squared returns and the estimation of GARCH effects in U.S. data.

There is a large literature incorporating housing into macroeconomic models beginning with the contributions of Benhabib et al. (1991) and Greenwood and Hercowitz (1991). These models modify the standard real business cycle (RBC) framework to include housing and home production. Davis and Heathcote (2005) extend this framework to allow for housing specific productivity shocks but find that the model underpredicts the volatility of house prices and predicts a counterfactually negative correlation between house price growth and new home construction. Iacoviello (2005)and Iacoviello and Neri (2010) incorporate housing as a transmission mechanism for monetary policy in a New-Keynesian framework and find that preference shocks are important for explaining house price volatility. While much work has been done incorporating housing into standard macroeconomic models, these models still struggle to explain the volatility of housing prices given observed fundamentals. This observation motivates the current paper as well as other recent work, e.g. Miao et al. (2014) who amplify housing prices in a model where housing is used as collateral by firms engaged in production.

A second strand of literature attempts to explain house prices using supply and demand models with city-specific shocks and search frictions beginning with the work of Wheaton (1990) and Krainer (2001). Recent contributions include: Glaeser and Gyourko (2006); Head et al. (2012). These models use a wide variety of fundamentals to explain the cross section of house price volatility across cities, e.g. shocks to local amenities and income. The goals of this paper differ, in that I am concerned about explaining the macroeconomic time series of house prices and returns, specifically focusing on return predictability and time varying volatility while this literature is concerned about the cross section and autocorrelation of price growth across metropolitan areas.

My paper also relates to the literature that models housing within an asset pricing framework. Piazzesi et al. (2007) model housing jointly as an asset and a consumption choice in an otherwise standard consumption based asset pricing model. They show that housing increases the risk premium and predicts excess returns in equity markets. Lustig and Nieuwerburgh (2005) reach a similar conclusion in a model where housing is an important source of collateral. Flavin and Nakagawa (2008) explore how the illiquidity of housing influences the stochastic discount factor in a consumption based asset pricing model. Ayuso and Restoy (2006) apply the asset pricing framework of Restoy and Weil (2011) and show that a large part of the fluctuations in Spanish house prices can not be explained with observed fundamentals. The present paper uses the asset pricing framework of Restoy and Weil (2011) to explain house prices, but differs from the above papers by focusing on the volatility and predictability of housing returns and considering a learning based model of expectation formation.

Many recent papers have tried to explain the recent U.S. housing boom and bust in rational expectations models using various institutional features and frictions in the housing market. For example, Chu (2014) uses the fall in down payment requirements as an explanation for the increase in house prices during the boom period. However, the model is unable to explain why interest rates remained low at the same time. Similar mechanisms are explored in Chambers et al. (2009) and Iacoviello and Pavan (2013) with the former examining the impact of down payment requirements on the rise in homeownership rates during the boom and the later arguing a tightening of credit constraints can lead to a large drop in home production though these papers take the path of house prices as given. Corbae and Quintin (2013) and Garriga and Schlagenhauf (2009) explore how leverage lead to an increase in foreclosures during the house price bust, but treat the path of prices as exogenous. Titman et al. (2014) and Chatterjee and Eyigungor (2009) argue overbuilding was an important contributor to the bust in housing prices, however they have difficulty explaining the positive correlation between house prices and residential investment. Favilukis et al. (2013) combine foreign capital inflows, relaxed credit constraints and financial market liberalization to explain the fluctuations in the price-to-rent ratio during the boom and bust. The model has considerable success explaining the volatility of house prices and returns and generates predictability in housing returns as well. However, agents are aware of this predictability and therefore expectations of future returns are low when the price to rent ratio is high. This result is in contrast to expected returns in my model and data on survey expectations discussed below.

One of the first papers to examine data on house price expectations is Case and Shiller

(2003). They find that home buyers have unrealistic expectations concerning future house price increases, predicting double digit increases annually over the next ten years. Households also are unlikely to view housing as a risky investment. Case et al. (2012b) replicate these results and argue that long run expectations (10-year) house price expectations the primary driver of the house price boom and present evidence of extrapolative behavior in agents expectations formation. Piazzesi and Schneider (2009) argue for the presence of momentum traders in the housing market, agents who are always optimistic about price changes, based on data from the Michigan Survey of Consumers. Foote et al. (2012) present evidence that even industry financial analysts were bullish about house prices even at the 2006 house price peak.

Based on this empirical evidence many authors have examined the implication of relaxing rational expectations for house price dynamics. In fact, given the shortcomings of rational expectations models to match the volatility of house prices Glaeser and Gyourko (2006) and Glaeser et al. (2008) argue that deviation from rational expectations and models of learning may be fruitful avenues of research. Glaeser and Nathanson (2015) study a model where agents neglect to consider the forecasts of other agents when forecasting future prices. They show that this mechanism generates extrapolation in agents price forecasts explaining short run momentum and medium term reversion in price changes. My paper differs in considering the impact of beliefs for time varying volatility of housing returns and also modeling learning which allows agents to abandon models which look unlikely given the data.

A few papers have modeled house price expectations using learning models. Burnside et al. (2011) consider a model where, as in the current paper, learning about long run fundamentals is essential. Though in their paper learning comes from social dynamics as opposed to observation of fundamentals. Bolt et al. (2014) generated boom and busts in house prices through an heterogeneous agent model where the agents endogenously switch between different price forecast rules. Adam et al. (2012) consider a model where agents learn about house price growth in an open economy model. Gelain and Lansing (2013) considers learning in an asset pricing based model of housing where agents learn about rent growth using a misspecified model and extrapolative expectations. All these models increase volatility of the price to rent ratio and the Gelain and Lansing paper also generates predictability in house price returns. However, my paper differs from these in important ways. First, the paper seeks to endogenously explain both the predictability and the time varying volatility in housing returns as well as amplifying volatility.[1] Secondly, I present a

[1] In Gelain and Lansing (2013) for example, fundamentals exhibit exogenous time-varying volatility.

novel model of learning where agents are unsure about the true process for fundamentals and change their beliefs based on how accurately each model captures the data. Finally, agents make significant mistakes about their long run expectations (as opposed to their short run expectations) consistent with the results in Case et al. (2012b,a).[2]

The rest of the paper proceeds as follows. Section two discusses the data and the key empirical facts. Section three outlines the model and section four explains its calibration. Section five gives the main model results and section six demonstrates the robustness of the results to alternative parameter specification. Section seven concludes.

2 Data

Data come from Davis et al. (2008).[3] Data begin in 1960:Q1 and end in 2013:Q1. Data on rents and house prices are obtained from the Decennial Census of Housing from 1960 to 2000. Data on rents are interpolated between census dates using the Bureau of Labor Statistics (BLS) index for the rent of primary residences. Data on house prices are interpolated using the Freddie Mac (CMHPI) series repeat-sales house prices index after 1970 and the median price of new homes sold index before 1970. The Macromarkets LLC national house price index, formally known as the Case-Shiller-Weiss index, is used after 2000 to construct house prices. Prices are deflated using the CPI.

Moments for the data are given in table 1. The expected return on housing, given by $E \frac{q_t}{q_{t-1}-\xi_{t-1}}$ where q_t is the house price at time t and ξ_{t-1} is the rent at time $t-1$, is equal to 6.4% on an annual basis. The standard deviation of the annual return is 6%. Rent growth averages 1% per year with a standard deviation of 2.3%. These data indicate the presence of an excess volatility puzzle with returns being almost three time as volatile as the underlying fundamental rents. The standard deviation of the log price-to-rent ratio is 15% and the standard deviation of the log HP-filtered housing price is 3.7%.[4]

Examining autocorrelations of the data at one to four quarterly lags, we see that the price-to-rent ratio is highly persistent with all autocorrelation coefficients above 0.95. The autocorrelation of returns decline from 0.84 at one lag to 0.51 at four lags. The existence

[2]Nguyen (2014) uses a similar learning model to explain serially correlated house price forecast errors and house price volatility in a model in which housing is allocated by a central planner who does not know the true process for housing preference shocks.

[3]Data are available at: http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp.

[4]The price-to-rent ratio is calculated as q_t/ξ_t. There is no need to take a past average of rents as rents (as opposed to equity dividends) are quite smooth in the data.

of positive momentum in the housing market has previously been documented by Case and Shiller (1989) among others.

Additionally, there is evidence of return predictability in the housing data. The price-to-rent ratio is negatively correlated with the cumulative return over the next four years $r_{t+1} + ... + r_{t+16}$ with a coefficient of -0.74. However the current period return is positively correlated with the same cumulative return with a coefficient of 0.19. Finally the PE ratio is negatively correlated with future rent growth, $\ln \xi_{t+16} - \ln \xi_t$ with a coefficient of -0.44. This long term mean reversion in housing prices is also noted by Glaeser et al. (2014).[5]

As a further examination of return predictability in the data I estimate the following Campbell-Shiller [Campbell and Shiller (1988)] style regressions:

$$r_{t+1} + ... + r_{t+16} = \alpha + \beta^{return} \ln(price - to - rent) + \epsilon_t$$
$$\ln \xi_{t+16} - \ln \xi_t = \alpha + \beta^{rent} \ln(price - to - rent) + \epsilon_t$$

Results for the regressions are presented in table 2. We find that $\beta^{return} = -0.8$. This coefficient implies that a 10% increase in the price to rent ratio predicts cumulative returns will be -8% lower over the next for years or about -2% per year. The R-squared of this regression is -0.5 suggesting cumulative returns over the next 4 years are explained fairly well using the price-to-rent ratio. On the other hand, $\beta^{rent} = -0.12$ and the R-squared of the regression is only 0.2. This suggests that rent growth is less predictable than return growth and importantly high price-to-rent ratios do not seem to forecast periods of high demand for housing. Quite the oppositive, if anything, they predict we are entering a period of low demand for housing.

There is also substantial evidence of time varying volatility in the data. I report skewness $E\frac{(x-\mu)^4}{\sigma^4}$ and kurtosis $E\frac{(x-\mu)^3}{\sigma^3}$ of the price to rent ratio. Housing returns are right skewed with a skewness of -2 while the price to rent ratio is left skewed with a skewness of 2.04. Both series also demonstrate high levels of kurtosis of around 7. If the series were normally distributed they would exhibit a kurtosis of 3. The high level of kurtosis is evidence of the existence of fat-tails in the distribution, i.e. increased frequency of extreme values relative to a normal distribution.

As further evidence of time varying volatility I examine the autocorrelation of squared returns and the autocorrelations of squared residuals from an AR(1) return regression. If large returns and residuals are more likely followed by large returns and residuals, as would

[5]Results are qualitatively similar for a variety of horizon windows from 3 years onward.

be the case with time varying volatility, we would expect to see positive autocorrelation of squared returns and residuals. That is indeed what we see. The autocorrelation of squared returns ranges from 0.7 at one lag to 0.27 and four lags. Similarly, the autocorrelation of squared residuals ranges from 0.1 to 0.39.[6]

For additional evidence of time varying volatility in the aggregate U.S. housing data, I estimate a GARCH(1,1) model on the quarterly return series. The GARCH(1,1) model is:

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + a_1 \varepsilon_{t-1}^2.$$

In this model the variance of ε_t in the AR(1) regression $r_t = \alpha + \rho r_{t-1} + \varepsilon_t$ is varying over time. Positive γ_1 and a_1 are evidence of time varying volatility with data that will exhibit periods of particularly high volatility. For the quarterly return data I estimate $\gamma_1 = 0.73$ and $a_1 = 0.27$. Both estimates are highly statistically significant. Furthermore the Engle test (Engle (1982)) rejects the null of no GARCH effects at the 95% confidence level.

3 Model

3.1 Housing Choice

A representative consumer can consume or buy units of housing h_t at a price q_t. The household is subject to stochastic shocks to their preference for housing ξ_t. The household's problem then is to choose

$$\max_{c_t, h_t} E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \xi_t \ln h_t \right]$$

subject to the constraint

$$c_t + q_t h_t = q_t h_{t-1} + y_t \tag{1}$$

Here y_t is income at time t, and c_t is consumption at time t. The first order conditions for the consumer's optimal choice are:

$$c_t : c_t^{-\gamma} - \lambda_t = 0 \tag{2}$$

[6]Under the null hypothesis of zero autocorrealtion, standard errors are calculated as $\frac{1}{\sqrt{T}} = 0.07$, making these estimates statistically significant. See Hamilton (1994) pp. 111.

$$h_t : \frac{\xi_t}{h_t} - \lambda_t q_t + \beta E_t \left[\lambda_{t+1} q_{t+1}\right] = 0 \tag{3}$$

where λ_t is the Lagrange multiplier on the budget constraint.

Combing these two equations one gets

$$q_t = \frac{\xi_t}{h_t \lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1}\right]$$

I will assume that housing is in fixed supply so that $h_t = 1$ for all t. From the first order condition we have

$$q_t = \frac{\xi_t}{\lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1}\right]$$

Letting $d_t = \frac{\xi_t}{\lambda_t}$ we can write

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t}\right]$$

This is a highly non-linear condition. But, following Campbell (1993) and Restoy and Weil (2011) it can be linearized in two steps. First assuming that housing returns and consumption are conditionally homoskedastic and jointly log-normally distributed and second by linearizing $\ln(q_t - d_t)$ around its mean. The appendix first shows that:

$$ln(q_t - d_t) = \ln\beta + E_t\left[-\gamma \Delta \ln c_{t+1} + \ln q_{t+1}\right] + \frac{1}{2}\left[\gamma^2 \sigma_c^2 + \sigma_q^2 - 2\gamma\sigma_{c,q}\right] \tag{4}$$

where $\sigma_c^2 = Var_t \Delta \ln c_{t+1}, \sigma_q^2 = Var_t \ln q_{t+1}$ and $\sigma_{c,q} = Cov_t(\Delta \ln c_{t+1}, \ln q_{t+1})$.

Then I linearize $ln(q_t - d_t)$ about the mean rent to price ratio and obtain:

$$\ln(q_t - d_t) \approx k + (1-\delta)\ln q_t + \delta \ln d_t$$

where $k = \ln(1 - \exp(\bar{x})) + \frac{\exp(\bar{x})}{1-\exp(\bar{x})}\bar{x}$, $\delta = \frac{-\exp(\bar{x})}{1-\exp(\bar{x})}$ and $\bar{x}{=}E\ln(\frac{d_t}{q_t})$.

Letting $\sigma = \frac{1}{2}\left[\gamma^2\sigma_c^2 + \sigma_q^2 - 2\gamma\sigma_{c,q}\right]$we have

$$\ln q_t \approx \frac{1}{1-\delta}\left[\sigma + \ln\beta - k - \delta \ln d_t + E_t(-\gamma\Delta \ln c_{t+1} + \ln q_{t+1})\right]$$

Iterating forward

$$\ln q_t \approx \frac{k - \ln\beta - \sigma}{\delta} - \frac{\delta}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \ln d_{t+s} - \frac{\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \Delta \ln c_{t+s+1}$$

Since $d_t = \frac{\xi_t}{\lambda_t}$ we obtain $\ln d_t = \ln \xi_t - ln\lambda_t = ln\xi_t + \gamma lnc_t$. Therefore we can solve for the following closed form solution for the log house price:

$$\ln q_t \approx \frac{k - \ln \beta - \sigma}{\delta} - \frac{\delta}{1-\delta} E_t \sum_{s=0}^{\infty} \left[\frac{1}{(1-\delta)^s} \ln \xi_{t+s} - \frac{\gamma}{(1-\delta)^s} \ln c_{t+s} \right] - \frac{\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \Delta \ln c_{t+s+1}$$

The appendix gives expressions for the sums and the conditional variances as a function of the underlying processes for consumption growth and housing preferences listed below.

Iacoviello (2010) argues that institutional rigidities lead to sluggish adjustment in the housing market. For example, listing agents often used "comps" or comparable sales when setting prices. These are listings which are similar to the property for sale and have sold recently. Likewise, banks often will not grant a mortgage on a property that does not appraise for the sale price. I model these institutional features in a reduced form way, assuming rigidity in the price such that the market price q^* is given by:

$$lnq_t^* = \lambda lnq_t + (1-\lambda) lnq_{t-1}^* \tag{5}$$

where λ is a measure of the degree of institutional price rigidity in the market. The smaller is $\lambda$the more sluggishly prices adjust in the housing market.

To close the model is is necessary to specify the exogenous processes for consumption and housing preference. In order to focus on the role of future expectations of housing fundementals for driving house prices, I assume that consumption growth is i.i.d. and uncorrelated with the housing preference process. As a results only future expectations of housing fundamentals will drive house prices. Again, to focus about learning about future housing fundamentals I allow the consumer to know the exact process for consumption.

In contrast, there is uncertainty about the nature of the housing preference process. The housing preference process will be a trend stationary process. However, the consumer does not know the true form of the preference process ξ_t. Specifically, he is uncertain if the preference process is stationary or not. He believes that:

$$\xi_t = \rho_0^s + \beta t + \rho_1^s \xi_{t-1} + ... + \rho_p^s \xi_{t-p} + \varepsilon_t^s \tag{6}$$

with probability p_t^s and that

$$\Delta\xi_t = \alpha + \rho_1^{ns} \Delta\xi_{t-1} + ... + \rho_p^{ns} \Delta\xi_{t-p} + \varepsilon_t^{ns} \tag{7}$$

with probability $p_t^{ns} = 1 - p_{t.}^s$ The next section describes how the consumer updates his beliefs.

In this model, I am deliberately vague about what the housing preference process represents. Since it is the key driver of house prices it is a reduced form representation of the various shocks which can affect house prices. It could represent interest rates, credit availability as well as a strict preference shock to the demand for housing. It can also represent macroeconomic conditions and housing supply conditions which affect house prices. I capture all of these effects with one parameter to better focus on the role learning and expectations can have in driving house prices.

However, it is worth noting that the emphasis on housing preference shocks is supported by the literature which seeks to explain house price fluctuations. Iacoviello and Neri (2010) find a large fraction of the variation in house prices can be attributed to preference shocks, even controlling for a wide variety of fundamentals like income, interest rates, availability of sub prime mortgages and mortgage fees. Iacoviello (2010) emphasizes this point noting that many press articles explain changes in house prices with changes in the nature of what housing consumers are looking for, e.g. looking for larger homes or viewing homes as an investment vehicle.

3.2 Beliefs

I use the methods of Cogley and Sargent (2005) to calculate the parameters of each model of housing preference and the probability weights on the stationary and non-stationary model. Their model uses Bayesian methods to recursively update the parameters on each model and then uses the likelihood of each model to calculate a probability weight on each model. For a given model (i.e. the stationary or non-stationary) indexed by $i = \{s, ns\}$, and a housing preference history Ξ^{t-1}, we assume that agents prior beliefs about the model parameters are distributed normally according to:

$$p(\Theta_{i,t-1}|\sigma_i^2, \Xi^{t-1}) = N(\Theta_{i,t-1}, \sigma_i^2 P_{t-1}^{-1})$$

and their prior beliefs concerning the model residual variance are given by:

$$p(\sigma_{i,t-1}^2|\Xi^{t-1}) = IG(s_{t-1}, v_{t-1})$$

Here N represents the normal distribution function and IG represents the inverse-gamma distribution function. P_{t-1} is the precision matrix that captures the confidence the agent has in his belief for $\Theta_{i,t-1}$, σ_i^2 is the estimate of the variance of the model residuals, s_{t-1} is an analogue to the sum of squared residuals, and v_{t-1} is a measure of the degrees of freedom to calculate the residual variance such that the point estimate of $\sigma_{i,t-1}^2$ is given by s_{t-1}/v_{t-1}. After observing the housing preference ξ_t the agent's posterior beliefs are given by:

$$\begin{aligned} p(\Theta_{i,t}|\sigma_i^2, \Xi^t) =& N(\Theta_{i,t}, \sigma_i^2 P_t^{-1}) \\ p(\sigma_i^2|\Xi^t) =& IG(s_t, v_t) \end{aligned}$$

Cogley and Sargent (2005) gives the following recursion to update the parameters of the beliefs:

$$\begin{aligned} P_t =& P_{t-1} + x_t x_t' \\ \theta_t =& P_t^{-1}(P_{t-1}\theta_{t-1} + x_t y_t) \\ s_t =& s_{t-1} + y_t^2 + \theta_{t-1}' P_{t-1}\theta_{t-1} - \theta_t' P_t \theta_t \\ v_t =& v_{t-1} + 1 \end{aligned}$$

Here x_t is the vector of right hand side variables for the model at time t and y_t is the left hand side variable for the model at time t. This recursion gives the parameters of each model. Now it is necessary to calculate the probability weight on each model.

Given a set of model parameters: $\{\Theta_i, \sigma_i\}$ we can calculate the conditional likelihood of the model as:

$$L(\Theta_i, \sigma_i^2, \Xi^t) = \prod_{s=1}^{t} p(y_s|x_s, \Theta_i, \sigma_i^2)$$

where y_s and x_s are the left and right hand side variables of the model at time s and Ξ^t is the housing preference history up to time t. Based on this likelihood, one can write the marginalized likelihood of the model by integrating over all possible parameters:

$$m_{it} = \iint L(\Theta_i, \sigma_i^2, \Xi^t) p(\Theta_i, \sigma_i^2) d\Theta_i d\sigma_i^2$$

Then we have the probability of the model given the observed data $p(M_i|\Xi^t) \propto m_{i,t} p(M_i) \equiv w_{i,t}$. Here we have defined the weight on model i, $w_{i,t}$ and $p(M_i)$ is the prior probability on model i.

Cogley and Sargent (2005) show that Bayes's rule implies

$$m_{it} = \frac{L(\Theta_i, \sigma_i^2, \Xi^t)p(\Theta_i, \sigma_i^2)}{p(\Theta_i, \sigma_i^2|\Xi_t)}$$

and therefore

$$\frac{w_{i,t+1}}{w_{i,t}} = \frac{m_{i,t+1}}{m_{i,t}} = p(y_{i,t+1}|x_{i,t}, \Theta_i, \sigma_i^2)\frac{p(\Theta_i, \sigma_i^2|\Xi_t)}{p(\Theta_i, \sigma_i^2|\Xi_{t+1})}$$

We assume that regression residuals are normally distributed allowing us to use the normal p.d.f to calculate $p(y_{i,t+1}|x_{i,t}, \Theta_i, \sigma_i^2)$. Cogley and Sargent (2005) show that $p(\Theta_i, \sigma_i^2|\Xi_t)$ is given by the normal-inverse gamma distribution and provide the analytical expressions for this probability distribution. Any choice of Θ_i, σ_i^2 will give the same ratio of weights; I use the posterior mean in my calculations.

This recursion implies the following recursion for model weights.

$$\frac{w_{s,t+1}}{w_{ns,t+1}} = \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}\frac{w_{s,t}}{w_{ns,t}}$$

Since housing preference shocks are an exogenous process, the model will eventually put all the weight on the true process. To allow for perpetual learning, I adapt the concept of constant gain learning from the least squares learning literature to the current setup. I introduce a gain parameter (g) that over-weights current observations. The gain probability can be interpreted as the probability of a structural break in the economy, such that the history of the housing preference process no longer has any bearing on the current process generating housing preferences, hence the previous weight ratio is set to one.

$$\frac{w_{s,t+1}}{w_{ns,t+1}} = (1-g)\frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}\frac{w_{s,t}}{w_{ns,t}} + g\frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}$$

Finally, to calculate the model probabilities, the consumer normalizes the weights to one, and therefore the weight on the stationary model is given by:

$$p_{s,t} = \frac{1}{1 + w_{ns,t}/w_{s,t}}$$

Using the estimated probabilities, he can then calculate the price by:

$$lnq_t^L = p_{s,t}lnq_t^S + (1 - p_{s,t})lnq_t^{NS} \tag{8}$$

where lnp_t^S and lnp_t^{NS}represent the log prices conditional on the stationary and non-stationary models being true.[7]

4 Calibration and Simulation

Time is quarterly and I set the discount rate in the model $\beta = 0.9975$. This implies a 3% annual real interest rate slightly higher than average rates on 10-year Treasury inflation protected securities (TIPS).[8] Low discount rates are consistent with the evidence in Giglio et al. (2014) who find very low discount rates when comparing the prices on housing with temporary ownership contracts versus permanent ownership contracts in the U.K. and Singapore. In the sticky price version of the model I set $\lambda = 0.25$ to better match the positive autocorrelation of returns. I set the lag length of the stationary and non stationary model of housing preferences to 4. I set $\gamma = 1$. Robustness to these parameters are explored in section 6.

I also need to calculate the prior beliefs of the agent, however these do not matter much for the results because I simulate the model for 2,000 periods and keep only the last 212 = (2013-1960)*4 observations to match the length of my data. The initial prior on the stationary model $p_{s,0} = 0.5$. To calibrate the initial beliefs for the stationary and non-stationary processes (Θ_o) I estimate ordinary least squares regression on the log housing rents series from Davis et al. (2008) deflated with the CPI. I assume that ε_t^s and ε_t^{ns} are distributed $N(0, \sigma_i^2)$ where σ_i^2 is estimated from the residuals in the previous estimation. I set the initial precision matrix $P_o = 0.01 * I$. This is a fairly defuse prior setting the standard error of the coefficients to 10 times the standard deviation of the regression residuals. I set s_o to the variance of the regression residuals and set the initial degrees of freedom (v_o) equal to 1. Finally, I assume that consumption growth is i.i.d, and estimate the process using real per-capita consumption of non-durables and services from the national income and product accounts.

I set the gain parameter (g) equal to 0.005 which is the minimum value of the gain that allows for perpetual learning. To see this examine figure 1. I plot the median probability on

[7]Importantly I make a standard assumption from the learning literature, that of anticipated utility (Kreps (1998)), i.e. the agent makes decisions assuming his future beliefs will be the same as his current beliefs. This includes his beliefs about both the likelihood of each process and the implied covariances. However, beliefs can and do change in the future.

[8]The real interest rate in the model is given by $(1 + g^c)^\gamma/\beta$ where g^c is the growth rate of consumption which is calibrated to be 0.5% per quarter.

the stationary model across 20 trials of length 5,000 keeping the last 4,000 observations. I use the stationary process, as calibrated in the previous paragraph, as the true process. We see that for a gain of 0.001 and 0.0025, the probability on the stationary model eventually converges to one. However, for a gain equal to 0.005 we do observe perpetual learning. So I choose this value for the gain, the minimal value that still allows for perpetual learning. Robustness of the results for higher values of the gain are considered in section 6.

To evaluate the model I assume the true preference process is the stationary process and simulate 500 trials of length 2,000 keeping the last 212 observations to match the length of my data set. I then report median statistics across the trials. Initial housing preferences are normalized to the steady state value of the stationary model when t equals zero and consumption is initialized to be twice this value in line with U.S. CPI data which suggests housing represents 30% of the U.S. consumption basket, though I find this initialization does not affect the results.

I am motivated to make the true process the stationary process by a variety of concerns. The first is that the survey evidence outlined in the introduction suggests that individuals overreacted to the run up in house prices and extrapolated current price changes far into the future. This evidence supports a true process for fundamentals being one with temporary deviations from trend and agents overrating to these temporary deviations by assuming they are permanent. Additionally, in the U.S. housing market temporary increases in prices may be persistent because of a slow response of supply. However, eventually supply can respond to bring prices down. A model where agents believe temporary shocks are permanent is consistent with neglecting the long run supply response of housing.[9] Finally, Shiller (2005) and Reinhart and Rogoff (2009) argue that individuals often attribute new-era stories of fundamental change to justify high valuations or current booms as being permanent instead of temporary. This view of the world is consistent with my modeling. Of course, it would be possible to have the underlying process be a true switching process and agents form rational beliefs about what state they are in. However, I believe the spirit of that model is different than my goal in this paper. In that model agents are as likely to underreact as overeact. But here I try to capture the general notion that in speculative bubbles agents are overreacting in their long run expectations and neglecting the tendency of fundamentals to return to long run trends.

[9] Fuster et al. (2012) argue that neglecting long run mean reversion is a key psychological bias that is useful for understanding equity market puzzles.

5 Results

In this section I compare results from the learning model with a rational expectations benchmark. The benchmark model is one in which the preference shock follows a stationary process and the household knows this. Results are reported for the flexible price model where $\lambda = 1$ so that prices immediately adjust to the fundamental value and then for $\lambda = 0.25$ so price adjust more slowly.

5.1 Model Mechanism

To provide intuition for the main mechanism of the model I examine a single simulated housing preference series and the implied path of rents and beliefs. Examining figure 2 we see plotted the probability the learning model puts on the stationary model being true for a single simulated housing preference series from the stationary model. We see that on average the model puts more weight on the stationary model than the non-stationary model. However, this weight is not constant. Around time 75 we see the beliefs drift to the stationary model where the agent goes from putting 75% weight on the stationary model to only putting 60% weight on the stationary model.

Recall that beliefs are endogenous here and depend on the realized housing preference and implied rent series. Figure 3 plots the rent series that corresponds to the simulated housing preference series. At time 75, we can see the growth rate of rents increases resulting in a housing fundamental series that is persistently above trend. Because the series is not reverting to trend, the agent begins to put more and more weight on the possibility that the housing preference series is non-stationary, revising his beliefs. Finally, around time 100 growth slows down, and the agent revises his beliefs, going back to putting 75% of his weight on the stationary model.

Figure 4 plots the price-to-rent ratio under the learning model (dashed line) versus a rational expectations benchmark where the agent knows the true process is stationary. We see a large spike up in the price-to-rent ratio, increasing 12% relative to a a slight fall under the rational expectations benchmark. There is a temporary housing boom while the agents believe that there has likely been a permanent increase in housing fundamentals which is then reversed with an abrupt fall in the price-to-rent ratio once the agent reverses his belief around time 100.

This mechanism is responsible for the main results of this paper. Agents overreact to news when the world looks as if it may be non-stationary. The overreaction is corrected once the fundamental begins to mean revert. This mechanism results in predictability of returns. Additionally, when the agent believes the world may be non-stationary he reacts strongly to news results in a higher volatility of returns. These reactions generate time varying volatility in returns.

5.2 Main Moments

Results on the performance of the model in explaining the main moments in the data are presented in table 3. Examining the flexible price case first, note that both models imply a 3% annual average return on housing and a 3% growth rate of rent. In the data, these number are 6.4% and 1% respectively.

The standard deviation of the log price-to-rent ratio $\sigma(\frac{P_t}{R_t}) = 15\%$ in the data. Both the rational expectations benchmark and the learning model generate 1/5 this volatility, predicting a standard deviation of 3%. The learning model better matches the standard deviation of the HP-filtered price (P_t^{HP}) which was 3.7% in the data. The learning model predicts 2% versus only 0.8% for the rational expectations benchmark. I obtain a similar result for the standard deviation of returns, $\sigma(r_t)$, with the learning model predicting 6% versus the 6% in the data. The rational expectations benchmark model predicts only 3%.

Recall, that both the price-to-rent ratio and returns are highly positively autocorrelated. Both models are consistent with the first fact, however since the housing price is modeled as an asset price neither model can explain the positive autocorrelation of returns.

Finally, I examine the ability of the model to explain the predictability of housing returns. The price-to-rent ratio is negatively correlated with future housing returns, $\rho(\frac{P_t}{R_t}, r_{t+1} + ... + r_{t+16}) = -0.74$. The rational expectations benchmark predicts a small positive correlation.[10] The learning model however predicts a coefficient of -0.41. Similarly, the price-to-rent ratio is negatively correlated with future rent growth $\rho(\frac{P_t}{R_t}, lnrent_{t+16} - lnrent_t)$. The learning model predicts a correlation coefficient of -0.24 vs. -0.44 in the data. The rational expectations benchmark obtains the wrong sign for the correlation, prediciting a positive correlation. Finally, neither model can match the fact that the current return is positively correlated with future returns.

Campbell-Shiller style regressions tell a similar story to the raw correlations. The rational expectations model predicts no predictability of returns. The coefficient on returns β^{return}=0.04 and the R^2=0.04. In contrast, the learning model generates a coefficient equal

[10] Small sample bias leads to a slight negative correlation instead of a value of zero.

to -0.73 versus -0.8 in the data with an appreciable higher R^2 of 0.16. Similarly, the rational expectations model poorly explains rent growth predictability. The coefficient on rent growth in the Campbell-Shiller regression $\beta^{rentgrowth}{=}-0.12$ however the rational expectations model predicts a coefficient equal to 0.4. Because fundamentals are mean reverting, a high price-to-rent ratio predicts future rent growth. However, in the learning model a high price-to-rent ratio is consistent with overreaction to growth in fundamentals. As a result, a high price-to-rent ratio forecasts lower rental growth in the future not higher rental growth. This mechanism allows the learning model to matche the data much more closely. It predicts a coefficient of $\beta^{rentgrowth} = -0.23$ and an $R^2 = 0.08$.

Next, I examine the ability of the sticky price model to match the data, highlighting the differences relative to the flexible price model. Sticky prices lower the volatility of the HP filtered price for the learning model, 1% vs. 2% before, and returns 3% vs. 6% as before. In both cases the learning model still increases volatility over the rational expectations benchmark.

Sticky prices have little effect on the predictions for the autocorrelations for the price-to-rent ratio. However, the model is now able to generate positive autocorrelation of returns. The rational expectations model predicts autocorrelations ranging from 0.73 at one lag to 0.27 at four lags vs. 0.84 to 0.51 in the data. The learning model also predicts positive autocorrelation ranging from 0.44 to 0.15.

Finally, the rational expectations model predicts a small negative correlation between the price-to-rent ratio and future returns and a positive correlation between the price-to-rent ratio and rent growth. In the data these correlations are strongly negative. The learning model generates negative correlations consistent with the data. Indeed it predicts a correlation between the price-to-rent ratio and future returns equal to -0.42 versus -0.74 in the data and between the price-to-rent ratio and future rent growth equal to -0.25 versus -0.44 in the data. But models now are consistent with the positive correlation between the current return and future returns. The rational expectations model predicts 0.27 and the learning model predicts 0.15 vs. 0.32 in the data.

Campbell-Shiller regressions for the sticky price model show a similar result. The rational expectations model again predicts no predictability of returns. The coefficient on returns $\beta^{return}{=}-0.06$ and the $R^2{=}0.03$. In contrast, the learning model generates a coefficient equal to -0.78 versus -0.8 in the data with an R^2 of 0.19. Similarly, the rational expectations model again does not explain rent growth predictability. The coefficient on rent growth $\beta^{rentgrowth}{=}-0.12$ however the rational expectations model predicts a coefficient equal to

0.37. The learning model matches the data much more closely. It predicts a coefficient of $\beta^{rentgrowth} = -0.33$ and an $R^2 = 0.11$.

5.3 Time Varying Volatility

Neither model can explain the skewness in the data, however the learning model generates higher kurtosis than the rational expectations benchmark. For the price-to-rent ratio, the learning model generates kurtosis equal to 3.3 vs 2.2 for the rational expectations benchmark. For returns, the learning model generates return kurtosis equal to 3.9 vs. 2.9 for the rational expectations model. In the data kurtosis is about 7. In the sticky price case the learning model generates more kurtosis than the rational expectations model and comes close to matching the data on kurtosis of returns with a value of 5.

The flexible price results indicate that the rational expectations model does not generate any autocorrelation in squared returns $\rho(r_t^2, r_{t-1}^2)$ or residuals $\rho(\varepsilon_t^2, \epsilon_{t-1}^2)$. In contrast, model learning allows for endogenous time varying volatility, though in the flexible price case the magnitudes of the correlations predicted by the model are smaller than in the data. The learning model predicts and autocorrelation of squared returns equal to 0.06 vs 0.5 in the data; the learning model predicts and autocorrealtion of 0.08 on average for AR(1) return residuals versus 0.2 in the data. In the sticky price case, the rational expectations benchmark has autocorrelation in squared returns but not in the squared residuals. However, the learning model predicts positive autocorrelation for both squared returns and residuals. Additionally, for the squared residuals the magnitude predicted by the learning model is approximately correct, about 0.15 for the model versus 0.2 for the data.

Results from estimating GARCH models on the simulated data, give a similar result. [11] There is no evidence of GARCH effects in the rational expectations benchmark. However, we consistently find significant GARCH effects in the learning model data and of a similar magnitude to the data when we allow for sticky prices. For the sticky price model the GARCH parameter equals 0.65 versus 0.73 in the data, while the ARCH parameter equals 0.22 versus 0.27 in the data. For the flexible price model, the median GARCH and ARCH paramaters are zero. However, even with flexible prices the learning model shows more evidence of GARCH effects. The Engle test for GARCH effects reject 42% of the time for

[11] To estimate the GARCH parameters, I first run an Engle test for the null hypothesis of no conditional heteroscedasticity on the simulated data. I estimate the GARCH parameters only if the test rejects, otherwise I assign zeros for the GARCH parameters. This procedure is required because absent GARCH effects I am unable to identify the GARCH parameters using the maximum likelihood procedure.

the learning model but only 7% of the time for the rational expectations model.

5.4 Expected Returns

One of the important features of the model is that it generates predictability in housing returns without generating time varying expected returns.[12] This in contrast to models with time varying risk which generate predictability in returns which are expected by investors. Applying these models to the housing market one would find that when the price-to-rent ratio is high investors would expect lower returns in the future. While these models are able to explain a negative correlation between the price-to-rent ratio and subsequent housing returns, they are at odds with an increasing large literature on survey expectations. For equity markets, survey results indicates that investors' expectations regarding future returns seems to be increasing in past stock market performance. As a results, high price to earnings ratios are correlated, if anything, with higher expectations about future returns not lower. See for example: Fisher and Statman (2002); Shiller (2000); Greenwood and Shleifer (2013); Vissing-Jorgensen (2004). Similarly, in the housing market, Case and Shiller (2003); Piazzesi and Schneider (2009); Shiller (2007); Case et al. (2012b) all find that expectations about future returns were increasing during the housing boom of the 2000s not declining. While, I am unable to generate increasing expectations of future returns when the price-to-rent ratio rises, I am able to explain predictability in housing returns without time varying expected returns. In this manner, my results are more inline with the survey evidence than models which require low expected returns when the price-to-rent ratio is high.

6 Robustness

The model has a small number of free parameters and therefore is straightforward to calibrate. However, I did set the AR lag length, the gain level g, the risk aversion coefficient γ and the sticky price parameter λ. Table 5 gives the results from varying each of these parameters one at a time, while keeping the others at their original calibrated value. Except when varying the sticky price parameter λ, I use the sticky price model as the benchmark and set $\lambda = 0.25$. Table 5, therefore, demonstrates the robustness of the results to the various parameter choices.

[12] One period ahead expected returns are given by: $E_t[r_{t+1}]=E_t\frac{q_{t+1}}{q_t-d_t} = \exp[E_t(lnq_{t+1}) + \frac{1}{2}\sigma_q^2]/(q_t - d_t)$. This quantity is approximately constant in the model and equal to $(1+g^c)^\gamma/\beta$ where g^c is the growth rate of consumption.

There is little effect on the results of varying the AR lag length. I consider and AR length 2 and 6. We see that the learning model exhibits slightly more volatility when the AR lag length is 2 versus 6. For example the standard deviation of returns is now 0.04 versus 0.03. The predictability correlations are all of the same sign and same magnitude under the alternative AR lag length calibrations as they are under the baseline calibration. The evidence of time varying volatility remains; the autocorrelation of residuals from the AR(1) return regression are all positive and of similar magnitude.

Similarly increasing the gain from 0.005 to 0.02 slightly amplifies volatility. Again the standard deviation of returns is 0.04 versus 0.03 for the baseline calibration. Additionally, the predictability of returns and rent growth increase and become more in line with the data. The correlation of the price-to-rent ratio with future returns is -0.53 versus -0.42 for the baseline calibration. Similarly the correlation of the price-to-rent ratio with future rent growth is equal to -0.44 versus -0.25 for the benchmark calibration.

Increasing γ from 1 to 3 has very little effect on the results. Again we see a slight increases in the level of volatility. At a level of $\gamma = 3$ the model exhibits a small degradation in its ability to explain kurtosis of returns, predictability of future rent growth, and the autocorrelation of squared return residuals. However, it is still clear that the learning model improves over the benchmark rational expectations model in this case as well. Finally, increasing λ, the sticky price parameter, from 0.25 to 0.5 and reducingit from 0.25 to 0.1 has little effect on the results. We only see that with more flexible prices the model has slightly more difficulty explaining the predictability of future returns using current returns and the autocorrelation of square return residuals.

7 Conclusion

Motivated by the large recent swing in U.S. house prices and the dramatic impact the housing crash had on real economic activity this paper has sought to explain key moments in the U.S. macroeconomic time series on house prices and rents and specifically the role expectations may have played in generating these empirical facts. Given that the hosuing markets boom and bust was similar to booms and busts that have occurred in equity markets in the U.S. and beyond we have focused on data moments that have received considerable attention in the analysis of equity markets.

The paper has documented that the price-to-rent ratio and housing returns are substantially more volatile than the underlying rent fundamentals. Both the price-to-rent ratio and housing returns exhibit momentum effects with strong positive autocorrelation in both the price-to-rent ratio and housing returns. Returns on housing are predictable with current returns forecasting higher returns in the future, while the price-to-rent ratio negatively forecasts both future returns and future rent growth. Finally, housing returns exhibit time varying volatility as evidenced both my autocorrelation in squared returns and significant GARCH effects.

I show that a standard rational expectations benchmark is unable to match these facts. I then modify the standard model in two ways. I first allow for sticky prices so that house prices slowly adjust to their fundamental value. Then, I incorporate learning about the true nature of the housing preference process, specifically is the process in trend stationary (so shocks are temporary) or difference stationary (so shocks are permanent).

I find that these modifications substantially improve the fit of the model. They amplify the volatility of prices and returns and explain the positive autocorrelation of returns. They also allow the model to explain the ability of the price-to-rent ratio to predict future returns and rent growth and help the model generate time varying volatility similar to what is observed in the data.

This paper suggests that modeling expectations, particularity outside a strict rational expectations framework is key to understanding the determinates of aggregate U.S. house prices, especially in periods of booms and busts. The paper suggests that non-rational expectations should be incorporated into a wide variety of housing models and could significantly improve the models fit with the data.

References

Klaus Adam, Pei Kuang, and Albert Marcet. House Price Booms and the Current Account. *NBER Macroeconomics Annual*, 26(1):77 – 122, 2012.

Juan Ayuso and Fernando Restoy. House prices and rents: An equilibrium asset pricing approach. *Journal of Empirical Finance, Elsevier*, 13(3):371–388, June 2006.

Jess Benhabib, Richard Rogerson, and Randall Wright. Homework in Macroeconomics: Household Production and Aggregate Fluctuations. *Journal of Political Economy*, 99(6): 1166–87, December 1991.

Wilko Bolt, Maria Demertzis, Cees Diks, Cars Hommes, and Marco van der Leij. Identifying booms and busts in house prices under heterogeneous expectations. DNB Working Papers 450, Netherlands Central Bank, Research Department, December 2014.

Craig Burnside, Martin Eichenbaum, and Sergio Rebelo. Understanding Booms and Busts in Housing Markets. NBER Working Papers 16734, National Bureau of Economic Research, Inc, January 2011.

John Y Campbell. Intertemporal Asset Pricing without Consumption Data. *American Economic Review*, 83(3):487–512, June 1993.

John Y. Campbell and Robert J. Shiller. The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies*, 1(3):195–228, 1988.

Karl E Case and Robert J Shiller. The Efficiency of the Market for Single-Family Homes. *American Economic Review*, 79(1):125–37, March 1989.

Karl E. Case and Robert J. Shiller. Is There a Bubble in the Housing Market? *Brookings Papers on Economic Activity*, 34(2):299–362, 2003.

Karl E. Case, Robert J. Shiller, and Anne K. Thompson. What Have They Been Thinking? Homebuyer Behavior in Hot and Cold Markets. *Brookings Papers on Economic Activity*, 45(2 (Fall)):265–315, 2012a.

Karl E. Case, Robert J. Shiller, and Anne K. Thompson. What Have They Been Thinking? Homebuyer Behavior in Hot and Cold Markets. *Brookings Papers on Economic Activity*, 45(2 (Fall)):265–315, 2012b.

Matthew Chambers, Carlos Garriga, and Don E. Schlagenhauf. Accounting For Changes In The Homeownership Rate. *International Economic Review*, 50(3):677–726, 08 2009.

Satyajit Chatterjee and Burcu Eyigungor. Foreclosures and house price dynamics: a quantitative analysis of the mortgage crisis and the foreclosure prevention policy. Technical report, 2009.

Yongqiang Chu. Credit constraints, inelastic supply, and the housing boom. *Review of Economic Dynamics*, 17(1):52–69, January 2014.

John H Cochrane. How big is the random walk in gnp? *Journal of Political Economy*, 96 (5):893–920, October 1988.

Timothy Cogley and Thomas J. Sargent. The conquest of us inflation: Learning and robustness to model uncertainty. *Review of Economic Dynamics*, 8(2):528–563, April 2005.

Dean Corbae and Erwan Quintin. Leverage and the Foreclosure Crisis. NBER Working Papers 19323, National Bureau of Economic Research, Inc, August 2013.

Morris A. Davis and Jonathan Heathcote. Housing And The Business Cycle. *International Economic Review*, 46(3):751–784, 08 2005.

Morris A. Davis, Andreas Lehnert, and Robert F. Martin. The rent-price ratio for the aggregate stock of owner-occupied housing. *Review of Income and Wealth*, 54(2):279–284, 06 2008.

Angus Deaton. *Understanding Consumption*. Oxford University Press, 1992.

Robert F Engle. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4):987–1007, July 1982.

Jack Favilukis, Sydney C. Ludvigson, and Stijn Van Nieuwerburgh. The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium. NBER Working Papers 15988, National Bureau of Economic Research, Inc, May 2013.

K.L. Fisher and M. Statman. Blowing Bubbles. *Journal of Psychology and Financial Markets*, 3:53–65, 2002.

Marjorie Flavin and Shinobu Nakagawa. A model of housing in the presence of adjustment costs: A structural interpretation of habit persistence. *American Economic Review*, 98(1): 474–95, 2008.

Christopher L. Foote, Kristopher S. Gerardi, and Paul S. Willen. Why Did So Many People Make So Many Ex Post Bad Decisions? The Causes of the Foreclosure Crisis. NBER Working Papers 18082, National Bureau of Economic Research, Inc, May 2012.

Andreas Fuster, Benjamin Hebert, and David Laibson. Natural expectations, macroeconomic dynamics, and asset pricing. *NBER Macroeconomics Annual*, 26(1):1 – 48, 2012.

Carlos Garriga and Don E. Schlagenhauf. Home Equity, Foreclosures and Bailouts. Working paper, May 2009.

Paolo Gelain and Kevin J. Lansing. House prices, expectations, and time-varying fundamentals. Working Paper 2013/05, Norges Bank, February 2013.

Stefano Giglio, Matteo Maggiori, and Johannes Stroebel. Very Long-Run Discount Rates. NBER Working Papers 20133, National Bureau of Economic Research, Inc, May 2014.

Edward L. Glaeser and Joseph Gyourko. Housing Dynamics. NBER Working Papers 12787, National Bureau of Economic Research, Inc, December 2006.

Edward L. Glaeser and Charles G. Nathanson. An Extrapolative Model of House Price Dynamics. NBER Working Papers 21037, National Bureau of Economic Research, Inc, March 2015.

Edward L. Glaeser, Joseph Gyourko, and Albert Saiz. Housing supply and housing bubbles. *Journal of Urban Economics*, 64(2):198–217, September 2008.

Edward L. Glaeser, Joseph Gyourko, Eduardo Morales, and Charles G. Nathanson. Housing dynamics: An urban approach. *Journal of Urban Economics*, 81(C):45–56, 2014.

Jeremy Greenwood and Zvi Hercowitz. The Allocation of Capital and Time over the Business Cycle. *Journal of Political Economy*, 99(6):1188–214, December 1991.

Robin Greenwood and Andrei Shleifer. Expectations of Returns and Expected Returns. Nber working papers, National Bureau of Economic Research, Inc, January 2013.

James Hamilton. *Time Series Analysis*. Princeton University Press, 1994.

Allen Head, Huw Lloyd-Ellis, and Hongfei Sun. Search, Liquidity and the Dynamics of House Prices and Construction. Working Papers 1276, Queen's University, Department of Economics, September 2012.

Matteo Iacoviello. House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle. *American Economic Review*, 95(3):739–764, June 2005.

Matteo Iacoviello. Housing in dsge models: Findings and new directions. In Olivier de Bandt, Thomas Knetsch, Juan Peï¿œalosa, and Francesco Zollino, editors, *Housing Markets in Europe*, pages 3–16. Springer Berlin Heidelberg, 2010.

Matteo Iacoviello and Stefano Neri. Housing Market Spillovers: Evidence from an Estimated DSGE Model. *American Economic Journal: Macroeconomics*, 2(2):125–64, April 2010.

Matteo Iacoviello and Marina Pavan. Housing and debt over the life cycle and over the business cycle. *Journal of Monetary Economics*, 60(2):221–238, 2013.

John Krainer. A Theory of Liquidity in Residential Real Estate Markets. *Journal of Urban Economics*, 49(1):32–53, January 2001.

David Kreps. *Anticipated Utility and Dynamic Choice, a 1997 Schwartz Lecture*. Frontiers of Research in Economic Theory, 1998.

Hanno N. Lustig and Stijn G. Van Nieuwerburgh. Housing Collateral, Consumption Insurance, and Risk Premia: An Empirical Perspective. *Journal of Finance*, 60(3):1167–1219, 06 2005.

Jianjun Miao, Pengfei Wang, and Tao Zha. Liquidity premia, price-rent dynamics, and business cycles. NBER Working Papers 20377, National Bureau of Economic Research, Inc, 2014.

Huong D Nguyen. Uncertainty in the Housing Market:A Learning Model and Evidence from Household Survey Data. Ph.d. thesis, ch 1, Brandeis University, May 2014.

Monika Piazzesi and Martin Schneider. Momentum Traders in the Housing Market: Survey Evidence and a Search Model. *American Economic Review*, 99(2):406–11, May 2009.

Monika Piazzesi, Martin Schneider, and Selale Tuzel. Housing, consumption and asset pricing. *Journal of Financial Economics*, 83(3):531–569, March 2007.

Carmen Reinhart and Kenneth Rogoff. *This Time is Different: Eight Centuries of Financial Folly*. Princeton University Press, 2009.

Fernando Restoy and Philippe Weil. Approximate equilibrium asset prices. *Review of Finance*, 15(1):1–28, 2011.

R.J. Shiller. Measuring Bubble Expectatations and Investor Confidence. *Journal of Psychology and Financial Markets*, 1:49–60, 2000.

Robert Shiller. *Irrational Exuberence*. Princeton University Press, 2005.

Robert J. Shiller. Understanding recent trends in house prices and homeownership. *Proceedings - Economic Policy Symposium - Jackson Hole*, pages 89–123, 2007.

James H. Stock. Confidence intervals for the largest autoregressive root in u.s. macroeconomic time series. *Journal of Monetary Economics*, 28(3):435–459, December 1991.

Sheridan Titman, Ko Wang, and Jing Yang. The dynamics of housing prices. NBER Working Papers 20418, National Bureau of Economic Research, Inc, 2014.

Annette Vissing-Jorgensen. Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions. In *NBER Macroeconomics Annual 2003, Volume 18*, NBER Chapters, pages 139–208. National Bureau of Economic Research, Inc, 2004.

William C Wheaton. Vacancy, Search, and Prices in a Housing Market Matching Model. *Journal of Political Economy*, 98(6):1270–92, December 1990.

A House Price Derivation

A.1 Calculation of Log Price

From the first order condition we have

$$q_t = \frac{\xi_t}{\lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right]$$

Letting $d_t = \frac{\xi_t}{\lambda_t}$ we have

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t} \right]$$

If we assume that housing returns and consumption are conditionally homoskedastic and jointly log-normally distributed we can take logs of both sides

$$\begin{aligned} 0 &= \ln\beta + E_t\left[\Delta\ln\lambda_{t+1} + \ln q_{t+1} - \ln(q_t - d_t)\right] + \frac{1}{2}Var_t\left[\Delta\ln\lambda_{t+1} + \ln q_{t+1}\right] \\ ln(q_t - d_t) &= \ln\beta + E_t\left[\Delta\ln\lambda_{t+1} + \ln q_{t+1}\right] + \frac{1}{2}\left[\sigma_\lambda^2 + \sigma_q^2 + 2\sigma_{\lambda,q}\right] \end{aligned}$$

Where $\sigma_\lambda^2 = Var_t\Delta\ln\lambda_{t+1}, \sigma_q^2 = Var_t\ln q_{t+1}$ and $\sigma_{\lambda,q} = Cov_t(\Delta\ln\lambda_{t+1}, \ln q_{t+1})$ Now since $\Delta\ln\lambda_{t+1} = -\gamma\Delta\ln c_{t+1}$we have

$$ln(q_t - d_t) = \ln\beta + E_t\left[-\gamma\Delta\ln c_{t+1} + \ln q_{t+1}\right] + \frac{1}{2}\left[\gamma^2\sigma_c^2 + \sigma_q^2 - 2\gamma\sigma_{c,q}\right]$$

We linearize $ln(q_t - d_t)$ about the mean rent to price ratio

$$\begin{aligned} &\ln(q_t - d_t) = \ln[q_t(1 - \frac{d_t}{q_t})] \\ &\ln(q_t - d_t) = \ln q_t + \ln(1 - \exp\left[\ln(\frac{d_t}{q_t})\right]) \\ &\ln(q_t - d_t) \approx \ln q_t + \ln(1 - \exp(\bar{x})) - \frac{\exp(\bar{x})}{1 - \exp(\bar{x})}\left[\ln(\frac{d_t}{q_t}) - \bar{x}\right] \\ &\ln(q_t - d_t) \approx k + (1 - \delta)\ln q_t + \delta\ln d_t \end{aligned}$$

where $\bar{x}$=$E\ln(\frac{d_t}{q_t})$, $k = \ln(1 - \exp(\bar{x})) + \frac{\exp(\bar{x})}{1-\exp(\bar{x})}\bar{x}$, and $\delta = \frac{-\exp(\bar{x})}{1-\exp(\bar{x})}$. Letting $\sigma =$

$\frac{1}{2}\left[\gamma^2\sigma_c^2+\sigma_q^2-2\gamma\sigma_{c,q}\right]$we have

$$\ln q_t \approx \frac{1}{1-\delta}\left[\sigma+\ln\beta-k-\delta\ln d_t+E_t(-\gamma\Delta\ln c_{t+1}+\ln q_{t+1})\right]$$
$$\ln q_t \approx \frac{k-\ln\beta-\sigma}{\delta}-\frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\ln d_{t+s}-\frac{\gamma}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\Delta\ln c_{t+s+1}$$

Now recall $d_t=\frac{\xi_t}{\lambda_t}$ so $\ln d_t=\ln\xi_t-ln\lambda_t=ln\xi_t+\gamma lnc_t$. Therefore

$$\ln q_t \approx \frac{k-\ln\beta-\sigma}{\delta}-\frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\left[\frac{1}{(1-\delta)^s}\ln\xi_{t+s}-\frac{\gamma}{(1-\delta)^s}\ln c_{t+s}\right]-\frac{\gamma}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\Delta\ln c_{t+s+1}$$

A.2 Calculation of sums and conditional variances.

For the stationary rent process we have $\vec{\xi}_{t+1}=\Phi^s\vec{\xi}_t+\vec{\varepsilon}_t^s$ where $\vec{\xi}_t=[\begin{array}{ccccc}1 & t & \xi_{t-1} & \dots & \xi_{t-p}\end{array}]'$ and $\vec{\varepsilon}_t^s=[\begin{array}{cccc}0 & 0 & \varepsilon_t^s & 0\end{array}]'$ and for the non-stationary process we have $\vec{\Delta\xi}_{t+1}=\Phi^{ns}\Delta\vec{\xi}_t+\vec{\varepsilon}_t^{ns}$ where $\Delta\vec{\xi}_t=[\begin{array}{ccccc}1 & \xi_{t-1} & \Delta\xi_{t-1} & \dots & \Delta\xi_{t-p}\end{array}]'$ and $\vec{\varepsilon}_t^{ns}=[\begin{array}{cccc}0 & \varepsilon_t^{ns} & \varepsilon_t^{ns} & 0\end{array}]'$

For the rent part of the sum assuming the trend stationary process we can calculate, letting $\tilde{\Phi}=\Phi^S/(1-\delta)$

$$\frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\ln\xi_{t+s}=\frac{\delta}{1-\delta}e'_{3,p+2}[I-\tilde{\Phi}]^{-1}\vec{\xi}_t$$

where $e'_{3,p+2}$ is a row vector of length $p+2$ with zeros in all places except row 3.

We can also calculate the conditional variance of this sum as

$$\sigma_d^2=\left(\frac{\delta}{1-\delta}[I-\tilde{\Phi}]^{-1}_{3,3}\right)^2\sigma_s^2$$

For the non-stationary model the analogous results are:

$$\begin{aligned}\frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\ln\xi_{t+s} &= \frac{\delta}{1-\delta}e'_{2,p+2}[I-\tilde{\Phi}]^{-1}\Delta\vec{\xi}_t\\ \sigma_d^2 &= \left[\frac{\delta}{1-\delta}\left([I-\tilde{\Phi}]^{-1}_{2,2}+[I-\tilde{\Phi}]^{-1}_{2,3}\right)\right]^2\sigma_{ns}^2\end{aligned}$$

For the consumption process we have $\vec{\Delta c}_{t+1}=\Theta\Delta\vec{c}_t+\vec{\varepsilon}_t^c$ where $\Delta\vec{c}_t=[\begin{array}{ccccc}1 & c_{t-1} & \Delta c_{t-1} & \dots & \Delta c_{t-p}\end{array}]'$ and $\vec{\varepsilon}_t^s=[\begin{array}{cccc}0 & \varepsilon_t^{ns} & \varepsilon_t^{ns} & 0\end{array}]'$

For the first consumption sum we get, letting $\tilde{\Theta} = \Theta/(1-\delta)$

$$\frac{\delta\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \ln c_{t+s} = \frac{\delta\gamma}{1-\delta} e'_{2,p+2}[I - \tilde{\Theta}]^{-1} \Delta \vec{c}_t$$

For the second consumption sum we obtain

$$\frac{\gamma}{1-\delta} E_t \sum_{s=0}^{\infty} \frac{1}{(1-\delta)^s} \Delta \ln c_{t+s+1} = \frac{\gamma}{1-\delta} e'_{3,p+2}[I - \tilde{\Theta}]^{-1} \Theta \Delta \vec{c}_t$$

The conditional variance of the sum of these two sums is then given by

$$\sigma^2_{csum} = \left[\frac{-\delta\gamma}{1-\delta} \left([I - \tilde{\Theta}]^{-1}_{2,2} + [I - \tilde{\Theta}]^{-1}_{2,3} \right) - \frac{\gamma}{1-\delta} \left(\left[[I - \tilde{\Theta}]^{-1} \theta \right]_{3,3} \right) \right]^2 \sigma^2_c$$

Finally to complete our calculation for the price we need to calculate the conditional covariance between consumption growth and the housing price. Since housing preference shocks are independent of consumption shocks we get:

$$\sigma_{c,q} = \left[\frac{-\gamma\delta}{1-\delta} \left([I - \tilde{\Theta}]^{-1}_{2,2} + [I - \tilde{\Theta}]^{-1}_{2,3} \right) - \frac{\gamma}{1-\delta} \left(\left[[I - \tilde{\Theta}]^{-1} \theta \right]_{3,3} \right) \right] \sigma^2_c$$

Figure 1: Gain Calibration: gain = 0.001 dashed, gain = 0.0025 dotted, gain = 0.005 solid

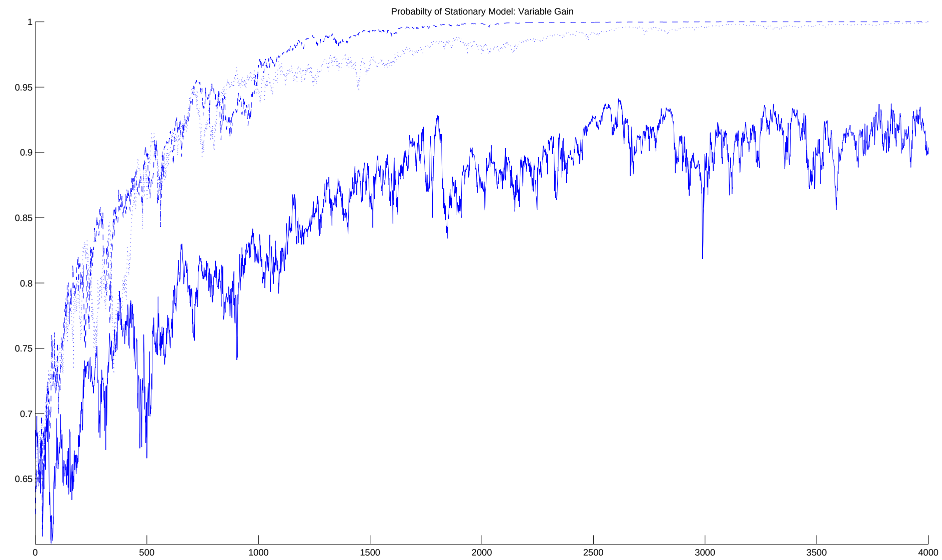

Figure 2: Probability of the Stationary Model

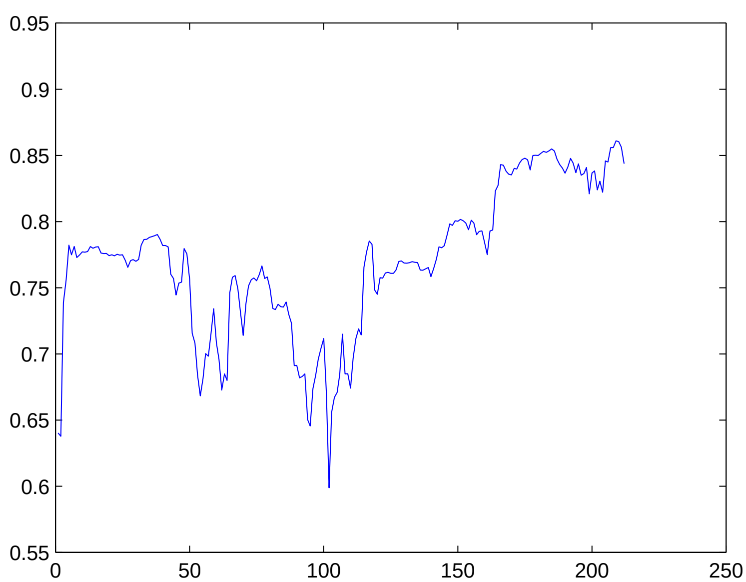

Figure 3: Rent

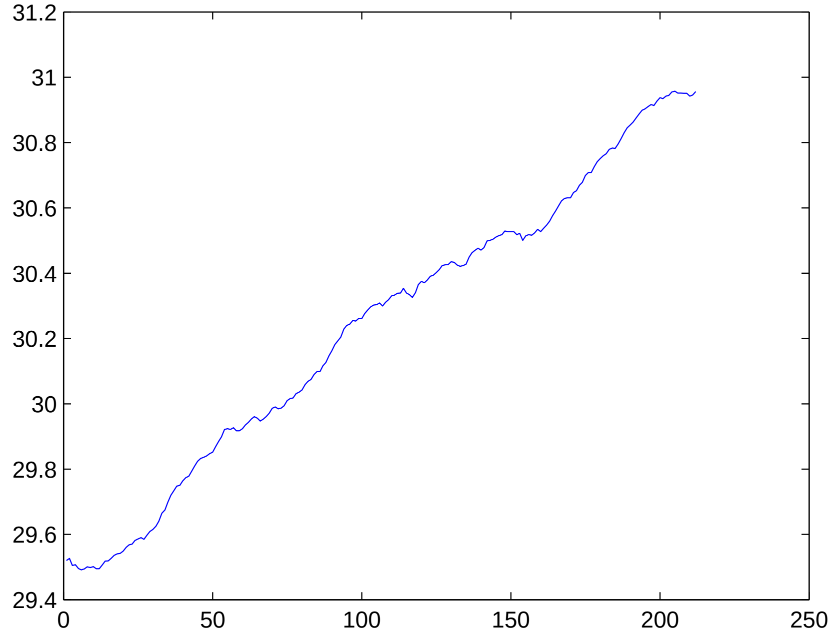

Figure 4: Price-to-rent Ratio: Learning Model (dashed), RE model (solid)

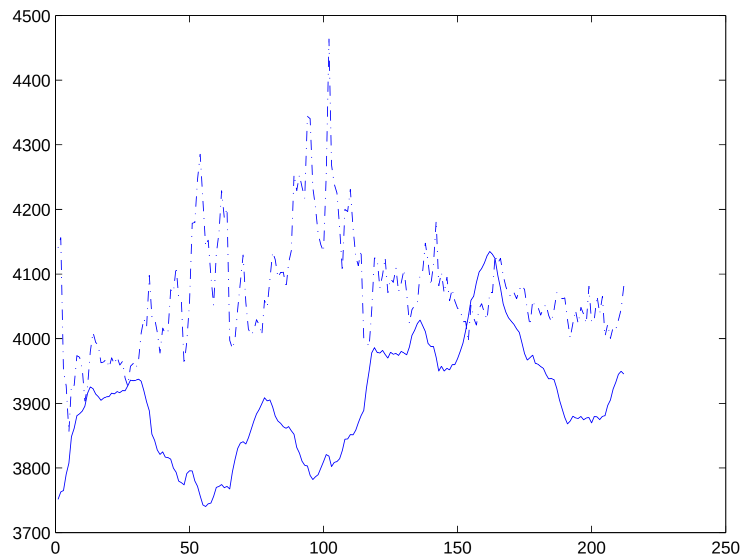

Table 1: Data Moments

Means and Standard Deviations		Skewness and Kurtosis	
$E(r_t)$	0.064	$skew(P_t/R_t)$	2.04
$E(\Delta ln(rent_t))$	0.009	$skew(r_t)$	-1.7
$\sigma(P_t/R_t)$	0.15	$kurtosis(P_t/R_t)$	7.1
$\sigma(P^{HP}_t)$	0.037	$kurtosis(r_t)$	7.5
$\sigma(r_t)$	0.06	Predictability	
$\sigma(\Delta ln(rent_t))$	0.023	$\rho(P/R_t, r_{t+1} + \ldots r_{t+16})$	-0.74
		$\rho(r_t, r_{t+1} + \ldots r_{t+16})$	0.32
		$\rho(P/R_t, ln(rent_{t+16}) - ln(rent_t))$	-0.44
Autocorrelations		Squared Autocorrelations	
$\rho(P_t/R_t)$	0.99	r^2_t	0.7
	0.98		0.56
	0.95		0.45
	0.92		0.27
$\rho(r_t)$	0.84	$(\varepsilon^r_t)^2$	0.14
	0.73		0.39
	0.66		0.1
	0.51		0.32
		GARCH Model	
		Υ_1 (GARCH)	0.73
			(0.06)
		a_1 (ARCH)	0.27
			(0.07)
		p-value Engle test	0.04

Note: This table provides the key moments on house prices, rents and price to rent ratio.

Table 2: Campbell-Shiller Regressions

Future Return Regression ($r_{t+1} + \ldots + r_{t+16}$)

β	R^2
-0.8	0.5
(0.06)	

Future Rent Growth Regression ($\ln \xi_t + \ldots \ln \xi_{t+16}$)

β	R^2
-0.12	0.2
(0.02)	

Note: This table presents results from running Campbell-Shiller regressions on the housing data. The price-to-rent ratio is used to predict returns and rent growth over the next four year. Standard errors are in parentheses.

Table 3: Model Results -- Main Moments

		Flex Price		Sticky Price	
	Data	RE	Learn	RE	Learn
Means, Stdev					
$E(r_t)$	0.064	0.03	0.03	0.03	0.03
$E(\Delta \ln(rent_t))$	0.009	0.03	0.03	0.03	0.03
$\sigma(P_t/R_t)$	0.15	0.03	0.03	0.03	0.03
$\sigma(P^{HP}{}_t)$	0.037	0.008	0.02	0.005	0.01
$\sigma(r_t)$	0.06	0.03	0.06	0.01	0.03
$\sigma(\Delta \ln(rent_t))$	0.023	0.03	0.03	0.03	0.03
Autocorrelations					
$\rho(P_t/R_t)$	0.99	0.99	0.91	0.97	0.96
	0.98	0.97	0.84	0.95	0.91
	0.95	0.94	0.78	0.91	0.87
	0.92	0.91	0.73	0.87	0.83
$\rho(r_t)$	0.84	-0.01	-0.01	0.73	0.44
	0.73	-0.01	-0.1	0.53	0.33
	0.66	0	0	0.38	0.22
	0.51	0	0	0.27	0.15
Predictability					
$\rho(P/R_t, r_{t+1} + \ldots r_{t+16})$	-0.74	0.04	-0.41	-0.08	-0.42
$\rho(r_t, r_{t+1} + \ldots r_{t+16})$	0.32	-0.03	-0.05	0.27	0.15
$\rho(P/R_t, \ln(rent_{t+16}) - \ln(rent_t))$	-0.44	0.27	-0.24	0.25	-0.25
Campbell Shiller Regression					
β^{return}	-0.8	0.04	-0.73	-0.06	-0.78
R^2 (return regression)	0.5	0.04	0.16	0.03	0.19
$\beta^{rent\ growth}$	-0.12	0.4	-0.23	0.37	-0.33
R^2 (rent growth regression)	0.2	0.08	0.08	0.07	0.11

Note: This table gives the model predicitons for the main data moments.

Table 4: Model Results -- Time Varying Volatility

	Data	Flex Price RE	Flex Price Learn	Sticky Price RE	Sticky Price Learn
Skewness and Kurtosis					
skew(P_t/R_t)	2.04	0.05	0.39	0.11	0.33
skew(r_t)	-1.7	0.01	0.04	-0.02	0.08
kurtosis(P_t/R_t)	7.1	2.2	3.3	2.3	2.8
kurtosis(r_t)	7.5	2.9	3.9	2.9	5
Squared Autocorrelations					
r^2_t	0.7	-0.01	0.07	0.72	0.4
	0.56	-0.01	0.07	0.52	0.3
	0.45	-0.01	0.06	0.37	0.2
	0.27	-0.01	0.05	0.26	0.17
$(\varepsilon^r_t)^2$	0.14	-0.02	0.08	-0.01	0.21
	0.39	-0.01	0.09	-0.01	0.16
	0.1	-0.01	0.08	-0.01	0.16
	0.32	0.01	0.09	-0.01	0.15
GARCH Model					
γ_1 (GARCH)	0.73	0	0	0	0.65
	(0.06)				
a_1 (ARCH)	0.27	0	0	0	0.22
	(0.07)				
p-value Engle test	0.04				

Note: This table gives the model predicitons for the key data moments.

Table 5: Robustness

Moment	Data	AR	4	2	6	Υ	1	2	3
$\sigma(P_t/R_t)$	0.15		0.03	0.05	0.03		0.03	0.03	0.03
$\sigma(P^{HP}_t)$	0.037		0.01	0.02	0.01		0.01	0.01	0.02
$\sigma(r_t)$	0.06		0.03	0.04	0.02		0.03	0.03	0.04
kurtosis(P_t/R_t)	7.1		2.8	2.6	2.6		2.8	2.9	3.1
kurtosis(r_t)	7.5		5	4	5.2		5	2.5	3.8
$\rho(P/R_t, r_{t+1} + \ldots r_{t+16})$	-0.74		-0.42	-0.49	-0.32		-0.42	-0.37	-0.36
$\rho(r_t, r_{t+1} + \ldots r_{t+16})$	0.32		0.15	0.18	0.16		0.15	0.18	0.19
$\rho(P/R_t, \ln(rent_{t+16}) - \ln(rent_t))$	-0.44		-0.25	-0.41	-0.04		-0.25	-0.12	-0.07
Autocorrelation: $(\varepsilon^r_t)^2$	0.14		0.21	0.15	0.2		0.21	0.2	0.16
	0.39		0.16	0.15	0.18		0.16	0.16	0.13
	0.1		0.16	0.12	0.16		0.16	0.15	0.13
	0.32		0.15	0.12	0.14		0.15	0.14	0.13
Moment	Data	g	0.005	0.01	0.02	λ	0.25	0.1	0.5
$\sigma(P_t/R_t)$	0.15		0.03	0.03	0.04		0.03	0.03	0.03
$\sigma(P^{HP}_t)$	0.037		0.01	0.02	0.02		0.01	0.01	0.02
$\sigma(r_t)$	0.06		0.03	0.04	0.04		0.03	0.02	0.04
kurtosis(P_t/R_t)	7.1		2.8	3.5	3.5		2.8	2.9	3.1
kurtosis(r_t)	7.5		5	5.1	5		5	6	4.7
$\rho(P/R_t, r_{t+1} + \ldots r_{t+16})$	-0.74		-0.42	-0.5	-0.53		-0.42	-0.52	-0.4
$\rho(r_t, r_{t+1} + \ldots r_{t+16})$	0.32		0.15	0.13	0.14		0.15	0.29	0.06
$\rho(P/R_t, \ln(rent_{t+16}) - \ln(rent_t))$	-0.44		-0.25	-0.42	-0.44		-0.25	-0.12	-0.27
Autocorrelation: $(\varepsilon^r_t)^2$	0.14		0.21	0.21	0.22		0.21	0.24	0.15
	0.39		0.16	0.18	0.19		0.16	0.19	0.14
	0.1		0.16	0.17	0.19		0.16	0.18	0.13
	0.32		0.15	0.18	0.16		0.15	0.18	0.13

Note: This table reports the robustness of the results from table 3 when varying some of the parameters.

Long Run Expectations, Learning and the U.S. Housing Market

Daniel Tortorice, Economics Department, Brandeis University

Working Paper Series

2015 | 85