

Long-term outcome of precommercial thinning in northwestern New Brunswick: growth and yield of balsam fir and red spruce

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Abstract: A study was established between 1959 and 1961 to study the long-term responses of balsam fir (*Abies balsamea* (L.) Mill.) and red spruce (*Picea rubens* Sarg.) to precommercial thinning. Three nominal spacings of 4 ft (1.2 m), 6 ft (1.8 m), and 8 ft (2.4 m) were compared with an unthinned control in a randomized complete block design with five replicates. At the time of thinning, natural regeneration averaged 16 years of age, 8 years after harvest. Although thinning had minimal effect on gross total volume production over a 42 to 44 year observation period, actual spacings between 2.1 and 2.5 m produced an average of 360 m³·ha⁻¹ gross merchantable volume (GMV), representing a 21% gain over unthinned stands. The same spacings produced quadratic mean diameters of 21 and 23 cm, respectively, compared with 18 cm in the unthinned stands. These size increases translated to individual stem volume gains of 33% and 62%, significantly reducing the age at which thinned stands would meet a specified minimum requirement for merchantability or habitat. The mean annual increment of GMV ranged from 6 m³·ha⁻¹·year⁻¹ in unthinned stands, to more than 7 m³·ha⁻¹·year⁻¹ in the thinned stands, and had not yet culminated an average of 50 years postharvest.

Résumé : Une expérience a été établie entre 1959 et 1961 pour étudier la réaction à long terme du sapin baumier (*Abies balsamea* (L.) Mill.) et de l'épinette rouge (*Picea rubens* Sarg.) à la suite d'une éclaircie précommerciale. Trois espacements nominaux de 4 pi. (1,2 m), 6 pi. (1,8 m) et 8 pi. (2,4 m) ont été comparés à un témoin non éclairci dans le cadre d'un plan expérimental en blocs aléatoires complets comportant cinq répétitions. Au moment de l'éclaircie, huit ans après la coupe du peuplement, les arbres qui composaient la régénération naturelle avaient en moyenne 16 ans. Quoique l'éclaircie n'ait eu qu'un effet minimal sur la production brute en volume total après une période d'observation de 42 à 44 ans, les espacements réels variant entre 2,1 et 2,5 m ont produit un volume marchand brut (VMB) de 360 m³·ha⁻¹, ce qui représente un gain de 21 % par rapport aux peuplements non éclaircis. Les mêmes espacements ont produit des diamètres moyens quadratiques de respectivement 21 et 23 cm comparativement à 18 cm dans les peuplements non éclaircis. Cet accroissement se traduisait chez les tiges individuelles par des gains en volume de 33 % et 62 %, ce qui réduit significativement l'âge auquel les peuplements éclaircis atteindraient les dimensions minimales requises pour avoir une valeur commerciale ou servir d'habitat. L'accroissement annuel moyen en VMB variait de 6 m³·ha⁻¹·an⁻¹ dans les peuplements non éclaircis à plus de 7 m³·ha⁻¹·an⁻¹ dans les peuplements éclaircis et n'avait pas encore atteint sa valeur maximale 50 en moyenne ans après la coupe de régénération.

[Traduit par la Rédaction]

Introduction

Precommercial thinning (PCT) is used in eastern North America as a means of reducing the density of young conifer stands that have developed from prolific natural regeneration. Objectives can vary, but often include control over stand composition, crop-tree selection, preparation of stand structure for commercial thinning, increased stand vigor and health, reduced time to merchantability, increased product yield per tree and (or) per hectare, and reduced harvesting and processing costs. Records indicate that approximately 2 000 000 ha of young forests have been precommercially thinned from Ontario eastward during the past 15 years, and efforts continue in this region at a rate approaching 200 000 ha·year⁻¹ (Canadian Council of Forest Ministers 2005).

With thinned forests increasingly contributing to future wood supply, forest managers are in need of corresponding long-term growth and yield data to accurately forecast mean tree size and volume per hectare, and help establish correct current allowable harvest levels (B. English, Newfoundland Forest Service, personal communication). Several studies have documented early to midterm responses to PCT (e.g., Ker 1987; Lavigne and Donnelly 1989; Karsh et al. 1994; Zarnovican and Laberge 1996), but there is a shortage of longer-term data to validate projections and increase confidence in wood-supply forecasts.

The Green River thinning trials, installed in northwestern New Brunswick by the Canadian Forest Service (then the Department of Forestry) between 1959 and 1961 (Basker-

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Table 1. Conifer establishment statistics for blocks involved in the Green River thinning trials.

Block name	Latitude	Longitude	Year of harvest	Year of plot establishment and thinning	Pretreatment stem density (stems·ha ⁻¹ >0.3 m tall)	Pretreatment stem density (stems·ha ⁻¹ >1.3 m tall)
Upper Belone Bk.	47°46'39.479678"	68°15'12.415019"	1953	1959	8 760	5 960
Lower Belone Bk.	47°46'1.088624"	68°14'43.444504"	1953	1960	12 875	5 050
Summit Road	47°45'21.159543"	68°18'14.423267"	1946	1960	22 425	8 670
Lower Chisholm Bk.	47°50'40.801669"	68°21'59.841319"	1955	1961	17 400	10 075
Upper Chisholm Bk.	47°48'35.807682"	68°22'23.301018"	1955	1961	20 050	8 950
Mean					15 050 ^a	7 450 ^b
<i>p</i> (treatment) ^c					0.94	0.24

^aValues shown are for conifer. A mean of 7000 stems·ha⁻¹ of hardwoods were also present.

^bA mean of 3150 stems·ha⁻¹ of hardwoods were also present.

^cFrom ANOVA, testing for pretreatment differences among plots.

ville 1959, 1961; Akerley 1961), represent the oldest known, replicated PCT experiments in eastern North America. The trials consist of five experimental blocks, each containing three thinning spacings (4 ft (1.2 m), 6 ft (1.8 m), and 8 ft (2.4 m)) and an unthinned control, applied to balsam fir (*Abies balsamea* (L.) Mill.)–red spruce (*Picea rubens* Sarg.) regeneration an average of 8 years postharvest. The trials are particularly relevant to today's growth and yield needs because they encompass typical operational thinning intensities and intervention times. Similar studies with balsam fir–spruce mixes, of near comparable vintage, offer results for much later intervention times (e.g., 16 years postharvest in the 30-year-old Austin Pond study (Daggett and Wagner 2002); 20 years postharvest in a 20-year-old Quebec study (Pothier 2002); and 30 years postharvest in a 30-year-old Michigan study (Day 1972)). The last formal reporting of the Green River thinning trials took place following the 20 year post-thinning measurements (Ker 1981). The 25 year assessments were presented at a New Brunswick PCT workshop in 1987 (Ker 1987), and the 30 year assessments remain unpublished.

In spring 2004, 42–44 growing seasons after establishment, we revisited the Green River plots and conducted a full measurement of the study's 48 permanent sample plots, and stem analysis on 120 balsam fir trees. In this paper, we report on the long-term effects of PCT on the growth and yield of these stands and make recommendations for their future study. This paper presents near rotation-length empirical data that, we hope, forest managers will ultimately find useful in supporting and enhancing their silviculture applications.

Methods

Study area

The study area is located in the Green River watershed of northwestern New Brunswick, approximately 48 km north of the town of Edmundston (Baskerville et al. 1960). This region is classified by Rowe (1972) as the Gaspé section (B.2) of the Boreal Forest Region, and by Loucks (1962) as the Green River Site District of the Gaspé–Cape Breton Ecoregion. Topography is strongly rolling, with occasional steep areas, and elevations between 300 and 450 m. Soils are predominantly stony loams and silt-loams derived from underlying Paleozoic slates and argillites (Loucks 1962). The area receives approximately 100 cm of precipitation an-

nually, nearly half of which falls between June and September. The annual frost-free period is 110 days, with a mean monthly summer temperature of 15 °C.

Five blocks that had been clearcut harvested (horse logged) for softwood pulpwood between 1946 and 1955 were selected for the study between 1959 and 1961 (Table 1). Before harvest, these areas were dominated by balsam fir, with minor components of red spruce and white birch (*Betula papyrifera* Marsh.). At the time of site selection, they contained abundant natural regeneration of balsam fir and red spruce, largely from preharvest advanced regeneration. Minor components of white birch and shrub species, including pin cherry (*Prunus pensylvanica* L. f.), showy mountain ash (*Sorbus decora* (Sarg.) C.K. Schneid.), mountain maple (*Acer spicatum* Lamb.), red elderberry (*Sambucus racemosa* L.), and service berry (*Amelanchier* spp.) had regenerated following harvest.

Each block was divided into four approximately equal-sized treatment plots of at least 2 ha. One of the blocks (Upper Belone Bk.) was large enough to accommodate eight such plots. Within the approximate center of each half of each treatment plot, a 28.45 m × 28.45 m (0.081 ha) permanent sample plot (PSP) was located (total of 48 PSPs or subsamples). These plots typically encompassed representative portions of the harvest area and extraction trails, characteristic of the harvest method used, and the blocks were chosen to represent the range of early development stages presenting opportunities for PCT. Although not specifically protected, these blocks were included as part of large-scale spruce budworm (*Christoneura fumiferana* Clem.) aerial spray programs conducted to prevent tree mortality in the region between 1953 and 1984 (Wayne MacKinnon, Canadian Forest Service, personal communication). Spruce budworm protection and defoliation affected all treatments equally.

Thinning treatments and monitoring

With the experimental objective of identifying the optimal thinning spacing to increase yield per tree and reduce time to merchantability (i.e., promote stands in a long-term harvest queue), three nominal spacings were tested: 4 ft (1.2 m, 6727 stems·ha⁻¹), 6 ft (1.8 m, 2990 stems·ha⁻¹), and 8 ft (2.4 m, 1682 stems·ha⁻¹). Each of these three spacings and an unthinned control were assigned, at random, to the treatment plots within each of the five blocks. Thinning

took place on the blocks late in the growing season, immediately after plot establishment (Table 1). The same crews (seasonal loggers, hired during their off-season) were used each year in the study, equipped with axes and McCulloch Brushmaster circular saws. Thinning was conducted in a manner that favored uniform tree spacing, by leaving the best spruce or fir tree at or nearest each spacing coordinate and removing all other shrubs and trees (Baskerville 1959). No attempts were made to compensate for the area occupied by extraction trails or natural openings within the plots. In 1972, the thinned plots were cleaned to remove small trees that were lost in the slash at the time of thinning and then found to be forming a potentially competitive understorey in the main stand (Baskerville 1965a).²

Immediately after plot establishment and before thinning, the number of stems situated within each PSP was tallied in 15 cm height classes, beginning at 0.3 m, by species. Following thinning, each tree remaining within the boundaries of the PSPs was assigned a unique tree number and identified with a metal tag. Each tree was then measured for diameter at breast height (DBH), outside bark, using a diameter tape (DBH, nearest 0.5 cm, 1.3 m above ground level), total height (HT, nearest 0.3 m) using a height pole, live crown length (LCL, nearest 0.3 m; base of live crown being identified as the first whorl with at least three live branches), and crown width (CW, nearest dm). These measurements were repeated for all living trees at post-thinning years 5 through 30, at 5 year intervals. Subsequent to 1972, ingrowth trees reaching breast height were also tagged and measured.

Owing to the large number of trees occurring in unthinned plots at the time of establishment (>22 000 stem·ha⁻¹), complete PSP enumerations were not conducted at the initial and year 5 measurements. Instead, the number of stems was tallied in 15 cm height classes, beginning at 0.3 m, by species, with full tagging and measurement following at year 10 and beyond.

Before the growing season in 2004 (42–44 years post-establishment), we conducted a seventh post-thinning measurement on all 48 PSPs. All standing trees were measured for DBH and health status (live or dead). On 25 live balsam fir and 5 live spruce, randomly selected through the range of DBH occurring in each PSP, we measured HT, LCL, and CW using a LaserAce 300[®] equipped with a digital compass (see www.mdl-laser.com).

Stem analyses

With the goals of (i) quantifying the age and site index, for balsam fir in each of the study blocks, (ii) detecting and quantifying treatment effects on stem form and taper, and (iii) verifying the suitability of the Honer et al. (1983) volume equation for the study's balsam fir, we undertook destructive sampling of 120 trees in the fall of 2004. Five trees were selected for stem analysis from within each treatment plot. Preidentified tree sizes were targeted for sampling in each plot based on the identification of five uniformly spaced DBH classes, ranging from and including 10 cm, through to the 95th percentile of DBH for the plot. With some variation, these were typically near the 10, 14,

18, 22, and 26 cm sizes. Balsam fir trees were selected for sampling on the basis of being

- at least one tree length from the boundaries of the treatment plot and its two PSPs (i.e., outside the PSPs but inside the treatment plot),
- within ± 1 cm of the target DBH sizes defined for the plot,
- free of exterior defects and internal rot, and
- with a sound, unforked top.

Once trees were felled, they were measured for HT, CW, and LCL (all measurements to the nearest cm). Disks (approximately 5 cm thick) were then cut from the base (stump height), DBH, and base of live crown. Trees greater than 16 m in height were also sectioned every 2 m through to a stem height of 10 m and every 1 m above 10 m; shorter trees were sectioned every 1 m. Each disk was labeled with the plot number, tree number, cardinal direction, and height above ground level. The disks were individually wrapped in cellophane, placed in plastic bags (one tree per bag), and maintained in frozen storage from the point of field collection until measurements were complete.

In the laboratory, the following measurements were collected and recorded:

- (1) Outside bark diameter of the DBH disk using a diameter tape (DBH, nearest mm).
- (2) Outside bark diameter of all disks at their widest (major) axis using an engineer's scale (DOB_{h1} nearest mm), where h is the height of the disk above ground level (nearest cm).
- (3) Outside bark diameter of all disks, 90° to the major axis (minor axis), through the pith of the tree, using an engineer's scale (DOB_{h2} nearest mm).
- (4) Age of the stump-height and DBH disks.

For age determinations, two radii of the major axis were shaved with a scalpel, and the rings were counted under a dissecting microscope (one serving as a check for the other). In the case of false rings and compressed rings, additional shaving of the disk was undertaken, as needed, to verify (cross-date) ring counts.

Data preparation

Stem section data were prepared for analysis by computing the geometric mean outside bark diameters for each disk (Husch et al. 1972), for example:

$$[1] \quad DOB_h = \sqrt{DOB_{h1} \times DOB_{h2}}$$

where DOB_h is the geometric mean diameter of the disk removed h m above ground level. The outside bark cross-sectional areas of each disk were computed from the averaged diameters. Stem volumes were estimated from the disk measurements of each tree using Smalien's formula (Husch et al. 1972). In these estimates, the stump and tip sections of the stem were approximated by a cylinder and cone, respectively.

PSP data collected between plot establishment and year 30 were error checked, and anomalous values were field verified and (or) corrected, where possible, during the 2004 measurements. The relationships between HT and DBH for

² A likely result of using axes to thin, as it was difficult to be as thorough with an axe as it was with a spacing saw.

the 25 balsam fir and 5 spruce sampled in each PSP in 2004 were quantified using the Chapman–Richards function (Richards 1959; Zeide 1993):

$$[2] \quad HT = 1.3 + \alpha(1 - e^{-\beta DBH})^\delta + \varepsilon$$

where α , β , and δ are parameters estimated through non-linear regression, e is Euler's constant (2.7183), and ε are random, independent, and normally distributed errors. For balsam fir, separate equations were estimated for each block–spacing combination ($n = 50$; $n = 100$ at Upper Belone Bk.) because of significant ($p \leq 0.05$) reductions in the mean square error when the “full” model (containing blocks and treatments) was compared with “reduced” models (averaging over blocks and (or) treatments) (the steps required in the general regression significance test can be found in Draper and Smith (1998)). For spruce, thinning treatments and blocks were combined wherever the reduced model explained as much variation as the larger model ($p > 0.05$ and $n \geq 20$). The resulting relationships were then used to estimate tree height from DBH across the entire 2004 data set. A similar process was used to estimate 2004 CW and LCL, through linear relationships established between these variables and DBH.

For each tree, basal area (BA), live crown ratio (LCR = $[(LCL/HT)100]$), crown area (CA, m^2 , assuming circular form), HT/DBH ratio (HDR, %), gross total volume (GTV, $m^3 \cdot \text{stem}^{-1}$), and gross merchantable volume (GMV₈, and GMV₁₅, $m^3 \cdot \text{stem}^{-1}$) were calculated. For the volume estimates, Honer's species-specific equations were used (Honer et al. 1983); GMV₈ was based on a stump height of 15 cm and a minimum top diameter of 8 cm; GMV₁₅ was based on the same stump height and a minimum top diameter of 15 cm. With a scale-up factor of 6.1774 ($10\,000 \text{ m}^2 \cdot \text{ha}^{-1}$ divided by 2 PSPs $\times 28.45 \text{ m} \times 28.45 \text{ m}$), individual-tree variables within each PSP were expressed as per hectare values and summed for each treatment plot: stems·ha⁻¹, BA, CA, GTV, GMV₈, and GMV₁₅. Mean annual volume increment (MAI) was then calculated for each measurement period as the GMV₈·ha⁻¹, divided by the number of years since harvest. Similarly, periodic mean annual volume increment (PAI) was calculated for each adjacent pair of measurement periods as the change in GMV₈·ha⁻¹, divided by the number of years between measurements. In addition, quadratic mean DBH (QDBH) was computed for each plot as the diameter of the tree of mean BA. Finally, the within-plot standard deviation of DBH (S_DBH) and dominant tree height (DHT, mean of the tallest 10 trees in each of the two PSPs per plot (representing approximately 120 stems·ha⁻¹)) were computed as additional variables of interest.

Stand-table data (stem counts by height class), collected pretreatment and in the unthinned plots for years 0 and 5, were used to approximate the variables described above, so that pretreatment and early unthinned data could conform to the larger data set. To do this, the Chapman–Richards function [2] was used to quantify the height–diameter relationships found in the year 0 data of the measured plots, by block and species. These relationships were used to estimate stem diameter associated with each of the height classes in the stand tables; the resulting tree size and frequency data were then used to approximate the same suite of variables used to describe the other plots.

Analytical approach

Stem analysis

The stem section data were used to determine the mean and range of tree ages on each block (*i*) for all trees sampled ($n = 20$; $n = 40$ at Upper Belone Bk.) and (*ii*) for the largest two trees sampled from each treatment plot ($n = 8$ or 16). The latter sample was then used to generate dominant tree height over breast-height age curves to quantify site index for each of the study blocks.

To assess treatment effects on tree taper and stem form, the stem profile model presented by Zakrzewski (1999) was fit to the stem analysis data from each treatment plot ($n = 5$ trees) to estimate stem outside-bark cross-sectional area at any relative tree height along a stem. These treatment-plot parameter estimates were then subjected to analysis of variance (ANOVA), incorporating the underlying randomized complete block experimental design (four treatments \times five blocks). PROC MIXED of the SAS[®] system (Littell et al. 1996) was used to accommodate the additional four treatment plots at Upper Belone Bk. in the analysis and compute the correct least-squares treatment means and their standard errors. Orthogonal contrasts were used to specifically compare the parameter estimates of (*i*) unthinned plots with the mean of the thinned plots, (*ii*) 4 ft spaced plots with the mean of the wider spacings, and (*iii*) 6 ft with 8 ft spaced plots. This analysis confirmed that Zakrzewski's (1999) model was sufficiently flexible to represent the range of densities in the experiment, and so the model was refit to the complete stem analysis data set and a single set of parameter estimates was obtained. The nature of density effects on tree taper was then explored graphically. Details of this analysis are provided in Appendix A.

Volume estimates for the stem analysis data were then produced from (*i*) the integrated form of Zakrzewski's (1999) equation, (*ii*) Honer et al.'s (1983) equation for balsam fir, and (*iii*) a fit of the natural-log-transformed version of the model described by Schumacher and Hall (1933) to the data. The resulting estimates were then compared with actual values through the bias evaluation procedures detailed in Appendix B, and the “best” equation was selected to recalculate balsam fir GTV, GMV₈, and GMV₁₅ for the larger PSP data set. In all of the above analyses, model residuals were tested to ensure that the assumptions of homogeneity of variance and normality were met.

Analyses of permanent sample plot data

Limiting our focus to the conifer content of the plots (hardwoods represented a small component of the stands), treatment plot level response variables reflecting the PSP data of the 2004 measurements were subjected to ANOVA, based on the underlying randomized complete block design, as previously described. These analyses included variables describing (*i*) overall stand dynamics, including SPH, BA, LCR, CW, CA, and GTV, (*ii*) the merchantable component of the stand, including SPH, BA, GMV₈, GSV₁₅, and MAI/PAI; and (*iii*) the mean tree characteristics, including DHT, QDBH, GMV₈, GMV₁₅, and HDR. In each case, the same set of orthogonal contrasts used on the stem analysis data were used to make a priori, end-point comparisons among the thinning treatments. In addition, the mean and variability

Fig. 1. Condition of the Green River plots in June 2004, as exemplified by plots in three of the five blocks.



Upper Belone Bk., unthinned



Upper Belone Bk., 6' spacing



Upper Chisholm Bk., 4' spacing



Lower Belone Bk., 8' spacing

(QDBH and S_DBH) of the end-point diameter distributions of the different treatments were compared using “stability analysis” (Bowley 1999).

Although we generally do not report comparative statistics for the earlier measurements, parallel analyses were conducted for each post-thinning measurement and the standard errors of the least-squares means from these analyses were represented wherever response variables were graphically depicted over time. We did not feel that repeated measures analyses were necessary for interpretation of the results.

Finally, we repeated the end-point analyses of the individual-tree variables, censoring the data for the largest

1000 stems·ha⁻¹ found in each treatment plot (i.e., the 162 trees with the largest DBH). These analyses were done to avoid deflation of the unthinned plot means by small, undesirable trees, as approximately 1000 stems·ha⁻¹ are typically harvested when natural stands reach maturity (Pothier 2002). The variables included in these analyses were QDBH, S_DBH, DHT, HDR, and GMV₈.

In all analyses of the PSP data, PROC MIXED of the SAS[®] system (SAS Institute Inc. 1990) was used and model residuals were tested to ensure that the assumptions of homogeneity of variance and normality were met. Data transformations were not necessary.

Table 2. Age statistics from the fall 2004 stem analysis of 119 trees in the Green River study.

	2004 Stump-height age (years)					2004 Breast-height age (years)					<i>n</i>
	Min.	Mean	SD	Dominant ^a	Max.	Min.	Mean	SD	Dominant ^a	Max.	
Upper Belone Bk.	43	58	7.4	60	77	34	48	4.3	50	56	40
Lower Belone Bk.	44	61	9.5	68	77	37	48	6.1	53	58	20
Summit Rd.	50	66	7.2	68	75	45	55	5.3	58	65	19 ^b
Lower Chisholm Bk.	45	57	10.0	56	75	40	48	5.8	49	61	20
Upper Chisholm Bk.	47	57	7.5	63	76	40	46	3.8	48	53	20
Mean	45	60	8.2	62	76	38	49	4.9	51	58	

^aAge of two dominant stems in each treatment plot.

^bOne stem sampled at Summit Rd. was aged at 126 years and excluded from the analysis.

Results

Plot condition in 2004

As of 2004, treatment plots and PSPs of the Green River study were generally in good condition (Fig. 1). All standing trees were found numbered and tagged, and the PSP corners were well monumented. An ice storm, occurring between the year 20 and 25 assessments (1984), resulted in considerable top breakage on many of the balsam fir, although recovery has since been good. An area immediately to the north of the Summit Rd. site was harvested following the year 30 assessment, and considerable blowdown ensued in the 6 ft spaced plot nearest the cut boundary. As a result of this anomaly, we excluded 2004 data from this plot in all analyses except stand age.

Stem analyses

Age

Stem analyses of 119 trees sampled in autumn of 2004 revealed that balsam fir in the Green River study averaged 60 years of age, ranging from 57 years at the two Chisholm Bk. sites to 66 years at Summit Rd. (Table 2). One tree sampled at Summit Rd., aged at 126 years, was a veteran from the previous stand and was excluded from these figures and subsequent analyses. Based on the harvest and thinning dates for each of the blocks (Table 1), these ages place the advanced regeneration fir at an average of 8 years at harvest and 16 years at the time of thinning. It typically took these trees 11 years to reach breast height, or 3 years to reach breast height following harvest. Dominant trees were, on average, 2 years older than the overall stand means. The data indicated that balsam fir on these sites continued to ingress over a 31 year period, approximately centered on the time of harvest.

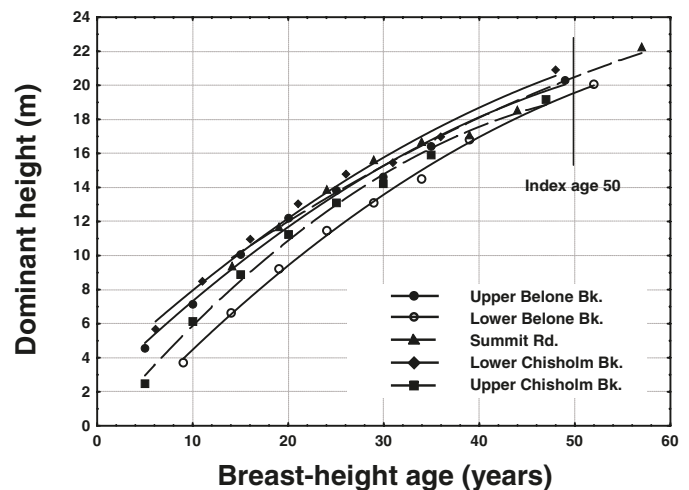
Site index

A plot of the block-mean dominant height values over actual breast-height age (Fig. 2) suggests that site index (dominant tree height at breast-height age 50) for balsam fir on the Green River blocks ranges between 19 and 21 m. In relative terms, this study encompasses what might be considered a fairly narrow range of some of the better sites for balsam fir in New Brunswick (Ker and Bowling 1991).

Stem taper

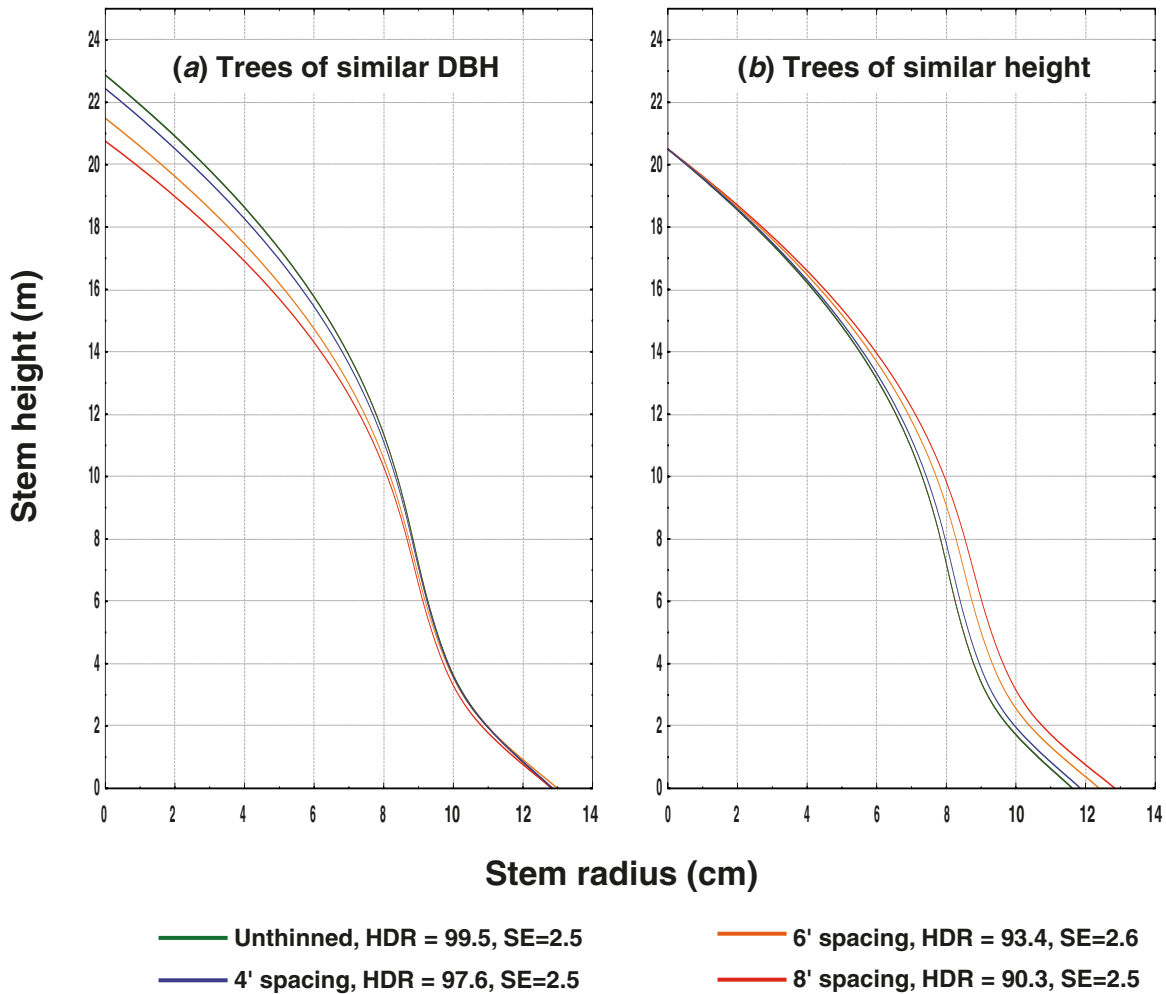
A fit of the Zakrzewski's (1999) stem profile model (model [A1] in Appendix A) to the stem analysis data from

Fig. 2. Mean dominant height at breast-height tree age for each experimental block. Ages were determined from stem analyses of the two largest stems sampled from each treatment plot (a total of 8 trees per block and 16 trees at Upper Belone Bk.). Mean dominant tree heights were determined from the 10 tallest trees in each permanent sample plot at each sample time (a total of 80 trees per block and 160 trees at Upper Belone Bk.).



each treatment plot revealed no spacing-related differences among the estimated parameters ($p \geq 0.39$). A test of the HDRs of the stem analysis trees, however, revealed clear increases in stem taper with increases in initial spacing, ranging from 99.5% in unthinned trees to 90.3% in 8 ft spaced trees (thinned versus unthinned and 4 versus 6 and 8 ft, $p < 0.01$; 6 versus 8 ft, $p = 0.09$). Thus, Zakrzewski's (1999) model appeared capable of compensating for the different stand densities in the study through tree-specific expressions of HDR ($s = 1 + \text{HDR}$) (see model [A1], Appendix A). A fit of this model to the complete stem analysis data set produced the parameter estimates -1.8969 and 1.0300 for β and γ , respectively, both parameter estimates being significantly different from 0, $p < 0.05$, and the approximate R^2 value equal to 0.97 ($1 - \text{residual sum of squares}/\text{corrected total sum of squares}$). Predicted values describing stem profile from this model are translated into stem radii for hypothetical trees of similar (i) diameter and (ii) height, based on treatment mean values of HDR, in Fig. 3. Spaced trees tended to be shorter and larger in diameter than unspaced trees, the extent of this effect being directly proportional to thinning intensity.

Fig. 3. Predicted values for stem profile from model [A1] (Appendix A) under treatment-specific mean values for HDR (height/DBH ratio) for trees of (a) similar DBH and (b) similar height. These curves illustrate thinning effects on stem taper observed from stem analysis of a stratified sample of 115 trees.



Stem volume

Volume predictions from the integrated form of Zakrzewski's (1999) model averaged 1.1% less than actual volume values ($n = 115$), with the absolute value of the errors averaging 5.3%. The relationship between actual and predicted stem volume had a slope and intercept of 1.071 and -0.009 , both significantly different from 1 and 0 ($p < 0.01$), respectively, suggesting that this model does exhibit a slight bias related to tree size with this data set. Model prediction errors were not associated with the thinning treatments ($p \geq 0.58$).

In contrast, the Honer et al. (1983) balsam fir equation, applied to the same trees, underestimated stem volume by an average of 11.7% (mean absolute value also averaged 11.7%). The relationship between actual and predicted volume had a slope of 1.170 ($p < 0.01$; intercept = -0.005 , $p = 0.12$) indicating that the equation has a strong bias related to tree size (i.e., greater underestimation in larger-sized trees) with these data. Model prediction errors were also associated with the thinning treatments, with strong differences between the errors of thinned and unthinned trees ($p = 0.02$), indicating that there may be some aspects of thinning-related tree form that the model, based on unthinned natural stands (Honer et al. 1983), does not adequately compensate for.

Combined, these results suggest that Honer's published equation may not be particularly suited to the balsam fir in the Green River study.

A fit of model [B1] (Appendix B) to 100 random drawings of half the stem analysis trees produced a mean error of -0.3% and a mean absolute error of 4.4%, when actual stem volumes were compared with those predicted for the remaining trees. The relationships between actual and predicted volume for the validation sets had a mean slope and intercept of 0.990 and 0.002, respectively. With standard deviations of 0.002 and 0.003, these means do not differ from 1 and 0, respectively ($p \geq 0.59$), suggesting that model [B1] (Appendix B) does not exhibit bias related to tree size. The p values for contrasts among treatments averaged over 0.50 and were >0.05 at least 95% of the time in the 100 validations, suggesting that model [B1] (Appendix B) is also capable of representing the range of stem forms found in the Green River study. A fit of model [B1] (Appendix B) to all 115 trees in the data set produced the equation:

$$[3] \quad \widehat{GTV} = (e^{-10.11114} \text{DBH}^{1.86935} \text{HT}^{1.11392}) 1.001386$$

with a root-mean square error (corrected by eq. B4 (Ap-

pendix B)) of $0.016 \text{ m}^3\text{-tree}^{-1}$ (coefficient of variation (CV) = 6.03%) and a R^2 value equal to 0.99. Residual plots of the log-log model suggest a very good fit, with strong homogeneity of variance and normality ($p = 0.65$). Although the integrated form of model [A1] (Appendix A) would likely perform equally well, we chose model [3] to recalculate GTV for balsam fir in the larger data set because of its simplicity and slightly better fit to the stem analysis data. The same deductions applied by Honer et al. (1983) were used to obtain GMV_8 and GMV_{15} .

Permanent sample plot data

Stand dynamics

Considering all conifer stems >1.3 m tall, initial stand densities on the Green River sites ranged from 5050 stems·ha⁻¹ at Lower Belone Bk. to just over 10 000 stems·ha⁻¹ at Lower Chisholm Bk., with a mean of 7450 stems·ha⁻¹ ($p = 0.24$) (Table 1 and Fig. 4a) (note that before thinning, stands also contained a mean of 3150 stems·ha⁻¹ of hardwoods). Over time, unthinned plots peaked at 8000 stems·ha⁻¹, approximately 10 years after adjacent plots were spaced, and then declined steadily as they underwent self-thinning. According to the maximum size density line established for balsam fir in New Brunswick (Penner et al. 2006), self-thinning initiated in the Green River plots as relative density index (RDI) increased above 50. This point was reached in the thinned plots approximately 15, 20, and 25 years after 4, 6, and 8 ft spacing, respectively. The mean minimum intertree spacing reached in the plots was 1.1 m (3.7 ft), 1.7 m (5.5 ft), 2.1 m (6.9 ft), and 2.5 m (8.3 ft) in each of the unthinned and thinned plots, respectively. By 2004, a mean of 43 years after thinning, all stands were well into the zone of “imminent competition-related mortality” for balsam fir (RDI = 78), with unthinned densities converging on those of the thinned plots, although statistical differences between the treatments still existed (p thinned vs. unthinned < 0.01, p 4 vs. 6 and 8 ft < 0.01, and p 6 vs. 8 ft = 0.06).

Consistent with the density trends, unthinned plots carried greater BA than thinned plots through approximately 20 years after thinning (Fig. 4b). After this period, high rates of natural mortality (RDI = 72) caused BA to fall to the point where the thinned plots carried greater BA by year 43 ($p < 0.01$). Through the first 25 years after thinning, the 8 ft spacing carried the lowest BA, undoubtedly reflecting an understocked condition (RDI < 50). By 2004, the highest BA (46 m²·ha⁻¹) was found in the 6 ft spaced plots (p 6 vs. 8 ft = 0.05). In each treatment, the inflection point where BA increases at a decreasing rate appeared to coincide with the onset of self-thinning (RDI > 50).

In contrast, differences between the treatments with respect to GTV production are generally not very strong (Fig. 4c). With the exception of reduced total volume in 8 ft spaced plots during the understocked phase following thinning, there were no notable differences among treatments (e.g., year 25, p thinned vs. unthinned = 0.13) until after major self-thinning took place in the unthinned plots. By 2004, thinned plots carried 15% more GTV than unthinned plots ($p < 0.01$), and differences among the thinning treatments were marginal (less than 8%, $p \geq 0.06$).

The mean live-crown ratio of unthinned trees declined steadily over the monitoring period, beginning at near 80% and falling below 30%, 40 years after adjacent plots were spaced (Fig. 4d). In contrast, the LCR in spaced plots increased until approximately 10 years after thinning—the likely point at which full crown closure was reached in these plots. An average of 43 years after thinning, the LCRs were 30, 32, and 34 in each of the respective treatments (p thinned vs. unthinned < 0.01, p 4 vs. 6 and 8 ft < 0.01, and p 6 vs. 8 ft = 0.01).

Irrespective of treatment, mean crown diameter increased steadily over time, as surviving trees expanded to fill voids created by the death of neighboring trees. Reflective of the different resultant densities, crown diameter was strongly proportional to thinning intensity throughout the observation period, with 2004 values averaging 2.5, 2.7, 3.1, and 3.5 m in the unthinned and thinned plots, respectively ($p < 0.01$). Individual tree crown areas, summed as a measure of site occupancy, show rapid increases through to about 15 000 m²·ha⁻¹ (even higher in the unthinned plots), suggesting a high degree of canopy layering and interlocking during early stand development (10–20 years after thinning). Again, coincident with self-thinning, total crown area values declined shortly after RDI values exceeded approximately 50. By year 30, the stands had converged on approximately 10 000 m²·ha⁻¹, as crowns fully occupied the main canopy.

Merchantable stand characteristics

When only stems >9 cm DBH are considered, stands did not reach their maximum densities until 20 years after thinning (Fig. 5a). The mean minimum intertree spacing reached in the plots was 2.2 m (7.3 ft), 2.1 m (7.1 ft), 2.4 m (7.9 ft), and 2.9 m (9.4 ft) in each of the unthinned and thinned plots, respectively. By 2004, unthinned and 4 ft spaced plots averaged just over 1500 stems·ha⁻¹ (2.6 m or 8.4 ft spacing), whereas the 6 and 8 ft spaced plots averaged 1360 stems·ha⁻¹ (2.7 m or 8.9 ft spacing) and 1100 stems·ha⁻¹ (3.0 m or 9.9 ft spacing), respectively (p thinned vs. unthinned = 0.03, p 4 vs. 6 and 8 ft < 0.01, and p 6 vs. 8 ft = 0.02). Were it not for large differences in the number of small trees (≤ 9 cm DBH, Fig. 4), the unthinned and 4 ft spaced plots would look very similar in terms of merchantable stand density throughout the observation period.

In contrast with trends observed with total BA, thinned plots generally carried greater merchantable BA than unthinned plots throughout the observation period (Fig. 5b). In 2004, spaced plots averaged 44 m²·ha⁻¹ of merchantable BA, 12% more than unthinned plots ($p < 0.01$). Again, the 6 ft spaced plots held the greatest BA at 46 m²·ha⁻¹ (p 6 vs. 8 ft = 0.05). Similarly, merchantable volumes (Fig. 5c) exhibited early thinned versus unthinned separation, with differences increasing over time. The 2004 GMV_8 values were greatest in the 6 and 8 ft spaced plots (363 m³·ha⁻¹ and $p = 0.57$), an 8% gain over the 4 ft spacing ($p = 0.04$) and a 21% gain over the unthinned plots (p thinned vs. unthinned < 0.01). With GMV_{15} (Fig. 5d), gains from the 6 and 8 ft spacings ($p = 0.33$) increased to 26% over the 4 ft spacing ($p < 0.01$) and 46% over unthinned plots (p thinned vs. unthinned < 0.01).

The mean annual increment (MAI) of GMV_8 increased steadily in all plots during the observation period (Fig. 6). Clear distinction between thinned and unthinned plots was

Fig. 4. Least-squares means and their standard errors for overall conifer stand attributes (all stems >1.3 m tall). Plotted values represent the means of five blocks, each containing at least two 0.08 ha permanent sample plots. Results of ANOVA contrasts (*p* values) are shown for the final measurement.

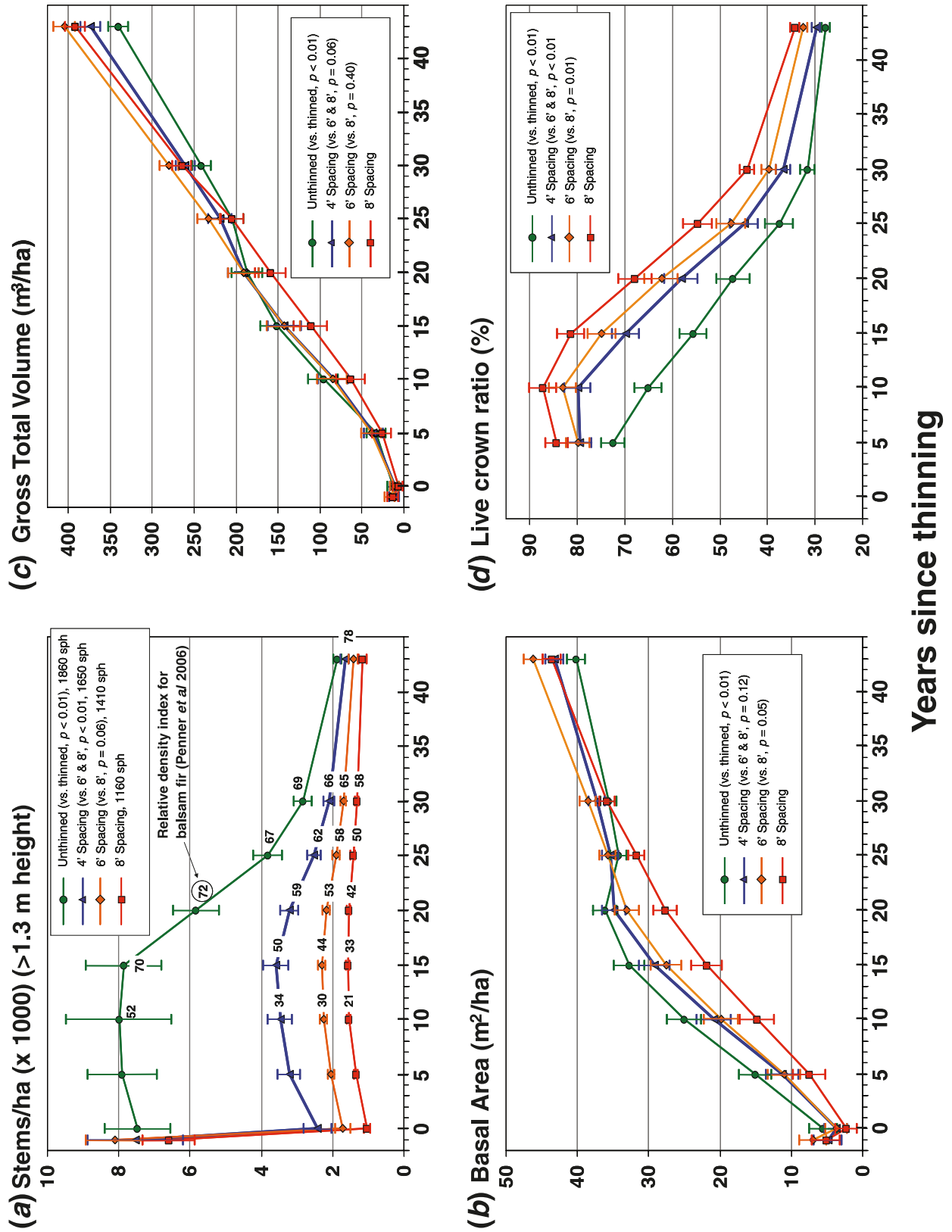


Fig. 5. Least-squares means and their standard errors for merchantable conifer stand attributes (all stems >9 cm DBH). Plotted values represent the means of five blocks, each containing at least two 0.08 ha permanent sample plots. Results of ANOVA contrasts (*p* values) are shown for the final measurement.

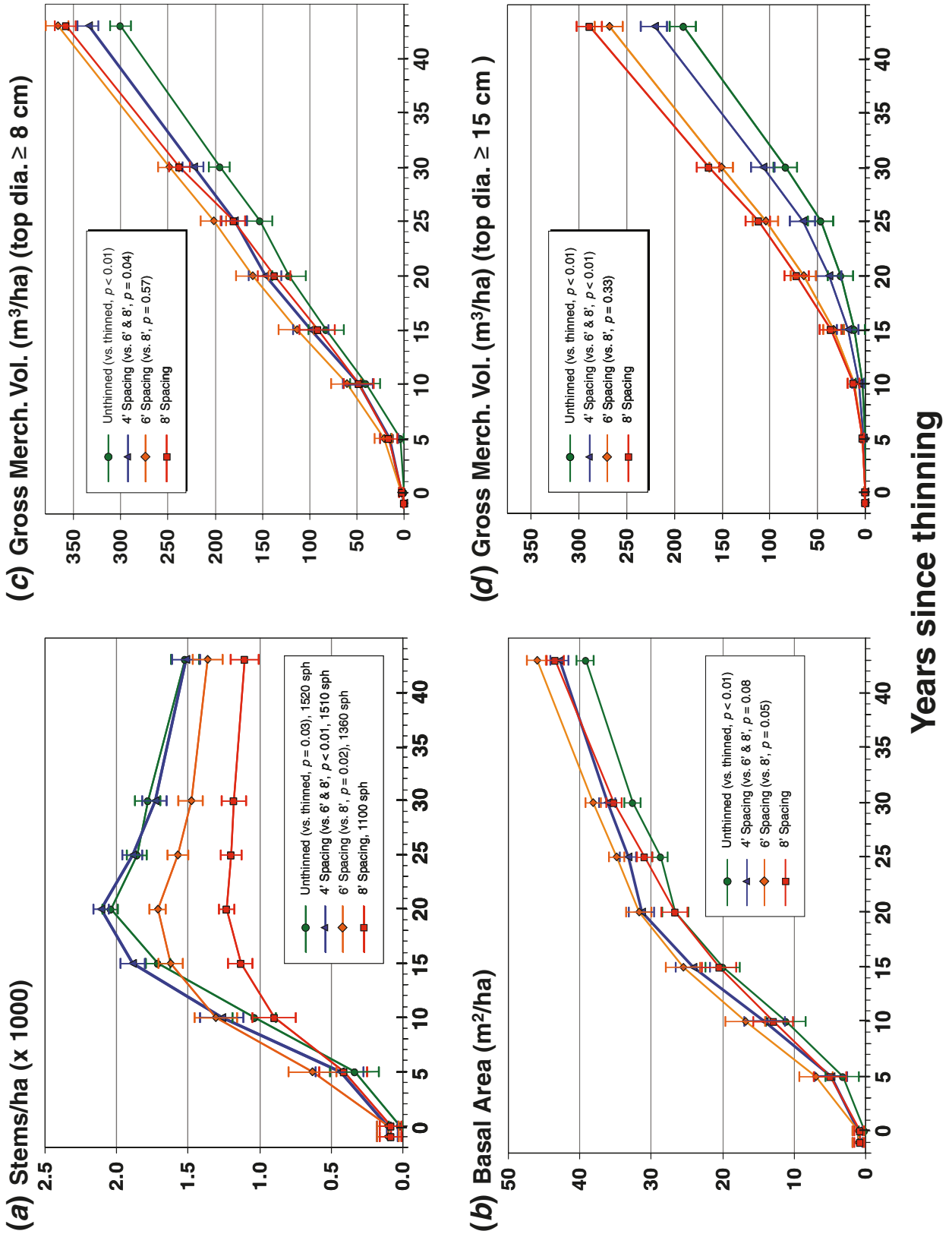
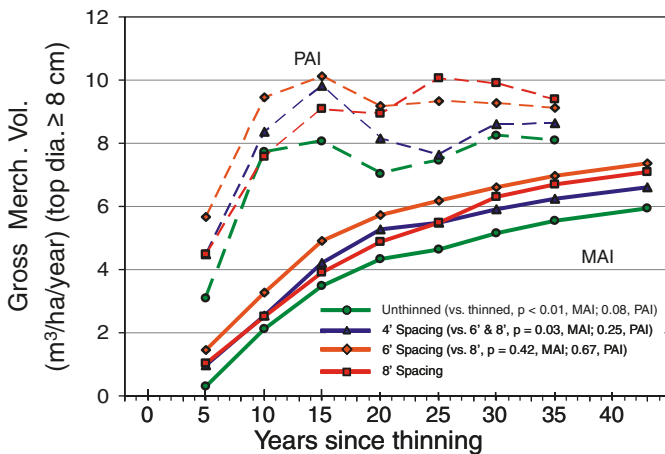


Fig. 6. Mean annual increment (MAI) and periodic mean annual increment (PAI) for gross merchantable conifer volume ($\text{m}^3 \cdot \text{ha}^{-1} \cdot \text{year}^{-1}$). Plotted values are means of five replicate blocks, each containing at least two 0.08 ha permanent sample plots. Results of ANOVA contrasts (p values) are shown for the final measurement.



maintained over time, with thinned plots typically providing 18% greater growth rates than unthinned plots ($p < 0.01$). Towards the end of the observation period, as trees in the 8 ft spacing achieved full site occupancy, the 6 and 8 ft spacings provided the fastest growth rates ($7.2 \text{ m}^3 \cdot \text{ha}^{-1} \cdot \text{year}^{-1}$) and a 9% gain over the 4 ft spacing ($p = 0.03$). As of 2004, MAI had not yet culminated (MAI and PAI intersect) in any of the plots, but trends suggest that this may begin taking place at approximately year 50, or at a stand age of 60 years postharvest.

Mean tree characteristics

Thinning did not affect the mean height of dominant and (or) codominant trees in the plots (Fig. 7a; $p \geq 0.16$). A small dip in year 25 DHT coincides with the crown-damaging ice storm that occurred in 1984. However, thinning did have a dramatic effect on all other aspects of mean tree size measured.

Quadratic mean diameter for all surviving merchantable stems (QDBH) illustrates a typical increase in mean tree size immediately following thinning because of the removal of small trees ("chainsaw effect") (Fig. 7b), with subsequent clear treatment separation immediately after thinning and differences increasing over time. By 2004, thinning produced 4%, 14%, and 23% gains in QDBH over unthinned plots ($p < 0.01$), the size of the gain directly proportional to thinning intensity. This thinning response is illustrated for the entire mean diameter distribution of each treatment in Fig. 8. Essentially, thinning resulted in a 2 cm shift in the diameter distribution for every 2 ft increase in nominal spacing. Although thinning did not appreciably reduce the standard deviation (S_{DBH}) of the overall diameter distribution (p thinned vs. unthinned = 0.09), thinning did reduce its CV (Fig. 9). Unthinned stands had high variability in tree size and small mean tree size, potentially undesirable and costly conditions from an operational perspective. In contrast, thinned stands tended to have larger trees with proportionally reduced variability in size; the 6 and 8 ft spacings exhibited the best characteristics in this regard.

Height:DBH ratio (HDR) values, which reflect thinning ef-

fects on stem taper, show a clear treatment separation throughout the observation period (Fig. 7c), with end-point values averaging 98% in the unthinned plots and 96%, 90%, and 85% in the three spacings, respectively (p thinned vs. unthinned < 0.01 , p 4 vs. 6 and 8 ft < 0.01 , and p 6 vs. 8 ft < 0.01). These end-point values differ slightly from those summarized from stem analysis (Fig. 3) because of the stratified sampling system used for the latter.

Similarly, treatment separations in mean piece size, as measured by GMV_8/tree (Fig. 7d), were distinct and increase over time. Thinning produced 10%, 33%, and 62% gains in GMV_8/tree over unthinned plots ($p < 0.01$), the size of the gain being exponentially proportional to thinning intensity.

Parallel end-point analyses conducted on the largest 1000 stems $\cdot \text{ha}^{-1}$ found in each treatment plot (Table 3) confirmed all of the above conclusions drawn for the mean individual-tree attributes. One minor exception was S_{DBH} , which increased in the 8 ft spacing relative to the other treatments because of some of these plots sitting at or very near the 1000 stems $\cdot \text{ha}^{-1}$ mark and all stems in the plots being included in the analysis. Censoring the data in this manner narrowed the range for all variables studied except dominant height and may have removed potential carryover related to the chainsaw effect initially induced by the thinning treatments, making tests for thinning effects more conservative. Thus, if one were only interested in the largest 1000 stems $\cdot \text{ha}^{-1}$ at the time of harvest, thinning still provided significant gains in terms of piece size (QDBH and GMV_8/tree , $p < 0.01$).

Discussion

Precommercial thinning (PCT) is a silvicultural practice used to allocate the growing resources of a site to selected stems during juvenile stand development, when none of the felled trees are of merchantable size (Smith 1986). It has been widely theorized that, with careful application, PCT provides the forester with a means of manipulating species composition and stand density, to strike a desired balance between achieving increased growth on individual stems and maintaining adequate density for maximizing volume per hectare production and stem quality (e.g., Smith 1962; Lavigne et al. 1987; Ziede 2001). Redistribution of the site's volume production onto fewer stems should provide the benefits of increased stand value and reduced time to merchantability (e.g., Piene 1982; Seymour 1992; Daggett and Wagner 2002; Pothier 2002). Although the narrow range of sites and single species composition comprising the Green River thinning trial limit its inference potential, the study's age and relevance to today's operational thinning intensities and intervention times provide us with a unique opportunity for regional, long-term, empirical validation of such conventional thinning theory. In our opinion, a major contribution of the Green River trial lies in the confidence it may lend to forecasted outcomes of PCT on other northeastern sites, with other species, until such time as we have similar long-term data for the different combinations of species and sites that we manage.

Production

Although we did not measure gross productivity in the

Green River stands, gross total stemwood production, including cumulative mortality, may be viewed as a partial surrogate (Fig. 10). If we included mortality, we observed no difference among the treatments with respect to total stem fiber production an average of 50 years postharvest ($p = 0.22$). However, like the trends observed in live-stem basal area and gross total volume production (Figs. 4b and 4c), earlier treatment comparisons with this variable suggested understocking of the thinned stands until about 20–25 years post-treatment. This result supports Baskerville's (1965b) theory that thinning will cause a temporary reduction in total fiber production per unit area, until remaining trees increase in size to the point of regaining full site occupancy. From the point of such recovery in the Green River plots, the data appear to support the theory that production is relatively constant through a range of stand densities (Langsater 1941; Mar:Moller 1954; Smith et al. 1997; Ziede 2001).

Yield

Cumulative stem volume losses through mortality in unthinned stands were reduced by approximately 25% with each nominal increase in thinning intensity ($p < 0.01$; Fig. 10). Such mortality over approximately 50 years postharvest amounted to a mean of $100 \text{ m}^3\text{-ha}^{-1}$, compared with $<30 \text{ m}^3\text{-ha}^{-1}$ in the 8 ft spaced stands. This, coupled with the mean tree size gains we observed in response to thinning, strongly affected the potential amount of usable wood, or "yield," of the Green River stands. Depending on utilization standards, actual spacings between 2.1 and 2.5 m (6.9–8.3 ft) suggest yield gains ($\text{GMV}\cdot\text{ha}^{-1}$) of 21%–46% over unthinned stands, an average of 43 years after thinning. A similar conclusion was drawn in the two previously cited long-term studies in Maine and Quebec (Daggett and Wagner 2002; Pothier 2002). Thus, unless one is capable of capturing and using total stand production in a series of thinnings, there is rather strong empirical evidence that PCT does increase long-term yield.

Rotation age

In simple terms, PCT accelerates natural thinning processes in the stand, reducing the age at which trees reach target size (Smith 1986). Thinning clearly increased piece size at Green River, as reflected by stem diameter, diameter distribution, and stem volume. Actual spacings of 2.1 and 2.5 m produced 14% and 23% gains in QDBH, respectively, and 33% and 62% gains, respectively, in GMV_8 , to a minimum 8 cm top diameter. Consistent with the original goal of these studies being to produce harvestable volume at an earlier age, the "technical rotation age" for a target piece volume of $200 \text{ dm}^3\text{-stem}^{-1}$, for example, would have been achieved approximately 30 years after thinning to 2.5 m, more than 10 years ahead of the unthinned stands (Fig. 7d). Although there would likely be a reduction in $\text{GMV}_8\text{-ha}^{-1}$ realized through such an early harvest (17%; Fig. 5c), the forest-level benefits of maintaining an even flow of usable wood may outweigh individual stand-level losses. Equally attain-

able through PCT, the wildlife and other nontimber benefits associated with the early production of large-diameter trees (mature conifer habitat) may be coincident with, or even override, timber objectives in some jurisdictions (e.g., Homyack et al. 2004). An average of 43 years after thinning, plots supported similar nontimber plant diversity to unthinned plots (S. Newmaster, unpublished data), similar numbers of snags ($31 \text{ stems}\cdot\text{ha}^{-1} > 18 \text{ cm DBH}$ and $p = 0.24$), and the wider spacings held two- to three-fold more stems 30 cm in diameter and larger than the unthinned plots (Fig. 8; $p = 0.03$).

Although thinning clearly reduced technical rotation age at Green River, it does not seem to be having an effect on the age at which volume production will be maximized (intersection of MAI and PAI). Trends suggest that this may take place approximately 60 years after harvest, and the relative difference between MAI and PAI values among the treatments hints that these rotation ages may be similar (Fig. 6).

Value

An underlying objective of thinning will always be to increase future stand "value" (Smith et al. 1997). That the Green River plots produced greater volume in larger-sized trees through PCT is a strong indicator that this objective was achieved, but more work is required to fully quantify the different aspects of value. At this time, we have no direct measures of wood quality in the Green River plots, and this is clearly a research need. Smith et al. (1997) alleviate fears somewhat by arguing that the notion that thinning-induced, fast-grown wood lacks strength is false. They state that thinning creates increased volume of the same kind of wood that might have been laid down in the absence of release, and therefore, the more outer wood that can be added by increased diameter growth, the greater the inherent value of the stem. Thinning did increase stem taper, the extent being proportional to thinning intensity (Fig. 3), but it is questionable, under today's mill-recovery standards, whether the amount of taper found would lead to increased waste wood. Ford (1994), however, found increased branch diameter (and potentially reduced wood quality) on 8 ft spaced trees in a subsample of the Summit Rd. plots and recommended the 6 ft spacing as a good potential compromise between increased growth and reduced branch diameter. There has also been unsubstantiated suspicion that PCT may increase the incidence of butt rots and therefore reduce long-term wood quality and yield (Tian 2000).³ Although we see no evidence that this is the case at Green River, further study of this is also warranted.

Perhaps the greatest potential increases in value resulting from PCT come from reduced costs per unit value of the final product (i.e., increased utilization and reduced harvesting, handling, and processing costs (e.g., Tong et al. 2005)). In this paper, we have refrained from conducting economic analyses because of the plethora of different cost–price assumptions and stand-level versus forest-level costs and (or) benefits that must be considered, opting instead to strive to

³G. Warren and B. English. 2001. Root and butt rots in semi-mature, precommercially thinned stands of balsam fir in Newfoundland. Poster presentation at (1) IUFRO Root and Butt Rot Working Group 7.02.01, 16–22 September 2001, Quebec City and (2) Forest Pest Control Forum 2001, 27–29 November 2001, Ottawa.

Fig. 7. Least-squares means and their standard errors for individual conifer tree attributes (stems >9 cm DBH). Plotted values represent the means of five blocks, each containing at least two 0.08 ha permanent sample plots. Results of ANOVA contrasts (*p* values) are shown for the final measurement.

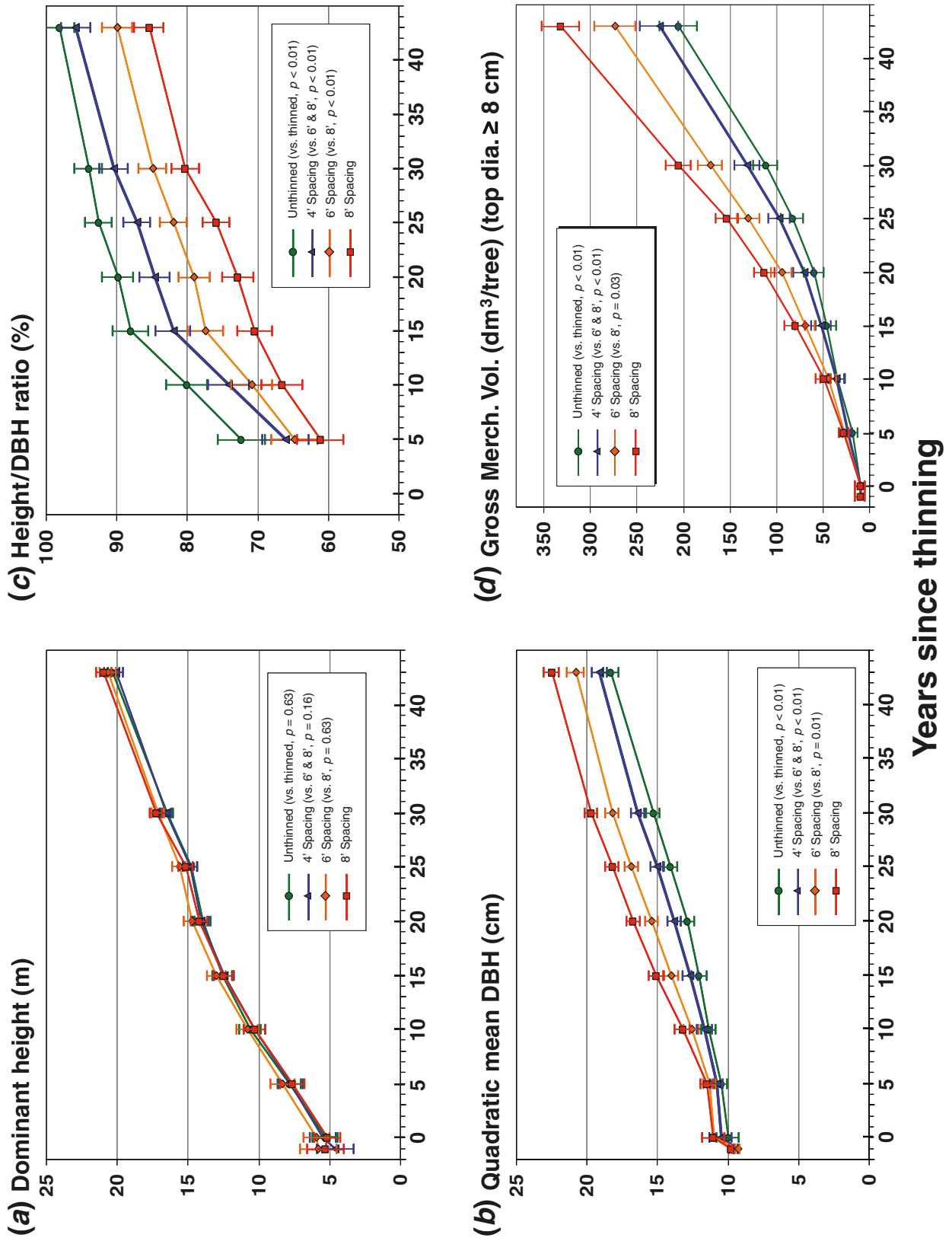


Fig. 8. Diameter distributions for live trees 42–44 years after thinning. Crosshatched portions of the bars represent balsam fir and darkened portions represent red spruce. Plotted values are averaged over five replicate blocks, each containing at least two 0.08 ha permanent sample plots. Each of the distributions is approximated by a normal curve to aid treatment with treatment comparisons.

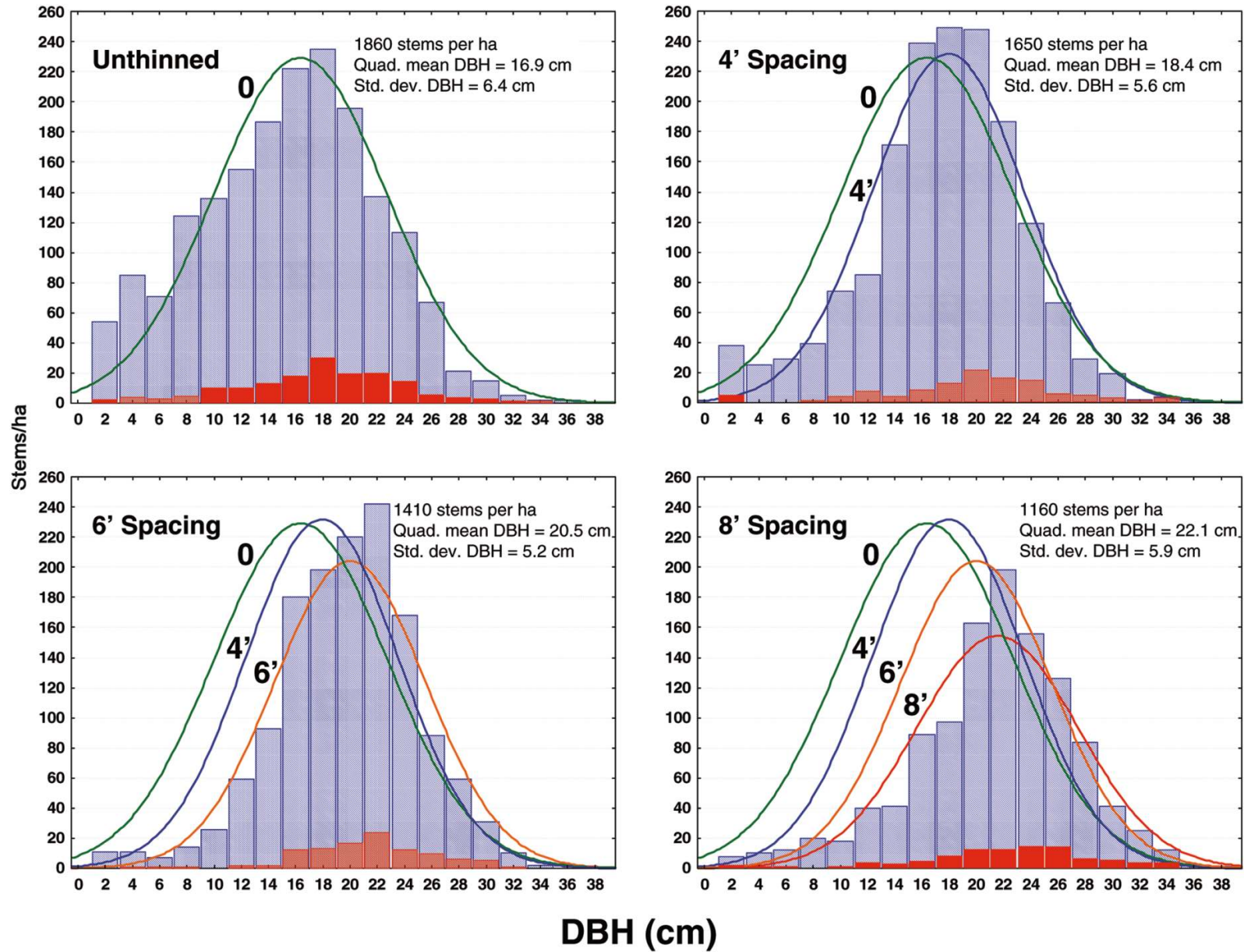
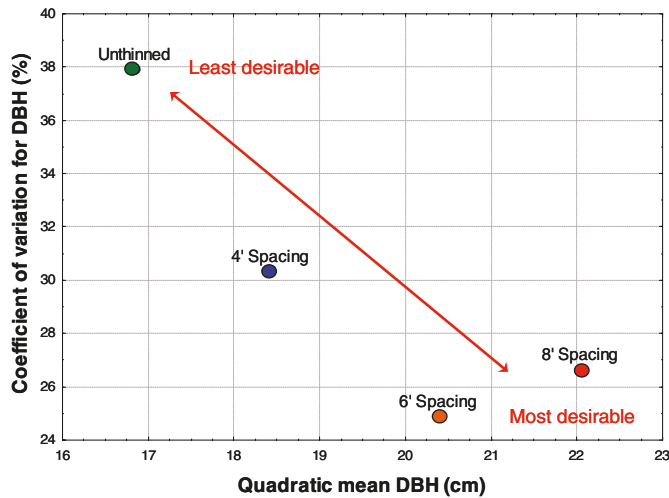


Fig. 9. Stability analysis for diameter at breast height (DBH). Stands that are most desirable from an operational perspective have large-diameter trees with minimum variability in tree size. Plotted values are means of five replicate blocks, 42–44 years after thinning, each containing at least two 0.08 ha permanent sample plots.



offer the reader data needed to conduct their own analyses based on their particular circumstances. Reduced coefficient of variation in diameter distribution, increased piece size, and fewer small dead stems to work around ($p < 0.01$) all make the thinned stands more attractive from an operating efficiency perspective. These stand factors, coupled with the forest-level benefits of regulating long-term volume flow, very likely make PCT an economically favorable silvicultural treatment.

Recommendations

Yield forecasts

We are often asked the question “Should precommercially thinned areas be ascribed greater volume in wood supply forecasts than unthinned areas?” The long-term results of the Green River thinning trials illustrate that the extent to which thinning may increase yield is highly dependent on both the utilization standard imposed and the amount of time elapsed post-thinning. Although not tested in this study, site quality likely plays an important role as well. Forecasted yield responses should therefore be made within the context of desired future products, their expected utilization standards, and the quality of sites chosen for intensive silviculture investments such as PCT.

Optimum spacing

A nominal spacing of 6 ft (1.8 m), or an actual spacing of 7 ft (2.1 m), appeared to offer a near-optimum balance between maximizing individual stem growth and maintaining adequate density for maximizing per hectare production (as measured by BA and $GMV_8 \cdot ha^{-1}$). A nominal spacing of 8 ft (2.4 m), or an actual spacing of 8.3 ft (2.5 m), produced larger trees earlier, but the combined evidence of understocking, increased taper, larger branch diameters (Ford 1994), and slightly reduced per hectare production suggest caution in its widespread use. However, as thinning intensity has a direct effect on reducing the time to merchantability,

in many jurisdictions, it may be wise to maintain some flexibility over this parameter so that individual stands can be promoted in a harvest or habitat queue as needed.

Commercial thinning opportunities

In natural stands, PCT is increasingly being used to create future commercial thinning opportunities by promoting root and crown development in crop trees so that they may withstand windthrow and possess the physiological capacity to respond following future thinning (Brunsdon and Pelletier 1995; Smith et al. 1997; Ruel et al. 2000). In the Green River study, although unthinned plots averaged live crown ratios (LCRs) $>50\%$ through to about 28 years after harvest, it is unlikely that these trees would have had adequate root development to resist windthrow following a commercial thinning on these sites and should probably never be considered as potential candidates for such activity (McKinnon et al. 2006). Thinned plots, on the other hand, would have been more likely to develop adequate root systems and appear to have moved through a window of opportunity for commercial thinning ($>50\%$ LCR and $<80\%$ HDR) somewhere between 15 and 25 years after treatment, the wider spacings offering later intervention times. The data suggest that between 30 and 75 $m^3 \cdot ha^{-1}$ of mortality might have been captured in these plots through a commercial thinning (Fig. 10). The wider spacings tested would offer increased volume recovery in a fixed, 30%–35% BA removal, and therefore, potentially offer more biologically effective and cost-efficient commercial thinning opportunities.

Existing and emerging tools offer managers assistance in making decisions such as the timing and extent of thinning operations (Piene 1982; Newton 1997; Wilson et al. 1999; Bégin et al. 2001; Penner et al. 2006). We have referenced key turning points in stand development against Penner et al.'s (2006) maximum size density line, and there was a strong association between the onset of self-thinning events and relative density index (RDI) values >50 . If the height/diameter ratio (HDR) values plotted in Fig. 7c are related back to the RDI values shown in Fig. 4a, it can be seen that each of the treatments crosses 80% HDR at approximately the same point that the RDI values move from the zone of maximum growth (RDI = 55) into the zone of imminent competition-related mortality. Thus, HDR, being a relatively easy parameter to measure, might be a simple index for monitoring and regulating stand density and useful for identifying the commercial thinning opportunities described above.

Future research at Green River

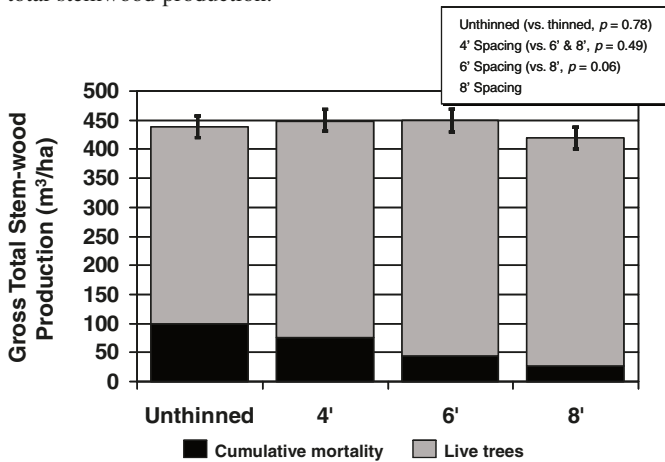
With respect to further research on the Green River plots, detailed stem analyses are currently being conducted with Windendro[®] (Regent Instruments Inc.) for the purpose of examining individual tree growth responses over time to competition and environmental factors, such as spruce budworm. Given that all plots were well into the zone of imminent competition-induced mortality (RDI = 78) in 2004, it is likely that these stands are on the cusp of breaking up. In light of the previously noted knowledge gaps, we recommend that three of the blocks be harvested, with the goals of conducting full treatment comparisons of (i) wood production, quality, and value from both pulp and paper and solid-wood producer's perspectives, (ii) gross biomass pro-

Table 3. Individual-tree statistics for the largest 1000 stems·ha⁻¹ found in each treatment plot in 2004, including least-squares means, standard errors (SE), and *p* values, from ANOVA.

Treatment	QDBH (cm) ^a		S_DBH (m) ^b		DHT (m) ^c		HDR (%) ^d		GMV ₈ (dm ³ ·tree ⁻¹) ^e	
	Mean	SE	Mean	SE	Mean	SE	Mean	SE	Mean	SE
0	20.5	0.30	3.7	0.3	20.4	0.61	90.8	1.9	265.7	15.3
4 ft	21.2	0.30	3.2	0.3	20.1	0.61	88.5	1.9	286.6	15.3
6 ft	22.4	0.33	3.4	0.3	20.7	0.64	85.3	1.9	326.6	16.6
8 ft	23.3	0.30	4.2	0.3	20.9	0.61	83.2	1.9	356.5	15.3
<i>p</i> treatment ^f	<0.01		0.08		0.44		<0.01		<0.01	
<i>p</i> thinned vs. unthinned	<0.01		0.90		0.64		<0.01		<0.01	
<i>p</i> 4 vs. 6 and 8 ft	<0.01		0.08		0.16		<0.01		<0.01	
<i>p</i> 6 vs. 8 ft	0.06		0.06		0.63		0.02		0.14	
Means spread (largest 1000 stems·ha ⁻¹)	2.8		1.0		0.8		7.6		90.8	
Means spread (all stems >9 cm DBH)	4.2		1.3		0.8		12.7		126.5	

^aQuadratic mean diameter at breast height (DBH).
^bStandard deviation of DBH.
^cDominant tree height.
^dHeight:diameter ratio.
^eGross merchantable volume to a minimum top diameter of 8 cm.
^fFrom ANOVA, testing for differences among thinning treatments.

Fig. 10. Gross total stemwood production (cumulative mortality plus live trees) an average of 50 years postharvest. Plotted values are least-squares means and their standard errors, representing five replicate blocks, each containing at least two 0.08 ha permanent sample plots. Results from ANOVA contrasts (*p* values) are shown for total stemwood production.



duction and carbon sequestration potential, (iii) incidence and effects of butt and stem decay, and (iv) long-term productivity (by re-establishing the same treatments and plots directly on top of the existing ones). There is also scientific merit in monitoring long-term succession and stand dynamics in these stands, and we therefore recommend that the remaining blocks or plots be preserved for this purpose.

Conclusions

As we look toward prime-site intensive silviculture as a means of maintaining wood supply in the face of ever-shrinking land bases, long-term managed-stand data, such as those offered by the Green River thinning trial, are critical for proper risk–benefit analyses in decision making and planning, as well as substantiating the sustainability of specific silvi-

cultural practices. This long-term study has confirmed that timely PCT can increase potential yield, particularly in the larger size-classes, reduce the time to merchantable tree size and mature conifer habitat, and offer potential to reduce harvesting and processing costs through greater stand uniformity and increased piece size. Such gains have both stand- and forest-level advantages that must be carefully weighed against the cost of PCT. In his establishment report for the Green River thinning trials, Baskerville (1959) forecasted that “with cleaning and thinning, it should be possible to produce 30 cords per acre in 40 years.” It is with some degree of satisfaction, and growing confidence that there is room for silviculture to enhance the productivity and competitiveness of our northern forests, that we can say, 40 years later, that his forecast fell short by twofold!

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Appendix A. Stem analysis of taper.

To assess treatment effects on tree taper and stem form, the stem profile model presented by Zakrzewski (1999) was fit to the stem analysis data from each treatment plot ($n = 5$ trees) to estimate stem outside-bark cross-sectional area (ca_z) at any relative tree height z (tree top, $z = 0$; base, $z = 1$) along a stem:

$$[A1] \quad ca_z = \left(\frac{C(z_0 - s)}{z_0^2 + \beta z_0^3 + \gamma z_0^4} \right) \left(\frac{z^2 + \beta z^3 + \gamma z^4}{z - s} \right) + \varepsilon$$

where C is the the cross-sectional area at breast height, HT is the total tree height, β and γ are parameters estimated by nonlinear least squares, $s = 1 + HT/DBH$, z_0 is relative breast height ($1 - 1.3/\text{height}$), and ε are random, independent, and normally distributed errors. PROC NLIN of the SAS[®] system was used for these regressions (SAS Institute Inc. 1990). The treatment-plot parameter estimates for β and

γ were then used as raw data in analyses of variance (ANOVA) that incorporated the underlying randomized complete block experimental design (four treatments \times five blocks). PROC MIXED of the SAS[®] system (Littell et al. 1996) was used to accommodate the additional four treatment plots at Upper Belone Bk. in the analysis and compute the correct least-squares treatment means and their standard errors. Orthogonal contrasts were used to specifically compare the parameter estimates of (i) unthinned plots with the mean of the thinned plots; (ii) 4 ft spaced plots with the mean of the wider spacings; and (iii) with 6 ft with 8 ft spaced plots.

A lack of statistically significant differences ($p > 0.05$) in parameter estimates among the treatments was used to confirm the flexibility of model [A1] across the experimental range of densities and to offer evidence that a single equation could represent the entire stem analysis data set. Model [A1] was then refit to the complete stem analysis data set, and a single set of parameter estimates for β and γ was obtained. As model [A1] is designed to compensate for stand density through changes in the s parameter or HDR (Zakrzewski 1999), treatment plot means for s were subjected to ANOVA to verify the presence of thinning effects on the HDR. The nature of density effects on tree taper were then explored graphically by plotting the predicted values from model [A1] under different treatment-mean values of s (see Fig. 3 in the text).

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Appendix B. Stem volume equation development and validation.

As model [A1] (Appendix A) can be integrated over the entire stem to produce volume estimates (Zakrzewski 1999), the resulting integrated form of the equation was used to estimate the volume (GTV) of each tree in the stem analysis data set. The overall relationship between actual and predicted volume was examined for bias related to tree size by regressing actual volumes on predicted values and then assessing the significance of the resulting intercept and slope parameter estimates against 0 and 1, respectively. To again verify that the model adequately compensated for the different stand densities represented in the sample, the presence of bias related to the thinning treatments was tested by calculating the absolute value of the individual tree prediction errors [$(|\text{predicted volume} - \text{actual volume}|/\text{actual volume})100$] and subjecting the treatment-plot mean values to analysis of variance (ANOVA), as described above.

To benchmark the precision and accuracy of the volume estimates of Zakrzewski's model, and to determine the suitability of the Honer et al. (1983) equation for the Green River balsam fir, Honer's predicted values for the stem anal-

yses data set were subjected to the same bias evaluation procedures.

For further comparison, the natural-log-transformed version of the model described by Schumacher and Hall (1933) was also fit to the stem analysis data:

$$[\text{B1}] \quad \ln(\text{GTV}) = \ln \alpha + \beta \ln(\text{DBH}) + \delta \ln(\text{HT}) + \ln \varepsilon$$

where DBH is the diameter at breast height, HT is the total tree height, and ε are random, independent, and normally distributed errors, and α , β , and δ are parameters to be estimated by ordinary least-squares regression. Predicted values from the back-transformed parameter estimates of this model were corrected for bias by multiplying by the correction factor

$$[\text{B2}] \quad \text{CF} = e^{(\text{MSE}/2)}$$

(Baskerville 1972), where MSE is the mean square error from the log-linear regression and e is Euler's constant (2.7183). For example, predictions made from model [B1], in arithmetic units, were computed from

$$[\text{B3}] \quad \widehat{\text{GTV}} = (e^{b_0} \text{DBH}^{b_1} \text{HT}^{b_2}) \text{CF}$$

where b_0 , b_1 , and b_2 are parameter estimates for α , β , and δ , respectively, of model [B1]. The RMSE (square-root mean square error) value of model [B1] was recomputed in arithmetic units from the predicted values of model [B3]:

$$[\text{B4}] \quad \text{RMSE}' = \sqrt{\frac{\sum_{i=1}^n (\text{GTV}_i - \widehat{\text{GTV}}_i)^2}{n-3}}$$

where RMSE' is the model square root MSE in arithmetic units, GTV_i is the GTV of the i th tree in arithmetic units, $\widehat{\text{GTV}}_i$ is the bias-corrected predicted GTV for the i th tree

from the log-linear model, and n is the number of trees involved in the regression.

In contrast with Zakrzewski's and Honer's models, model [B1] involved a direct fitting of stem volume to the DBH and HT data. As such, bias evaluation for model [B1] had to be conducted via bootstrapping methods (Chernick 1999). Specifically, we randomly selected half of the sample trees for model construction and used the remaining half for bias evaluation, repeating this process 100 times. The resulting means and standard deviations for the bias-evaluation statistics were compared with those of the above two models. Based on these comparisons, the best model was chosen and used to recalculate balsam fir GTV, GMV_8 (gross merchantable volume based on a stump height of 15 cm and a minimum top diameter of 8 cm), and GMV_{15} (gross merchantable volume based on the same stump height and a minimum top diameter of 15 cm) for the larger permanent sample plot (PSP) data set. In all of these analyses, model residuals were tested to ensure that the assumptions of homogeneity of variance and normality were met.

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