

Long-wave radiation at the ground

II. Geometry of interception by slopes, solids, and obstructed planes

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(Received 23 April 1974; revised 22 August 1974)

SUMMARY

At the earth's surface, long-wave radiation received from the atmosphere in the absence of cloud may be expressed as the sum of the flux density from an isotropic source and the flux density due to anisotropy of atmospheric radiation. Separation of these components facilitates calculations of the radiative flux received by obstructed horizontal surfaces (near walls or in valleys), by sloping planes, by solid cylinders, and by spheres and prisms. Radiation received from adjacent surfaces is also considered.

As the angular distribution of radiation is the same for overcast skies as for clear skies, the relationships for clear skies may be extended to derive climatological mean values for the long-wave irradiances of slopes, solids, etc.

1. INTRODUCTION

The apparent emissivity of the atmosphere may be defined as the ratio of incoming long-wave radiation at the ground to black-body radiation at screen temperature. Several series of measurements reported in the literature have demonstrated that the emissivity of an atmospheric column at zenith angle Z is a linear function of the optical path length for water, i.e.

$$\varepsilon = a + b \ln(u \sec Z) \quad (1)$$

where u is the reduced depth of precipitable water (Robinson 1947, 1950; Deacon 1970). Unsworth and Monteith (1975) established ranges and mean values of a and b for their own measurements in the English Midlands and for published measurements made at other sites. This paper is a sequel, exploring the implications of Eq. (1) for obstructed and sloping surfaces and for solid objects receiving long-wave radiation both from the sky and from the ground. Previous attempts to calculate the long-wave exchange of obstructed and sloping surfaces have been based on empirical formulae for *net* long-wave radiation (Linke 1931; Lauscher 1934). Calculations based on Eq. (1) are more rigorous and allow the separate components of the long-wave exchange to be distinguished. The relationships derived in this way have numerous practical applications in building science, in urban climatology, in the climatic physiology of humans and other animals and in plant science.

2. RADIATION ON A HORIZONTAL SURFACE WHEN THE SKY IS PARTLY OBSCURED

For radiation from an entirely *unobscured* hemisphere Unsworth and Monteith (1975) showed that the apparent atmospheric emissivity ε_a , defined as the ratio of the hemispherical irradiance on a horizontal surface to black-body radiation at screen temperature, is identical to the apparent emissivity of a column at $Z = 52.5^\circ$, i.e.

$$\varepsilon_a = a + b(0.5 + \ln u) \quad (2)$$

Knowledge of the angular distribution of atmospheric radiation enables the radiation falling on a horizontal plane to be calculated when part of the sky is obscured.

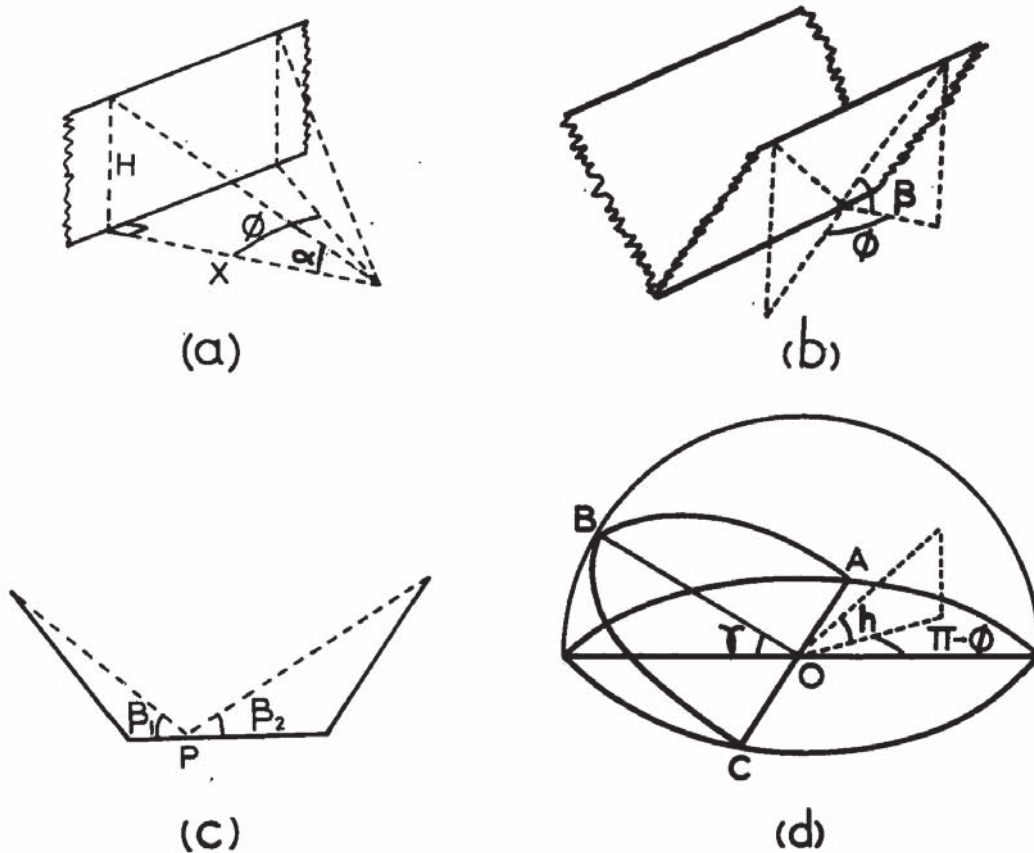


Figure 1. Geometry of radiation interception: (a) Horizontal plane near a vertical wall; (b) Floor of V-shaped valley; (c) Point on floor of valley whose walls do not intersect; (d) Sloping plane.

(a) *Radiation near a vertical plane*

Consider an infinitely long vertical plane, height H (Fig. 1(a)). The apparent emissivity of the atmosphere at a point on a horizontal plane distance x from the base of the wall is the sum of the apparent emissivity of the unobscured half-hemisphere, i.e. $0.5\epsilon_a$, and an emissivity which depends on the angle α defined by $\alpha = \tan^{-1}(H/x)$. It may be shown that the apparent emissivity is given by

$$\epsilon(\alpha) = 0.5 \epsilon_a + \int_{-\pi/2}^{+\pi/2} d\phi \int_{\alpha'}^{\pi/2} \frac{\epsilon(h)}{\pi} \sinh \cosh dh \quad (3)$$

where $\alpha' = \tan^{-1}((H \cos \phi)/x)$.

In this and similar expressions in this paper, the elevation angle h is more convenient than the zenith angle $Z = \frac{\pi}{2} - h$.

From Eqs. (1) and (2) we may write

$$\epsilon(h) = \epsilon_a - b(0.5 - \ln \operatorname{cosech} h) \quad (4)$$

and hence Eq. (3) may be partially evaluated to give

$$\epsilon(\alpha) = \epsilon_a(0.5 + I_1(\alpha)) - 0.5 b I_2(\alpha) \quad (5)$$

where

$$I_1 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{1 + \psi^2} d\phi \quad \text{and} \quad \psi = (H \cos \phi)/x$$

and

$$I_2 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \frac{\psi^2}{1 + \psi^2} \ln \frac{1 + \psi^2}{\psi^2} d\phi.$$

Thus the apparent emissivity may be expressed as the sum of an isotropic emissivity depending on ϵ_a and a term depending on b which accounts for the anisotropy of the long-wave flux.

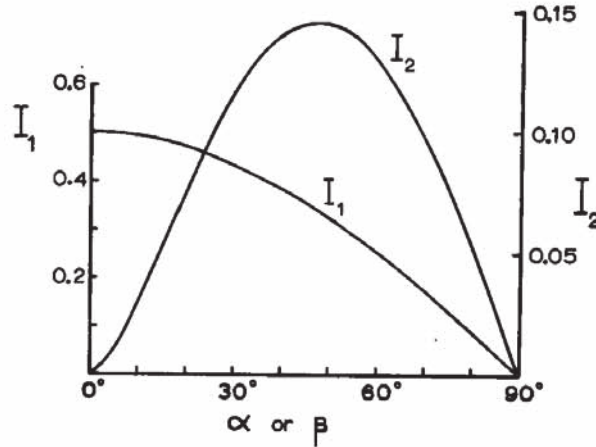


Figure 2. Dependence of integrals I_1 and I_2 on α or β .

The integral I_1 may be evaluated analytically and is $0.5 \cos \alpha$; and I_2 was evaluated numerically on a computer. Fig. 2 shows the variation of I_1 and I_2 with α . When $\alpha = 0$, $\epsilon(\alpha) = \epsilon_a$ and when $\alpha = 90^\circ$, $\epsilon(\alpha) = 0.5 \epsilon_a$ as expected.

Radiation also reaches the horizontal surface from the wall. If the wall has radiative temperature T_w and emissivity ϵ'_w , it can be shown that the emissivity of the wall ϵ_w , expressed as a fraction of σT_a^4 , is

$$\epsilon_w = 0.5 \epsilon'_w (1 - \cos \alpha) \frac{\sigma T_w^4}{\sigma T_a^4} \quad \cdot \quad \cdot \quad \cdot \quad (6)$$

The total long-wave irradiance of the horizontal surface is then

$$L_\alpha = \sigma T_a^4 (\epsilon(\alpha) + \epsilon_w(\alpha)) \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

(b) Radiation at the floor of a valley

Fig. 1(b) represents an infinitely long V-shaped valley with plane walls at an angle β to the horizon. The irradiance of an infinitesimal horizontal plane at the intersection of the walls is the sum of radiation from the atmosphere and radiation from the valley walls. The apparent emissivity of the atmosphere is given by

$$\epsilon(\beta) = 2 \int_{-\pi/2}^{\pi/2} d\phi \int_{\beta'}^{\pi/2} \frac{\epsilon(h)}{\pi} \sinh \cosh dh \quad \cdot \quad \cdot \quad (8)$$

where $\beta' = \tan^{-1} (\tan \beta \cos \phi)$

Substituting for $\epsilon(h)$, Eq. (8) reduces to

$$\epsilon(\beta) = 2\epsilon_a I_3 - b I_4 \quad \cdot \quad \cdot \quad \cdot \quad (9)$$

where
$$I_3 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{1 + \psi_2^2} d\phi; \quad I_4 = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \frac{\psi_2^2}{1 + \psi_2^2} \ln \left(\frac{1 + \psi_2^2}{\psi_2^2} \right) d\phi$$

and $\psi_2 = \tan\beta \cos\phi$.

Clearly the integrals I_3 and I_4 are identical to integrals I_1 and I_2 writing $\alpha = \beta$. Consequently Eq. (9) may be evaluated from Fig. 2 for any value of β . Fig. 2 shows that the apparent emissivity of the atmosphere decreases only slightly when β is less than 20° .

If the walls have temperature T_w and emissivity ϵ'_w , the emissivity of the walls ϵ_w expressed as a fraction of σT_a^4 is

$$\epsilon_w(\beta) = \epsilon'_w(1 - \cos\beta) \frac{\sigma T_w^4}{\sigma T_a^4} \quad \cdot \quad \cdot \quad \cdot \quad (10)$$

The total long-wave irradiance at the floor of the V-shaped valley is then

$$L_\beta = \sigma T_a^4(\epsilon(\beta) + \epsilon_w(\beta)) \quad \cdot \quad \cdot \quad \cdot \quad (11)$$

In a valley whose walls do not intersect, the irradiance of an infinitesimal horizontal area at P depends on the angles β_1 and β_2 subtended by the walls (Fig. 1(c)). By comparison with Eqs. (5)–(11), treating the radiation from each wall separately, the total irradiance at P is

$$L(\beta_1, \beta_2) = \sigma T_a^4 \left\{ \epsilon_a(I_1(\beta_1) + I_1(\beta_2)) - 0.5 b(I_2(\beta_1) + I_2(\beta_2)) + \epsilon'_w \frac{\sigma T_w^4}{\sigma T_a^4} (1 - 0.5(\cos\beta_1 + \cos\beta_2)) \right\} \quad (12)$$

which for the case $\beta_1 = \beta_2$ reduces to Eq. (11) and for the case $\beta_1 \neq 0, \beta_2 = 0$ reduces to Eq. (7). The mean irradiance of the valley floor depends on the ratio of valley width to valley depth and may be obtained for particular cases by integration of Eq. (12) across the width of the valley.

3. RADIATION ON SLOPING PLANES

Atmospheric radiation and radiation from the ground both contribute to the irradiance of a sloping plane. Fig. 1(d) shows the geometrical details. Atmospheric radiation falling on the upper surface of the plane OABC comes from the whole hemisphere excluding a region limited by the curve ABC. It may be shown from spherical co-ordinate geometry (Kondratyev 1969) that if $\epsilon(\gamma)$ is the apparent emissivity of the sky referred to a plane at γ to the horizontal, the appropriate integral is

$$\epsilon(\gamma) = 2 \int_0^\pi d\phi \int_{h_3}^{h_4} \frac{\epsilon(h)}{\pi} f(\gamma, h, \phi) dh \quad \cdot \quad \cdot \quad (13)$$

where $f = (\sin\gamma \cos\phi \cosh h + \cos\gamma \sinh h) \cosh h$ and the limits of the integral are

$$h_3 \begin{cases} = \cos^{-1} \frac{\cos\gamma}{\sqrt{1 - \sin^2\gamma \sin^2\phi}} & \gamma \leq \frac{\pi}{2} & 0 \leq \phi \leq \frac{\pi}{2} \\ = 0 & \gamma \leq \frac{\pi}{2} & \frac{\pi}{2} \leq \phi \leq \pi \\ = 0 & \gamma > \frac{\pi}{2} & 0 \leq \phi \leq \pi \end{cases}$$

The mean irradiance of the body is therefore

$$L = \frac{\sigma T_a^4 \int \epsilon(\gamma) dA_\gamma}{\int dA_\gamma}$$

and the apparent emissivity of the atmosphere to the body may be defined by

$$\epsilon = \frac{\int \epsilon(\gamma) dA_\gamma}{\int dA_\gamma} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (16)$$

(a) *Cylinder*

Consider a cylinder with a horizontal axis of length l and circular cross-section of radius r . Atmospheric radiation on the curved surface and on the plane ends can be represented by an emissivity ϵ_c , i.e.

$$\epsilon_c = 0.5\epsilon_a + \frac{0.19bM + 0.35b}{M + 1} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (17)$$

where $M = l/r$.

Priestley (1957) estimated the radiation falling on a sheep by considering it to be a horizontal cylinder. Neglecting the contribution of the ends, and making several approximations concerning the angular distribution of radiation, he estimated that the flux incident on the animal was $0.52L_a$ which corresponds to $\epsilon_c = 0.52\epsilon_a$. For an infinite cylinder (i.e. no contribution from the ends) Eq. (17) reduces to

$$\epsilon_c = 0.5\epsilon_a + 0.19b \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (18)$$

which, for a typical value $b = 0.090$ (Unsworth and Monteith 1975), gives $\epsilon_c = 0.5\epsilon_a + 0.017$, which is very close to Priestley's value over the range $0.6 < \epsilon_a < 1$. For an isotropic hemisphere ϵ_c would be $0.5\epsilon_a$.

(b) *Sphere*

Dividing the surface of a sphere into conic zones of constant γ and integrating over γ gives for the apparent atmospheric emissivity ϵ_s

$$\epsilon_s = 0.5\epsilon_a + 0.25b \quad (19)$$

(c) *Solid with vertical and horizontal surfaces*

For a solid with area A_v of vertical surface and A_h of horizontal surface exposed to the sky, e.g. a building, or a vertical cylinder or prism, the atmospheric emissivity ϵ_p is

$$\epsilon_p = 0.5\epsilon_a(1 + N) + 0.35b(1 - N) \quad (20)$$

where $N = \frac{A_h}{A_v + A_h}$

5. LINEARIZATION

Eqs. (5), (9), (14), (17), (19) and (20) show that the apparent emissivity of the sky can in general be expressed as

$$\epsilon_s = h\epsilon_a + g$$

where the functions h and g depend on the geometry of the situation. Similarly the apparent emissivity of the ground may be expressed by

$$\epsilon_g = j\epsilon'$$

and the total long-wave irradiance is given by

$$L_T = \sigma T_a^4 (\epsilon_s + \epsilon_g) = hL_d + (g + j\epsilon')\sigma T_a^4 \quad (21)$$

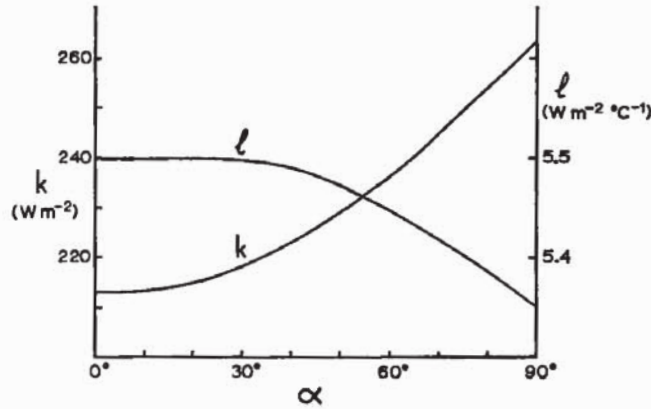


Figure 4. Dependence on α of the coefficients k and l in the equation $L_T = k + lT_a$.

Unsworth and Monteith (1975) showed that measurements of L_d from cloudless skies in Britain were fitted by the linear equation

$$L_d = 213 + 5.5T_a \quad (22)$$

where L_d was in $W m^{-2}$ and T_a was in $^{\circ}C$. The standard error of a value of L_d predicted from Eq. (22) was $\pm 30 W m^{-2}$. Over a small range of temperature the function σT_a^4 can also be adequately represented by a linear function of T_a , and over the range $-5 < T_a < 25^{\circ}C$ corresponding to the measurements of L_d in Britain the relation is

$$\sigma T_a^4 = 313 + 5.2T_a \quad (23)$$

The first order error in Eq. (23) is $6 W m^{-2}$ at the extremes of the temperature range. Eq. (21) may thus be written

$$L_T \simeq k + lT_a \quad (24)$$

Figs. 4, 5 and 6 show the variation of k and l with angle for radiation near walls, in V-shaped valleys and on slopes for the special but common case $T_g = T_a$, $\epsilon' = 1$. A mean value of

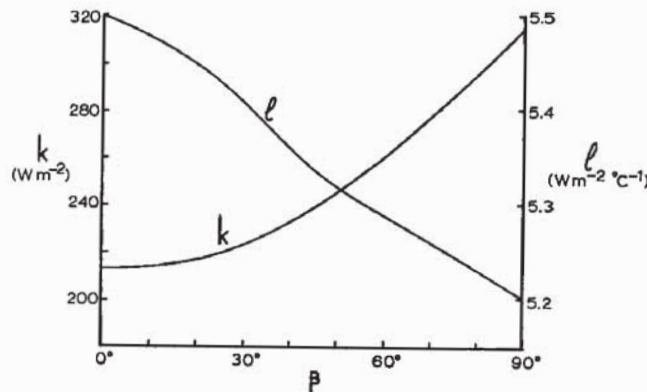


Figure 5. Dependence on β of the coefficients k and l in the equation $L_T = k + lT_a$.

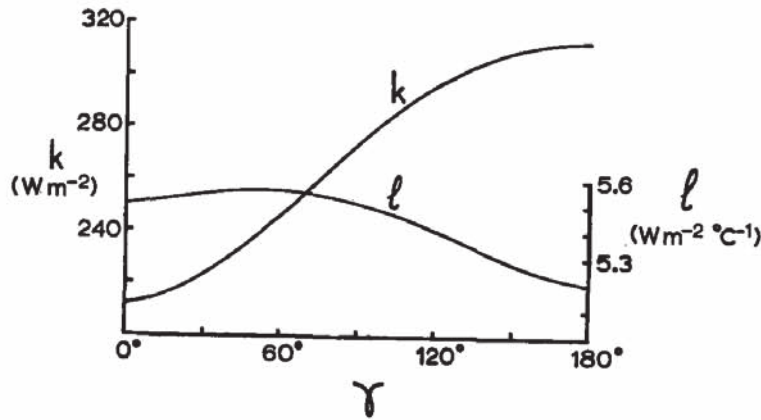


Figure 6. Dependence on γ of the coefficients k and l in the equation $L_T = k + lT_a$.

TABLE 1. VALUES OF THE CONSTANTS k AND l IN THE FORMULA $L_T = k + lT_a$

Shape	k (W m^{-2})	l ($\text{W m}^{-2} \text{ degC}^{-1}$)
Cylinder radius r , axis horizontal, length l $l/r = M$	$268 + \frac{5}{M+1}$	$5.43 + \frac{0.08}{M+1}$
Sphere	270	5.47
Prism standing on ground, vertical surface area A_v horizontal surface area A_h $\frac{A_h}{A_v + A_h} = N$	$273 - 60N$	$5.51 - 0.01N$

$b = 0.090$ was assumed (Unsworth and Monteith 1975). Table 1 shows the values of k and l for the same conditions for horizontal cylinders, spheres and prisms.

Values of L_T calculated from Eq. (24) agree to within $\pm 6 \text{ W m}^{-2}$ with values calculated by the longer method based on Eq. (1) using mean values $a = 0.70$; $b = 0.090$. Other linear equations with a similar accuracy are derived easily for situations where surface temperatures differ from air temperature.

6. COMPARISON WITH OTHER WORK

Previous studies of long-wave radiation geometry have been confined to the measurement and calculation of *net* long-wave radiation under cloudless skies assuming that surface temperature was equal to screen temperature (Linke 1931; Lauscher 1934). Lauscher tabulated the net long-wave radiation on sheltered or inclined surfaces as a fraction of the equivalent net long-wave radiation to the zenith $L_N(\pi/2)$. His calculations were based on the empirical relationship derived by Linke, that the variation of L_N with elevation angle h can be described by

$$L_N(h) = L_N(\pi/2)\sin^v h$$

where v is a constant which varies with vapour pressure (e). Lauscher's calculations assumed $v = 0.3$ which corresponds to $e = 7 \text{ mb}$.

TABLE 2. COMPARISON OF CALCULATED VALUES OF $L_N/L_N(\pi/2)$ WITH CALCULATIONS OF LAUSCHER (1934)

		L = Lauscher's values					U = Present calculations				
Angle		0	20	30	45	60	75	90	120	150	180
Slope	U	1.00	0.95	0.90	0.77	0.65	0.52	0.38	0.15	0.03	0.00
	L	1.00	0.95	0.90	0.80	0.67	0.53	0.40	0.17	0.03	0.00
Wall	U	1.00	0.98	0.95	0.87	0.76	0.63	0.50	—	—	—
	L	1.00	0.98	0.95	0.88	0.77	0.64	0.50	—	—	—

The general equations derived in this paper may be compared with Lauscher's work for two geometries: (a) radiation near a wall (section 2); (b) radiation on a sloping plane (section 3). For the comparisons, surface temperature and T_a were taken as 10°C , and $b = 0.090$. Table 2 shows that the agreement for both cases was good. Absolute values of net long-wave radiation may be immediately calculated from equations in this paper, but to obtain absolute values from Lauscher's methods requires the intermediate calculation or measurement of incoming long-wave radiation on a horizontal surface.

7. EFFECT OF CLOUD

Assuming that over periods of several weeks the distribution of cloud over the sky is uniform, Unsworth and Monteith (1975) showed that the mean apparent emissivity of the atmosphere at elevation h is

$$\varepsilon(h, c) = (1 - pc)\varepsilon(h, 0) + pc \quad (25)$$

where $\varepsilon(h, 0)$ is the apparent emissivity of cloudless sky and c is fractional cloudiness. Mean values of p for periods of three months based on a long series of measurements in Britain (Dines and Dines 1927) may be derived from Unsworth and Monteith (1975) and range from 0.81 to 0.87. The annual mean of p was 0.84.

Substituting for $\varepsilon(h, 0)$ from Eq. (1), Eq. (25) becomes

$$\varepsilon(h, c) = a' + b'\ln(ucosech) \quad (26)$$

where $a' = a + pc(1 - a)$ and $b' = b(1 - pc)$.

It follows that the mean apparent emissivity of a hemisphere of fractional cloudiness c is

$$\varepsilon_a(c) = a' + b'(0.5 + \ln u) \quad (27)$$

Consequently the methods used to derive the long-wave irradiance of slopes, solids, etc., when the sky is cloudless are also applicable for calculating long-term mean irradiances under cloudy skies.

8. CONCLUSIONS

Long-wave radiation from the sky received by sloping surfaces or by horizontal surfaces partly screened from the sky may conveniently be expressed as the sum of the radiant flux from an isotropic source of radiation and a flux which expresses the degree of anisotropy. The first component may be derived from measurements or estimates of the flux received by an unobstructed horizontal surface. The second component can be calculated from the measurable coefficient b , tabulated for a number of sites by Unsworth and Monteith (1975). Similar constants may be used for average conditions of cloud. Little error is intro-

duced by expressing the long-wave flux received by obstructed or sloping surfaces as a *linear* function of air temperature. Climatological measurements of temperature and cloudiness can therefore be used to calculate the receipt of long-wave radiation by sloping ground, by the walls and roofs of buildings, and by isolated objects such as trees or animals. Knowing the emissivity and average temperature of the surface receiving radiation, the net long-wave exchange may be readily calculated.

ACKNOWLEDGMENTS

I am grateful to Dr. K. Gregson for assistance in computing, to Professor J. L. Monteith for encouragement and advice in the preparation of this paper, and to a referee for helpful suggestions in the treatment of section 2(b).

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