Longevity and Environmental Quality in an OLG Model

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Abstract

1 Introduction

The relations between demography and environment are complex and multiple. The most famous is likely to be that illustrated by the tragedy of the commons: with fixed natural resources, free riding individuals tend to adopt a too high fertility rate. An extension of that idea if longevity is assumed to be endogenous is that individuals would tend also to live too long. Another well-known relation is that between pollution and mortality. There exists plenty of evidence that environmental degradation increases mortality and that this external effect is not internalized by market. What is interesting is that these two external effects can partially offset each other. Pollution hurts longevity, which is desirable if longevity decisions don't internalize their effect on environment. This is what we study in this paper.

2 The model

We consider an overlapping generations model with endogenous life longevity and unpriced pollution.

2.1 Firms and pollution

We assume the existence of a neo-classical production sector using a quantity K of capital and L of labor. We assume that capital fully depreciates during the process of production. At each time t, firms produce a good (Y_t) with a well-behaved production function,

$$Y_t = F(K_t, L_t) \tag{1}$$

Within the framework of a competitive equilibrium, each firm, at time t, chooses the quantity of capital and labor which will maximize its profit,

$$\pi_t = F(K_t, L_t) - R_t K_t - w_t L_t \tag{2}$$

At equilibrium, the levels of return from the inputs correspond respectively to their marginal productivity:

$$R_t = F_K(K_t, L_t) \tag{3}$$

$$w_t = F_L(K_t, L_t) \tag{4}$$

where R_t is the interest rate for savings and w_t the wage rate.
At each period, the flow of pollution emission is equal to a proportion η of current production,

$$E_t = \eta F(K_t, L_t) \tag{5}$$

The dynamics of the stock of pollution at time t, P_t, is defined by

$$P_t = (1 - \delta)P_{t-1} + E_t \tag{6}$$

where δ the natural level of pollution absorption, $0 \leq \delta \leq 1$.

2.2 Agents

We suppose that the population is constant, and that at each time t, N identical agents are born. Each agent lives through two life periods. He or she works during the first period of life and is a pensioner for the second period of life.
The first period is of unitary length; the second period lasts a period h, inferior to 1, that can be increased through health spending (like primary prevention in first period), x_t, in the second period and decreased by industrial pollution, P_{t+1} (see Evans and Smith, JEEM 2005). The longevity function, $h(x_t, P_{t+1})$, is strictly concave with $h_x > 0$, $h_{xx} < 0$, $h_P < 0$, $h_{PP} > 0$.
Any agent born in period t derives utility from consumption, c_t and the amount of space or land per person, q_t, in her/his first period of life and from consumption, d_{t+1}, and the amount of space or land per person, q_{t+1}, in her/his second period of life.

The preferences of the agents are represented by a function of utility, $U(c_t, q_t)$ $+h(x_t, P_{t+1})U(d_{t+1}, q_{t+1})$ where $q_t = \overline{Q}/N(1 + h(x_{t-1}, P_t))$ with $\overline{Q}$ the given total quantity of space. $U(.)$ is supposed to be strictly concave with $U_i > 0$ (for $i = c, d$), $U_q > 0$ and the cross derivative is assumed to be non negative, $U_{iq} \geq 0$[1].

During the first period of life, each agent supplies one inelastic unit of labor and receives the wage, w_t, which he or she consumes, c_t and saves s_t in the form of capital and spend x_t for health.
Let us introduce a tax ξ on health spending and a tax τ in capital income, along with a lump-sum subsidy T. The budget constraint in the first period of life is:

$$w_t + T = c_t + s_t + (1 + \xi)x_t \tag{7}$$

In the second period of life, the agents receive the return of their saving, $R_{t+1}s_t$ and consume d_{t+1} during $h(x_t, P_{t+1})$. The budget constraint in the second period of life of an agent born in t is therefore:

$$h(x_t, P_{t+1})d_{t+1} = (R_{t+1} - \tau)\, s_t. \tag{8}$$

The problem of each individual is thus to maximize:

$$U(c_t, q_t) + h(x_t, P_{t+1})U(d_{t+1}, q_{t+1})$$

subject to (7) and (8) by choosing x_t and s_t. What is crucial is that in choosing x_t and s_t individuals don't perceive the effect of their decisions on the environmental variables: pollution and space. Since the individual does not see the effect on savings on pollution, nor the effect of health spending on total population $N(1 + h(x_t, P_{t+1}))$. One obtains thereby the first order conditions for savings

$$-U_{c_t} + (R_{t+1} - \tau)\, U_{d_{t+1}} = 0, \tag{9}$$

and for health spending

$$-(1 + \xi)U_{c_t} + h_{x_t}U(d_{t+1}, q_{t+1}) - d_{t+1}h_{x_t}U_{d_{t+1}} = 0 \tag{10}$$

[1]This assumption converts a complementary effect of space and consumption. An alternative assumption is that space and consumption are substitutable, a negative crossed derivative. For an in-depth discussion of these assumptions we refer the reader to Michel and Rotillon (1995).

3 Equilibrium and steady state

3.1 Intertemporal equilibrium

The intertemporal equilibrium is defined, by a sequence of prices, individual variables and aggregate variables satisfying all the equilibrium conditions. Firms maximizes their profit (conditions (19) and (20) hold) and consumers their utility (conditions (9) and (10) hold).
The capital stock is equal to savings,

$$K_t = Ns_{t-1} \tag{11}$$

The market of labor clears,

$$L_t = N, \tag{12}$$

as well the market of goods,

$$Y_t = F(K_t, N) = Nc_t + Nx_t + Nh(x_t, P_{t+1})d_t + K_{t+1} \tag{13}$$

In addition, the dynamic equation of pollution holds.

$$P_t = (1-\delta)P_{t-1} + \eta F(K_t, N) \tag{14}$$

At time 0, consumption of the retirees satisfies:

$$h(x_{-1}, P_1)d_0 = R_0 s_{-1}. \tag{15}$$

Further we take for given the initial capital stock $K_0 = Ns_{-1}$, the pollution stock P_{-1} and health expenditure x_{-1}.

3.2 Stationary equilibrium

At the steady state with a given policy, the stock of capital is given by the sum of saving, $K = Ns$ and the economy's resource constraint per young is,

$$f(k) = c + x + h(x, P)d + k \tag{16}$$

where $k = K/N$ is the capital per young and $f(k) = F(k, 1)$. Long-run equilibrium pollution is defined by,

$$P = \frac{\eta}{\delta} N f(k) \tag{17}$$

The amount of space or land per person is defined by,

$$q = \frac{\overline{Q}}{N\left[1 + h(x, P)\right]} \tag{18}$$

Using relation (2), the interest factor (with tax) and the wage rate are,

$$R = f'(k) \tag{19}$$

and

$$w = f(k) - kf'(k) \tag{20}$$

The optimal consumers' decision are given by,

$$-U_c + (R - \tau)\, U_d = 0 \tag{21}$$

and for health spending

$$-(1+\xi)U_c + h_x U(d, q) - dh_x U_d = 0 \tag{22}$$

4 Social Optimum and optimal policy

We now turn to the analysis of the social optimum and optimal policy. At the long-run equilibrium, we are looking for the maximum possible utility in the economy.

4.1 Social optimum

In a centralized economy, the objective of the central planner is to maximize the welfare of the agents by choosing the level of consumptions (c, d), health spending (x) and the level of capital (k), under constraints of resources, pollution and space per person.

$$\max_{c,d,x,k} U(c, q) + h(x, P)U(d, q)$$

$$s.t.: \begin{cases} f(k) = c + x + h(x, P)d + k \\ P = \frac{\eta}{\delta} N f(k) \\ q = \frac{\overline{Q}}{N[1+h(x,P)]} \end{cases}$$

Denoting by λ the Lagrangian multiplier of resource constraint (16), the Lagrangian $\mathcal{L}$ is defined by

$$\begin{array}{c} \mathcal{L}(c, d, x, k) = U\left(c, \frac{\overline{Q}}{N\left[1+h(x,\frac{\eta}{\delta}Nf(k))\right]}\right) \\ +h\left(x, \frac{\eta}{\delta}Nf(k)\right) U\left(d, \frac{\overline{Q}}{N\left[1+h(x,\frac{\eta}{\delta}Nf(k))\right]}\right) \\ +\lambda\left[f(k) - c - x - h(x, \frac{\eta}{\delta}Nf(k))d - k\right] \end{array} \tag{23}$$

One obtains thereby the first order conditions,

$$U_c = \lambda \tag{24}$$

$$U_d = \lambda \tag{25}$$

$$h_x U(d,q) - \frac{N\overline{Q}h_x}{\left(N\left[1+h(x,P)\right]\right)^2}\left(U_q(c,q)+h(x,P)U_q(d,q)\right) = \lambda(1+h_x d) \tag{26}$$

and

$$\begin{aligned} & h_P \frac{\eta}{\delta} N f'(k) U(d,q) - \frac{N\overline{Q}h_P \frac{\eta}{\delta} N f'(k)}{(N\left[1+h(x,P)\right])^2}(U_q(c,q)+h(x,P)U_q(d,q)) \\ = \; & \lambda(1+h_P \frac{\eta}{\delta} N f'(k) d - f'(k)) \end{aligned} \tag{27}$$

From (24) and (25) we obtain,

$$U_c = U_d \tag{28}$$

Using (28), (26) and (27), we have

$$\begin{aligned} U_c\left(c,q\right)\left(1+h_x(x,P)d\right) &= h_x U(d,q) - h_x \frac{\overline{Q}}{N\left[1+h(x,P)\right]^2}(U_q(c,q) \\ & \quad +h(x,P)U_q(d,q)) \end{aligned} \tag{29}$$

and

$$\begin{aligned} f'\left(k\right) &= 1 + f'\left(k\right) h_p \frac{\eta}{\delta} N \left[\frac{\overline{Q}}{N(1+h)^2}\left(U_q\left(c,q\right)+hU_q\left(d,q\right)\right)\right. \\ & \quad \left. - \left(\frac{u(d,q)}{u_d} - d\right)\right]. \end{aligned} \tag{30}$$

Note that without the environmental variables, these two optimal conditions would reduce to:

$$U_c\left(c,q\right) = h_x\left[\frac{U(d,q)-d}{U_q(d,q)}\right]$$

and

$$f'\left(k\right) = 1.$$

The first equation is equivalent to (22) with $\zeta = 0$. As to the second, it is the Golden rule (population growth is here 0).

With the environmental variable, we have some externalities. Starting with (29) associated with health spending, the externality comes from the fact that individuals don't internalize in their decisions the effect of increased longevity on the number of inhabitants on a finite earth.

The choice of investment (eq. (30)) entails two externalities. Keeping in mind that investment, production and pollution are closely related, more capital means more pollution and thus less population, which is good for the capital quality of the environment. The second externality is negative: more pollution means lower longevity and thus lower utility in the second period of live. This is the key idea of this paper.

4.2 Optimal policy

Contrasting the market solution with the first-best optimum, one sees that the social optimum can be decentralized with appropriate choices of "Pigouvian taxes" ξ and τ for the environmental damage:

$$\xi = \frac{h_x \overline{Q}}{N\left[1+h(x,P)\right]^2} \frac{(U_q(c,q)+h(x,P)U_q(d,q))}{U_c} > 0 \tag{31}$$

and for saving

$$\begin{aligned}\tau = f'(k)\,h_P - \frac{\eta}{\delta}N &\left[\frac{\overline{Q}}{N(1+h)^2}\left(U_q(c,q)+hU_q(d,q)\right)\right. \\ &\left. - \left(\frac{u(d,q)}{u_d} - d\right)\right] \lesseqgtr 0.\end{aligned} \tag{32}$$

To these two taxes one should add an intergenerational transfer device that leads to the golden rule: $R = 1$.

5 Second-best policy

Notation:

ξ : tax on health spending
a : lump-sum transfer in the first period
$n = 0$: zero population growth.

Resource constraint:

$$\begin{aligned} f(k) &= c + x + hd + k \\ f'(k)\,k + w &= f'(k)\,k + c + x(1+\xi) + s - a = c + x + hd + k. \end{aligned}$$

We know that: $s = k$ and $sR = hd$. Hence

$$(f'(s) - R)\,s + \xi x = a.$$

In other words, the resource constraint and the revenue constraint imply each other. Either one is sufficient to express the Lagrangean expression. We will use the second one here below.[2]

$$\begin{aligned}
\mathcal{L}(R,\tau,\xi,a) &= u\,(c,q) + h\left(x,\frac{\eta}{\delta}f\,(s)\right)u\,(d,q) \\
&\quad +\gamma\,[\tau s + \xi x - a] \\
&\quad -\mu\left[q - \frac{\bar{Q}}{1+h\left(x,\frac{\eta}{\delta}f(s)\right)}\right].
\end{aligned}$$

We maximize $\mathcal{L}$ with respect to τ,ξ and a, our tax parameters and with respect to q, an adjustment variable.

The FOC's are given by:

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial a} = u_c + \gamma\left[\tau\frac{\partial s}{\partial\tau} + \xi\frac{\partial x}{\partial\tau}\right] - \mu\frac{\bar{Q}}{(1+h)^2}\left[h_x\frac{\partial x}{\partial\tau} + h_q\frac{\eta}{\delta}f'\,(s)\,\frac{\partial s}{\partial a}\right] \\
+hp\frac{\eta}{\delta}f'\,(s)\,\frac{\partial s}{\partial a}u\,(d,q) = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\tau} = -u_c\frac{s}{R} + \gamma\left[\tau\frac{\partial s}{\partial\tau} + \xi\frac{\partial x}{\partial\tau} + s\right] - \mu\frac{\bar{Q}}{(1+h)^2}\left[h_x\frac{\partial x}{\partial\tau} + h_p\frac{\eta}{\delta}f'\,(s)\,\frac{\partial s}{\partial\tau}\right] \\
+h_p\frac{\eta}{\delta}f'\,(s)\,\frac{\partial s}{\partial\tau}u\,(d,q) = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial\tau} = -u_c + \gamma\left[\frac{\partial s}{\partial\xi} + \xi\frac{\partial x}{\partial\xi} + x\right] - \frac{\mu\bar{Q}}{(1+h)^2}\left[h_x\frac{\partial x}{\partial\xi} + h_p\frac{\eta}{\delta}f'\,(s)\,\frac{\partial s}{\partial\xi}\right] \\
+u_q\,(c,q) + hu_q\,(d,q) - \mu = 0 \qquad (33)
\end{aligned}$$

As usual with this type of problem, we express the tax formula in compensated terms. In other words, an increase in either τ or ξ is compensated by an increase in a and we want to know the effect of such compensated increase on social welfare. We use the superscript c to denote the compensated effects. The multiplier μ in (33) expresses the social value of environmental quality.

[2] Note that with the instruments τ,ζ and a we cannot reach the first-best. To do so we need some type of intergenerational redistribution device that leads to the golden rule.

Also note that for the consumer R is considered as given.

$$
\begin{aligned}
\frac{\partial \mathcal{L}^c}{\partial \tau} &= \gamma \overset{[1]}{\left[\tau \frac{\partial s^c}{\partial \tau} + \xi \frac{\partial x^c}{\partial \tau}\right]} + \overset{[2]}{\gamma \frac{s}{R}\left(R-1\right)} - \overset{[3]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2} h_x \frac{\partial x^c}{\partial \tau}} \\
&\quad - \left[\overset{[4]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2}} \overset{[5]}{- u\left(d,q\right)}\right] h_q \frac{\eta}{\delta} f'\left(s\right) \frac{\partial s^c}{\partial \tau} = 0
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}^c}{\partial \xi} &= \gamma \overset{[1]}{\left[\tau \frac{\partial s_c}{\partial \xi} + \xi \frac{\partial x^c}{\partial \xi}\right]} - \overset{[3]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2} h_x \frac{\partial x^c}{\partial \tau}} \\
&\quad - \left[\overset{[4]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2}} \overset{[5]}{- u\left(d,q\right)}\right] h_p \frac{\eta}{\delta} f'\left(s\right) \frac{\partial s^c}{\partial \xi} = 0
\end{aligned}
\tag{35}
$$

We distinguish among five terms in those two conditions:

1. the traditional Ramsey formula,
2. the gap between the rate of interest and the population growth rate,
3. the crowding effect induced by longevity-enhancing investment,
4. the decrowding effect induced by pollution.
5. the utility loss arousing from shorter lifetime.

To get further insight, we make a quite extreme assumption: the cross-derivatives are negligible. In other words, τ has no influence over x and ξ over s. Then we have:

$$
\begin{aligned}
\frac{\partial \mathcal{L}^c}{\partial \tau} &= \overset{[1]}{\gamma \tau \frac{\partial s^c}{\partial \tau}} + \overset{[2]}{\gamma \frac{s}{R}\left(R-1\right)} - [\overset{[4]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2}} - \overset{[5]}{u(d,q)]h_q \frac{\eta}{\delta} f'\left(s\right) \frac{\partial s^c}{\partial \tau}} = 0 \\
\frac{\partial \mathcal{L}^c}{\partial \xi} &= \overset{[1]}{\gamma \zeta \tau \frac{\partial s^c}{\partial \xi}} - \overset{[3]}{\mu \frac{\bar{Q}}{\left(1+h\right)^2} h_x \frac{\partial x^c}{\partial \xi}}\,.
\end{aligned}
$$

Starting with τ, we have two positive terms: [2] and [5] and two negative ones [1] and [4]. The terms [4] and [2] are standard. The term [5] reflects the fact that by increasing τ, there is less capital accumulation and thus less pollution. This leads to an increased longevity and thus to an increased utility in period [2]. The term [4] shows also that a tax on saving has a positive effect on longevity but longevity has a crowding effect on the fixed environmental quality $\bar{Q}$.

Turning to ξ, we have a negative effect [1] and a positive one [3]. The first is standard; the second says that by taxing health care, people don't live as long as without such tax and this has a relief effect on the fixed quality of environment.

Admittedly, the result obtained both in the first and the second best, that health care ought to be taxed is a bit surprising and has to be qualified. Subsidizing health care that is often recommended arises from other considerations: redistribution, externality,... The negative effect underlined in this paper is likely to be dominated by these other considerations.

The two external effects of production and pollution are quite interesting. On prior grounds, one cannot say whether saving ought to be taxed or subsidized.

6 A numerical application

Let us now consider the implications of the present model for public policy in the light of a numerical example. For that purpose, we shall first introduce the functional forms postulated for production, longevity and preferences. Then, for those forms, we shall solve the agents' optimization problem, and characterize intertemporal equilibria and steady-states. That short exercise allows us to discuss geometrically the issues of existence and (local) stability of steady-states. We are then able to explore, with several numerical examples, the dynamics of capital accumulation, pollution and longevity in the economy under study, first under *laissez-faire* and, then, under public intervention.

6.1 Functional forms

6.1.1 Production

Throughout this section, we shall assume that production takes a Cobb-Douglas form:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{36}$$

with $0 < \alpha < 1, A > 0$.

The labour force is supposed to be constant over time (i.e. $L_t = N$). Output can be rewritten in per worker terms as follows:

$$y_t = Ak_t^{\alpha} \tag{37}$$

Moreover, it is assumed that perfect competition prevails, so that factors are paid at their marginal productivities:

$$\begin{aligned} R_t &= A\alpha k_t^{\alpha-1} && (38) \\ w_t &= A(1-\alpha)k_t^{\alpha} && (39) \end{aligned}$$

6.1.2 Longevity

In the general model, the length of the second period of life h_{t+1} depends on the health expenditures when being young x_t, as well as on the aggregate pollution when being old, that is, P_{t+1}.

In the rest of this section, we shall keep those two major determinants of longevity - health expenditures and pollution - but we shall depart slightly from the assumed timing regarding pollution, and suppose that longevity $1 + h_{t+1}$ depends on pollution through the stock of pollution prevailing when individuals are *young*, and not when they are old.

The reason why we make this slight change is to capture the *intergenerational* dimension of pollution: individuals born at time t, even though they can be perfectly rational and perfectly informed, can only choose their length of life within a range allowed by previous generations, because some determinants of longevity - here the social component P_t - result from past decisions on which they can have no influence.

One should notice that this slight change does not modify the normative conclusions drawn in the previous section, because these conclusions concerned the steady-state, where, by definition, P_{t+1} and P_t are equal.

Moreover, for the conveniency of the presentation, we shall express, within the longevity production function, the stock of pollution in terms of pollution per worker, denoted by p_{t+1}, defined as P_{t+1}/N. Given that each cohort is of constant size over time, that notational change has no consequence.

Regarding the functional form for h_{t+1}, we shall here suppose that longevity is affected by health spendings x_t and by pollution flow p_t as follows:

$$h_{t+1} = \frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(1 - \frac{1}{1+\phi x_t}\right) \tag{40}$$

where $0 < \lambda < 1$, while the parameters ϕ and ψ are supposed to be positive.

The above functional form exhibits some interesting properties.[3] If there is no pollution and no health expenditures, h_{t+1} is equal to λ, which can be regarded as the 'natural' longevity level. In the case where health expenditures tend to infinity, h_{t+1} is equal to $\frac{1}{1+\psi p_t}$, which is equal to 1 (i.e. limit-longevity) if there is no pollution, but tends to 0 if there is infinite pollution (in that case even infinite health expenditures cannot help).

6.1.3 Preferences

Individual utility is supposed to be additive over time, and, for each period, logarithmic in consumption and in the available quantity of space:[4]

$$u_t = \theta \ln (c_t) + (1-\theta) \ln (q_t) + \beta h_{t+1} [\theta \ln (d_{t+1}) + (1-\theta) \ln (q_{t+1})] \quad (41)$$

where β is a discount factor, whereas θ, which lies between 0 and 1, reflects the relative importance of consumption and environmental quality. That preference parameter is supposed to be constant across periods, which is a non-negligible simplification, as old individuals may be more or less sensitive to environmental quality than young individuals.

Another non-negligible assumption is that pollution does not enter individual utility directly, but, only indirectly, through its influence on longevity h_{t+1}.

6.2 Equilibrium

6.2.1 Optimal savings and health expenditures

Let us now turn to individual decisions. Individuals are supposed to choose their savings s_t and their health expenditures x_t in such a way as to maximize their lifetime welfare.[5]

It is supposed that individuals, when making their decisions, anticipate the impact of their health expenditures on their longevity, *except* in so far as the available space q_{t+1} is concerned.

[3] That functional form satisfies the properties mentioned in Section 2: $h_x > 0$, $h_{xx} < 0$, $h_p < 0$ and $h_{pp} > 0$.

[4] Under that functional form, the cross derivative U_{cq} is equal to zero.

[5] One should notice that, given that the present model is a model of partial overlapping (the second period being of length $0 \leq h \leq 1$), one must here postulate some device allowing the payment of the return on saving while the old are still alive. For instance, one can assume that the old are paid in advance, and can consume their entire revenue from the production process (which lasts a period of unitary length as usual) before dying.

While there exists several ways to model such a behaviour, we shall make the simplifying postulate that, as far as future available space per head is concerned, individuals suppose that h_{t+1} is merely equal to the natural longevity level λ.

Thus, if one substitutes for consumption when being young and old, individuals' problem consists of choosing s_t and x_t in such a way as to maximize:

$$\begin{aligned} u_t &= \theta \ln (w_t - s_t - x_t(1+\xi) + T) + (1-\theta) \ln \left(\frac{\bar{Q}}{N(1+h_t)} \right) \\ &\quad + \beta h_{t+1} \left[\theta \ln \left(\frac{(R_{t+1} - \tau) s_t}{h_{t+1}} \right) + (1-\theta) \ln \left(\frac{\bar{Q}}{N(1+\lambda)} \right) \right] \end{aligned} \tag{42}$$

where ξ is the tax rate on health spendings, τ is a tax rate on savings, while T is a lump-sum subsidy.

Optimization yields:

$$s_t = \frac{\beta h_{t+1}}{1+\beta h_{t+1}} (w_t - x_t(1+\xi) + T) \tag{43}$$

$$\begin{aligned} \frac{\theta(1+\xi)}{w_t - s_t - x_t(1+\xi) + T} &= \beta \left(\frac{(1-\lambda)\phi}{(1+\psi p_t)(1+\phi x_t)^2} \right) \\ &\quad \left(\theta \ln \left(\frac{(R_{t+1} - \tau) s_t}{h_{t+1}} \right) + (1-\theta) \ln \left(\frac{\bar{Q}}{N(1+\lambda)} \right) - 1 \right) \end{aligned} \tag{44}$$

Combining those expressions yields:

$$\begin{aligned} \frac{\theta(1+\xi)(1+\beta h_{t+1})}{w_t - x_t(1+\xi) + T} &= \beta \left(\frac{(1-\lambda)\phi}{(1+\psi P_t)(1+\phi x_t)^2} \right) \\ &\quad \left(\theta \ln \left(\frac{(R_{t+1} - \tau) s_t}{h_{t+1}} \right) + (1-\theta) \ln \left(\frac{\bar{Q}}{N(1+\lambda)} \right) - 1 \right) \end{aligned} \tag{45}$$

While that expression does not allow us to express savings s_t and health expenditures x_t independently from each other, it is nonetheless possible to use the above expression in order to characterize all possible combinations (x_t, p_t, k_t) that can prevail in the economy under study.

Actually, given that longevity h_{t+1} is a function of $h(x_t, p_t)$, whereas the wage w_t is a function of capital, and savings is a function $s(x_t, p_t, k_t)$, it follows that the above expression can be rewritten as:

$$\left(\frac{\theta(1+\xi)\left[1+\beta h(x_t,p_t)\right]}{A(1-\alpha)k_t^{\alpha}-x_t(1+\xi)+T}\right)\left(\frac{(1+\psi p_t)(1+\phi x_t)^2}{\beta(1-\lambda)\phi}\right)$$
$$= \theta\ln\left(\frac{A\alpha\left(s\left(x_t,p_t,k_t\right)\right)^{\alpha}-\tau\left(s\left(x_t,p_t,k_t\right)\right)}{h(x_t,p_t)}\right)+(1-\theta)\ln\left(\frac{\bar{Q}}{N(1+\lambda)}\right) \quad (46)$$

That expression defines a plane in the three dimensional space (x_t, p_t, k_t), which provides us the level of health expenditures x_t that is optimal for the individual *given* the pollution and the capital that are currently prevailing at time t. If individuals choose their savings and health expenditures in such a way as to maximize their lifetime welfare in the postulated way, it is certain that the economy must always lie on that plane.

Figure 1a below provides an numerical example of such planes in the three dimensional space (x_t, p_t, k_t).[6] Clearly, the x-plane gives us, for each combination of capital k_t and pollution p_t, the level of health expenditures x_t chosen by individuals (i.e. satisfying the above first-order conditions). As suggested by the shape of the x-plane, x_t is, for a given pollution level, increasing with k_t. Moreover, for a given capital level, x_t is here (slightly) decreasing with p_t. Naturally, the precise shape of the x-plane depends on various parameters. If, for instance, a larger impatience is assumed (i.e. a lower discount factor β), then, as shown by Figure 1b, the level of health expenditures x_t will now tend to be lower *ceteris paribus*, which is not surprising, given that health expenditures concern the length of the second period.[7]

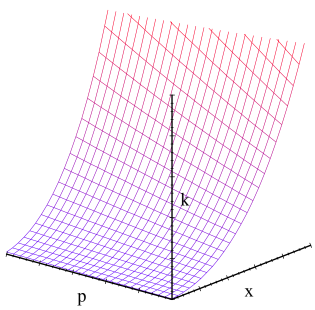


Figure 1a: x-plane

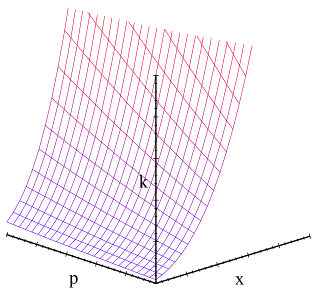


Figure 1b: x-plane under a lower β

[6] Figure 1a is based on the following parameters: $A = 20$, $\alpha = 0.5$, $\beta = 0.5$, $\theta = 0.5$, $\lambda = 0.2$, $\phi = 0.2$, $\psi = 0.1$, and $\bar{Q}/N = 10000$ (based on a population density of 100 persons per square-kilometer, i.e., per 10^6 square-meters). Public policy parameters T, ξ and τ are fixed to 0.

[7] Figure 1b is based on the same parameters as Figure 1a, except that β is here equal to 0.20 instead of 0.50 as in Figure 1a.

The shape of the x-plane is crucial, because, given the postulated preferences, the economy must always lie on that plane and nowhere else in the (x_t, p_t, k_t) space.

6.2.2 Intertemporal equilibrium

Given variables from the previous period $\{s_{t-1}, x_{t-1}, p_{t-1}\}$, and constants $\left\{N, \bar{Q}, \xi, T, \tau\right\}$, an intertemporal equilibrium can now be characterized as variables $\{k_t, h_t, L_t, y_t, w_t, R_t, s_t, x_t, p_t\}$ satisfying the following properties:

$$
\begin{aligned}
k_t &= s_{t-1} && (47)\\
h_t &= \frac{\lambda}{1+\psi p_{t-1}} + \left(\frac{1-\lambda}{1+\psi p_{t-1}}\right)\left(1 - \frac{1}{1+\phi x_{t-1}}\right) && (48)\\
L_t &= N && (49)\\
y_t &= A k_t^{\alpha} && (50)\\
w_t &= A(1-\alpha) k_t^{\alpha} && (51)\\
R_t &= A\alpha k_t^{\alpha-1} && (52)\\
s_t &= \frac{\beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(\frac{\phi x_t}{1+\phi x_t}\right)\right)}{1+\beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(\frac{\phi x_t}{1+\phi x_t}\right)\right)}\left(w_t - x_t(1+\xi) + T\right) && (53)\\
\frac{\theta(1+\xi)}{w_t - s_t - x_t(1+\xi) + T} &= \beta\left(\frac{(1-\lambda)\phi}{(1+\psi p_t)(1+\phi x_t)^2}\right) \\
&\quad \left(\theta \ln\left(\frac{(\alpha A s_t^{\alpha-1} - \tau)s_t}{h_{t+1}}\right) + (1-\theta)\ln\left(\frac{\bar{Q}}{N(1+h_t)}\right) - \sigma\right) && (54)\\
p_t &= (1-\delta)p_{t-1} + \eta A k_t^{\alpha} && (55)\\
q_t &= \frac{\bar{Q}}{N(1+h_t)} && (56)\\
N\tau s_{t-1} + \xi N x_t &= NT && (57)
\end{aligned}
$$

6.2.3 Steady-states

Let us now focus on the steady-states of the economy under study. Given that, for particular levels of capital and pollution, there exists a *unique* level of health expenditures (represented by the x-plane), it follows that, if both capital and pollution are constant, it must be the case that health expenditures are constant, and, hence, that longevity must also be constant. Therefore, discussions on the existence of a steady-state can concentrate exclusively on the constancy of capital and pollution.

For that purpose, we shall first present, in the 3-dimensional space (x_t, p_t, k_t), the kk locus and the pp locus, along which capital per worker and pollution per worker are constant, and, then, discuss the possibility of their intersection on the x-plane.

The kk locus Given that $k_{t+1} = s_t$, it follows that:

$$k_{t+1} = \frac{\beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(1 - \frac{1}{1+\phi x_t}\right)\right)}{1 + \beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(1 - \frac{1}{1+\phi x_t}\right)\right)} \left(A(1-\alpha)k_t^{\alpha} - x_t(1+\xi) + T\right) \tag{58}$$

Given that this expression provides k_{t+1} as a function of k_t, p_t and x_t, it is, in principle, possible, by assuming $k_{t+1} = k_t$, to derive the kk locus, defined as the set of combinations (x_t, p_t, k_t) such that k_t is constant. Unfortunately, there is no possibility to express that kk locus analytically, but, only a possibility to represent it geometrically for particular values of parameters.

Such a possibility is illustrated by Figure 2.[8] It is important to notice here that, in Figure 2, there exists, for each combination of x_t and p_t, not one, but two capital levels at which capital is constant for those values of x_t and p_t. However, given that the x-plane will only intersect the inferior kk locus at $k = 0$, we should not be too much worried about the inferior kk locus, and focus on the superior one only.[9]

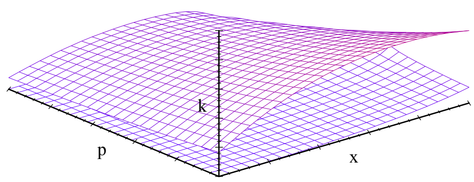


Figure 2: The kk locus

[8] Figure 2 is based on the following parameters: $A = 20$, $\alpha = 0.5$, $\beta = 0.5$, $\theta = 0.5$, $\lambda = 0.2$, $\phi = 0.2$, $\psi = 0.1$, and $\bar{Q}/N = 10000$. Public policy parameters T, ξ and τ are fixed to 0.

[9] Actually, the inferior locus suggests only that, if capital is equal to zero, then capital will remain at that level, whatever the levels of pollution and health spendings are. That piece of information is not really central, so that we can concentrate on the superior locus.

It is crucial to notice that, if the economy lies below the (superior) kk locus, capital will tend to grow, whereas, if the economy lies above the (superior) kk locus, capital will tend to fall. Thus, vertical arrows are oriented upwards below the (superior) kk locus, and oriented downwards above the (superior) kk locus.

The pp locus Regarding pollution, substituting, within the pollution equation, for emissions $e_{t+1} = \eta A k_{t+1}^{\alpha}$ and converting it as a function of p_t, k_t and x_t yields:

$$
\begin{aligned}
p_{t+1} &= (1-\delta)p_t + e_{t+1} \\
&= (1-\delta)p_t + \eta A \left(\frac{\beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(\frac{\phi x_t}{1+\phi x_t}\right)\right)\left(A(1-\alpha)k_t^{\alpha} - x_t(1+\xi) + T\right)}{1+\beta\left(\frac{\lambda}{1+\psi p_t} + \left(\frac{1-\lambda}{1+\psi p_t}\right)\left(1-\frac{1}{1+\phi x_t}\right)\right)} \right)^{\alpha} \quad (59)
\end{aligned}
$$

Given that this expression defines p_{t+1} as a function of k_t, p_t and x_t, it is, in principle, possible, by assuming $p_{t+1} = p_t$, to derive the pp locus, defined as the set of combinations (x_t, p_t, k_t) such that p_t is constant. Unfortunately, there is no possibility to express that pp locus analytically, but, there exists, here again, a possibility to represent it geometrically for particular values of parameters. Figure 3 shows the pp locus for the values of the parameters used in Figures 1a and 2, and for δ equal to 0.9 and η equal to 0.1.

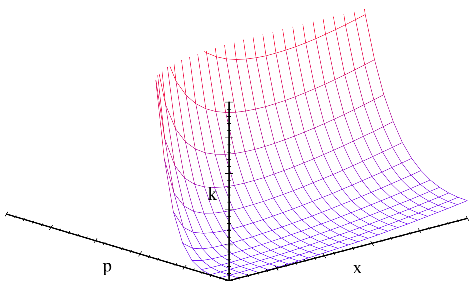


Figure 3: The pp locus

One should notice here that, if the economy lies on the right of the pp locus, it follows that pollution will grow, whereas, if the economy is on the left of the pp locus, pollution will tend to fall. It is only on the pp locus that pollution is, for given values of x_t and k_t, constant. For instance, Figure 3 suggests that, if initial pollution is zero and initial health expenditure is zero, only a zero capital level can maintain pollution at such a level; otherwise, any strictly positive capital level will tend to make pollution grow.

Steady-state: existence, uniqueness and stability Having introduced the x-plane, as well as the kk and pp loci, we can now consider the issue of the existence of a non-trivial steady-state.[10] Given that the loci cannot be expressed analytically, we shall here confine ourselves to discuss informally the existence, uniqueness and stability of a non-trivial steady-state.

For that purpose, it is crucial to remind first that the economy must always lie on the x-plane. Thus, the existence of a steady-state depends here on whether the kk locus and the pp locus intersect at a point that belongs to the x-plane. Actually, at such a point, both capital and pollution will be constant, so that, given the resulting constancy of health expenditures, longevity must also be constant.

Unfortunately, given that the present model only admits geometrical representation, there is no possibility to derive general conditions under which such a non-trivial steady-state would necessarily exist. There seem to be no obvious reason why such a steady-state would always exist, as it might be the case that the two loci do not intersect on the x-plane. Hence, while Figure 4 below presents a case where such a non-trivial steady-state exists, the existence of such an equilibrium in general cannot be taken for granted.[11] Similarly, little can be said as far as the uniqueness of such a steady-state is concerned.

[10] Given that $(0, 0, 0)$, which is on the x-plane, lies at the intersection of the (inferior) kk locus and the pp locus, it is definitely a steady-state.

[11] Figure 4 is based on the following parameters: $A = 20$, $\alpha = 0.5$, $\beta = 0.5$, $\theta = 0.5$, $\lambda = 0.2$, $\phi = 0.2$, $\psi = 0.1$, and $\bar{Q}/N = 10000$. Public policy parameters T, ξ and τ are fixed to 0. $\delta = 0.90$ and $\eta = 0.10$.

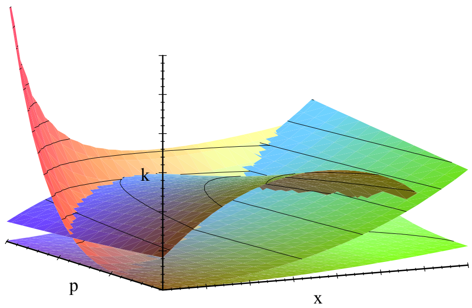


Figure 4: Existence of a steady-state

Let us now use the numerical example of Figure 4 to discuss the issue of the (local) stability of a non-trivial steady-state.

When such an equilibrium exists, the kk and pp loci divide the x-plane - the only relevant area in the (x_t, p_t, k_t) space - into four distinct areas (the other areas being incompatible with individual utility maximization). In order to identify those four areas, let us take the example provided by Figure 4, where a steady-state exists (i.e. the two distinct divisions of the x-plane intersect at some point).

If the economy lies initially on the part of the x-plane that lies below the kk locus and above the pp locus, it must be the case that capital and pollution will grow. Given that the economy must remain on the x-plane, that move with conjoint growth of capital and pollution must consist of a move towards the interior of Figure 4, which implies a rise in health expenditures. That rise can only stop at the point of the x-plane where the two loci intersect, that is, at the steady-state (if it exists, as on Figure 4).

Alternatively, if the economy lies initially on the part of the x-plane that lies below the kk locus and below the pp locus, then it must be the case that capital will grow, while pollution will fall, so that health expenditures will have to grow. Such a move will make the economy move to the area previously considered (below the kk locus and above the pp locus).

If the economy lies initially in the part of the x-plane that is above the kk locus and below the pp locus (i.e. in the upper right corner of Figure 4), then both capital and pollution will fall. This consists in a move oriented towards the origin of axis (leading to a fall in x_t), which will, here again, send the economy back to the first area considered.

Finally, if the economy lies initially in the part of the x-plane that is above the kk locus and above the pp locus, it must be the case that capital will fall and pollution will grow. This move involves a strong fall in health expenditures, and is likely to bring the economy to the area just considered.

Therefore, whereas little can be said regarding the existence and uniqueness of a non-trivial steady-state, the above informal discussion seems to suggest that the steady-state, if it exists and is unique, could be locally stable. Actually, the economy seems to have a tendency to come back in the area of the x-plane where both capital and pollution grow, area where the economy tends to converge towards the steady-state. But that informal discussion should not be regarded as suggesting more than the mere *possibility* of local stability of a unique steady-state. Obviously, only a more formal analysis could allow us to state the necessary local stability of such an equilibrium.

Thus, those geometrical, informal discussions do not allow us to say much about the issues of existence, uniqueness and stability. However, that preliminary analysis, by stating the possible local stability of a steady-state when it exists and is unique, constitutes a necessary preliminary stage before considering numerically what public intervention should be in that economy.

6.3 Optimal public policy: a numerical application

Given the existence of various imperfections in the economy under study, it is not straightforward to discuss the issue of the optimal intervention of the government. There exist various effects at work, and the present - non exhaustive - subsection does not claim to be able to provide a complete answer to the question of the design of the optimal policy, but, more modestly, to emphasize the main difficulties involved in the present numerical exercise, as well as the impact of these on the issue of optimal policy making.

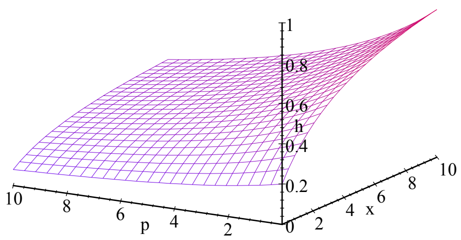
6.3.1 Calibration

Demographic parameters A major difficulty in OLG models with endogenous longevity consists of the selection and the calibration of an adequate longevity production function. While it seems plausible to suppose that health expenditures have a positive impact on longevity, whereas pollution has a negative impact, the precise *strength* of each determinant remains quite hard to calibrate.

However, the particular calibration of the longevity production function may play a crucial role as far as the study of optimal public policy is concerned: as suggested by expressions (34) and (35), the desirability of a rise in a public policy parameter τ or ξ depends significantly on the derivative of longevity with respect to health expenditures and pollution.

Hence, it makes sense to postulate not one, but several distinct scenarii as far as the values of the parameters λ, ψ and ϕ. Throughout this subsection, we shall suppose that λ equals 0.20, which amounts to assume a 'natural' longevity (i.e. without health expenditures and without pollution) equal to $65 + 0.20(40) = 73$ years.

Regarding ϕ and ψ, we shall take two distinct cases, represented below on Figures 5a and 5b. On Figure 5a, where ϕ equals 0.20 and ψ equals 0.15, pollution has a negative, but relatively weak effect on longevity. On the contrary, on Figure 5b, ψ is fixed to 0.50, reflecting the existence of a pollution process damaging health and longevity to a larger extent (while ϕ remains equal to 0.20).



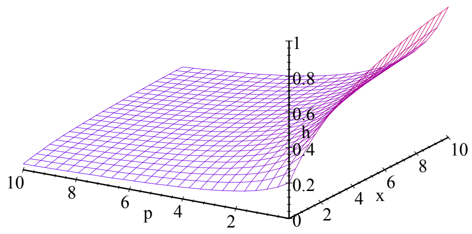


Figure 5a: Longevity ($\phi = 0.2, \psi = 0.15$)Figure 5b: Longevity ($\phi = 0.2, \psi = 0.50$)

On each graph, health expenditures x_t increase longevity h_{t+1} at a decreasing rate, while pollution reduces longevity. One should notice that, when pollution tends to be larger, the marginal effect of health expenditures on longevity tends to fall *ceteris paribus*. But Figures 5a and 5b differ significantly regarding the size of the impact of a marginal rise in health expenditures on longevity. Clearly, a rise in x_t has a larger impact on h_{t+1} when ψ is low than when ψ is large, and that difference is larger the higher pollution is.

Given that this may not be without influence on the impact, in welfare terms, of a given public policy, we shall consider, throughout this subsection, those two different scenarii, illustrated by Figures 5a and 5b.

Preference parameters The discount factor β is here supposed to be equal to 0.30, which amounts to quite low impatience, given the length of 40 years of a normal period.

The preference parameter θ, which captures the relative importance of consumption *versus* available space in the determination of individual welfare, is likely to play a significant role as far as the definition of optimal public intervention is concerned.

But independently from the calibration of θ, the present numerical exercise raises the question of the choice of a unit of measurement for the available space per individual q_t. Should that space be measured in square-kilometers or square-meters? In order to answer that question, it is necessary to define a kind of 'critical' level of space beyond which the addition of some surface adds almost no welfare, but below which life can become extremely unpleasant.

Defining such a critical level is far from straightforward. We shall here suppose that individual welfare becomes seriously affected once the available surface per person becomes inferior to about 0,001 square-kilometers (i.e. 1000 square-meters). To give an idea of what that level consists of, one can remind that the population density of France is about 100 habitants per square-kilometer, which corresponds to 10000 square-meters per person. The postulated critical level corresponds approximately to the observed density in Bangladesh (equal to 1018 habitants per square-kilometer), and slightly below the one in Malta (equal to 1245 habitants per square-kilometer).[12]

Throughout this subsection, we shall postulate that θ is equal to 0.50. Figure 6 illustrates the level of the temporal utility derived from enjoying a particular space (expressed in square-meters), under θ equal to 0.25 (thin), 0.50 (bold) and 0.75 (dots). Clearly, postulating θ equal to 0.75 would amount to assign a too large weight to an increase of available space even when the available space is already large, while a θ equal to 0.25 tends to minimize the sentitivity of individual welfare to the available space around the critical level of 1000 square-meters per person. Thus, θ equal to 0.50 seems to be a reasonable assumption.

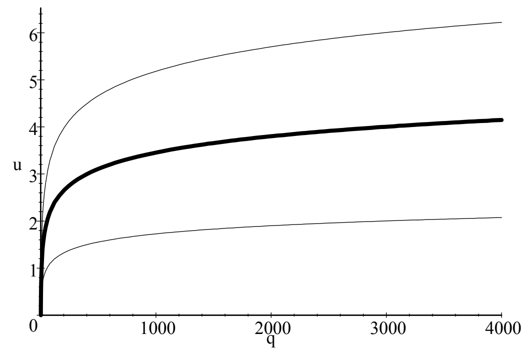


Figure 6: Space component of welfare

[12]Sources: INSEE: *Population, densité et part de la population urbaine des principaux pays du monde*, Octobre 2006.

Naturally, this is not straightforward to calibrate the total available space. We shall suppose that the constant population N is such that the available space per person is about 3000 square-meters (i.e. approximately the density in Belgium). But, if we really want to make the 'number problem' play in future numerical applications, we may have to depart from that assumption, and postulate a higher population density, for instance the one prevailing in Bangladesh or Malta, where a rise in the population size through ageing is really likely to affect individual welfare.

Pollution The calibration of the pollution parameters δ and η depends on the particular pollution process under study. Obviously, there can be as many calibrations as there exist different pollution processes.[13]

Given that each period is supposed to be of length 40 years, it makes sense to suppose that δ is relatively high, but its precise level depends on what pollution consists of. We shall, as a benchmark case, suppose that δ is equal to 0.9, which amounts to assume that 9/10 of the pollution have vanished naturally after a time lag of 40 years.

Regarding η, we shall suppose that it is equal to 0.10. That value is large (i.e. one tenth of the output), but this is deliberately made for the purpose at hand in this paper. As shown by Figure 7, the choice of η is crucial, as it determines the location of the pp locus, and, hence, the steady-state level of pollution.

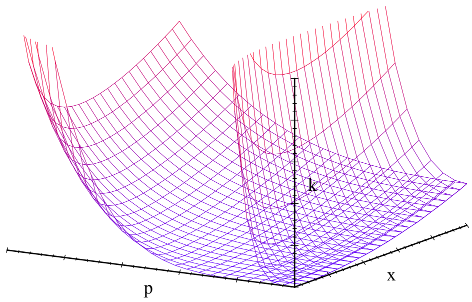


Figure 7: The pp locus: $\eta = 0.1$ and $\eta = 0.5$

Under η equal to 0.5, the steady-state level of pollution is likely to be larger than under η equal to 0.1, as the kk locus is independent from η.

[13] However, it should be stressed that the assumed dynamic expression for pollution may not cover many existing pollution processes, whose dynamics is far more complex.

Production Finally, we shall, in the Cobb-Douglas production function, fix the productivity parameter A to 20 and α to 0.50.

6.3.2 The dynamics of capital, longevity and pollution: an example

Let us now present the numerical example of two economies A and B, which are characterized by the same preferences, the same production, and same pollution, but by two different longevity production functions: in A, pollution is less damaging for longevity (i.e. $\psi = 0.15$), whereas pollution is more noxious for health in B (i.e. $\psi = 0.50$).

It is supposed that the two economies are characterized by the same initial conditions: capital per worker is equal to 0.10, so that initial pollution is equal to about 0.632.[14] Figures 8a-8d show the evolution, for the first ten periods, of capital per worker, pollution (in per worker terms), health expenditures and longevity.

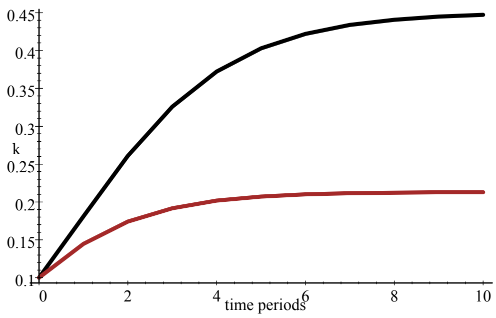


Figure 8a: Capital per worker

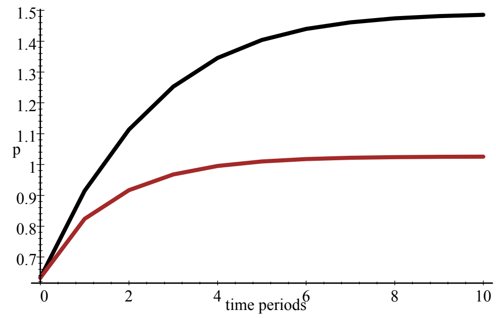


Figure 8b: Pollution per worker

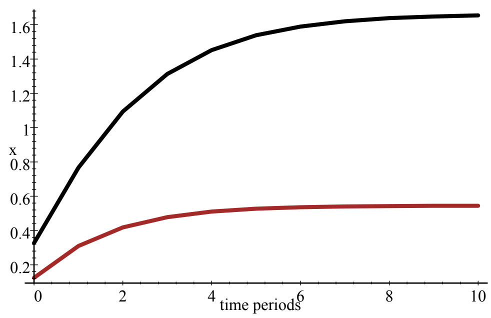


Figure 8c: Health expenditures

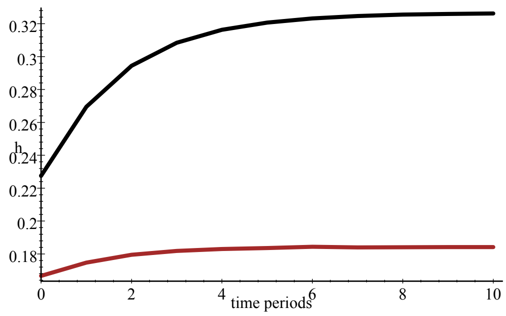


Figure 8d: Longevity

[14] However, the initial level of health expenditures differs across countries, as this depends on the noxiousness of pollution.

It appears clearly that the two economies, which are characterized by the same initial capital level, tend to grow over time, despite the pollution process. However, the economy where pollution is more noxious for longevity tends to grow more slowly, and reaches a much lower steady-state capital level. The same observation holds for the evolution of pollution, which is not surprising, given the connection between pollution and production. At the steady-state, the pollution level is higher in economy A than in B, that is, higher where it is the least damageable for longevity.

The initial gap between health expenditures in the two economies tends also to grow over time (Figure 8c). But given that pollution is much larger in A than in B, and that more is spent on health in economy A, this raises the question of where longevity is the highest. Figure 8d suggests that the answer to that question is unambiguous: as one may expect, longevity is always higher where pollution is the least damageable. Even after ten periods of capital accumulation, longevity in economy B is always lower than its initial level in A, and this comes clearly from the more noxious nature of pollution in economy B: economic development cannot help, as this constitutes nothing else than the source of pollution.

Finally, one should also notice, regarding the dynamics, that the transition towards the steady-state takes a much longer time for the economy where pollution is less noxious than for the economy with more noxious pollution. In the latter, the steady-state is reached after about 6 periods, whereas the transition lasts about 10 periods in economy A. This reflects that, under a more noxious pollution, the environmental constraints on economic expansion are stronger, so that the steady-state is reached more quickly as the constraint is met earlier.

6.3.3 The impact of public intervention

Let us now consider the desirability of taxing savings in the two economies under study. For that purpose, Figures 9a-9d contrast the evolutions of capital, pollution, health expenditures and longevity under *laissez-faire* (in doted lines) with their evolution under a tax rate τ fixed to 40 percents (in thin lines). The tax rate on health expenditures ξ is here fixed to zero, whereas the lump-sum transfer T corresponds exactly to the fiscal revenue from the tax on savings (i.e. the government's budget is balanced).

Unambiguously, such a policy contributes to promote capital accumulation. While this result may appear surprising, this is due to the fact that the savings decision is here independent from the net rate of return (because of the logarithmic utility), so that taxing the old's capital and returning the revenues to the young can only favour economic development. As a consequence, the introduced savings tax tends to increase - rather than decrease - steady-state pollution. But given the substantial rise in health expenditures, longevity will also be higher under public intervention.

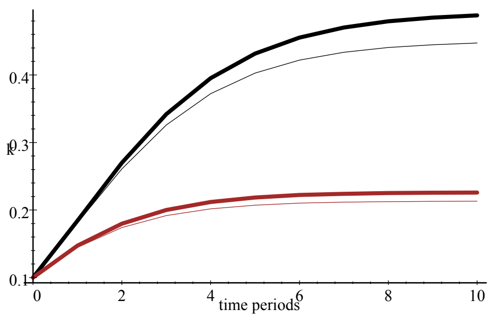


Figure 9a: Capital per worker

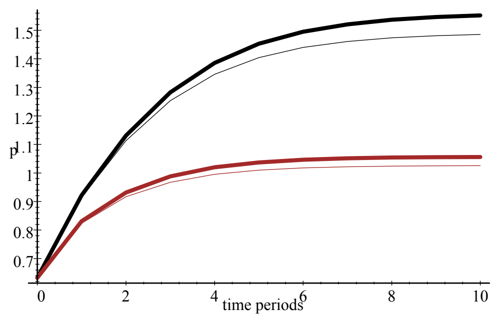


Figure 9b: Pollution per worker

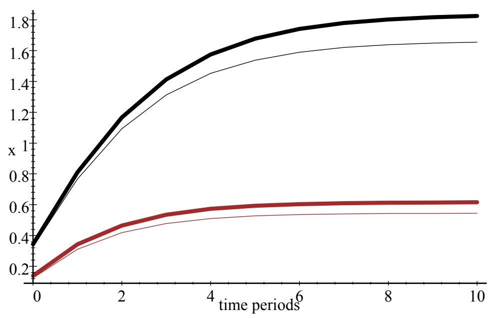


Figure 9c: Health expenditures

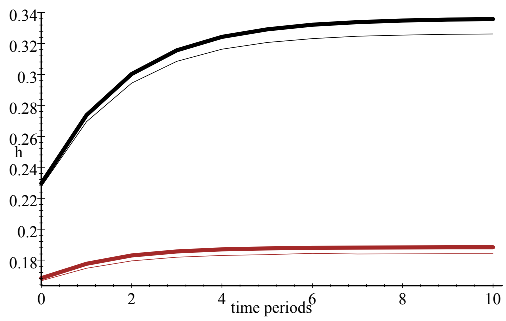


Figure 9d: Longevity

One can also notice that the impact of public intervention is stronger on the steady-state than during the transition, and is also stronger in the case of the economy where pollution is the least noxious.

In welfare terms, the introduction of a tax on savings can be quite beneficial, as suggested by Table 1 below, which contrasts the steady-states prevailing in the two economies under *laissez-faire* and under the tax on savings.[15] However, while the welfare gain resulting from such a tax is substantial in

[15] While Table 1 focuses on the steady-state lifetime utility, it is clear that all generations - also the ones living during the transition towards the steady-state - are better off under the tax on savings. But this does not imply that such a tax is politically feasible, as generation 'minus 1' would definitely be against the introduction of such a tax at period 0.

both economies, it tends to be larger, in absolute and relative terms, in the economy where pollution is less noxious for longevity.

Economies	k^*	p^*	h^*	u^*
Economy A, $\tau = 0$	0.447	1.486	0.326	5.148
Economy A, $\tau = 40\%$	0.485	1.546	0.336	5.188
Economy B, $\tau = 0$	0.213	1.025	0.184	4.899
Economy B, $\tau = 40\%$	0.226	1.055	0.188	4.922

This suggests that the desirability of introducing some taxation on savings is likely to depend, to some extent, on the precise pollution-longevity relationship that is postulated. Given the central role played by longevity in the present model, this does not really constitute a surprise.

However, it cannot be overemphasized here that the above numerical exploration is only a preliminary stage before focussing on the - more difficult - question of the definition of optimal public intervention in the present economy. The complexity of that issue comes from the number of potential combinations of public instruments (τ, ξ, T), but, also, from the plausible dependency of the optimal policy mix on parameters of various natures (environmental, economic, demographic). A such, this will require to focus on some particular economies, whose precise features will allow us to make appear the various effects at work. Hence, much work remains to be done in the future, to clarify what optimal public intervention is in an OLG economy with endogenous longevity and pollution.