

Loop-wire medium for investigating plasmons at microwave frequencies

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We present numerical simulations and microwave measurements on a loop-wire structure that acts as an effective medium exhibiting a well-defined bulk plasma frequency in the microwave regime, with an effective negative dielectric function below this plasma frequency. The dependence of this plasmonic response on the self-inductance of the constituent wire elements is made explicit. A finite structure, approximately spherical, composed of this inductive medium is studied, and reveals subwavelength surface plasmon resonances below the bulk plasma frequency. © 1999 American Institute of Physics. [S0003-6951(99)03936-4]

Photonic band gap (PBG) structures, originally proposed as a means of controlling the electromagnetic properties of matter at optical wavelengths,^{1,2} have now found unique applications at much longer wavelengths, including the infrared and microwave regimes.³⁻⁵ At these longer wavelengths, metals can naturally be incorporated into periodic structures without incurring unacceptable loss tangents,⁶ and lead to interesting and unique electromagnetic effects, such as anomalously large photonic band gaps,⁷ and the appearance of bulk plasmon-like modes associated with negative dielectric constants.⁸⁻¹⁰ In this letter, we introduce an array of inductive metal scatterers that responds to electromagnetic radiation at microwave frequencies in the same manner that an ideal metal responds to electromagnetic energy at optical frequencies, supporting both bulk and surface plasmon modes.

In 1996, Pendry *et al.*⁸ demonstrated that a square array of closely spaced thin wires could have an effective plasma frequency determined by the ratio of the wire spacing d to the wire radius r . While thin wires can successfully yield composite structures with plasmon-like behavior at microwave frequencies,^{9,10} the requisite ratio of d/r (on the order of 300–1000) makes the structure challenging both to simulate and fabricate.

Plasmonic behavior can be understood by assuming that the effective dielectric constant of the composite structure is determined by a plasma frequency, according to

$$K(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \quad (1)$$

We propose an effective plasmon structure based on the recognition that the effective dielectric constant of the thin wire structure is due to the self-inductance of the constituent wires, and that other more feasible geometries can be engineered that yield a self-inductance of similar or greater magnitude.

Assume a finite section of conducting wire with cross sectional area A_w , and with a self-inductance L , in a spatially uniform and harmonically time-varying electric field of magnitude E_0 . As currents flow in the wire, the inductive component will lead to an electromotive force (EMF) countering the applied field, such that the local electric field E_{loc} will

differ from the applied field by $E_{loc} = E_0 - E_L = E_0 - (A_w/l)i\omega Lj$, where l is the length of the repeated section of wire with loop and j is the current density in each wire. E_L is the electric field associated with the induced EMF. From Ohm's law ($j = \sigma E_{loc}$), we can find an effective conductivity for the inductive wire, and calculate an effective dielectric function analogous to Eq. (1), but with the plasma frequency directly related to the inductance of the unit cell according to $\omega_p^2 = l/(d^2 L \epsilon_0)$. Thus, by raising the self-inductance L , the plasma frequency can be lowered. This can be accomplished by making the wire radius very thin or by numerous other geometrical configurations, such as wire loops or coils.

An easily constructed geometry that exhibits a bulk microwave plasma frequency is the loop-wire array depicted in Fig. 1 (inset). In our experiments and simulations, these wires are spaced 5 mm apart in a square array, and we restrict our attention to cases where the electric field is polarized along the direction of the straight segments of the wires. To estimate the inductance of the loop structure, we make the initial assumption that there is no interaction between the loops, and that the self-inductance per unit cell can be approximated by

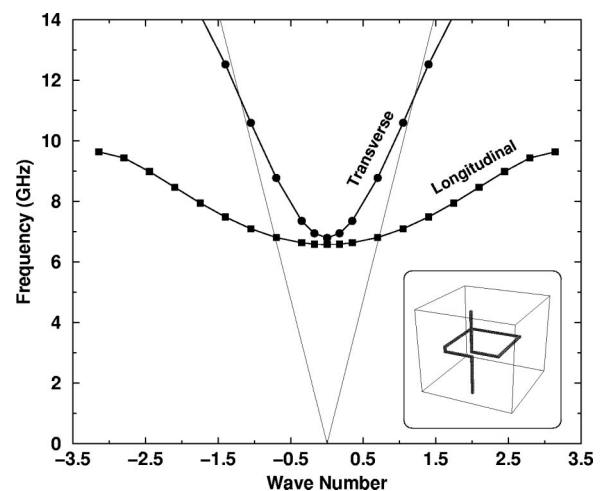


FIG. 1. Plot of the simulated longitudinal and transverse dispersion curves for the infinite structure based on the unit cell shown in the inset (unit cell length=5 mm, loop side length=3 mm). The straight lines are coincident with the light cone ($\omega = ck$). The theory [Eq. (2)] predicts a plasma frequency of 7.0 GHz.

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$$\frac{L}{l} = \mu_0 \frac{R}{l} [\ln(8R/r) - 7/4] + \mu_0 \frac{\ln(d/r)}{2\pi},$$

where R is the radius of the loop and r is the wire radius. The first term is due to the loop¹¹ while the second term is due to the straight wire segments.^{8-10,12} Thus we find, using the above equations,

$$\omega_p^2 = \frac{2\pi(c^2/d^2)}{(2\pi R/l)[\ln(8R/r) - 7/4] + \ln(d/r)}. \quad (2)$$

To demonstrate the plasmon effect, we simulated, using commercial finite-difference code,¹³ the periodic infinite metallic structure consisting of wire loop elements. To ease the computational requirements, a square loop (rather than circular) was used with each side of length 3 mm. From Eq. (2), a plasma frequency of 7.0 GHz is predicted. To confirm that a plasma mode did indeed occur at the predicted frequency, longitudinal and transverse dispersion curves were calculated for the infinite structure, as shown in Fig. 1. These curves, consistent with a medium of effective dielectric constant⁹ as given in Eq. (1), both begin at about 6.8 GHz, easily within the error introduced by our approximations and numerical procedure. Note that the dispersion curve for the transverse modes flattens out at higher frequencies; this is due to unavoidable capacitive coupling that complicates the model.

A useful probe of the plasmon dispersion relation is to measure transmittance of a model structure. We performed such a measurement at microwave frequencies in a two-dimensional scattering chamber, described in detail previously.¹⁴ The scattering structure consisted of a two-dimensional array of 1 cm high vertical wire elements standing between the opposing metal plates of the chamber. Each wire element was constructed from a single strand of 26 gauge copper wire ($r=0.2$ mm) with a single loop of radius $R=1.3$ mm centered vertically between the plates with the loop plane parallel to the plates [shown schematically in Fig. 2(b)]. The wires were inserted into a 5 mm thick Styrofoam plate in a square lattice pattern, lattice spacing $d=5$ mm. The resulting lattice had five rows perpendicular to incident radiation, and 30 rows in the transverse direction.

The transmittance versus frequency of the loop wire structure is shown in Fig. 2(a). Applying Eq. (2) with the parameters used in the experiment leads to an effective plasma frequency of 10.7 GHz, in good agreement with the data. For comparison, the transmittance of a second structure composed of a lattice of straight wires 1 cm long was measured, and is also shown in Fig. 2.

Having demonstrated an effective plasmonic medium at microwave frequencies, we might now hope to construct from this medium a finite structure exhibiting a surface plasmon resonance. Colloidal metal spheres such as silver and gold with subwavelength dimensions have surface plasmon resonances spanning an optical frequency range from $\omega_p/\sqrt{3}$ (corresponding to the dipole mode) to $\omega_p/\sqrt{2}$ (corresponding to a surface mode), where $\omega_p/2\pi \sim 10^{15}$ Hz. In particular, the electric dipole mode of a sphere ($\omega_p/\sqrt{3}$) is a particularly important example because the electric field within the sphere is nearly uniform, and it is the only mode that couples to a uniformly applied electric field.

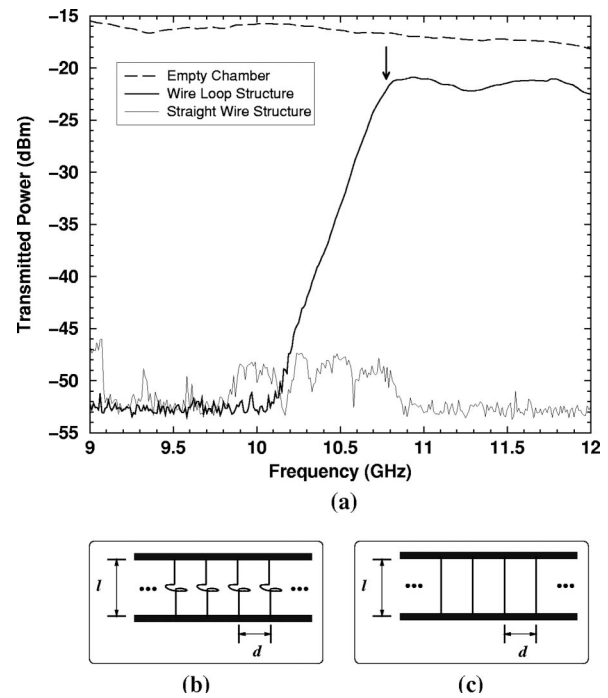


FIG. 2. (a) Measured transmittance of a loop-wire structure and a straight wire structure, performed at microwave frequencies in a two-dimensional (2D) scattering chamber. The plasma frequency for the loop-wire structure is calculated to occur at 10.7 GHz, denoted by the arrow in the figure. (b), (c) Schematic representation of a single row out of an array of the loop-wire elements (b) and the straight wire elements (c).

We constructed a wire element sphere out of 26 gauge ($r=0.2$ mm) copper wire. Twenty-one vertical wire elements, each with planar loops ($R=1.1$ mm) wound at 5 mm intervals, were placed in a square lattice array with a $d=5$ mm lattice constant, such that the whole structure was symmetrically confined within a spherical volume with a diameter of 2.5 cm. The wires were held in place by boring out variable depth holes in a Styrofoam block, and loading the wire elements into these holes. Each of the wire elements

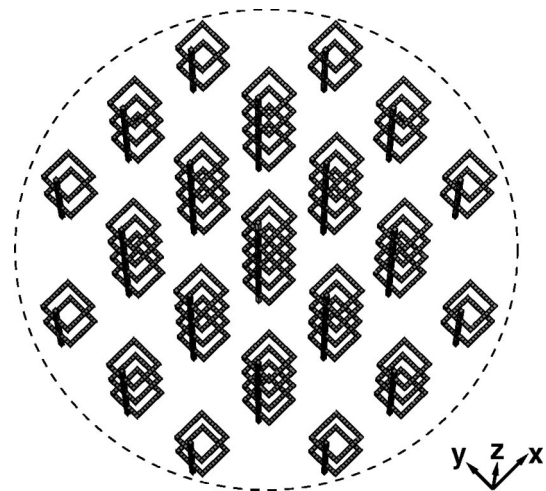


FIG. 3. Wire element sphere composed of units with 2, 3, 4, or 5 loops, each with a vertical spacing of 5 mm in between. The elements are arranged in a square lattice with lattice spacing 5 mm, such that they are centered vertically. The resulting structure approximates a sphere, confined within a region of diameter 2.5 cm. This square loop structure was used in the simulations. An equivalent structure with circular loops was constructed for experimental verification of the surface plasmon resonance.

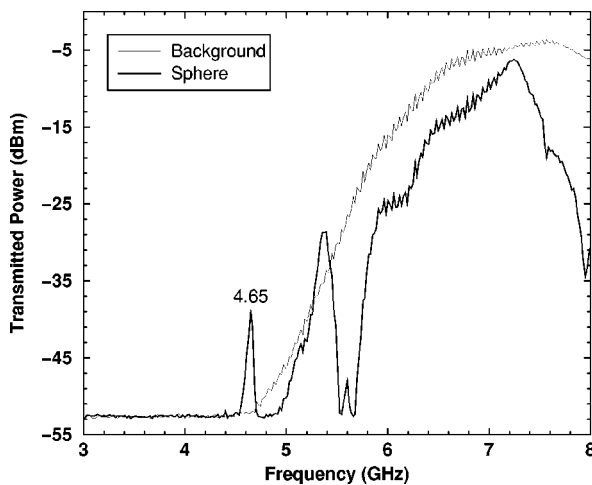


FIG. 4. Transmission spectrum of an experimental sphere structure patterned after the model in Fig. 3. The background transmission spectrum (no sphere) is shown for reference. The shape of the background is due to the use of X-band couplers that have a cutoff frequency of ~ 8 GHz. The electric field for this measurement was polarized along the direction of the wires. Turning the sphere by ninety degrees, such that the electric field was perpendicular to the wires, resulted in a signal equivalent to the background (no scattering).

had 2, 3, 4, or 5 loops, depending on the overall length of the element. Figure 3 shows a computer representation of the experimental sphere used for the corresponding simulations.

The loop-wire sphere was placed between a pair of X-band coaxial-to-waveguide couplers, which served as the excitation and detection for the transmission spectrum shown in Fig. 4. The mode at 4.6 GHz was particularly easy to couple to, and could be excited by a variety of methods, including using coaxial antennas. Other higher order modes producing the structure at higher frequencies were very dependent on the coupling, and will not be analyzed further here.

A finite-difference simulation was carried out on the loop-wire configuration of Fig. 3. A dipole mode was found at 4.48 GHz, the electric field pattern of which is shown in Fig. 5. While there is clearly capacitive coupling between the wires and loops toward the periphery of the structure, the mode does behave as expected toward the interior region of the sphere; that is, the electric field tends to be uniform from one pole of the sphere to the other, with opposite sign to that of the field emanating from the poles away from the sphere. This is consistent with a buildup of charge at the poles along the z axis. Other simulated modes did not have dipole symmetry, were not localized to the structure, or had internal fields that pointed directly away from and into the wire elements.

We have demonstrated a localized surface plasmon resonance at microwave frequencies in a structured artificial medium. Because capacitance can enter in a very different man-

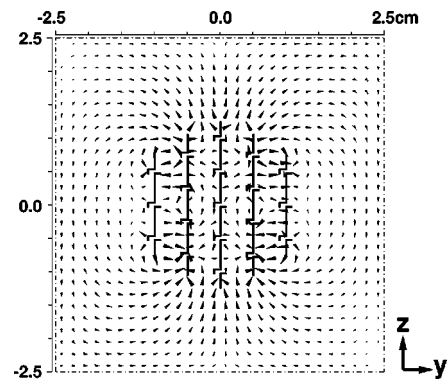


FIG. 5. Arrow plot (in the $x=0$ plane) of the electric field pattern from a simulation of the structure shown in Fig. 3. The frequency of this mode occurred at 4.48 GHz. To illustrate the dipole character of the mode, the data are plotted on a log scale, such that the plotted arrow size F' is related to the field strength F by $F' = [2 + \log(F)]/2$, where $F=1$ corresponds to the maximum field strength. Magnetic boundary conditions (tangential $H=0$) were used on the surfaces of the bounding walls.

ner in such a test structure than in an actual metal, one must be cautious when using the artificial medium as a plasmon analog. However, we expect that refinement to the loop-wire structure will result in a useful medium to study the important and complex problem of plasmon resonances, in a manner that will isolate the electromagnetic effects from the chemical effects associated with colloidal particles and thin metal films used at optical frequencies.

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- ¹E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).
- ²S. John, Phys. Rev. Lett. **58**, 2486 (1987).
- ³D. R. Smith, S. Schultz, N. Kroll, M. Sigalas, K. M. Ho, and C. M. Soukoulis, Appl. Phys. Lett. **65**, 645 (1994).
- ⁴E. Ozbay, B. Temelkuran, M. Sigalas, G. Tuttle, C. Soukoulis, and K. M. Ho, Appl. Phys. Lett. **69**, 3797 (1996).
- ⁵H. Contopanagos, N. G. Alexopoulos, and E. Yablonovitch, IEEE Trans. Microwave Theory Tech. **46**, 1310 (1998).
- ⁶W. Y. Leung, G. Tuttle, M. M. Sigalas, R. Biswas, K. M. Ho, and C. M. Soukoulis, J. Appl. Phys. **84**, 4091 (1998).
- ⁷D. F. Sievenpiper, E. Yablonovitch, J. N. Winn, S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **80**, 2829 (1998).
- ⁸J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Youngs, Phys. Rev. Lett. **76**, 4773 (1996).
- ⁹J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, J. Phys.: Condens. Matter **10**, 4785 (1998).
- ¹⁰G. Guida, D. Maystre, G. Tayeb, and P. Vincent, J. Opt. Soc. Am. B **15**, 2308 (1998).
- ¹¹L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon, Oxford, 1987), p. 124.
- ¹²G. Guida, D. Maystre, G. Tayeb, and P. Vincent, J. Electromagn. Waves Appl. **12**, 1153 (1998).
- ¹³T. Weiland, AIP Conf. Proc. **297**, 291 (1994).
- ¹⁴D. R. Smith, R. Dalichaouch, N. Kroll, S. Schultz, S. L. McCall, and P. M. Platzman, J. Opt. Soc. Am. B **10**, 314 (1993).