

# Lorentz Force Effects on the Orbit of a Charged Artificial Satellite: A New Approach

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**Abstract:** A charged artificial satellite moving relative to a magnetic field accelerates in a direction perpendicular to its velocity and the magnetic field due to the Lorentz force. The geomagnetic field is considered as a multipole potential field and the satellite electrical charge is supposed to be constant. The study is provided to compute Lorentz force acceleration of a charged satellite in Earth's magnetic field as a function of the orbital elements of the satellite. Periodic perturbations in the orbital elements of the satellite are derived using Lagrange planetary equations. Numerical results for a chosen satellite orbit shows the most effects of Lorentz force are in semi major axis, eccentricity, and the longitude of the satellite, but there aren't any effects of the force on the inclination and the argument of the perigee of the satellite elements.

## INTRODUCTION

The important quantity which determines the magnitude of the effect is the satellite's electrical charge. The surface of a satellite is charged to a negative potential (see Al'pert et al., 1964) and in the first approximation behaves like a spherical condenser with respect to the ionosphere vicinity.

The relationship between the plasma environment and spacecraft potential must be taken into account for the importance of mission role and spacecraft configuration in evaluating absolute and differential charging effects. The build-up of large potentials on spacecraft relative to ambient plasma is not, of itself, a serious electrostatic discharged design concern. However, such charging enhances surface contamination, which degrades thermal properties. These and other charging effects can produce potential differences between spacecraft surface or between spacecraft surface and spacecraft ground. When a breakdown threshold is exceeded, an electrostatic discharge can occur. Vehicle torquing or wobble can also be produced when multiple discharging occur (Carolyn K. Purvis et al. 1984). Anderson (et al. 1994) examined the relationship between the plasma environment and spacecraft potential for the Dynamics Explorer 2 (DE 2) spacecraft in an attempt to improve the accuracy of ion drift measurements by the retarding potential analyzer (RPA). The geomagnetic field

had considerable effect on the spacecraft potential due to magnetic field confinement of the electrons as well as to the (Lorentz force)  $V \times B$  electric field resulting from the movement of the spacecraft across magnetic field lines. They derived an algorithm for determining the spacecraft potential (at the location of the RPA on the spacecraft) for any point of the DE 2 orbit. Juhasz Antal and Mihaly Horanyi (1997) studied the Degrading objects in orbit around the Earth as well as solid rocket motors generate micron- and submicron-sized space debris. They shown, the motion of these particles is dictated by gravity, solar radiation pressure, and electromagnetic forces, since these grains collect electrostatic charges and become vulnerable to the electric and magnetic fields in the Earth's magnetosphere. They shown that magnetosphere effects tend to reduce the lifetime of these grains either by forcing them onto elliptical orbits to collide with the Earth or by swiftly ejecting them into the interplanetary space.

Several attempts were already made to assess the effects of Lorentz force to show in principle its negligible value with respect to some other, pre-important effects (e.g. Sehna, 1969). However, the necessary high precision of orbital determination of some proposed space experiments (Ciufolini, 1987), requires a full knowledge of the electrodynamic effects connected with the Lorentz force, which we shall try to study in detail. VokRouhlicky (1989) determined the orbital effects of the Lorentz force on the motion of an electrically charged artificial satellite moving in the Earth's magnetic field. In his case the influence of the geomagnetic field manifests itself predominantly by Lorentz force. Peck, (2005) found out the components of the Lorentz force in the spherical coordinate system. He evaluated the use of Lorentz force as a means of orbit control for finite bodies, including small spacecraft.

In the present work, the components of the Lorentz force acceleration of a charged satellite in the Earth's magnetic field are computed as a function of orbital elements of the satellite in two directions (radial and tangential). Periodic perturbations in orbital elements of the satellite due to the Lorentz force are derived using Lagrange planetary equations. Numerical applications for the LAGOS Satellite are introduced.

### EQUATIONS of MOTION

Let us consider the magnetic field of the earth to be given as a multipole potential field and the satellite electrical charged is supposed to be constant.

The components of the disturbing force (Lorentz force) we are looking for will be the components of a vector

$$\mathbf{F}_L = Q\mathbf{V} \times \mathbf{B} \quad (1)$$

where  $\mathbf{V}$  is the velocity of the satellite in the orbit,  $\mathbf{B}$  is the vector of the magnetic field intensity of the Earth,  $Q$  is the satellite's electrical charge. The components of the vector  $\mathbf{F}_L$ , from equation (1) in the direction  $\hat{\mathbf{r}}$  (the direction of the position vector) and  $\hat{\mathbf{t}}$  (the tangential direction, which normal to  $\mathbf{r}$  and the orbital angular momentum). As in Peck (2005) Augmented with the Lorentz Force, Newton's law of gravitation for a satellite of mass  $m$  moving in  $r^{-2}$  gravitational field of the Earth with point mass  $M$  becomes

$$m \frac{{}^N d^2}{dt^2} \mathbf{r} = -m \frac{\mu}{r^2} \hat{\mathbf{r}} + Q \left( \frac{{}^N d}{dt} \mathbf{r} - \boldsymbol{\omega}_e \times \mathbf{r} \right) \times \mathbf{B}, \quad (2)$$

where, the superscript N indicates a derivative taken with respect to a Newtonian, or inertial frame,  $\mathbf{r}$  is the vector position (magnitude  $r$  and direction  $\hat{\mathbf{r}}$ ) of the satellite relative to the system barycentre,  $\mu = MG$  where  $G$  is the universal gravitational constant,  $\boldsymbol{\omega}_e$  is the Earth's angular velocity vector fixed frame with the respect to an inertial frame. This expression acknowledges that it is the satellite's velocity relative to the magnetic field  $\mathbf{V} = \frac{{}^N d}{dt} \mathbf{r} - \boldsymbol{\omega}_e \times \mathbf{r}$  that determines the Lorentz force. In the simplest model, the earth's magnetic field rotates with Earth. Br relativity, this time-varying magnetic field represents an electric field, which is the means by which work can be done on the Lorentz force. In a frame  $E$  the rotates with the Earth, the equation of motion in terms of the relative velocity  $\mathbf{V} = \frac{{}^E d}{dt} \mathbf{r}$  and a gravitational potential  $\Phi_{gr}$

is

$$\frac{{}^E d}{dt} \mathbf{V} = -\nabla \Phi_{gr} + \frac{Q}{m} \mathbf{V} \times \mathbf{B} - 2 \boldsymbol{\omega}_e \times \mathbf{V} + 2 \boldsymbol{\omega}_e \times (\boldsymbol{\omega}_e \times \mathbf{r}) \quad (3)$$

where, dividing through by  $m$  introduces the commonly used charge per mass  $\frac{Q}{m}$  as a parameter that determines the scale of the Lorentz force. Following Schaffer and Burns (1994), we project this equation onto  $\mathbf{V}$

$$\frac{{}^E d}{dt} \mathbf{V} \cdot \mathbf{V} = \frac{1}{2} \frac{d}{dt} V^2 = -\nabla \Phi_{gr} \cdot \frac{{}^E d}{dt} \mathbf{r} + \frac{d}{dt} \left( \frac{1}{2} r^2 \omega_e^2 \sin^2 \theta \right) \quad (4)$$

where  $\theta$  is a coordinate of a spherical coordinate system  $(r, \theta, \phi)$  with origin at the Earth's centre and associated with an E-fixed basis. Integrating between  $t_1$  and  $t_2$  shows that the total mechanical energy in the rotating fram  $H$  is constant:

$$H(t_2) - H(t_1) = 0, \quad (5)$$

Schaffer and Burns (1994) point out that this function is the appropriate Hamiltonian in the non-canonical variables  $(\mathbf{r}, \mathbf{p} = m\dot{\mathbf{r}})$  as demonstrated by Littlejohn (1982, 1979), who used these coordinates in a perturbation theory for highly charged particles in slowly varying electromagnetic fields. The existence of this constant Hamiltonian in a rotating frame suggests that only the rotation of the magnetic field, which causes the co-rotational electrical field, can do work in the inertial frame. By way of illustration, we consider the osculating elements of a restricted two-body orbit whose angular momentum is aligned with the Earth's magnetic moment (i.e., a magnetic equatorial orbit). The energy  $E$  per unit satellite mass in inertial frame is given by

$$E = -\frac{\mu}{2a}, \quad (6)$$

where  $a$  is the semimajor axis of the satellite's orbit. Its time rate of change is

$$\dot{E} = \dot{a} \frac{\mu}{2a^2} = \frac{N}{dt} \mathbf{r} \cdot d\mathbf{F}_L, \quad (7)$$

a familiar result in which the energy depends entirely on the semimajor axis. Work done if and only if the semimajor changed. The perturbing force  $d\mathbf{F}$  per unit mass is

$$d\mathbf{F}_L = F_r \hat{\mathbf{r}} + F_t \hat{\mathbf{t}} \quad (8)$$

In the basis of proposed directions of the Lorentz force, then

$$\frac{N}{dt} \mathbf{r} = \dot{r} \hat{\mathbf{r}} + r \dot{\phi} \hat{\mathbf{t}} \quad (9)$$

where  $\phi$  is the true anomaly of the orbit. Therefore,

$$\dot{a} \frac{\mu}{2a^2} = (\dot{r} \hat{\mathbf{r}} + r \dot{\phi} \hat{\mathbf{t}}) (F_r \hat{\mathbf{r}} + F_t \hat{\mathbf{t}}) \quad (10)$$

or

$$\dot{a} \frac{\mu}{2a^2} = r F_r + r \dot{\phi} F_t. \quad (11)$$

At the magnetic equator  $\mathbf{B} = B \hat{\mathbf{r}} \times \hat{\mathbf{t}}$ . In this case the radial component of the Lorentz force is

$$\mathbf{F}_r = \hat{\mathbf{r}} \cdot \frac{Q}{m} \left( \frac{N}{dt} \mathbf{r} - \boldsymbol{\omega}_e \times \mathbf{r} \right) \times \mathbf{B} = \frac{Q}{m} B r (\dot{\phi} - \omega_e), \quad (12)$$

And the tangential component of the Lorentz force is

$$\mathbf{F}_t = \hat{\mathbf{t}} \cdot \left[ \frac{Q}{m} \left( \frac{N}{dt} \mathbf{r} - \boldsymbol{\omega}_e \times \mathbf{r} \right) \times \mathbf{B} \right] = \frac{QB}{m} \dot{r}. \quad (13)$$

The signs of  $Q$  and  $B$  are important her: positive charge (due to, say electron emission) and negative (southward) magnetic fields cause a loss in orbital energy, as does negative charge in a positive (northward) magnetic field. Only when  $Q$  and  $B$  are of opposite sign is the energy change positive. Although this result would indicate that greater Lorentz force is available at higher altitude, in fact  $B$  drops off approximately with  $r^{-3}$ ; so that net effect of increased altitude is deleterious.

We shall make use the formulas

$$\begin{aligned} r \dot{\phi} &= a \sqrt{1-e^2} \dot{E}, & \dot{r} &= a e \sin E \dot{E}, \\ \sin \phi &= \sqrt{1-e^2} \frac{a}{r} \sin E, & \cos \phi &= \frac{a}{r} (\cos E - e) \end{aligned} \quad (14)$$

where  $E$  is the eccentric anomaly of the satellite's orbit,  $e$  is the eccentricity of the orbit. Using the relations in (14), we can rewrite the components of the Lorentz force  $\mathbf{F}_L$  in radial and tangential directions as the following:

$$\mathbf{F}_r = \frac{Q}{m} B \left[ a \sqrt{1-e^2} \dot{E} - r \omega_e \right], \quad (15)$$

$$\mathbf{F}_t = \frac{Q}{m} B a e \sin E \dot{E}. \quad (16)$$

The Lagrange Planetary equations:

$$\begin{aligned}
\frac{da}{dt} &= \frac{2}{na} \left( \frac{\partial F}{\partial M} \right), \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e} \left( \sqrt{1-e^2} \frac{\partial F}{\partial M} - \frac{\partial F}{\partial \omega} \right), \\
\frac{dI}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin I} \left( \cos I \frac{\partial F}{\partial \omega} - \frac{\partial F}{\partial \Omega} \right), \\
\frac{d\omega}{dt} &= \frac{1}{na^2} \left( -\frac{\cos I}{\sqrt{1-e^2} \sin I} \frac{\partial F}{\partial I} - \frac{\sqrt{1-e^2}}{e} \frac{\partial F}{\partial e} \right), \\
\frac{d\Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin I} \frac{\partial F}{\partial I}, \\
\frac{dM}{dt} &= n - \frac{1-e^2}{na^2 e} \frac{\partial F}{\partial e} - \frac{2}{na} \frac{\partial F}{\partial a},
\end{aligned} \tag{17}$$

are used to derive the perturbations in the orbital elements, where  $n, I, M, \omega, \Omega$  are the mean motion, inclination, mean anomaly, argument of perigee, ascending node of the satellite's orbit respectively. Therefore, the periodic perturbations in the Keplerian elements due to the Lorentz force are in the following shape.

$$\begin{aligned}
\Delta a &= \frac{da}{dt} = \left( \frac{Q}{m} \right) B \left( \frac{2ae}{nr} \right) \sin E \left[ \sqrt{1-e^2} - \omega_e + \cot E \dot{E} + e \sin E \right], \\
\Delta e &= \frac{de}{dt} = \left( \frac{Q}{m} \right) B \left( \frac{1-e^2}{nr} \right) \sin E \left[ \sqrt{1-e^2} - \omega_e + \cot E \dot{E} + e \sin E \right], \\
\Delta I &= \frac{dI}{dt} = 0, \\
\Delta \omega &= \frac{d\omega}{dt} = - \left( \frac{Q}{m} \right) B \left( \frac{\sqrt{1-e^2}}{na^2 e} \right) \left[ -\frac{2ae}{\sqrt{1-e^2}} \dot{E} + \frac{a\sqrt{1-e^2}}{n} \left( \frac{ae}{r} \sin^2 E - \cos E \right) + \right. \\
&\quad \left. + \omega_e a \cos \phi + a \sin E \dot{E} + \frac{a^2 e}{r} \sin E \cos E \dot{E} + \frac{a^2 e^2}{nr} \sin^3 E - \frac{ae}{n} \sin E \cos E \right], \\
\Delta \Omega &= \frac{d\Omega}{dt} = 0,
\end{aligned} \tag{18}$$

$$\Delta M = \frac{dM}{dt} = n - \left(\frac{Q}{m}\right) B \left\{ \left(\frac{1-e^2}{na^2e}\right) \left[ -\frac{2ae}{\sqrt{1-e^2}} \dot{E} + \frac{a\sqrt{1-e^2}}{n} \left( \frac{ae}{r} \sin^2 E - \cos E \right) + \right. \right. \\ \left. \left. + \omega_e a \cos \phi + a \sin E \dot{E} + \frac{a^2e}{r} \sin E \cos E \dot{E} + \frac{a^2e^2}{nr} \sin^3 E - \frac{ae}{n} \sin E \cos E \right] \right. \\ \left. - \frac{2}{na} \left[ \sqrt{1-e^2} \dot{E} + \frac{3}{2} n^2 a^3 \sqrt{1-e^2} \dot{E} - \omega_e \frac{r}{a} + e \sin E \dot{E} + \frac{3}{2} n^2 a^3 e \sin E \dot{E} \right] \right\}.$$

**RESULTS and DISCUSSION**

As we already stated, The LAGEOS orbital elements were taken as a basis for numerical examples. The electrical charge of the satellite was taken as  $Q = -3 \times 10^{-11} C$ , (see VokRouhlicky, 1989). The elements of the LAGEOS satellite are summarized in the table 1.

Table 1. Parameter and elements of LAGEOS satellite under the computation.

Semi-major axis	12270 km
Eccentricity	0.004
Inclination	109.9 deg
Mass of satellite	407 kg
Charge	$3 \times 10^{-11} C$

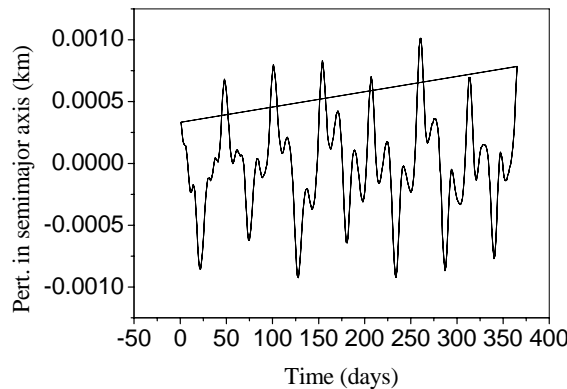


FIGURE 1. Perturbations in the semimajor axis due to Lorentz force.

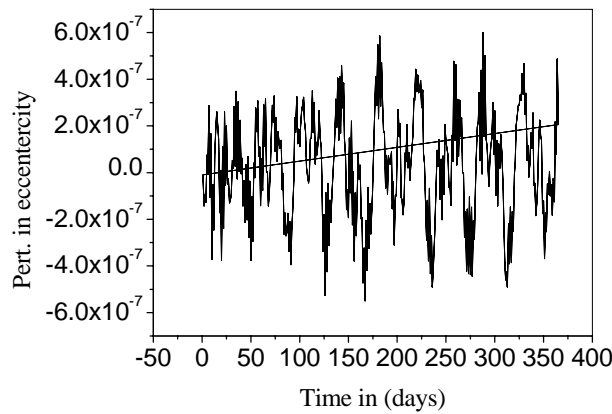


FIGURE 2. Perturbations in the eccentricity due to Lorentz force.

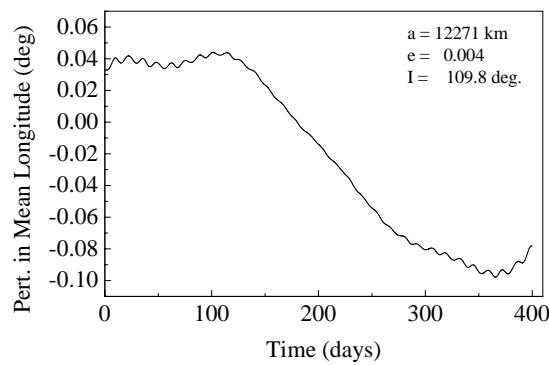


FIGURE 3. Perturbations in the longitude of the satellite orbit due to Lorentz force.

Figure 1, presents the periodic perturbations in the semi-major axis due to the Lorentz force, notice that Lorentz force change the semi-major axis between  $-0.001$  and  $0.001$  km in a period of 400 days, it may help us to explain the reason of the shift in LAGEOS position about 1 millimetre per day. Figure 2, presents the effects of the Lorentz force on the eccentricity, which describe these effects as a periodic perturbations in the eccentricity in a level between  $-6 \times 10^{-7}$  and  $6 \times 10^{-7}$  after 400 days. Figure 3, presents the influence of the Lorentz force on the mean longitude of the satellite  $\lambda = \omega + \Omega + M$ , which has secular perturbations from 0.04 to -0.1 deg after 400 days.

## CONCLUSION

The components of the Lorentz force acceleration of a charged satellite in the Earth's magnetic field are computed as a function of the orbital elements of the satellite in two directions (radial and tangential). Periodic perturbations in the orbital elements of the satellite due to the Lorentz force are derived using Lagrange planetary equations. Numerical applications for the LAGOS Satellite are introduced. These Numerical results shown that the most effects of Lorentz force are in the semimajor axis, eccentricity, and the longitude of the satellite, but there aren't any effects of the Lorentz force on the inclination and the argument of perigee of the orbit. The Lorentz force that derived in this work may help us to explain the reason of the shift in LAGEOS position about 1 millimetre per day.

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