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Lorenz dominance and non-welfaristic redistribution*

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Abstract

Given the decision to implement a non-welfaristic redistribution scheme, we analyze which mechanisms are favored by traditional Lorenz dominance and poverty dominance adherents. We show that for large classes of income functions Lorenz dominance results can be found in the comparison of two egalitarian equivalent mechanisms. Comparisons of different conditionally egalitarian mechanisms only yield poverty dominance results. Finally, certain egalitarian equivalent mechanisms can Lorenz dominate all conditionally egalitarian mechanisms. Our analysis stresses the need for accurate empirical estimates of the pre-tax income function and of the distributions of responsibility and compensation characteristics.

Keywords: non-welfaristic redistribution, Lorenz dominance
JEL classification: D31, D63, H21, I32

1 Introduction

Supported by numerous empirical applications (e.g. Kakwani (1984)), the traditional analysis of redistribution mechanisms has concentrated on the study of income distributions using concepts such as Lorenz dominance (following fundamental contributions of Atkinson (1970) and Shorrocks (1983)), the with Lorenz dominance related issue of progressivity (starting with Jakobsson (1976), Fellman (1976) and Kakwani (1977)) and poverty (following pioneering work by Sen (1976)).

Recently, non-welfaristic income redistribution schemes are presented (Fleurbaey (1994, 1995), Bossert (1995), Bossert and Fleurbaey (1996)). These axiomatically founded mechanisms are designed to fulfil to different degrees the fairness goal of compensating individuals for factors beyond their responsibility.

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In that, they follow ideas of political philosophers like Dworkin (1981a, 1981b), Arneson (1989) and Cohen (1989), who motivate egalitarians to criticize traditional welfaristic theory, where the objective of the government is to maximize a social welfare function based on the aggregation of individual utilities only, for ignoring the issue of personal responsibility. Following Cohen, different outcomes should only be equalized when they are due to factors beyond control of the individual. Conversely, differences due to differential effort are morally acceptable. These opinions have smoothed the path for non-welfaristic theory, where one has to collect non-utility information such as expended effort to express a social judgement. However, these equal opportunity-considerations have given rise to the compensation problem (extensively surveyed in Fleurbaey and Maniquet (2003)). Within the context of first best income redistribution and without separability assumptions on the pre-tax income function, no solution can at the same time assure equal incomes to individuals with equal responsibility characteristics, while guaranteeing equal transfers to individuals with equal compensation characteristics. The redistribution mechanisms of Bossert and Fleurbaey (1996), namely the *egalitarian equivalent mechanism* and the *conditionally egalitarian mechanism*, following Pazner and Schmeidler (1978) and Fleurbaey (1995), are derived from relaxing either one of these axioms.

Today, non-welfaristic theory still has a hard time convincing traditional analysts and hence takes up a relatively small share of normative public economics. However, both empirical work (e.g. Schokkaert (1999)) and real life examples show that these non-welfaristic concepts are interesting from a policy viewpoint, since in most western democracies people seem sensitive to the basis on which redistribution is performed. The unpopularity could be due to the fact that most of the theory is predominantly first best (for second best versions of the egalitarian equivalent and the conditionally egalitarian solutions, see Bossert et al. (1999)) or because the implications of the implementation of non-welfaristic redistribution schemes in terms of the traditional concepts are not yet clear.

Our paper aims at contributing to that latter need. Revolving around the comparison of income distributions, we start from the viewpoint of a social planner, primarily concerned about the size of mean income and how unequally incomes are distributed around this mean. However, instead of comparing real world income distributions, we study the income distributions resulting from the implementation of the egalitarian equivalent mechanism and the conditionally egalitarian mechanism. Clearly, the priorities of our social planner conflict those of her non-welfaristic opponent: while the former favors the non-welfaristic redistribution mechanism that implements the most equal income distribution, the latter considers income differences after non-welfaristic redistribution, representing the reward of showing higher responsibility, perfectly justifiable. In that sense, our analysis could be interpreted as a traditional look into non-welfaristic income redistribution: given the decision to implement a non-welfaristic redistribution scheme, we analyze which mechanisms are favored by traditional Lorenz dominance and poverty dominance adherents. However, since a comparison of different non-welfaristic redistribution schemes essentially amounts to a comparison of different reward patterns to responsibility (see Fleurbaey (1996) for a related discussion), we believe that also non-welfarists can benefit from this analysis.

This paper proceeds as follows. In section 2 we introduce the pre-tax income function, discuss the goal of non-welfaristic redistribution, present the egalitarian equivalent mechanism and the conditionally egalitarian mechanism and suggest the criterion of Lorenz dominance to compare different income distributions. In section 3 we impose some minimal assumptions on the pre-tax income function in order to keep the analysis as general as possible. Without making distributional assumptions on individual's responsibility and compensation parameters, we compare the income distributions of two different egalitarian equivalent mechanisms, two different conditionally egalitarian mechanisms and an egalitarian equivalent mechanism versus a conditionally egalitarian mechanism. We check whether under one of the two mechanisms both the poorest do not get poorer while the richest do not get richer. After taking this necessary condition for Lorenz dominance into consideration, only the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism remains undecided. Additional information is needed, so in section 4 we present a specific example, which illustrates the results derived in the previous section but at the same time allows us to draw more precise conclusions: possible poverty dominance between two conditionally egalitarian mechanisms is examined and also the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism is revisited. Section 5 concludes by pointing out which non-welfaristic redistribution mechanisms are attractive for traditional analysts.

2 Non-welfaristic redistribution schemes

The model presented here is taken from Bossert (1995). Suppose that in a population $N = \{1, ..., n\}$, $n \geq 2$, person i's ($i \in N$) individual characteristics vector $\mathbf{a}_i$ can be partitioned into two vectors: $\mathbf{a}_i^R \in \Omega_R$, representing the individual's responsibility characteristics, and $\mathbf{a}_i^S \in \Omega_S$, representing her compensation characteristics. The set of all possible characteristics vectors is $\Omega = \Omega_R \times \Omega_S$, where $\Omega_R \subseteq \mathbb{R}^r$, $\Omega_S \subseteq \mathbb{R}^s$ and Ω_R, $\Omega_S \neq \emptyset$. The characteristics profile is given by $\mathcal{A} = (\mathbf{a}_1, ..., \mathbf{a}_n) \in \Omega^n$. Denote $f : \Omega \to \mathbb{R}_{++} : \mathbf{a} = (\mathbf{a}^R, \mathbf{a}^S) \to f(\mathbf{a})$ an income function mapping, assigning pre-tax income to each possible characteristics vector and let $\mathcal{F}$ be the set of all possible pre-tax income functions. An economy is described by $e = (\mathcal{A}, f)$. A redistribution mechanism $F : (\Omega^n, \mathcal{F}) \to \mathbb{R}^n : e \to [F_1(\mathcal{A}), ..., F_n(\mathcal{A})]$ maps an economy into an income distribution.

The goal of non-welfaristic income redistribution is to preserve the effect of responsibility characteristics and to eliminate the influence of compensation characteristics. Revelatory work of Fleurbaey (1994, 1995) and Bossert (1995) elucidates the compensation problem: unless the income function is additively separable in $\mathbf{a}^R$ and $\mathbf{a}^S$, no redistribution mechanism can only but fully compensate for differentials in $\mathbf{a}^S$. Bossert and Fleurbaey (1996) relax some of the axioms underlying this impossibility result to derive the two solutions at the heart of our research, the *egalitarian equivalent* mechanism F^{EE} and the *conditionally egalitarian* mechanism F^{CE}. These mechanisms are designed to fully accomplish one of the two goals (respectively compensation and responsibility), while the other goal is only fulfilled for a so called 'reference' vector. F^{EE} and F^{CE} are first best redistribution schemes, i.e. redistribution is supposed to be possible without loss of total income: $\sum_{i=1}^{n} f(\mathbf{a}_i) = \sum_{i=1}^{n} F_i^{EE}(\mathcal{A}) = \sum_{i=1}^{n} F_i^{CE}(\mathcal{A})$.

Our purpose is to evaluate these redistribution mechanisms, using the traditional concept of Lorenz dominance. The following subsections explain F^{EE}, F^{CE} and the concept of Lorenz dominance in more detail.

2.1 The egalitarian equivalent mechanism

F^{EE} gives an individual k the following income:

$$Y_k^{EE} = f(\mathbf{a}_k^R, \tilde{\mathbf{a}}^S) - \frac{1}{n}\sum_{i=1}^{n}\left(f(\mathbf{a}_i^R, \tilde{\mathbf{a}}^S) - f(\mathbf{a}_i^R, \mathbf{a}_i^S)\right) \tag{1}$$

where $\tilde{\mathbf{a}}^S \in \Omega_S$ is the reference compensation characteristics vector.

With this mechanism, every agent has a post-tax income equal to the pre-tax income she would earn if her compensation characteristics were $\tilde{\mathbf{a}}^S$, plus a uniform transfer. Two egalitarian equivalent mechanisms only differ in the choice of $\tilde{\mathbf{a}}^S$. This choice determines the reward scheme for responsibility, i.e. it determines the magnitude of income differences due to differences in $\mathbf{a}^R$.

F^{EE} satisfies the strong compensation axiom of 'group solidarity in $\mathbf{a}^S$': any variation in some agent's compensation characteristics is equally borne by all agents. At the same time, F^{EE} only satisfies the mild responsibility axiom of 'equal transfer for reference $\tilde{\mathbf{a}}^S$'. This implies that there is no reason to perform any redistribution only when everybody's compensation characteristics are equal to the reference vector.

2.2 The conditionally egalitarian mechanism

F^{CE} gives an individual k the following income:

$$Y_k^{CE} = f(\mathbf{a}_k^R, \mathbf{a}_k^S) - f(\tilde{\mathbf{a}}^R, \mathbf{a}_k^S) + \frac{1}{n}\sum_{i=1}^{n} f(\tilde{\mathbf{a}}^R, \mathbf{a}_i^S) \tag{2}$$

where $\tilde{\mathbf{a}}^R \subseteq \Omega_R$ is the reference responsibility characteristics vector.

With this mechanism, every agent has a post-tax income equal to the average pre-tax income that would prevail if everyone in society has $\mathbf{a}^R = \tilde{\mathbf{a}}^R$. If the agent deviates from this reference level, she alone bears the resulting difference. Two conditionally egalitarian mechanisms only differ in the choice of $\tilde{\mathbf{a}}^R$. This choice determines the magnitude of differences in transfers due to differences in $\mathbf{a}^S$.

F^{CE} only satisfies the mild compensation axiom of 'equal income for $\tilde{\mathbf{a}}^R$': income equality is only required if all agents have responsibility characteristics equal to some reference value. F^{CE} also satisfies the strong responsibility axiom of 'individual monotonicity in $\mathbf{a}^R$'. This means that a change in one agent's responsibility characteristics only affects this person's post-tax income.

2.3 Lorenz dominance

Since total income does not change after the implementation of an egalitarian equivalent mechanism or a conditionally egalitarian mechanism, our social planner favors the non-welfaristic redistribution mechanism that implements the most equal income distribution. Therefore, the criterion of Lorenz dominance suggests itself for the analysis. An income distribution A Lorenz dominates an income distribution B when the bottom $100p$ per cent of income units in A have a greater share of total income than in B and this holds true for every p between 0 and 1. In other words, Lorenz dominance amounts to second order stochastic dominance of the cumulative income distribution function of A, $G^A(x)$, over the cumulative income distribution function of B, $G^B(x)$. Formally, $S(Y) = \int_{Y_{\min}}^{Y} (G^B(x) - G^A(x))dx \geq 0$, for all $Y \in [Y_{\min}, Y_{\max}]$. A necessary condition for Lorenz dominance requires that, at the same time, the poorest are not poorer and the richest are not richer under distribution A compared to distribution B. When results in terms of Lorenz dominance cannot be drawn, attention goes to the poorest. The criterion of poverty dominance, which searches Lorenz dominance over all incomes below some poverty line Z, may tell which distribution has less poverty. Formally, $S(Y) \geq 0$, for all $Y \in [Y_{\min}, Z]$.

3 Distributional analysis: general framework

In this section, we state Lorenz dominance results with respect to the different non-welfaristic redistribution schemes in general terms. First, we impose some properties on and define two large classes of pre-tax income functions which we use to define domains for redistribution mechanisms. Successively, we compare the income distributions of two egalitarian equivalent mechanisms with different reference compensation parameters, the income distributions of two conditionally egalitarian mechanisms with different reference responsibility parameters and the income distributions of an egalitarian equivalent mechanism and a conditionally egalitarian mechanism. We search for dominance results that hold for all economies in the domains defined. This implies that the results we obtain are valid over all distributions of responsibility and compensation parameters, since no explicit assumptions on these distributions are stated. We conclude this section with some remarks.

3.1 Assumptions on the distribution of a^R and a^S and on the pre-tax income function

We consider the case where a^R and a^S are scalars instead of vectors with support $[a^R_{\min}, a^R_{\max}]$ and $[a^S_{\min}, a^S_{\max}]$ respectively and $a^R_{\min} \neq a^R_{\max}$ and $a^S_{\min} \neq a^S_{\max}$. All distributions of characteristics that have these properties belong to Σ.

The pre-tax income function f is continuously differentiable to the required degree and monotonic in its arguments a^R and a^S.

Furthermore, let

$$\frac{\partial f}{\partial a^R} \geq 0 \tag{3}$$

and[1]

$$\frac{\partial f}{\partial a^S} \geq 0 \tag{4}$$

So for all pre-tax income functions having properties (3) and (4), the poorest have characteristics $(a^R_{\min}, a^S_{\min})$, while the richest have characteristics $(a^R_{\max}, a^S_{\max})$.

Denote by $\mathcal{F}^+$ the class of income functions that satisfy (3), (4) and

$$\frac{\partial^2 f}{\partial a^R \partial a^S} \geq 0 \tag{5}$$

Denote by $\mathcal{F}^-$ the class of income functions that satisfy (3), (4) and[2]

$$\frac{\partial^2 f}{\partial a^R \partial a^S} \leq 0 \tag{6}$$

The goal of this paper is to look for Lorenz dominance results for different non-welfaristic redistribution mechanisms over the following domains:

D^+ : the set of all economies with $\mathbf{a} \in \Sigma$ and whose income function f belongs to $\mathcal{F}^+$.

D^- : the set of all economies with $\mathbf{a} \in \Sigma$ and whose income function f belongs to $\mathcal{F}^-$.

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[1] Assumptions (3) and (4) are a matter of measurement. If, for example, we only have information on handicaps, assumption (4) can be satisfied by redefining the information in terms of '*lack of* ... (handicap)'.

[2] Remark that income functions that belong to both $\mathcal{F}^+$ and $\mathcal{F}^-$ are additively separable in a^R and a^S. In that case both the egalitarian equivalent mechanism and the conditionally egalitarian mechanism reduce to 'the natural redistribution mechanism', which assigns the income due to an individual's a^R entirely to that agent and divides the total income due to a^S equally among all agents (Bossert, 1995). The choice of a reference skill level or reference preferences becomes irrelevant.

3.2 Two egalitarian equivalent mechanisms

First, the following lemma identifies the poorest and richest individuals under any egalitarian equivalent mechanism:

Lemma 1 *: For all economies with* $\mathbf{a} \in \Sigma$ *and* f *satisfying (3), under any egalitarian equivalent mechanism, the poorest individuals have responsibility characteristic* $a^R_{\min}$ *and the richest individuals have responsibility characteristics* $a^R_{\max}$*, irrespective of their compensation characteristics* a^S*.*

Proof. : Since an egalitarian equivalent mechanism exists of an individual specific part $f(a^R_k, \tilde{a}^S)$ plus a uniform transfer, income is increasing in a^R. Income differences do not depend on differences in a^S. ∎

We state the following proposition:

Proposition 1 *: For all economies in* D^+ (D^-)*, an egalitarian equivalent mechanism with reference compensation characteristic* $\tilde{a}^S_1$ $(\tilde{a}^S_2)$ *Lorenz dominates an egalitarian equivalent mechanism with reference compensation characteristic* $\tilde{a}^S_2$ $(\tilde{a}^S_1)$ *if and only if* $\tilde{a}^S_1 \leq \tilde{a}^S_2$ *.*

Proof. : Compare two egalitarian equivalent mechanisms with reference compensation characteristics $\tilde{a}^S_1$ and $\tilde{a}^S_2$ ($\tilde{a}^S_1 \leq \tilde{a}^S_2$; $\tilde{a}^S_1, \tilde{a}^S_2 \in [a^S_{\min}, a^S_{\max}]$). Using (1), the income difference for an individual k between the two mechanisms equals:

$Y^{EE,\tilde{a}^S_1}_k - Y^{EE,\tilde{a}^S_2}_k = f(a^R_k, \tilde{a}^S_1) - f(a^R_k, \tilde{a}^S_2) + A = \phi(a^R_k)$

where $A = \left(\frac{1}{n}\sum_{i=1}^n f(a^R_i, \tilde{a}^S_2) - \frac{1}{n}\sum_{i=1}^n f(a^R_i, \tilde{a}^S_1)\right)$.

Redistribution in the first best means that:

$\sum_{i=1}^n \phi(a^R_i) = 0.$ $\quad (*)$

By (4) and ($\tilde{a}^S_1 \leq \tilde{a}^S_2$): $A \geq 0$ and $f(a^R_k, \tilde{a}^S_1) - f(a^R_k, \tilde{a}^S_2) \leq 0.$ $\quad (**)$

For D^+:

by (5), $\phi(a^R_k)$ is decreasing in a^R_k. $\quad (***)$

$(*)$, $(**)$ and $(***)$ make that $\exists a^R_+(\tilde{a}^S_1, \tilde{a}^S_2) \in \mathbb{R}$, $a^R_{\min} < a^R_+ < a^R_{\max}$:

$a^R_k \leq a^R_+ \Rightarrow \phi(a^R_k) \geq 0$: these individuals receive at least the same income under $\tilde{a}^S_1$ than under $\tilde{a}^S_2$.

$a^R_k \geq a^R_+ \Rightarrow \phi(a^R_k) \leq 0$: these individuals receive at most the same income under $\tilde{a}^S_1$ than under $\tilde{a}^S_2$.

So there are transfers of income from individuals with a high a^R to individuals with a low a^R. Since an egalitarian equivalent mechanism exists of an individual specific part $f(a^R_k, \tilde{a}^S)$ plus a uniform transfer, income is increasing in a^R. This implies that all transfers go from rich to poor. As a result the income distribution of the egalitarian equivalent mechanism with reference compensation characteristic $\tilde{a}^S_1$ Lorenz dominates the income distribution of the egalitarian equivalent mechanism with reference compensation characteristic $\tilde{a}^S_2$.

For D^-:

by (6), $\phi(a_k^R)$ is increasing in a_k^R. $\quad (****)$

$(*)$, $(**)$ and $(****)$ make that $\exists a_-^R(\tilde{a}_1^S, \tilde{a}_2^S) \in \mathbb{R}$, $a_{\min}^R < a_-^R < a_{\max}^R$:

$a_k^R \leq a_-^R \Rightarrow \phi(a_k^R) \leq 0$: these individuals receive at most the same income under $\tilde{a}_1^S$ than under $\tilde{a}_2^S$.

$a_k^R \geq a_-^R \Rightarrow \phi(a_k^R) \geq 0$: these individuals receive at least the same income under $\tilde{a}_1^S$ than under $\tilde{a}_2^S$.

So there are transfers of income from individuals with a low a^R to individuals with a high a^R, that is all transfers go from poor to rich. As a result the income distribution of the egalitarian equivalent mechanism with reference compensation characteristic $\tilde{a}_2^S$ Lorenz dominates the income distribution of the egalitarian equivalent mechanism with reference compensation characteristic $\tilde{a}_1^S$. ■

Note that proposition 1 is instructive for the choice of reference compensation characteristic $\tilde{a}^S$, since this choice determines the inequality in the resulting income distribution.

3.3 Two conditionally egalitarian mechanisms

First, the following lemma identifies the poorest and richest individuals under any conditionally egalitarian mechanism:

Lemma 2 : *For all economies in D^+, under any conditionally egalitarian mechanism, the poorest individuals have characteristics ($a_{\min}^R$, $a_{\max}^S$) and the richest individuals have characteristics ($a_{\max}^R$, $a_{\max}^S$). For all economies in D^-, under any conditionally egalitarian mechanism, the poorest individuals have characteristics ($a_{\min}^R$, $a_{\min}^S$) and the richest individuals have characteristics ($a_{\max}^R$, $a_{\min}^S$).*

Proof. : For D^+ or D^-, employing (3), $\frac{\partial Y_k^{CE}}{\partial a_k^R} = \frac{\partial f}{\partial a_k^R} \geq 0$ (see (2)), so the poorest have responsibility characteristic $a_{\min}^R$ and the richest have responsibility characteristic $a_{\max}^R$. For D^+, due to (5), for those individuals with $a_{\min}^R$, $\frac{\partial Y_k^{CE}}{\partial a_k^S} = \frac{\partial f}{\partial a_k^S}|_{a_{\min}^R} - \frac{\partial f}{\partial a_k^S}|_{\tilde{a}^R} < 0$, while for those individuals with $a_{\max}^R$, $\frac{\partial Y_k^{CE}}{\partial a_k^S} = \frac{\partial f}{\partial a_k^S}|_{a_{\max}^R} - \frac{\partial f}{\partial a_k^S}|_{\tilde{a}^R} > 0$, implying that both the poorest and the richest have compensation characteristics $a_{\max}^S$. For D^-, due to (6), for those individuals with $a_{\min}^R$, $\frac{\partial Y_k^{CE}}{\partial a_k^S} = \frac{\partial f}{\partial a_k^S}|_{a_{\min}^R} - \frac{\partial f}{\partial a_k^S}|_{\tilde{a}^R} > 0$, while for those individuals with $a_{\max}^R$, $\frac{\partial Y_k^{CE}}{\partial a_k^S} = \frac{\partial f}{\partial a_k^S}|_{a_{\max}^R} - \frac{\partial f}{\partial a_k^S}|_{\tilde{a}^R} < 0$, implying that both the poorest and the richest have compensation characteristics $a_{\min}^S$. ■

From lemma 2 and lemma 1, note that for all economies in D^+ or D^- the poorest and the richest under a conditionally egalitarian mechanism are also among the poorest and richest under an egalitarian equivalent mechanism.

We state the following proposition:

Proposition 2 *: There is no Lorenz dominance between two different conditionally egalitarian mechanisms for all economies in D^+ or D^-.*

Proof. : Compare two conditionally egalitarian mechanisms with reference preferences $\tilde{a}_1^R$ and $\tilde{a}_2^R$ ($\tilde{a}_1^R \leq \tilde{a}_2^R$; $\tilde{a}_1^R, \tilde{a}_2^R \in [a_{\min}^R, a_{\max}^R]$). Using (2), the income difference for an individual k between the two mechanisms equals:

$Y_k^{CE,\tilde{a}_1^R} - Y_k^{CE,\tilde{a}_2^R} = f(\tilde{a}_2^R, a_k^S) - f(\tilde{a}_1^R, a_k^S) + B = \varphi(a_k^S)$

where $B = \left(\frac{1}{n}\sum_{i=1}^n f(\tilde{a}_1^R, a_i^S) - \frac{1}{n}\sum_{i=1}^n f(\tilde{a}_2^R, a_i^S)\right)$.

Redistribution in the first best means that:

$\sum_{i=1}^n \varphi(a_i^S) = 0. \qquad (*)$

By (3) and ($\tilde{a}_1^R \leq \tilde{a}_2^R$): $B \leq 0$ and $f(\tilde{a}_2^R, a_k^S) - f(\tilde{a}_1^R, a_k^S) \geq 0. \qquad (**)$

For D^+:

by (5), $\varphi(a_k^S)$ is increasing in a_k^S. $\qquad (***)$

$(*)$, $(**)$ and $(***)$ make that $\exists a_+^S(\tilde{a}_1^R, \tilde{a}_2^R) \in \mathbb{R}$, $a_{\min}^S < a_+^S < a_{\max}^S$:

$a_k^S \leq a_+^S \Rightarrow \varphi(a_k^S) \leq 0$: these individuals receive at most the same income under $\tilde{a}_1^R$ than under $\tilde{a}_2^R$.

$a_k^S \geq a_+^S \Rightarrow \varphi(a_k^S) \geq 0$: these individuals receive at least the same income under $\tilde{a}_1^R$ than under $\tilde{a}_2^R$.

So there are transfers of income from individuals with a low a^S to individuals with a high a^S. From lemma 2, the poorest and the richest individuals gain from the transfers, making Lorenz dominance impossible.

For D^-:

by (6), $\varphi(a_k^S)$ is decreasing in a_k^S. $\qquad (****)$

$(*)$, $(**)$ and $(****)$ make that $\exists a_-^S(\tilde{a}_1^R, \tilde{a}_2^R) \in \mathbb{R}$, $a_{\min}^S < a_-^S < a_{\max}^S$:

$a_k^S \leq a_-^S \Rightarrow \varphi(a_k^S) \geq 0$: these individuals receive at least the same income under $\tilde{a}_1^R$ than under $\tilde{a}_2^R$.

$a_k^S \geq a_-^S \Rightarrow \phi(a_k^S) \leq 0$: these individuals receive at most the same income under $\tilde{a}_1^R$ than under $\tilde{a}_2^R$.

So there are transfers of income from individuals with a high a^S to individuals with a low a^S. From lemma 2, the poorest and the richest individuals gain from the transfers, making Lorenz dominance impossible. ■

Remark that for all economies in D^+ or D^- the poorest gain income from the change of a conditionally egalitarian mechanism with a higher $\tilde{a}^R$ to a conditionally egalitarian mechanism with a lower $\tilde{a}^R$. This suggest that results in terms of poverty dominance might be drawn, but more assumptions on the pretax income function are needed. We come back to this issue within our specific framework in paragraph 4.2.

3.4 EE versus CE

We state the following proposition:

Proposition 3 *: Apart from one unique exception*[3]*, a conditionally egalitarian mechanism does not Lorenz dominate an egalitarian equivalent mechanism for all economies in D^+ or D^-.*

Proof. : Compare an egalitarian equivalent mechanism with reference compensation characteristic $\tilde{a}^S$ with a conditionally egalitarian mechanism with reference preferences $\tilde{a}^R$ ($\tilde{a}^S \in [a^S_{\min}, a^S_{\max}]$, $\tilde{a}^R \in [a^R_{\min}, a^R_{\max}]$). Using (1) and (2), the income difference between the two mechanisms for an individual k equals:

$Y_k^{EE,\tilde{a}^S} - Y_k^{CE,\tilde{a}^R} = C + E = \psi(a_k^R, a_k^S)$

where $C = f(a_k^R, \tilde{a}^S) + f(\tilde{a}^R, a_k^S) - f(a_k^R, a_k^S)$ and $E = \frac{1}{n}\sum_{i=1}^n f(a_i^R, a_i^S) - \frac{1}{n}\sum_{i=1}^n f(a_i^R, \tilde{a}^S) - \frac{1}{n}\sum_{i=1}^n f(\tilde{a}^R, a_i^S)$.

Redistribution in the first best means that:

$\sum_{i=1}^n \psi(a_i^R, a_i^S) = 0$.

E is the same for all individuals. Without further assumptions on the pre-tax income function, E can be either positive or negative. Anyhow, the individuals that gain from the change of a conditionally egalitarian mechanism to an egalitarian equivalent mechanism have a larger C than those who lose from the regime change.

What happens with the income of the poorest and the richest under a conditionally egalitarian mechanism after an egalitarian equivalent mechanism is implemented? Divide the population in two groups: group 1 comprises all individuals with $a_k^S \leq \tilde{a}^S$, group 2 all individuals with $a_k^S \geq \tilde{a}^S$. From lemma 2, both belong to group 2 for all economies in D^+ and to group 1 for all economies in D^-.

For D^+:

by (5), $\frac{\partial C}{\partial a_k^R} = \frac{\partial f(a_k^R, \tilde{a}^S)}{\partial a_k^R} - \frac{\partial f(a_k^R, a_k^S)}{\partial a_k^R} \leq 0$ for group 2.

$\exists \grave{a}_+^R(\tilde{a}^R, \tilde{a}^S, a^S_{\max}) \in \mathbb{R}$:

$a_k^R \leq \grave{a}_+^R \Rightarrow \psi(a_k^R, a^S_{\max}) \geq 0$: these individuals receive at least the same income under *EE* than under *CE.*

$a_k^R \geq \grave{a}_+^R \Rightarrow \psi(a_k^R, a^S_{\max}) \leq 0$: these individuals receive at most the same income under *EE* than under *CE.*

[3] For D^+ (D^-), this unique exception is the case when the EE-mechanisms has $\tilde{a}^S = a^S_{\max}$ ($a^S_{\min}$) and the CE-mechanism has $\tilde{a}^R$ chosen such that $f(\tilde{a}^R, \tilde{a}^S) = E$. In this specific comparison, the income of the poorest and the richest remains unchanged under both mechanisms. In order to derive Lorenz dominance results, further assumptions on the distributions of responsibility and compensation characteristics have to be made. We illustrate this specific exception in the numerical example in paragraph 4.3.

For D^-:

by (6), $\frac{\partial C}{\partial a_k^R} = \frac{\partial f(a_k^R, \tilde{a}^S)}{\partial a_k^R} - \frac{\partial f(a_k^R, a_k^S)}{\partial a_k^R} \leq 0$ for group 1.

$\exists \grave{a}_-^R(\tilde{a}^R, \tilde{a}^S, a_{\min}^S) \in \mathbb{R}$:

$a_k^R \leq \grave{a}_-^R \Rightarrow \psi(a_k^R, a_{\min}^S) \geq 0$: these individuals receive at least the same income under *EE* than under *CE.*

$a_k^R \geq \grave{a}_-^R \Rightarrow \psi(a_k^R, a_{\min}^S) \leq 0$: these individuals receive at most the same income under *EE* than under *CE.*

If respectively $\grave{a}_+^R, \grave{a}_-^R \in [a_{\min}^R, a_{\max}^R]$, the poorest do not become poorer and the richest do not become richer under *EE* than under *CE.* Therefore, the necessary condition for Lorenz dominance of an egalitarian equivalent mechanism over a conditionally egalitarian mechanism is fulfilled, implying that the necessary condition for Lorenz dominance of a conditionally egalitarian mechanism over an egalitarian equivalent mechanism is violated, proving the proposition. However, this necessary condition cannot be assumed to hold generally without further assumptions on the pre-tax income function. If $\grave{a}_+^R, \grave{a}_-^R \notin [a_{\min}^R, a_{\max}^R]$, the incomes of the poorest and the richest change in the same direction making Lorenz dominance results between egalitarian equivalent mechanisms and conditionally egalitarian mechanisms impossible. We come back to this issue in paragraph 4.3. ■

3.5 Remarks

Remark that the validity of propositions 1-3 only holds for all economies in D^+ or D^-, in the case where the responsibility and compensation parameter is one-dimensional. Extending these limiting assumptions greatly enlarges the difficulty to obtain clear solutions.

Within the case where a^R or a^S are scalars, Lorenz dominance results are hard to establish. Extensions to the case where $\mathbf{a}^R$ and $\mathbf{a}^S$ are vectors lead to even fewer dominance results. Under multidimensional versions of (3), (4) and (5) or (6), requiring that the derivatives of the pre-tax income function with respect to every element in the $\mathbf{a}^R$- and $\mathbf{a}^S$-vectors are positive and the cross derivatives have equal sign, an unambiguous comparison of two different egalitarian equivalent mechanisms along the lines of proposition 1 can only be made when every element in the reference skill vector $\tilde{\mathbf{a}}^S$ changes in the same direction.

Restricting the analysis to income functions in $\mathcal{F}^+$ and $\mathcal{F}^-$ drives the positive result of proposition 1. A natural question is whether we can derive Lorenz dominance results for the comparison of two conditionally egalitarian mechanisms by restricting the analysis to other classes of income functions. Suppose that the income function still satisfies (3) and (4) but $\frac{\partial^2 f}{\partial a^R \partial a^S}$ changes sign: $\frac{\partial^2 f}{\partial a^R \partial a^S} \leq 0$ for small values of a^R, $\frac{\partial^2 f}{\partial a^R \partial a^S} \geq 0$ for large values of a^R and $\frac{\partial^3 f}{\partial^2 a^R \partial a^S} \geq 0$. Denote this class of income functions $\mathcal{F}^\pm$. The poorest gain income and the richest lose income if the $\tilde{a}^R$ of the new conditionally egalitarian mechanism is lower than $\tilde{a}^R$ of the old conditionally egalitarian mechanism. Indeed, the transfers going to the poorest increase as $\tilde{a}^R$ decreases, while the transfers going to the richest decrease as $\tilde{a}^R$ decreases. However, this result only holds if, under both mechanisms, the poorest have characteristics $(a^R_{\min}, a^S_{\min})$ and the richest have characteristics $(a^R_{\max}, a^S_{\max})$. This does not apply for all income functions in $\mathcal{F}^{\pm}$, making Lorenz dominance results only possible over suitably defined domains.

4 Distributional analysis: example

In this section, we illustrate our previous results with an example, based on a framework adopted in Schokkaert et al. (2003), following Atkinson (1995). Closed form solutions for the pre-tax income function and the different non-welfaristic redistribution schemes are derived. We show that the pre-tax income function belongs to $\mathcal{F}^{+}$, which automatically enables us to draw a corollary from proposition 1, since we assume that the distributional assumptions of domain D^+ also hold true. Despite the fact that we look at one particular income function and hence use very detailed information about it, we show that the no-dominance results of proposition 2 and proposition 3 remain. However, this example enables us to draw more precise results with respect to poverty dominance between two conditionally egalitarian mechanisms. In particular, an upper bound poverty line is determined below which poverty is unambiguously reduced under one of the two regimes. Finally, the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism is revisited. We show exactly which egalitarian equivalent mechanisms are eligible to Lorenz dominate which conditionally egalitarian mechanisms in our example. Additionally, a small numerical example proofs the necessity of making assumptions on the marginal distributions of individual characteristics.

4.1 Responsibility versus compensation

Suppose individuals differ in only two dimensions. The first dimension is their skill level w, assumed to be beyond their control as it is completely determined by their genetic endowment. The second variable is a preference parameter e, meant to capture a pure preference for leisure. Compensation is desirable for differences in w, while at the same time individuals can be held responsible for differences in e. We suppose that both variables have finite support: the preference parameter and the skill level are measured such that $0 < e_L \leq e \leq 1$ and $0 < w_L \leq w \leq 1$ respectively, but we exclude the possibility that $e_L = 1$ and $w_L = 1$. For convenience[4], we assume that e and w are distributed independently with density functions $g_w(w) : [w_L, 1] \to \mathbb{R}$ and $g_e(e) : [e_L, 1] \to \mathbb{R}$. All distributions of characteristics that satisfy these properties belong to Σ°, which is equivalent to Σ with a continuum of individuals and the additional assumption of independence.

[4]Furthermore, it is doubtful that agents should be held responsible for characteristics which are determined by characteristics that require compensation.

4.1.1 The pre-tax income function belongs to $\digamma^{+}$

In our first best setting, we rule out behavioural responses: every individual chooses her labor supply as if there were no redistribution, or alternatively the government has complete information on individual behaviour and is able to enforce this behaviour.

Suppose labor supply is iso-elastic:

$$L = e^{\varepsilon} w^{\varepsilon} \tag{7}$$

where ε is the constant elasticity of labor supply, a measure for the efficiency cost of the tax and assumed to be identical for all individuals.

Using (7), pre-tax income f as a function of e and w equals:

$$f(e, w) = wL = e^{\varepsilon} w^{\varepsilon+1} \tag{8}$$

Pre-tax income is increasing in both e and w, implying that the laziest, lowest skilled people (e_L, w_L) are poorest. The technical rate of substitution $\frac{\partial w}{\partial e}|_{f()=cte}$ equals $-\frac{\varepsilon}{(\varepsilon+1)}\frac{w}{e}$. Since $\frac{\partial f(e,w)}{\partial e}$, $\frac{\partial f(e,w)}{\partial w}$ and $\frac{\partial^2 f(e,w)}{\partial e \partial w}$ are positive, the pre-tax income function belongs to $\digamma^{+}$.

In this section, we search for Lorenz dominance results of different non-welfaristic redistribution mechanisms over the following domain:

D° : the set of all economies with $g_w(w)$, $g_e(e) \in \Sigma^{\circ}$ and $f(e, w)$ given by (8).

4.1.2 The egalitarian equivalent mechanism

Define the α-th moment of a variable x with support $[\underline{x}, \overline{x}]$ as:

$$\mu_{\alpha}(x) = \int_{\underline{x}}^{\overline{x}} x^{\alpha} g_x(x)\, dx \tag{9}$$

Using (8) and (9), the egalitarian equivalent mechanism can be rewritten in a continuous framework. An individual k receives an income:

$$\begin{aligned} Y_k^{EE} &= f(e_k, \tilde{w}) - \int_{e_L}^{1}\int_{w_L}^{1} \left(f(e, \tilde{w}) - f(e, w)\right) g_e(e) g_w(w) de dw \\ &= e_k^{\varepsilon} \tilde{w}^{\varepsilon+1} - \mu_{\varepsilon}(e) \tilde{w}^{\varepsilon+1} + \mu_{\varepsilon}(e) \mu_{\varepsilon+1}(w) \end{aligned} \tag{10}$$

where $\tilde{w}$ denotes the reference skill level, chosen by the social planner[5].

Y^{EE} is increasing in e but Y^{EE} is no longer a function of w. This implies that the laziest ($e = e_L$) are poorest and the hard working ($e = 1$) are richest, regardless their skill level (cfr. lemma 1).

From proposition 1 we have:

Corollary 1 *: For all economies in D°, an egalitarian equivalent mechanism with reference compensation characteristic $\tilde{w}_1$ Lorenz dominates an egalitarian equivalent mechanism with reference compensation characteristic $\tilde{w}_2$ if and only if $\tilde{w}_1 \leq \tilde{w}_2$.*

4.1.3 The conditionally egalitarian mechanism

Using (8) and (9), an individual k receives under a conditionally egalitarian mechanism an income:

$$\begin{aligned} Y_k^{CE} &= f(e_k, w_k) - f(\tilde{e}, w_k) + \int_{w_L}^{1} f(\tilde{e}, w) g_w(w) dw \\ &= e_k^\varepsilon w_k^{\varepsilon+1} - \tilde{e}^\varepsilon w_k^{\varepsilon+1} + \tilde{e}^\varepsilon \mu_{\varepsilon+1}(w) \qquad (11) \end{aligned}$$

where $\tilde{e}$ denotes the reference preference parameter, chosen by the social planner[6].

Y^{CE} is increasing in e. Since $\tilde{e} \geq e_L$, Y^{CE} is no longer increasing in w for all individuals. More precisely, the poorest are laziest ($e = e_L$) and highest skilled ($w = 1$), while the richest are hard working ($e = 1$) and highest skilled ($w = 1$)(cfr. lemma 2). The technical rate of substitution $\frac{\partial w}{\partial e}|_{Y^{CE}=cte}$ equals $-\frac{\varepsilon}{(\varepsilon+1)} w \frac{e^{\varepsilon-1}}{(e^\varepsilon - \tilde{e}^\varepsilon)}$, which is larger than the TRS of the pre-tax income function and is positive for all individuals i with $e_i < \tilde{e}$. Figure 1 depicts iso-Y^{CE} curves for a conditionally egalitarian mechanism with reference preference $\tilde{e}$ ($Y_1^{CE} < Y_2^{CE} < Y_3^{CE}$):

[5] Note that when $\tilde{w} = 0$, Y^{EE} is equally distributed. However, a non-welfaristic social planner will not choose $\tilde{w} = 0$, since this completely eliminates the impact of responsibility on received income, which clearly contrasts the goal of non-welfaristic redistribution. Throughout, we assume that $\tilde{w}$ is chosen between $[w_L, 1]$.

[6] We assume that $\tilde{e}$ is chosen between $[e_L, 1]$. It deserves mentioning that in order to avoid the delicate choice of $\tilde{w}$ or $\tilde{e}$, Fleurbaey (1995) and Bossert and Fleurbaey (1996) also propose average versions of the egalitarian equivalent mechanism and the conditionally egalitarian mechanism (F^{AEE} and F^{ACE} respectively). The idea is to use every value of $w \in [w_L, 1]$ or $e \in [e_L, 1]$ successively as $\tilde{w}$ or $\tilde{e}$ and to give each agent the average of the resulting incomes she would obtain under these different F^{EE} or F^{CE} mechanisms. However, it can be easily shown that in this example F^{AEE} is equivalent to an F^{EE}-mechanism with $\tilde{w} = \left(\mu_{\varepsilon+1}(w)\right)^{\frac{1}{\varepsilon+1}}$ and F^{ACE} is equivalent to a F^{CE}-mechanism with $\tilde{e} = (\mu_\varepsilon(e))^{\frac{1}{\varepsilon}}$.

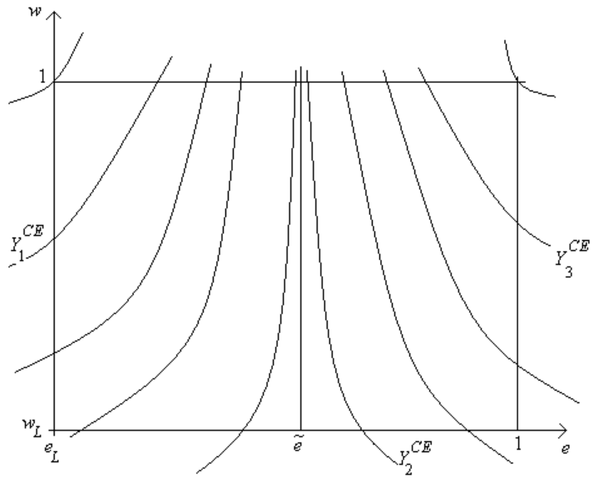

Figure 1 : Iso-Y^{CE} curves for $F_{\tilde{e}}^{CE}$

Note that Y^{CE} will never be equally distributed. Graphically, all individuals would have to lie on the same iso-Y^{CE} curve. This requires correlation between e and w, which violates our independence assumption[7].

Making an income distribution comparison between two conditionally egalitarian mechanisms $F_{\tilde{e}_1}^{CE}$ and $F_{\tilde{e}_2}^{CE}$ with different reference preferences $\tilde{e}_1$ and $\tilde{e}_2$ ($\tilde{e}_1 \leq \tilde{e}_2$) leads to the following conclusion:

- For all economies in D°, there is no Lorenz dominance between two conditionally egalitarian mechanisms with reference preferences $\tilde{e}_1$ and $\tilde{e}_2$.[8]

4.2 Poverty dominance between two conditionally egalitarian mechanisms

Denote $Y_{(e,\, w)}^{CE,\, \tilde{e}}$ the income an individual with characteristics (e, w) receives under a conditionally egalitarian mechanism $F_{\tilde{e}}^{CE}$ with reference preferences $\tilde{e}$. As suggested at the end of paragraph 3.3, the fact that $Y_{(e_L,\, 1)}^{CE,\, \tilde{e}_1} > Y_{(e_L,\, 1)}^{CE,\, \tilde{e}_2}$ suggests that we might arrive at a conclusion in terms of poverty dominance between two different conditionally egalitarian regimes. Specify Z as the poverty line: people with an income below Z are considered poor. A function $\theta(Y|Z)$ measures the poverty of someone with income Y, conditional on the chosen poverty line Z. Aggregate poverty under a certain regime F, given Z, can be written

[7]Remark that Y^{CE} is equally distributed when $e_i = \tilde{e}$ for all i. However, this distribution $\notin \Sigma^{\circ}$.

[8]Proof in Appendix

in canonical form[9] as $P(F|Z) = \int\limits_0^Z \theta(Y|Z) g_Y(Y) dY$. The following proposition can be stated:

Proposition 4 *: For all* $Z \leq e_L^{\varepsilon} \mu_{\varepsilon+1}(w)$*,* $P(F_{\tilde{e}_1}^{CE}|Z) \leq P(F_{\tilde{e}_2}^{CE}|Z)$ *if* $\tilde{e}_1 \leq \tilde{e}_2$*.*

Proof. : The proof amounts to showing that $F_{\tilde{e}_1}^{CE}$ Lorenz dominates $F_{\tilde{e}_2}^{CE}$ over the income-interval $[Y_{(e_L,\, 1)}^{CE,\, \tilde{e}_2},\, Z]$:

First, note that for all preferences within the interval $[e_L,\, \tilde{e}_1]$, the technical rate of substitution of the conditionally egalitarian regime with reference preferences $\tilde{e}_1$ is higher than the technical rate of substitution of the conditionally egalitarian regime with reference preferences $\tilde{e}_2$. Therefore, iso-income curves satisfy the single crossing property over the subspace $[e_L,\, \tilde{e}_1] \times [w_L,\, 1]$.

Second, remark that all individuals with characteristics $(e_L,\, (\mu_{\varepsilon+1}(w))^{\frac{1}{\varepsilon+1}})$ receive an income $Y_*^{CE} = e_L^{\varepsilon} \mu_{\varepsilon+1}(w)$, irrespective of which conditionally egalitarian regime is implemented. The iso-Y_*^{CE} curve is depicted in figure 2 for two conditionally egalitarian mechanisms $F_{\tilde{e}_1}^{CE}$ and $F_{\tilde{e}_2}^{CE}$ with reference preferences $\tilde{e}_1$ and $\tilde{e}_2$ ($e_L < \tilde{e}_1 < \tilde{e}_2 < 1$):

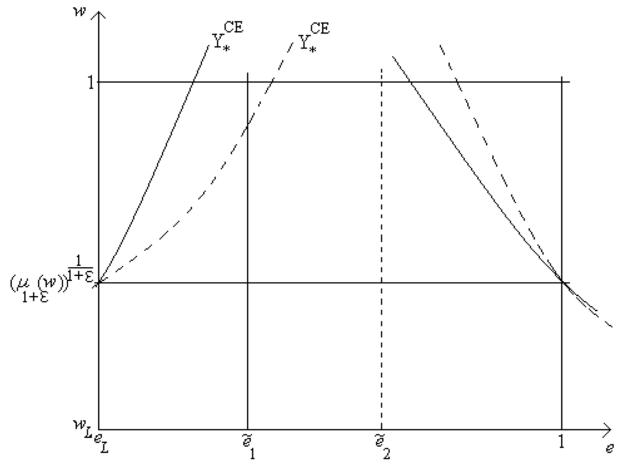

Figure 2: Iso-Y^{CE} curves for $F_{\tilde{e}_1}^{CE}$ and $F_{\tilde{e}_2}^{CE}$

Third, it remains to show that two equal iso-Y^{CE} curves of two different conditionally egalitarian regimes only cross when $Y^{CE} > Y_*^{CE}$. These crossings imply that assumptions about the marginal distributions of w and e have to be made to establish poverty dominance for income levels larger than Y_*^{CE}: we need to know exactly how many individuals are situated at each point in the $e \times w$

[9] The class of poverty indices of the form $P(F|Z)$ contains not only most widely-used indices as the headcount ratio and the income gap ratio. More generally, it includes among others all members of the Foster-Greer-Thorbecke family as well as the Watts index. See Lambert (2001) for an overview.

space. Define $w^{\tilde{e}_1}_{Y^{CE}}(e_L)$ as the skill level that an individual with preferences e_L should have to obtain an income Y^{CE} under a conditionally egalitarian regime with reference preferences $\tilde{e}_1$. Keeping the single crossing property in mind, two equal iso-Y^{CE} curves of two different conditionally egalitarian regimes cross over the subspace $[e_L, \tilde{e}_1] \times [w_L, 1]$ when:

$w^{\tilde{e}_2}_{Y^{CE}}(e_L) > w^{\tilde{e}_1}_{Y^{CE}}(e_L)$

$\Leftrightarrow \left(Y^{CE} - \tilde{e}_2^{\varepsilon}\mu_{\varepsilon+1}(w)\right)^{\frac{1}{1+\varepsilon}} \left(\frac{1}{e_L^{\varepsilon} - \tilde{e}_2^{\varepsilon}}\right)^{\frac{1}{1+\varepsilon}} > \left(Y^{CE} - \tilde{e}_1^{\varepsilon}\mu_{\varepsilon+1}(w)\right)^{\frac{1}{1+\varepsilon}} \left(\frac{1}{e_L^{\varepsilon} - \tilde{e}_1^{\varepsilon}}\right)^{\frac{1}{1+\varepsilon}}$ (using (11))

$\Leftrightarrow \left(Y^{CE} - \tilde{e}_2^{\varepsilon}\mu_{\varepsilon+1}(w)\right)(e_L^{\varepsilon} - \tilde{e}_1^{\varepsilon}) > \left(Y^{CE} - \tilde{e}_1^{\varepsilon}\mu_{\varepsilon+1}(w)\right)(e_L^{\varepsilon} - \tilde{e}_2^{\varepsilon})$ since $(e_L^{\varepsilon} - \tilde{e}_1^{\varepsilon}) < 0$ and $(e_L^{\varepsilon} - \tilde{e}_2^{\varepsilon}) < 0$

$\Leftrightarrow (\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon})Y^{CE} > (\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon})e_L^{\varepsilon}\mu_{\varepsilon+1}(w)$

$\Leftrightarrow Y^{CE} > e_L^{\varepsilon}\mu_{\varepsilon+1}(w) = Y_*^{CE}$ since $(\tilde{e}_2^{\varepsilon} - \tilde{e}_1^{\varepsilon}) > 0$

which completes the proof[10]. ■

4.3 EE versus CE revisited

Let us reconsider the comparison of an egalitarian equivalent mechanism with a conditionally egalitarian mechanism:

- For all economies in D°, apart from one unique exception[11], the income distribution of a conditionally egalitarian mechanism cannot Lorenz dominate the income distribution of any egalitarian equivalent mechanism[12].

Finally, the remaining question is whether an egalitarian equivalent mechanism can Lorenz dominate a conditionally egalitarian mechanism. A priori, one could think that the income distribution of the former Lorenz dominates the income distribution of the latter, since the egalitarian equivalent mechanism satisfies stronger compensation axioms and milder responsibility axioms than the conditionally egalitarian mechanism. Therefore, we examine whether all egalitarian equivalent mechanisms are eligible to Lorenz dominate all conditionally egalitarian mechanisms. However, this assumption proves to be too demanding, as summarized in the following proposition:

Proposition 5 *: For all economies in D°, the income distribution of an egalitarian equivalent mechanism can only Lorenz dominate the income distribution of all conditionally egalitarian mechanisms when $\tilde{w} \leq (\mu_{\varepsilon+1}(w))^{\frac{1}{\varepsilon+1}}$.*

[10] Analogously, it can be shown that equal iso-income curves of two different conditionally egalitarian regimes no longer cross for all income levels higher than the income of a person with characteristics $\left(1, (\mu_{\varepsilon+1}(w))^{\frac{1}{\varepsilon+1}}\right)$ (see also figure 2).

[11] The unique exception is the comparison of an EE-mechanism with $\tilde{w} = 1$ and a CE-mechanism with $\tilde{e} = (\mu_{\varepsilon}(e))^{\frac{1}{\varepsilon}}$. Under these two regimes the incomes of the poorest and the richest do not change and Lorenz dominance is still possible. Further distributional assumptions have to be made. We illustrate this exception in the numerical example at the end of this paragraph.

[12] The proof is analogous to the proof of proposition 5 with reversed inequality signs.

Proof. : Denote $Y^{EE,\,\tilde{w}}_{(e,\,w)}$ the income an individual with characteristics (e, w) receives under an egalitarian equivalent mechanism with reference skill level $\tilde{w}$. The necessary condition for Lorenz dominance that the poorest have at least the same income under F^{EE} as under F^{CE}, while the income of the richest should not be higher under F^{EE} than under F^{CE} requires that: $\forall \tilde{e}$:

$Y^{CE,\,\tilde{e}}_{(e_L,\,1)} \leq Y^{EE,\,\tilde{w}}_{(e_L,\,\cdot)}$

$\Leftrightarrow e_L^{\varepsilon} - \tilde{e}^{\varepsilon} + \tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w) \leq e_L^{\varepsilon}\tilde{w}^{\varepsilon+1} - \mu_{\varepsilon}(e)\tilde{w}^{\varepsilon+1} + \mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)$

$\Leftrightarrow \tilde{w}^{\varepsilon+1} \leq \frac{\tilde{e}^{\varepsilon}(\mu_{\varepsilon+1}(w)-1)+e_L^{\varepsilon}-\mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)}{(e_L^{\varepsilon}-\mu_{\varepsilon}(e))} = RHS(1)$

and

$Y^{CE,\,\tilde{e}}_{(1,\,1)} \geq Y^{EE,\,\tilde{w}}_{(1,\,\cdot)}$

$\Leftrightarrow 1 - \tilde{e}^{\varepsilon} + \tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w) \geq \tilde{w}^{\varepsilon+1} - \mu_{\varepsilon}(e)\tilde{w}^{\varepsilon+1} + \mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)$

$\Leftrightarrow \tilde{w}^{\varepsilon+1} \leq \frac{\tilde{e}^{\varepsilon}(\mu_{\varepsilon+1}(w)-1)+1-\mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)}{(1-\mu_{\varepsilon}(e))} = RHS(2)$

Note that $RHS(1)$ is increasing in $\tilde{e}$ since $\frac{(\mu_{\varepsilon+1}(w)-1)}{(e_L^{\varepsilon}-\mu_{\varepsilon}(e))} > 0$ while $RHS(2)$ is decreasing in $\tilde{e}$ since $\frac{(\mu_{\varepsilon+1}(w)-1)}{(1-\mu_{\varepsilon}(e))} < 0$. The shaded pentagon of Figure 3 depicts which values of $\tilde{w}$ and $\tilde{e}$ and corresponding F^{EE}- and F^{CE}-mechanisms fulfill the necessary condition. Since the dominance result has to hold for all values of $\tilde{e}$, only F^{EE}-mechanisms with $\tilde{w} \leq (\mu_{\varepsilon+1}(w))^{\frac{1}{\varepsilon+1}}$ are eligible to Lorenz dominate all F^{CE}-mechanisms.

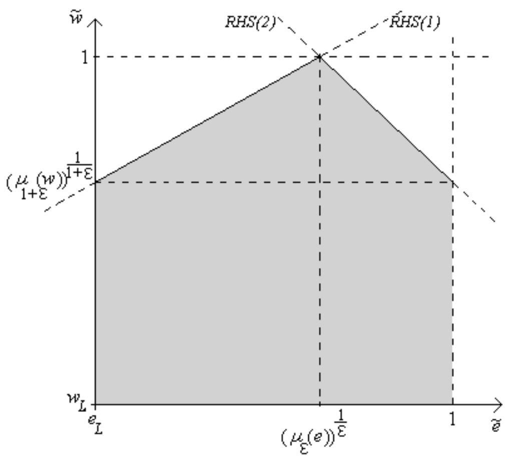

Figure 3: A necessary condition for LD

■

Now we know exactly which egalitarian equivalent mechanisms can Lorenz dominate which conditionally egalitarian mechanisms. The question is whether we can require the domain assumption that these Lorenz dominance results hold over all characteristics' distributions in Σ°, i.e. whether these dominance results do not depend on specific assumptions about exactly how many individuals have which characteristics. We proof this domain assumption is too demanding in our example, as stated in the following proposition:

Proposition 6 *: Lorenz dominance results of an egalitarian equivalent mechanism over a conditionally egalitarian mechanism cannot be drawn without assumptions about the marginal distributions $g_w(w)$ and $g_e(e)$.*

Proof. : Lorenz dominance of $F^{EE,\tilde{w}}$ over $F^{CE,\tilde{e}}$ amounts to second order stochastic dominance of the cumulative income distribution function $G^{EE,\tilde{w}}(x)$, over the cumulative income distribution function $G^{CE,\tilde{e}}(x)$: $S(Y) = \int_{Y_{\min}^{CE,\tilde{e}}}^{Y} (G^{CE,\tilde{e}}(x) - G^{EE,\tilde{w}}(x))dx \geq 0$, for all $Y \in [Y_{\min}^{CE,\tilde{e}}, Y_{\max}^{CE,\tilde{e}}]$[13]. From Figure 3 we see that all egalitarian equivalent mechanisms can Lorenz dominate a conditionally egalitarian mechanism with $\tilde{e} = (\mu_{\varepsilon}(e))^{\frac{1}{\varepsilon}}$. In the following simulation, whose set-up is given in Appendix, we test whether Lorenz dominance is found when e and w are uniformly distributed over the interval $[0.1, 1]$ and $\varepsilon = 1$ for respectively an egalitarian equivalent mechanism with $\tilde{w} = 0.5$, $\tilde{w} = 0.7$ and $\tilde{w} = 1$ over a conditionally egalitarian mechanism with $\tilde{e} = \mu(e) = 0.55$. The incomes of the poorest and the richest under the four regimes equal:

	$Y_{\min}$	$Y_{\max}$
1) $F^{CE,\tilde{e}=0.55}$	-0.2465	0.6535
2) $F^{EE,\tilde{w}=0.5}$	0.091	0.316
3) $F^{EE,\tilde{w}=0.7}$	-0.017	0.424
4) $F^{EE,\tilde{w}=1}$	-0.2465	0.6535

Figure 4 depicts $S(Y)$, where 1) vs 2): upper curve, 1) vs 3): middle curve and 1) vs 4): lower curve.

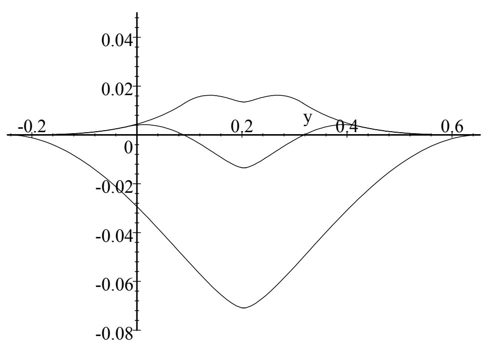

Figure 4: $S(Y)$

[13]Note that $S(Y_{\min}^{CE,\tilde{e}}) = 0$ and that $S(Y_{\max}^{CE,\tilde{e}}) = \mu^{EE}(Y) - \mu^{CE}(Y) = 0$.

Figure 4 shows that, when e and w are uniformly distributed, $F^{EE,\tilde{w}=0.5}$ Lorenz dominates $F^{CE,\tilde{e}=0.55}$. However, $F^{EE,\tilde{w}=0.7}$ does not Lorenz dominate $F^{CE,\tilde{e}=0.55}$. Ultimately, $F^{CE,\tilde{e}=0.55}$ Lorenz dominates $F^{EE,\tilde{w}=1}$, which illustrates the unique combination of $\tilde{e}$ and $\tilde{w}$ for which the conditionally egalitarian mechanism can Lorenz dominate the egalitarian equivalent mechanism[14]. ■

5 Conclusion

The implementation of a non-welfaristic redistribution mechanism not only requires a normative judgement for the choice of reference responsibility or compensation parameter. In order to trace out the implications of non-welfaristic redistribution, also empirical information is needed. Only with accurate estimations of the pre-tax income function and the distributions of responsibility and compensation characteristics, different non-welfaristic mechanisms can be thoroughly compared. If one lacks such detailed information, but at least some agreement on the properties of the pre-tax income function is reached, our analysis suggests that:

- Confronted with the choice which egalitarian equivalent mechanism to implement and depending on the economy concerned, Lorenz dominance and poverty dominance adherents choose either the egalitarian equivalent mechanism with the lowest (if the economy belongs to D^+) or with the highest (if the economy belongs to D^-) reference compensation parameter.

- Confronted with the choice which conditionally egalitarian mechanism to implement, Lorenz dominance adherents do not favor any particular mechanism. Poverty dominance adherents choose the conditionally egalitarian mechanism with the lowest reference responsibility parameter.

- If Lorenz dominance and poverty dominance adherents have the full choice which non-welfaristic mechanism to implement, they should not believe that any egalitarian equivalent mechanism Lorenz dominates all conditionally egalitarian mechanisms, since the former satisfies stronger compensation axioms and milder responsibility axioms than the latter. Depending on the economy concerned, only egalitarian equivalent mechanisms with a sufficiently low (if the economy belongs to D^+) or sufficiently high (if the economy belongs to D^-) reference compensation parameter are eligible to Lorenz dominate all conditionally egalitarian mechanisms.

[14]Indeed, note that the incomes of the poorest and the richest are unchanged under $F^{CE,\tilde{e}=0.55}$ and $F^{EE,\tilde{w}=1}$.

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7 Appendix

- Proof of the income distribution comparison of section 4.1.2:

We show that both the poorest and the richest have at least the same income under $F^{CE}_{\tilde{e}_1}$ than under $F^{CE}_{\tilde{e}_2}$.

Indeed, $Y^{CE,\,\tilde{e}_1}_{(e_L,\,1)} \geq Y^{CE,\,\tilde{e}_2}_{(e_L,\,1)}$
$\Leftrightarrow e^{\varepsilon}_L - \tilde{e}^{\varepsilon}_1 + \tilde{e}^{\varepsilon}_1\mu_{\varepsilon+1}(w) \geq e^{\varepsilon}_L - \tilde{e}^{\varepsilon}_2 + \tilde{e}^{\varepsilon}_2\mu_{\varepsilon+1}(w)$
$\Leftrightarrow (\tilde{e}^{\varepsilon}_2 - \tilde{e}^{\varepsilon}_1) \geq (\tilde{e}^{\varepsilon}_2 - \tilde{e}^{\varepsilon}_1)\mu_{\varepsilon+1}(w)$
which is the case since $\tilde{e}_2 \geq \tilde{e}_1$ and $1 > \mu_{\varepsilon+1}(w)$.
Analogously, $Y^{CE,\,\tilde{e}_1}_{(1,\,1)} \geq Y^{CE,\,\tilde{e}_2}_{(1,\,1)}$
$\Leftrightarrow 1 - \tilde{e}^{\varepsilon}_1 + \tilde{e}^{\varepsilon}_1\mu_{\varepsilon+1}(w) \geq 1 - \tilde{e}^{\varepsilon}_2 + \tilde{e}^{\varepsilon}_2\mu_{\varepsilon+1}(w)$
$\Leftrightarrow -\tilde{e}^{\varepsilon}_1(1 - \mu_{\varepsilon+1}(w)) \geq -\tilde{e}^{\varepsilon}_2(1 - \mu_{\varepsilon+1}(w))$ and since $1 > \mu_{\varepsilon+1}(w)$,
$\Leftrightarrow \tilde{e}^{\varepsilon}_1 \leq \tilde{e}^{\varepsilon}_2$
$\Leftrightarrow \tilde{e}_1 \leq \tilde{e}_2$
which is true by assumption.

- Numerical example set-up of section 4.3:

Take an arbitrary income level $x \in [Y^{CE,\tilde{e}}_{\min}, Y^{CE,\tilde{e}}_{\max}]$:

1) $G^{EE,\tilde{w}}(x)$:

$$G^{EE,\tilde{w}}(x) = P(e^{\varepsilon}\tilde{w}^{\varepsilon+1} - \mu_{\varepsilon}(e)\tilde{w}^{\varepsilon+1} + \mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w) \leq x)$$
$$= P\left(e \leq \left(\frac{x+\mu_{\varepsilon}(e)\tilde{w}^{\varepsilon+1}-\mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)}{\tilde{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}\right)$$
$$= \int_{e_L}^{\left(\frac{x+\mu_{\varepsilon}(e)\tilde{w}^{\varepsilon+1}-\mu_{\varepsilon}(e)\mu_{\varepsilon+1}(w)}{\tilde{w}^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}} g_e(e)de$$

2) $G^{CE,\tilde{e}}(x)$:

$$G^{CE,\tilde{e}}(x) = P(e^{\varepsilon}w^{\varepsilon+1} - \tilde{e}^{\varepsilon}w^{\varepsilon+1} + \tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w) \leq x)$$

i) $\forall x \in [Y^{CE,\tilde{e}}_{\min}, Y^{CE,\tilde{e}}_{(\tilde{e},\cdot)} = \tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w)]$:

$$= 1 - \left(\int\limits_{e^2(x)}^{1} g_e(e)de \right) - \left(\int\limits_{e^1(x)}^{e^2(x)} \int\limits_{w_L}^{\left(\frac{x-\tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w)}{e^{\varepsilon}-\tilde{e}^{\varepsilon}}\right)^{\frac{1}{\varepsilon+1}}} g_w(w)g_e(e)dwde \right) \text{ where}$$

$e^1(x) = \max\{e_L, \left(\frac{x-\tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w)+\tilde{e}^{\varepsilon}w_L^{\varepsilon+1}}{w_L^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}\}$ and $e^2(x) = \left(x - \tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w) + \tilde{e}^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$ and $g(e, w) = g_w(w)g_e(e)$ since w and e are independently distributed.

ii) $\forall x \in [Y^{CE,\tilde{e}}_{(\tilde{e},\cdot)}, Y^{CE,\tilde{e}}_{\max}]$:

$$= \left(\int\limits_{e_L}^{e^2(x)} g_e(e)de \right) + \left(\int\limits_{e^2(x)}^{e^3(x)} \int\limits_{w_L}^{\left(\frac{x-\tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w)}{e^{\varepsilon}-\tilde{e}^{\varepsilon}}\right)^{\frac{1}{\varepsilon+1}}} g_w(w)g_e(e)dwde \right) \text{ where } e^3(x) =$$

$\min\{\left(\frac{x-\tilde{e}^{\varepsilon}\mu_{\varepsilon+1}(w)+\tilde{e}^{\varepsilon}w_L^{\varepsilon+1}}{w_L^{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}, 1\}$

If we suppose that $e \sim U[e_L, 1]$, $w \sim U[w_L, 1]$ and $\varepsilon = 1$:
$g_e(e) = \frac{1}{1-e_L}$, $g_w(w) = \frac{1}{1-w_L}$, $\mu_1(e) = \mu(e) = \frac{1+e_L}{2}$ and $\mu_2(w) = \frac{(w_L^2+w_L+1)}{3}$

$S(Y) = \int_{Y^{CE,\tilde{e}}_{\min}}^{Y}(G^{CE,\tilde{e}}(x) - G^{EE,\tilde{w}}(x))dx$ then becomes a function of Y, e_L, w_L, $\tilde{e}$ and $\tilde{w}$ and is simulated for $e_L = 0.1$, $w_L = 0.1$, $\tilde{e} = \mu(e) = 0.55$ and $\tilde{w} = 0.5/0.7/1$.