Loss Aversion and the Uniform Pricing Puzzle for Media and Entertainment Products

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The uniform pricing puzzle for vertically differentiated media and entertainment products (movies, books, music, mobile apps, etc.) is that a firm with market power sells high- and low-quality products at the same price even though quality is perfectly observable and price adjustments are not costly. We resolve this puzzle by assuming that consumers have an uncertain taste for quality and accounting for consumer loss aversion in monetary and consumption utilities. The novelty of our approach is that the so-called reference transaction is endogenously set as part of a "personal equilibrium" and is based only on past purchases of same-quality products.

Key words: uniform pricing puzzle, vertically differentiated products, expectations-based loss aversion, personal equilibrium

1. Introduction

In media and entertainment markets, high-quality products sometimes sell at prices that are the same as (or similar to) those of low-quality products. For instance, the price of a movie ticket does not depend on observable signals that predict gross revenue, such as the movie's ratings or popularity, or on whether the movie is a sequel or a new release (*The Atlantic* 2012, Orbach and Einav 2007). Attempts at variable pricing in the industry have failed (*The Guardian* 2006). Similar pricing patterns for vertically differentiated products exist also in sports games (where ticket prices do not reflect the popularity of opponents) and theater season tickets (Chu et al. 2011) as well as in some media markets such as music, including CDs or downloadable tracks (Shiller and Waldfogel 2011), books, and mobile apps. A considerable amount of work addresses price uniformity, but the puzzle remains for some vertically differentiated products in media and entertainment. Under standard demand specifications, the profit-maximizing price should increase with quality. The puzzle is that observed prices either do not respond to quality (price uniformity) or respond insufficiently (price compression).

We provide a behavioral explanation that is based on consumer loss aversion and whereby a consumer has a *reference transaction* or benchmark, which accounts for both consumption utility and monetary payment, and reacts more strongly when an outcome falls short of this benchmark than otherwise. If we assume that there is a single reference transaction for all purchases, then loss aversion could explain price uniformity in certain contexts where prices do not respond to cost (Heidhues and Kőszegi 2008) or demand shocks (Chen and Cui 2013). Yet, several scholars have argued that, in the case of vertically differentiated products, this assumption is unrealistic because consumers observe quality: "All consumers are familiar with the concept of different prices for different products. [...] Thus, charging premiums or giving discounts for unique categories of movies is unlikely to be perceived as unfair. For example, given the unique characteristics and highly publicized production budgets of event movies, charging premiums for such movies probably would not violate fairness perceptions" (Orbach and Einav 2007: 145-46). Thus consumers should have different reference points corresponding to different product quality classes, in which case the simple argument in Kahneman et al. (1986)—which is based on perceived fairness with respect to a single reference point—does not apply. The critical component of our model is its conceptualization of a separate reference transaction for each quality class, which enables us to analyze rigourously the effect of loss aversion on consumers' purchase decisions.

Following Mussa and Rosen (1978), we assume that the consumer's valuation for a product of quality q is $v = v_0 + q\theta$, where v_0 is the baseline value and $\theta \in [\theta_0, \theta_1]$ is an idiosyncratic random preference shock that captures the consumer's taste uncertainty. In the movie context, for example, all consumers prefer movies that are popular which is captured by the quality dimension q. But a consumer also has an idiosyncratic preference level θ for each movie, which reflects the movie's "match value" uncertainty and is not resolved until the time of purchase. Leslie and Sorensen (2014) adopt the same formulation—in which the taste shock and product quality are complements—to study the demand for concerts. The match value component, which is learned at the time of purchase, is of more consequence for higher-quality products. So if the evaluation of critics reasonably approximates consumer valuations and if we can take a movie's budget as a proxy for its quality, then complementarity implies that there should be more variability in the critics' opinions of big-budget movies.

We approach the pricing problem at two levels. First we study the problem of pricing a given quality class. This is possible because, in our model, the consumer has a specific reference transaction for each quality class. Given the price, the consumer decides on which taste realizations to consume by applying the notion of *personal equilibrium* (hereafter PE) developed in Kőszegi and Rabin (2006, 2009). In other words, a loss-averse consumer compares current purchases with a reference point which is formalized as her recent expectations about past purchase outcomes in that quality class. A series of experimental and empirical papers support the notion of reference points as expectations held under uncertainty; examples include the effort provision experiments of Abeler et al. (2011), the exchange experiments of Ericson and Fuster (2011), and research by Pope and Schweitzer (2011) in the professional golf context.

The case *without* uncertainty in match value is trivial because the consumer faces no risk: she consumes if the valuation is above the price and otherwise does not. This means that the seller, when setting the price, need not account for consumer loss aversion. The case *without* loss aversion is standard: the consumer consumes when the taste draw is above a quality-dependent threshold. Yet if uncertainty and loss aversion are both present, then there are two new costs associated with consuming only when the taste draw exceeds a given threshold, which we illustrate with the movie example. For movies that are poor matches (low θ), the consumer suffers a loss when she compares the choice of not consuming with what she would receive from a movie with a better match (higher θ). The opposite holds for expenditure comparisons because the consumer perceives paying the price for a well-match movie as a loss when compared to not consuming. We derive a set of sufficient conditions such that full consumption—understood as consuming for all taste draws—is the preferred personal equilibrium (hereafter PPE), defined as the PE that yields the highest expected utility (Kőszegi and Rabin 2006). In addressing the case of multiple PEs, ours is one of the few papers to present a nontrivial analysis of PPE with taste uncertainty (see also Hahn et al. 2015).

To explicate the intuition underlying full-consumption outcome, we look at the consumer's utility from consuming only when her valuation is above a given consumption threshold. Although expected utility is an inverse U–shaped function of the consumption threshold under loss neutrality, even small amounts of loss aversion transform it into a U–shaped function—for which extreme consumption plans (always or never consume) dominate intermediate ones. The dominance follows because the net effect of loss aversion is significant losses associated with comparing consumption and expenditure outcomes for intermediate thresholds. If there are multiple PEs and the expected utility is U–shaped, then PEs with intermediate consumption thresholds will be dominated by those with extreme thresholds for which the consumer consumes most or least often.

Given these patterns of consumer behavior, we next investigate the firm's pricing decision for the given quality level. We establish that for moderate gain–loss parameter values in consumption and when quality is not too high, full consumption solves the firm's revenue maximization problem. Full consumption is a local minimum under risk neutrality but is a local maximum for sufficiently high monetary loss aversion—and this holds for any distribution of the taste draw. Under additional conditions, full consumption is the global maximum. Our analyses of the consumer's problem and the firm's problem complements the literature on loss aversion, which imposes full consumption;

see, for example, the "full coverage" assumption in Heidhues and Kőszegi (2008) and Karle and Peitz (2014). Full consumption accords with the casual observation that some consumers often watch movies and read books that are not necessarily good matches.

As a second level of analysis, we use the results on a single quality class to compare prices across quality classes and thereby address the uniform pricing puzzle. Because quality is multiplied by the lowest taste draw θ_0 in the characterization of the optimal price, it has a limited influence on the optimal price. Contrast this with the cases where the consumer is either loss neutral or certain about her taste. The price for a product of quality q sold to a loss-neutral consumer is $v_0 + q\theta^{\mathrm{LN}}$ (where θ^{LN} is the optimal loss-neutral consumption threshold) and is $v_0 + q\mathbb{E}\theta$ when taste is certain and normalized to $\mathbb{E}\theta$. For a consumer who faces considerable taste uncertainty, θ_0 will be much lower than either θ^{LN} or $\mathbb{E}\theta$; hence the optimal price is less responsive to changes in quality under loss aversion and taste uncertainty than it is when either of those conditions is absent. Price compression occurs when both loss aversion and taste uncertainty are present; uniform pricing obtains when $\theta_0 = 0$.

Price compression happens when full consumption is optimal, which is the case only for product classes below a threshold level of quality and for moderate levels of aversion to consumption loss. In this case, the consumer's surplus is increasing in quality, a feature consistent with the applications discussed previously. For product qualities above that threshold, full consumption is no longer optimal and prices respond to quality. These findings clarify the role (in the specification of Mussa and Rosen 1978) played by the complementarity between quality and the taste draw. Additive quality, for example, does not interact with the consumption threshold and cannot generate price compression.

When uniform pricing is optimal, deviating from it imposes a first-order loss on profits. To demonstrate this point, we show that the losses that result from wrongly assuming loss neutrality when the consumer is in fact loss averse, are always greater than the losses from making the opposite mistake. The reason is that, under loss neutrality, the profit function is flat around the optimal price. Under loss aversion, however, the optimal profit is achieved at the full-consumption corner, and a deviation from the optimal policy imposes a first-order loss. To illustrate the relevance of this point, we consider Siller and Waldfogel's (2011) study of iTunes uniform pricing at $0.99 per song. The authors argue that revenue could increase by at least a sixth and as much as a third if Apple abandoned uniform pricing. Yet, these figures do not account for loss aversion, which probably affects consumers' music purchases (Kahneman et al. 1986). Uniform pricing may be optimal once we account for the loss aversion costs associated with differential pricing of quality classes. Our analysis suggests that uniform pricing is more likely to be optimal for less popular songs.

The rest of the paper is organized as follows. In Section 2, we review the related literature and position our work accordingly. Section 3 presents the model and notation. Section 4 solves the consumer's decision-making problem and also the firm's revenue maximization problem. Section 5 presents our main results on the uniform pricing puzzle and discusses an application to the case of uniformly distributed consumer valuations. We conclude in Section 6.

2. Literature Review

An extensive literature studies the pricing of vertically differentiated products in a monopoly (see e.g., Anderson and Dana 2009, Moorthy 1984, Mussa and Rosen 1978) or a competitive setting (Liu and Zhang 2013). This literature has explained why prices increase with service quality (Iyer and Seetharaman 2003), printer speed (Deneckere and McAfee 1996), car attributes (Verboven 1999), airfare classes (Talluri and van Ryzin 2004), and numerous other quality dimensions. We contribute to this literature by studying the rather unusual case in which quality is *not* priced in a monopoly. We show that a combination of consumer loss aversion and random taste draws for quality can explain the phenomenon. An explanation based on loss aversion is compelling for the set of transactions considered in this paper because loss aversion has been shown to affect decisions of consumers who intend to consume (Novemsky and Kahneman 2005) and are emotionally attached to the product (Ariely et al. 2005). A random taste draw is plausible for media and entertainment products but possibly less so for durable goods (e.g., cars, printers), where pricing quality is the norm. In formalizing a non-trivial behavioral rationale for price uniformity, we exclude from the model such considerations as cost and competition; these factors help explain some pricing practices but are less relevant in the case of media and entertainment goods, where fixed costs and market power prevail.

Our paper is also part of a growing literature that addresses uniform pricing. A few studies have looked at the "branded variant" puzzle, which is that prices are remarkably uniform irrespective of brand popularity (Chen and Cui 2013). Eckert and West (2013) investigate why more popular brands of beer cost the same as less popular ones, and McMillan (2007) does likewise for soft drinks. These studies differ from ours in that branded variants are typically *horizontally* differentiated products. Although practitioners have remarked on the use of uniform pricing for media and entertainment products (*ArtsJournal* 2013, *The Atlantic* 2012), only a handful of academic studies have been devoted to this puzzle in the context of specific markets (Chu et al. 2011, Courty and Pagliero 2014, Orbach and Einav 2007, Shiller and Waldfogel 2011) and we are not aware of any formal modelling for such products. Informal explanations for the uniform pricing puzzle are reviewed in Orbach and Einav (2007) and Eckert and West (2013). Certainly other rationales that are based on asymmetric information or supply-side arguments—such as menu cost (McMillan

2007) and legal constraints on the relations between distributors and movie theatres (Orbach and Einav 2007)— do help explain the puzzle in some cases. However, we believe that a more general explanation is needed that can encompass the broad range of media and entertainment contexts that exhibit price compression. This paper presents a formal model that incorporates several features of media and entertainment markets (e.g., market power, uncertain taste for quality, multi-dimensional loss aversion) and explains how uniform pricing or price compression arises in these markets.

Finally, we contribute to the stream of behavioral research in operations, marketing, and economics that relates price uniformity to loss aversion. Nasiry and Popescu (2011) and Popescu and Wu (2007) show that if consumers reference transactions are exogenous and deterministic, then a monopolist tends to charge a uniform price in the long run. These two papers do not consider quality. Chen and Cui (2013) demonstrate that peer-induced fairness can result in unform pricing for horizontally differentiated products. Heidhues and Kőszegi (2005) show that prices do not respond to changes in the production cost when a monopolist sells to loss-averse consumers; Heidhues and Kőszegi (2008) and Karle and Peitz (2014) extend the same insight to competitive markets. We follow these papers in assuming endogenous reference points, but we focus on vertically differentiated products with negligible marginal cost. Although various approaches have been proposed for modeling the reference point (Carbajal and Ely 2012, Eliaz and Spiegler 2015, Zhou 2011), we adopt the framework proposed in Kőszegi and Rabin (2006) because it is natural to assume that consumers form lagged expectations using only past purchases of products of that quality class.[1] A similar structure for reference points has been applied to study other problems. For example, Herweg (2013) and Baron et al. (2015) study newsvendor ordering decisions, Yang et al. (2014) investigate queuing behavior, and Lindsey (2011) examines congestion pricing. Our model is related also to those of Herweg and Mierendorff (2013) and Hahn et al. (2015). The former paper assesses the optimality of two-part tariffs under multi-unit demand for a single product. In contrast, we restrict our analysis to a unit-demand model so that we can tackle the case of vertically differentiated products. Hahn et al. paper assumes endogenous product design and introduces screening, which is ruled out in our analysis.

3. Model: Preliminaries

Consider a firm that sells a product of quality q to a representative loss-averse consumer. In this section and Section 4, we characterize the consumer behavior as well as the firm's optimal pricing

[1] In Heidhues and Köszegi's (2008) model of horizontally differentiated products, all consumers share the same reference point and it corresponds to a random purchase. Although this approach is reasonable for horizontally differentiated products that are hedonistically substitutable, it is less reasonable for vertically differentiated products because quality draws natural boundaries between product classes.

policy for the quality q. In Section 5, we let the firm offer multiple quality classes and show how uniform pricing emerges when consumers are loss averse.

The consumer's valuation of a product has two components. First, the consumer is willing to pay $v_0 \geq 0$ regardless of product quality. In the case of movies, one can view v_0 as the inherent value of a night out with family and friends to watch a movie. Second, the consumer has a random marginal willingness to pay for quality that we model as follows. Let $\theta \in \boldsymbol{\Theta} = [\theta_0, \theta_1]$ denote the consumer's private taste draw for quality. We will sometimes refer to θ as the "state of nature" and further assume $\theta_0 \geq 0$; that is, the marginal effect of quality on a consumer's valuation is nonnegative. Now we can write the consumer's intrinsic utility for a product of quality q in state θ as $v = v_0 + q\theta$. Our characterization is similar to Mussa and Rosen (1978) and implies that quality q and the state of nature θ are complements. The same structure of consumer preferences is applied by Leslie and Sorensen (2014) in an empirical application to live concerts. Herweg and Mierendorff (2013) consider a similar structure in the loss aversion model that they use to study flat-rate tariffs. The taste draw θ has density $g(\theta)$, cumulative distribution $G(\theta)$, and survival function $\bar{G}(\theta) = 1 - G(\theta)$. Finally, in line with the applications discussed in the Introduction, we assume that there is no marginal cost associated with serving the consumer.[2]

Assumption 1. *The density function $g(\theta)$ is increasing for $\theta < \mathbb{E}\theta$, and $g(\mathbb{E}\theta - x) = g(\mathbb{E}\theta + x)$ for $x \in [0, \mathbb{E}\theta]$.*

Assumption 1 means that $g(\cdot)$ is single peaked and symmetric. This assumption holds for many distributions, including the truncated normal, the uniform, and any tent-shaped distribution.

According to prospect theory, the consumer experiences feelings of gain and loss when comparing her consumption outcome with a reference transaction (Tversky and Kahneman 1991). We assume that the reference transaction has a consumption component along with a monetary component and also that the consumer experiences gains and losses in both components. There is evidence that both components matter (Carbajal and Ely 2012), and we will show that both are sufficient to explain the uniform pricing puzzle. In line with the extant literature, we assume that the gain–loss utility is piecewise linear (see e.g., Heidhues and Kőszegi 2008, Herweg and Mierendorff 2013, Kőszegi and Rabin 2006). The consumer experiences a loss in consumption utility equal to $\lambda_c(v' - v)$ when her valuation v is lower than her reference valuation v', and she experiences a loss in monetary utility equal to $\lambda_p(p - p')$ if she spends more than the reference amount p'. Likewise, the consumer experiences a gain in consumption utility equal to $\beta_c(v - v')$ when she consumes more than expected (i.e., $v > v'$) and a gain in monetary utility equal to $\beta_p(p' - p)$ when she spends

[2] The analysis naturally follows when each product is produced at a fixed cost that increases with the product's quality.

less than expected (i.e., $p < p'$). Under prospect theory, consumers are loss averse; this means they dislike losses more than they like equal-sized gains, so $\lambda_c \geq \beta_c$ and $\lambda_p \geq \beta_p$.

The consumer decides on her consumption plan knowing the price and the quality but before the uncertainty regarding her taste for quality is resolved. To characterize the reference transaction in the uncertain decision-making environment, we apply the personal equilibrium concept developed in Kőszegi and Rabin (2006) whereby the reference transaction (or the PE) is a consumption plan such that the decision in each state is optimal given the reference transaction. Formally, we denote a consumption plan $\bar{\pi} = \{\pi(\theta)\}_{\theta \in \mathbf{\Theta}}$ in which $\pi(\theta)$ is the probability that the consumer consumes in state θ. The full-consumption plan is defined as $\pi(\theta) = 1$ for all θ. According to Kőszegi and Rabin (2006), $\bar{\pi}$ is a PE if and only if

$$u(\pi(\theta)|\bar{\pi}, \theta) \geq u(x|\bar{\pi}, \theta) \quad \forall x \in [0,1], \ \forall \theta \in \mathbf{\Theta}; \tag{1}$$

where $u(\pi(\theta)|\bar{\pi}, \theta)$ is the ex post realized utility of the consumer given the consumption plan $\pi(\theta)$, the reference transaction $\bar{\pi}$, and the taste draw θ. The condition expressed by (1) means that, after taste uncertainty has been resolved, the consumer has no incentive to deviate from the consumption plan given the reference point $\bar{\pi}$.

The consumer's ex ante expected utility is

$$\mathrm{EU}(\bar{\pi}) = \int_{\mathbf{\Theta}} u(\pi(\theta)|\bar{\pi}, \theta)\, dG(\theta). \tag{2}$$

The set of PEs is not necessarily a singleton, so Kőszegi and Rabin (2006) define the preferred personal equilibrium (PPE) as a PE that yields the highest expected utility. We follow Herweg and Mierendorff (2013) and impose a participation constraint: the consumer can commit *not* to participate, in which case she receives a utility normalized to zero. The participation constraint (henceforth PC) implies that the consumer adopts the PPE $\bar{\pi}$ only if her expected utility is nonnegative ($\mathrm{EU}(\bar{\pi}) \geq 0$). A PC is standard for models of price discrimination in contract theory (Bolton and Dewatripont 2005).[3]

To summarize, the timing of events is as follows. The firm sets the price p for a product of quality q. We rule out random prices (Heidhues and Kőszegi 2014) because they are not relevant in the applications we have in mind. The consumer decides whether or not to participate. If she does, then she forms expectations $\bar{\pi}$ about her probability of consuming in each state. Next, the consumer discovers her taste draw θ. Finally, the consumer makes a consumption decision $\pi(\theta)$ based on her reference transaction $\bar{\pi}$. In solving the model, we impose three conditions as follows.

[3] A challenge that arises in connection with the concept of personal equilibrium is that the PPE may yield a lower expected utility than does never consuming (i.e., $\pi(\theta) = 0$). Adding a PC is then equivalent to assuming that $\pi(\theta) = 0$ is always a PE. For models that do no impose PC, see Heidhues and Kőszegi (2014) and Eliaz and Spiegler (2015).

(i) *PE requirement:* The consumer follows through on her expectations; that is, the expectation $\bar{\pi}$ coincides with the actual consumption $\pi(\theta)$.

(ii) *PPE requirement:* The consumer selects the PE that yields her the most utility.

(iii) *PC requirement:* The consumer's expected utility is nonnegative.

The firm maximizes its revenue, $p\int_{\mathbf{\Theta}}\pi(\theta)\,dG(\theta)$, subject to the PE, the PPE, and the PC. Table 2 (in Appendix A) summarizes our notation. Throughout the paper, we illustrate the results assuming θ is uniformly distributed over $\mathbf{\Theta}=[0,1]$ and that $\beta_c=\beta_p=0$.

4. Model: Analysis

The analysis for a general distribution of states of nature is complex. Therefore, in Section 4.1 we analyze a two-state case $\theta\in\{\theta_l,\theta_h\}$ where $\theta_l<\theta_h$ and $\beta_p=\beta_c=0$. The consumer draws taste l with probability γ. This model uses a simple setup to showcase consumer's decision making problem and also the firm's pricing problem. In Section 4.2 we consider the general case. All proofs are given in Appendix B.

4.1. Benchmark: Two-State Case

In this section we use a simple model to demonstrate the consumer behavior model and to derive a set of sufficient conditions such that uniform pricing and price compression are optimal for the firm. We proceed by first deriving sufficient conditions such that consuming in both states is a PE, and then a PPE, and finally such that the firm's maximum revenue in this PPE (i.e., always consume) exceeds the revenue in any other PPE. The set of sufficient conditions derived here naturally extend to the general case.

In the benchmark case we ignore mixed strategies ($\pi\in(0,1)$),[4] which means that the consumer may either consume (C) or not (N). Hence there are four possible consumption plans (a "plan" must determine the consumer' behavior under each possible taste realization). The four candidate PEs are then $\{(C,C),(C,N),(N,C),(N,N)\}$, where the first (resp. second) element in each pair is the consumption plan in the low (resp. high) state. The plan (C,N)—which stipulates that the consumer consumes only in the low state—cannot be a PE,[5] and (N,N) can be ignored because of the participation constraint. Thus the only possible PEs are (C,C), when the consumer always consumes; and (N,C), when the consumer consumes only in the high state.

Table 1 reports the ex post state-dependent utilities when the consumer consumes (C) and when she does not (N) conditional on the reference point ($\bar{\pi}\in\{(C,C),(N,C)\}$). As an illustration, we discuss the consumer's utility when she consumes in the low-taste realization and when the

[4] In Section 4.2, we show that this restriction is without loss of generality.

[5] We prove in Section 4.2 that consumption is increasing in θ; that is, if the consumer consumes in a state then she will consume in all higher states, too.

reference plan is always to consume—that is, we derive $u(C|(C,C),l)$. The consumer receives an intrinsic utility equal to $v_0+q\theta_l$ and pays p. She also experiences a loss equal to $(1-\gamma)\lambda_c q(\theta_h-\theta_l)$ in consumption utility because, with probability $1-\gamma$, her valuation would have been higher in the high state. In other words, she compares what she does consume $(v_0+q\theta_l)$ with what she could have consumed had the high state realized $(v_0+q\theta_h)$.

	Payoff when reference point is:	
	$\bar{\pi}=(C,C)$	$\bar{\pi}=(N,C)$
$u(C\|\bar{\pi},l)$	$v_0+q\theta_l-\lambda_c(1-\gamma)q(\theta_h-\theta_l)-p$	$v_0+q\theta_l-\lambda_c(1-\gamma)q(\theta_h-\theta_l)-(1+\lambda_p\gamma)p$
$u(N\|\bar{\pi},l)$	$-\lambda_c(v_0+q\mathbb{E}\theta)$	$-\lambda_c(1-\gamma)(v_0+q\theta_h)$
$u(C\|\bar{\pi},h)$	$v_0+q\theta_h-p$	$v_0+q\theta_h-(1+\lambda_p\gamma)p$
$u(N\|\bar{\pi},h)$	$-\lambda_c(v_0+q\mathbb{E}\theta)$	$-\lambda_c(1-\gamma)(v_0+q\theta_h)$
$\mathrm{EU}(\bar{\pi})$	$\gamma u(C\|\bar{\pi},l)+(1-\gamma)u(C\|\bar{\pi},h)$	$\gamma u(N\|\bar{\pi},l)+(1-\gamma)u(C\|\bar{\pi},h)$
	$=v_0+q\mathbb{E}\theta-\lambda_c\gamma(1-\gamma)q(\theta_h-\theta_l)-p$	$=(1-\gamma)\left((v_0+q\theta_h)(1-\lambda_c\gamma)-(1+\lambda_p\gamma)p\right)$

Table 1 Reference point, state utility, and expected utility

The consumption plan (C,C) is a PE if $u(C|(C,C),s)\geq u(N|(C,C),s)$ for $s\in\{l,h\}$. Observe that $p_{CC}^{\mathrm{LA}}\triangleq(1+\lambda_c)(v_0+q\theta_l)$ is the highest price such that both conditions hold. Recall that we intend to derive sufficient conditions under which the full-consumption plan, (C,C), is the PPE. We are therefore interested in equilibrium prices that are lower than p_{CC}^{LA}, since any higher price would violate the PE conditions. The participation constraint holds for price p_{CC}^{LA} if $\mathrm{EU}(C,C)\geq 0$ or, equivalently, if

$$\lambda_c(v_0+q\theta_l)\leq q(1-\lambda_c\gamma)(\mathbb{E}\theta-\theta_l). \tag{3}$$

The other viable consumption plan, (N,C), is a PE if both $u(C|(N,C),h)\geq u(N|(N,C),h)$ and $u(N|(N,C),l)\geq u(C|(N,C),l)$ hold. It is straightforward to observe that the former inequality is implied by PC; that is, $\mathrm{EU}(N,C)\geq 0$. Hence (N,C) is a PE if both the latter inequality and the PC are satisfied. Simplifying these expressions reveals that the price must satisfy

$$p\in\left[\frac{1+(1-\gamma)\lambda_c}{1+\gamma\lambda_p}(v_0+q\theta_l),\frac{1-\gamma\lambda_c}{1+\gamma\lambda_p}(v_0+q\theta_h)\right].$$

If this interval is empty then (N,C) is never a PE; otherwise, (N,C) is a PE that is dominated by (C,C) if $\mathrm{EU}(C,C)\geq\mathrm{EU}(N,C)$. When $p=p_{CC}^{\mathrm{LA}}$, this condition is equivalent to

$$\frac{\lambda_p(1+\lambda_c)}{\lambda_c}\geq\frac{\gamma}{1-\gamma}. \tag{4}$$

In short, if the firm charges p_{CC}^{LA} then full consumption is the PPE when conditions (3) and (4) hold.

Next we turn to the firm's revenue maximization problem. Although the firm can charge prices other than p^{LA}_{CC}, the maximum that the firm can earn under (N, C) is $\frac{(1-\gamma\lambda_c)(v+q\theta_h)}{1+\gamma\lambda_p}$. That revenue is exceeded by p^{LA}_{CC} if

$$(1+\lambda_c)(1+\gamma\lambda_p)\left(\frac{1}{(1-\gamma)(1-\gamma\lambda_c)}-1\right) \geq \frac{q(\theta_h-\theta_l)}{v_0+q\theta_l}. \tag{5}$$

We conclude that, when inequalities (3)–(5) hold, the optimal price is p^{LA}_{CC} and full consumption is the PPE.

When there are multiple products of different qualities, we obtain uniform pricing; that is, $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} = 0$, when (3)–(5) hold and $\theta_l = 0$.[6] When $\theta_l > 0$, we say that price compression occurs if the price under loss aversion and taste uncertainty responds less to quality than when either of these conditions is absent. So conditional on (3)–(5) we have $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} = (1+\lambda_c)\theta_l$, which we compare next with the corresponding expressions in the certain-taste and loss-neutral cases.

Consider the case where a certain taste for quality is normalized to $\mathbb{E}\theta$. Observe that the optimal price is $p^C \triangleq v_0 + q\mathbb{E}\theta$ and that $\frac{\partial p^C}{\partial q} = \mathbb{E}\theta$. We therefore have price compression, defined as $\frac{\partial p^{\mathrm{LA}}_{CC}}{\partial q} < \frac{\partial p^C}{\partial q}$, as long as $\mathbb{E}\theta > (1+\lambda_c)\theta_l$—which holds if there is sufficient taste uncertainty (i.e., if $\mathbb{E}\theta \gg \theta_l$).[7]

Similarly, in the loss-neutral case the optimal price is $p^{\mathrm{LN}} \triangleq v_0 + q\theta_h$ provided $(1-\gamma)(v_0+q\theta_h) > v_0 + q\theta_l$; in other words, it is optimal to sell to the consumer only when the taste draw is h. This condition is satisfied whenever the taste differential, defined as $q(\theta_h - \theta_l)$, is greater than $\gamma(v_0+q\theta_h)$. Then $\frac{\partial p^{\mathrm{LN}}}{\partial q} = \theta_h$ and price compression obtains if $\theta_h > (1+\lambda_c)\theta_l$, which holds for high taste differentials.

To sum up, our second main result is that if conditions (3)–(5) hold, $\theta_l > 0$, and there is enough taste uncertainty or taste differential (as defined previously), then the loss-averse price responds less to quality than does the price for a certain-taste or for a loss-neutral consumer. Thus taste uncertainty and loss aversion are both necessary for price compression to occur. Furthermore, conditions (3)–(5) are more likely to hold when quality q is low, monetary loss aversion λ_p is large, and consumption loss aversion λ_c is not too large.

For the general case with gain–loss aversion and continuous taste draws, we shall derive sufficient conditions such that uniform pricing and price compression occur. We follow the same approach used for the two-draw case: (a) characterize the set of PEs for any price, (b) analyze the PC, (c) derive the PPE, and (d) derive sufficient conditions such that full consumption obtains at the firm's optimal price. We concentrate on the full-consumption PPE because our goal is to understand the uniform pricing puzzle. The conditions given by inequalities (3), (4), and (5) generalize respectively to Assumptions 2, 3, and 4 in the analysis that follows. We shall also discuss (in the main text and also in Appendix C) what happens when these assumptions are violated.

[6] Products are priced independently, an approach that we motivate (in Section 5) in the context of the applications relevant to the uniform pricing puzzle.

[7] In the two-state case, this condition is equivalent to $\theta_h > (1+2\lambda_c)\theta_l$.

4.2. The Consumer's Problem

Under loss neutrality, a consumer will consume if, when the taste uncertainty is resolved, her valuation is greater than the price. This decision procedure translates into a threshold consumption rule: the consumer consumes if the taste draw is above the threshold $\theta = \frac{p-v_0}{q}$ provided that $\frac{p-v_0}{q} \in \boldsymbol{\Theta}$. Otherwise, the consumer either always consumes (when $p \leq v_0 + q\theta_0$) or never consumes (when $p \geq v_0 + q\theta_1$). We first show that, under loss aversion, the consumer still adopts a threshold rule in equilibrium.

Lemma 1. *In a PE, $\pi(\theta) \in \{0,1\}$ almost everywhere and $\pi(\theta)$ is nondecreasing in θ.*

Nonrandomization follows because $u(\pi(\theta)|\bar{\pi},\theta)$ is linear in $\pi(\theta)$. Lemma 1 implies that the optimal consumption plan takes a threshold form; that is, for some $\theta^* \in \boldsymbol{\Theta}$, consume if $\theta \geq \theta^*$ but not if $\theta < \theta^*$. Define $u^1(\theta,\theta^*)$ as the ex post utility of consuming and $u^0(\theta,\theta^*)$ as the ex post utility of not consuming when the consumer's taste draw is θ and the threshold is θ^*. Following the same reasoning as in Table 1 (see also Table 3), we have:

$$u^0(\theta,\theta^*) = -\lambda_c \int_{\theta^*}^{\theta_1} (v_0 + q\theta')\, dG(\theta') + \beta_p p \bar{G}(\theta^*) \quad \text{for } \theta \leq \theta^*; \tag{6}$$

$$\begin{aligned} u^1(\theta,\theta^*) =& v_0 + q\theta - p - \lambda_c q \int_{\theta}^{\theta_1} (\theta' - \theta)\, dG(\theta') + \beta_c \left((v_0 + q\theta) G(\theta^*) + q \int_{\theta^*}^{\theta} (\theta - \theta')\, dG(\theta') \right) \\ & - \lambda_p G(\theta^*) p \quad \text{for } \theta \geq \theta^*. \end{aligned} \tag{7}$$

Because the utility from not consuming is independent of the taste draw, we denote it simply by $u^0(\theta^*)$. From (6) and (7) it follows that the net utility of consuming over not consuming, $u^1(\theta,\theta^*) - u^0(\theta^*)$, is increasing in θ. In other words, the higher the taste realization for quality, the more inclined the consumer is to consume. Any interior equilibrium must solve $u^1(\theta,\theta^*) = u^0(\theta^*)$. (For ease of exposition, hereafter we omit the asterisk when no confusion could result). It follows that $\theta \in (\theta_0,\theta_1)$ is an interior PE if and only if

$$V(\theta) \triangleq (v_0 + q\theta) L(\theta) = p, \tag{8}$$

where the function

$$L(\theta) \triangleq \frac{1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta)}{1 + \beta_p + (\lambda_p - \beta_p) G(\theta)} \tag{9}$$

is positive and decreasing in θ with $L(\theta_0) = \frac{1+\lambda_c}{1+\beta_p} > 1$ and $L(\theta_1) = \frac{1+\beta_c}{1+\lambda_p} < 1$. The numerator of $L(\theta)$ increases the consumer's willingness to pay and is known as the *attachment effect*. A consumer who consumes in states greater than θ suffers an "attachment" in states lower than θ for which she does not consume. Consumption loss aversion alone pushes toward greater consumption relative to the loss-neutral case. The denominator reduces the consumer's willingness to pay and is called the *comparison effect*. The consumer receives a net monetary benefit in states lower than θ because she saves p relative to the states above θ in which she consumes. Price loss aversion alone pushes toward less consumption relative to the loss-neutral case. We define $\tilde{\theta} = G^{-1}\left(\frac{\lambda_c - \beta_p}{\lambda_c - \beta_c + \lambda_p - \beta_p}\right)$ as the state in which the attachment and comparison effects are equal; thus $L(\tilde{\theta}) = 1$.

There may also be corner equilibria. A corner PE at $\theta = \theta_0$ exists if $u^1(\theta_0, \theta_0) \geq u^0(\theta_0)$ or (equivalently) if

$$p \leq \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0); \tag{10}$$

similarly, a corner PE at $\theta = \theta_1$ exists if $u^1(\theta_1, \theta_1) \leq u^0(\theta_1)$ or (equivalently) if

$$p \geq \frac{1+\beta_c}{1+\lambda_p}(v_0 + q\theta_1). \tag{11}$$

We use $\mathbf{\Theta}^{\mathrm{PE}}(p)$ to denote the set of PEs associated with price p. This set may include interior PEs and corner PEs. Equilibrium multiplicity arises when a corner PE exists simultaneously with another corner or an interior PE. Multiple interior PEs may also exist because equation (8) may admit multiple solutions. In the absence of loss aversion, there is a unique solution to $v_0 + q\theta = p$ that determines the consumption rule. With loss aversion, however, there is no standard restriction on $g(\cdot)$ to impose regular behavior on the function $V(\theta)$. Hence a PE exists every time $V(\theta)$ crosses p, which may occur multiple times. We prove that a PPE always exists.

Lemma 2. *A PPE always exists.*

When θ is uniformly distributed over $[\theta_0, \theta_1]$, there are at most two interior PEs in addition to two possible corner PEs. The solid curves in Figure 1 plot $V(\theta) = (v_0 + q\theta)L(\theta)$ against the loss aversion coefficient; the dashed diagonal lines plot $V(\theta) = v_0 + q\theta$ for a loss-neutral consumer (we have $L(\theta) = 1$ under loss neutrality). The horizontal dotted lines on Figures 1(b) and 1(c) illustrate the interior PEs corresponding to hypothetical price $p = \$26$. For sufficiently large loss aversion coefficients there are either multiple interior PEs, as in Figure 1(b), or no interior PEs, as in Figure 1(c). In the latter case, only the corner θ_1 is a PE.

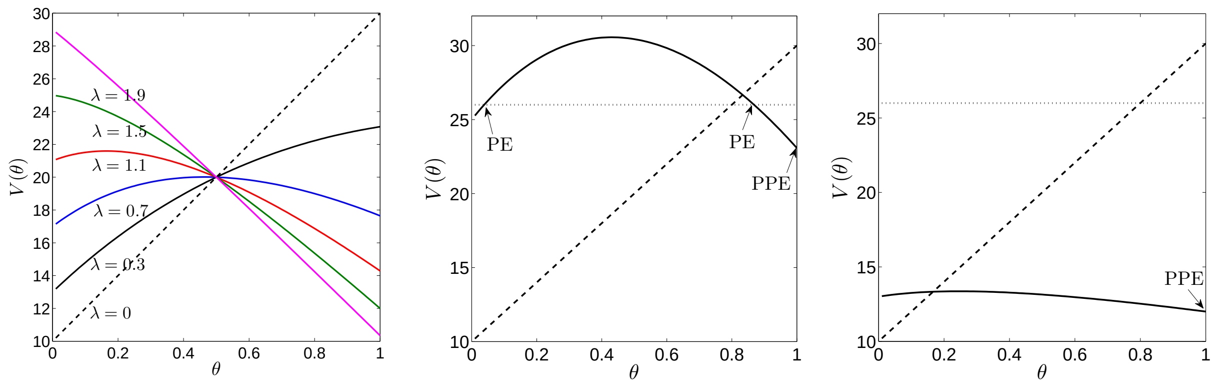
A consumer who consumes when her taste draw is above threshold θ derives the expected utility

$$\mathrm{EU}(\theta, p) = u^0(\theta)G(\theta) + \int_{\theta}^{\theta_1} u^1(\theta', \theta)\, dG(\theta'). \tag{12}$$

Our next result simplifies $\mathrm{EU}(\theta, p)$ and helps distinguish its components.

Lemma 3. *The consumer's expected utility from consuming in accordance with the threshold consumption rule θ is*

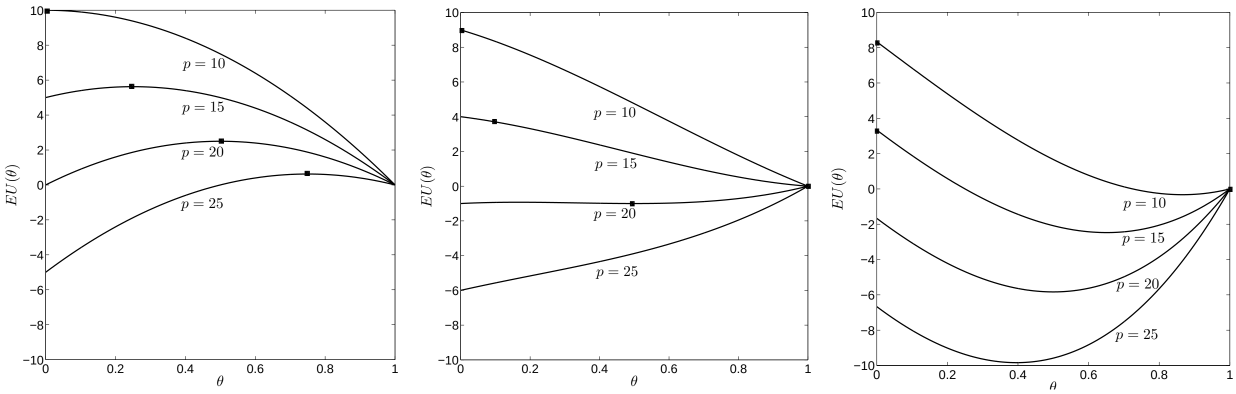
$$\begin{aligned} EU(\theta, p) = &\int_{\theta}^{\theta_1} (v_0 + q\theta' - p)\, dG(\theta') - (\lambda_c - \beta_c) \int_{\theta}^{\theta_1} (G(\theta') - \bar{G}(\theta'))(v_0 + q\theta')\, dG(\theta') \\ &- (\lambda_p - \beta_p) p G(\theta) \bar{G}(\theta). \end{aligned} \tag{13}$$



(a) $\lambda_c = \lambda_p = \lambda$ (b) $\lambda_c = 1.5, \lambda_p = 0.3$ (c) $\lambda_c = 0.3, \lambda_p = 1.5$

Figure 1 The solid curves plot $V(\theta) = (v_0 + q\theta)L(\theta)$. The dashed line plots $v_0 + q\theta$, the loss-neutral consumer's willingness to pay. The parameters used for this figure are $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$, and $\theta \sim U[0,1]$.

The expected utility in (13) has three components. The first term is the standard expected utility without loss aversion. The second term captures consumption loss aversion and is negative. The third term is the monetary loss aversion and is also negative. Although monetary loss aversion is zero under full consumption, the consumption loss aversion is not because the consumer compares consumption utility across taste draws. Figure 2 illustrates consumer expected utility under a uniform taste distribution for three (λ_c, λ_p) pairs. The black dots represent the PPEs. In Figure 2(a), the-loss neutral consumer $(\lambda_p = \lambda_c = 0)$ has a concave expected utility and the θ that maximizes expected utility is also the threshold θ (from Figure 1) such that $v_0 + q\theta = p$. Low levels of loss aversion eliminate the curvature of the expected utility as shown in Figure 2(b). For sufficiently large values of loss aversion coefficients, the expected utility is convex (see Figure 2(c)) and the PPE is achieved at a corner.



(a) $\lambda_c = 0, \lambda_p = 0$ (b) $\lambda_c = 0.3, \lambda_p = 0.3$ (c) $\lambda_c = 0.5, \lambda_p = 1$

Figure 2 Consumer's expected utility under loss neutrality and loss aversion. The PPEs are shown on the graphs with black rectangles. In this figure, $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$ and $\theta \sim U[0,1]$. The black dots represent the PPE θ corresponding to price $p = 10, 15, 20, 25$.

The set of PPEs associated with price p is

$$\mathbf{\Theta}^{\mathrm{PPE}}(p) = \left\{\theta \in \mathbf{\Theta}^{\mathrm{PE}}(p) \mid \mathrm{EU}(\theta, p) \geq \mathrm{EU}(\theta', p) \,\forall\, \theta' \in \mathbf{\Theta}^{\mathrm{PE}}(p)\right\}.$$

The set $\mathbf{\Theta}^{\mathrm{PPE}}(p)$ is nonempty (by Lemma 2) and it could have multiple elements if the consumer receives the same expected utility in multiple nondominated PEs. In this case we use the tie-breaking rule that the consumer selects the lowest PPE with nonnegative utility, which is the PPE that maximizes the firm's profits. With this convention, $\Theta^{\mathrm{PPE}}(p)$ has a unique element. Although a PPE exists for any price, the reverse is not true: there may exist thresholds that are not a PPE for any price. The set of implementable consumption thresholds is $\bigcup_{p>0} \mathbf{\Theta}^{\mathrm{PPE}}(p)$.

Characterizing $\mathbf{\Theta}^{\mathrm{PPE}}(p)$ for a given p consists of ranking PEs according to the expected utility criteria, so this ranking will depend on the shape of $\mathrm{EU}(\theta, p)$. The three panels of Figure 2 together suggest that there may not be any simple and general ranking rules. Take, for instance, the case $\lambda_p = \lambda_c = 0.3$ (Figure 2(b)) and $p = 20$; the expected utility is almost flat. If there were multiple interior PEs, then the PPE selection could change for arbitrarily small changes in p or λ. That being said, an interesting pattern appears in Figure 2(c): extreme consumption thresholds ($\theta_0 = 0$ or $\theta_1 = 1$) dominate intermediate ones. In fact, we can derive fairly general conditions such that if the full-consumption corner θ_0 is a PE then it is also the PPE. To proceed, we use the notation $\hat{\theta}(\theta)$ to signify the symmetric value of $\theta \in [\theta_0, \mathbb{E}\theta]$ relative to $\mathbb{E}\theta$; that is, $\frac{\hat{\theta}(\theta)+\theta}{2} = \mathbb{E}\theta$. For ease of exposition, we usually drop the argument in $\hat{\theta}(\cdot)$ and simply write $\hat{\theta}$.

Lemma 4. *Suppose Assumption 1 holds, let $v_0 + q\mathbb{E}\theta \geq p_0$, and assume θ is a PE in $[\theta_0, \mathbb{E}\theta]$ such that $\frac{\partial EU(\theta)}{\partial \theta} < 0$ and $EU(\theta) \geq 0$. Then θ dominates any PE in $(\theta, \hat{\theta}]$.*

The assumption $v_0 + q\mathbb{E}\theta \geq p_0$ implies that the expected surplus of a loss-neutral consumer under full consumption is nonnegative, or that the price is not too high. According to Lemma 4, if the expected utility is decreasing at a PE for which consumption happens frequently ($\theta < \mathbb{E}\theta$) then that PE dominates any PE with intermediate consumption frequencies. Our next result establishes when the inequality $\frac{\partial \mathrm{EU}(\theta)}{\partial \theta} < 0$ holds.

Lemma 5. *Assume that θ is a PE. Then $\frac{\partial EU(\theta)}{\partial \theta} < 0$ if and only if $G(\theta) < \frac{\lambda_p(1+\lambda_c) - \beta_c(1+\beta_p)}{\lambda_c - \beta_c + \lambda_p - \beta_p + 2(\lambda_p\lambda_c - \beta_c\beta_p)}$.*

The necessary and sufficient condition in Lemma 5 simplifies to the following assumption when applied to the corner PE at θ_0.

Assumption 2. *$\lambda_p(1 + \lambda_c) > \beta_c(1 + \beta_p)$.*

This assumption is not a restrictive one. It is implied, for example, by $\lambda_p \geq \beta_c$, which in turn is implied by Köszegi and Rabin's (2006) assumption that gain and loss coefficients are the same for consumption and money ($\lambda_c = \lambda_p$ and $\beta_c = \beta_p$). The assumption also holds if the gain part of the value function is assumed to be flat ($\beta_c = \beta_p = 0$). Put together, Lemmas 4 and 5 imply that—if Assumptions 1 and 2 hold and if θ_0 is a PE such that $\mathrm{EU}(\theta_0) \geq 0$—then θ_0 is a PPE. The reason is that the expected utility decreases at $\theta = \theta_0$ and never returns to that level for $\theta > \theta_0$. To see this clearly, consider the three terms in equation (13); the first term is inverse U–shaped with a peak at $\theta = \frac{p-v_0}{q}$; the last two terms are negative and have a unique minimum at $\mathbb{E}\theta$. If these last two terms are sufficiently large, then the expected utility is initially decreasing and has a U-like shape; see Figure 2(c).

Now, applying Lemma 4 to the corner at θ_0 requires further that we check the participation constraint. The consumer's expected utility from full consumption is

$$\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) = v_0 + q\mathbb{E}\theta - p_0^{\mathrm{LA}} - (\lambda_c - \beta_c)q\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta'). \tag{14}$$

Assumption 3 gives a sufficient condition for $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$.

Assumption 3. $q(\mathbb{E}\theta - \theta_0)\big(1 - \frac{\lambda_c - \beta_c}{2}\big) \geq \frac{\lambda_c - \beta_p}{1+\beta_p}(v_0 + q\theta_0)$.

Assumption 3 is independent of $G(\cdot)$, but clearly PC holds more generally. The consumer participates if her expected consumption utility dominates the net loss from comparing consumption utility across θs, which it will whenever there is no loss aversion in consumption ($\lambda_c = \beta_c = 0$). More generally, Assumption 3 is more likely to hold for $\mathbb{E}\theta - \theta_0$ large and λ_c small. We can now present the main result of this section: characterizing a set of sufficient conditions under which full consumption solves the consumer's problem.

Proposition 1. *Suppose Assumptions 1–3 hold and that* $p_0^{\mathrm{LA}} = \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0)$. *Then* θ_0 *is a PPE.*

4.3. The Firm's Problem

The firm's revenue can be written as a function of the consumption threshold θ. Consider the interior values of $\theta \in (\theta_0, \theta_1)$. The threshold θ is a PE for price $p = (v_0 + q\theta)L(\theta)$. One may then conclude that the consumer will buy at price p with probability $\bar{G}(\theta)$. However, this follows only if θ is also a PPE (i.e., $\theta \in \bigcup_{p>0} \mathbf{\Theta}^{\mathrm{PPE}}(p)$) *and* satisfies the PC, which for interior PEs is equivalent to $\mathrm{EU}(\theta, (v_0 + q\theta)L(\theta)) \geq 0$. The firm's revenues are then

$$R^{\mathrm{LA}}(\theta) = (v_0 + q\theta)L(\theta)\bar{G}(\theta). \tag{15}$$

The firm maximizes $R^{\mathrm{LA}}(\theta)$ subject to PC and PPE. In addition to choosing interior thresholds, the firm can choose the full-consumption corner θ_0. In that case, the firm's revenue is $p_0^{\mathrm{LA}} \triangleq R^{\mathrm{LA}}(\theta_0) =$

$\frac{1+\lambda_c}{1+\beta_p}(v_0+q\theta_0)$ if $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$; otherwise the revenue is the highest price such that $\mathrm{EU}(\theta_0, p) = 0$. We ignore the corner θ_1 because the firm earns zero revenue there. Denote by θ^{LA} the threshold that maximizes the firm's revenue.

Loss aversion changes the firm's objective function relative to the loss-neutral case in two ways: first, the objective function is weighted by $L(\theta)$; and second, not all consumption thresholds are feasible. The function $R^{\mathrm{LA}}(\theta)$ is not necessarily concave, and the set of thresholds that satisfy PPE and PC is not necessarily convex. We could not find general conditions to characterize the optimal solution for all parameter values of loss aversion. Our main result rests on the observation that the revenue function reaches a maximum at θ_0 for a general subset of parameter values. In particular, this will be the case when $R^{\mathrm{LA}}(\theta)$ is decreasing in θ or, equivalently, when

$$\frac{\frac{\partial}{\partial\theta}[(v_0+q\theta)\bar{G}(\theta)]}{(v_0+q\theta)\bar{G}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)}. \tag{16}$$

We make the standard assumption that the firm revenue under loss neutrality, $R^{\mathrm{LN}}(\theta) = (v_0 + q\theta)\bar{G}(\theta)$, is single peaked with a maximum at θ^{LN}. Because $L(\theta)$ is decreasing, inequality (16) always holds for $\theta \geq \theta^{\mathrm{LN}}$. We derive a sufficient condition such that $R^{\mathrm{LA}}(\theta)$ is decreasing for any θ. Define $\varepsilon_0 = (v_0+q\theta_0)\frac{g(\theta_0)}{q}$ as the price elasticity of a loss-neutral consumer at the corner $p = v_0 + q\theta_0$.

ASSUMPTION 4. $1+\bar{G}(\theta^{\mathrm{LN}})\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_c)(1+\lambda_p)}\right) \geq \varepsilon_0^{-1}$.

Since ε_0^{-1} increases with $\frac{q}{v_0}$, it follows that Assumption 4 is less likely to hold for high-quality products.

LEMMA 6. *Assumption 4 implies that $R^{\mathrm{LA}}(\theta)$ is decreasing in θ. For uniformly distributed taste uncertainty, a necessary and sufficient condition for $R^{\mathrm{LA}}(\theta)$ to be decreasing in θ is that $\frac{q}{v_0} < 1+\lambda_p+\frac{\lambda_c}{1+\lambda_c}$.*

Assumption 4 gives only a sufficient condition, and $R^{\mathrm{LA}}(\theta)$ can also be decreasing more generally. In particular, $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0 if the price elasticity of demand at θ_0, denoted $\varepsilon_0^{\mathrm{LA}}$, is greater than 1. We have $\varepsilon_0^{\mathrm{LA}} = \frac{\varepsilon_0}{1-\kappa\varepsilon_0}$, where $\kappa = \frac{\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p}{(1+\beta_p)(1+\lambda_c)}$. Although loss aversion increases the price elasticity of demand, monetary and consumption loss aversion do not have the same effects: $\kappa \to \infty$ with λ_p whereas $\lim_{\lambda_c\to\infty}\kappa = \frac{1+\lambda_p}{1+\beta_p}$. There is always a sufficiently large λ_p that $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0. Hence loss aversion transforms a standard inverse U–shaped revenue function into a function that is decreasing at θ_0 when $1+\kappa > \varepsilon_0^{-1}$ and decreasing everywhere when Assumption 4 holds. For a uniform taste distribution and $\beta_c = \beta_p = 0$, the inequality $1+\kappa > \varepsilon_0^{-1}$ is equivalent to the condition in Lemma 6 and so $R^{\mathrm{LA}}(\theta)$ decreasing at θ_0 implies that it is decreasing everywhere. This will be the case when monetary loss aversion is high or when product quality is low.

Proposition 2. *Suppose that Assumptions 1–4 hold. Then the full-consumption corner with associated price* p_0^{LA} *solves the firm's revenue maximization problem; that is,* $\theta^{\mathrm{LA}} = \theta_0$.

The intuition behind this proposition is as follows. Under loss neutrality, the firm must lower its price to $p_0^{\mathrm{LN}} = v_0 + \theta_0 q$ to induce the consumer to always buy. Such a low price is not necessary under loss aversion because the firm can charge $p_0^{\mathrm{LA}} > p_0^{\mathrm{LN}}$ provided the consumer participates (which, by Assumption 3, she does). That statement holds as long as the firm does not want to increase the price above p_0^{LA}. This will be the case if demand elasticity is not too large at θ_0 (Assumption 4).

5. The Uniform Pricing Puzzle

In this section, we assume that the firm sells multiple quality classes and study how the optimal price depends on quality. We follow Shiller and Waldfogel (2011) in ignoring demand interactions across quality classes. They argue that this approach is appropriate when the products' demands are "independent" and provide evidence that this condition is satisfied in the context of their application to the pricing of music songs. For a product of quality q, we denote by $p^{\mathrm{LA}}(q)$ the firm's optimal when selling to a loss-averse consumer. Similarly, let $p^{\mathrm{LN}}(q) = v_0 + q\theta^{\mathrm{LN}}(q)$ be the firm's optimal price when selling to a loss-neutral consumer, where $\theta^{\mathrm{LN}}(q)$ maximizes the loss-neutral revenue generated by a product of quality q. We make the standard assumption that there is a unique interior solution to the firm revenue maximization problem when the consumer is loss neutral, which implies that the firm's optimal price satisfies the comparative static $p_q^{\mathrm{LN}}(q) > 0$.[8] When taste is certain, we normalize it to $\mathbb{E}\theta$; then the optimal price, denoted $p^{\mathrm{C}}(q)$, is equal to $v_0 + q\mathbb{E}\theta$ (see Section 4.1).

Figure 3(a) plots $p^{\mathrm{LA}}(q)$ for $\lambda_p = 0.6$ and for different values of λ_c. The figure also shows the optimal price when consumers are loss neutral, $p^{\mathrm{LN}}(q)$, and when there is no taste uncertainty, $p^{\mathrm{C}}(q)$. The pricing schedules $p^{\mathrm{LA}}(q)$ have at most two kinks. The flat segments correspond to the case discussed previously: full consumption is optimal and the participation constraint holds. The parts to the right of these flat segments correspond to interior equilibria (i.e., where Assumption 4 is violated); the parts to the left correspond to a binding participation constraint (i.e., where Assumption 3 is violated).

Clearly, the pricing schedules under loss aversion are less steep than under loss neutrality and taste certainty. To formalize this observation, we need workable definitions of uniform pricing and price compression that are applicable to our setting. A strict interpretation of the uniform pricing puzzle is that the price does not respond to quality, $p_q^{\mathrm{LA}}(q) = 0$; we denote this property (P1).

[8] The unimodality of the loss-neutral revenue function $(v_0 + q\theta)\bar{G}(\theta)$ is implied, for example, by the log-concavity of $g(\cdot)$. Sufficient conditions for an interior solution are $\varepsilon_0 < 1$ (which is equivalent, for a uniform $\boldsymbol{\Theta} = [0, 1]$ distribution, to $q > v_0$), and $g(\theta_1) > 0$. We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$ and $\theta_q^{\mathrm{LN}} > 0$.

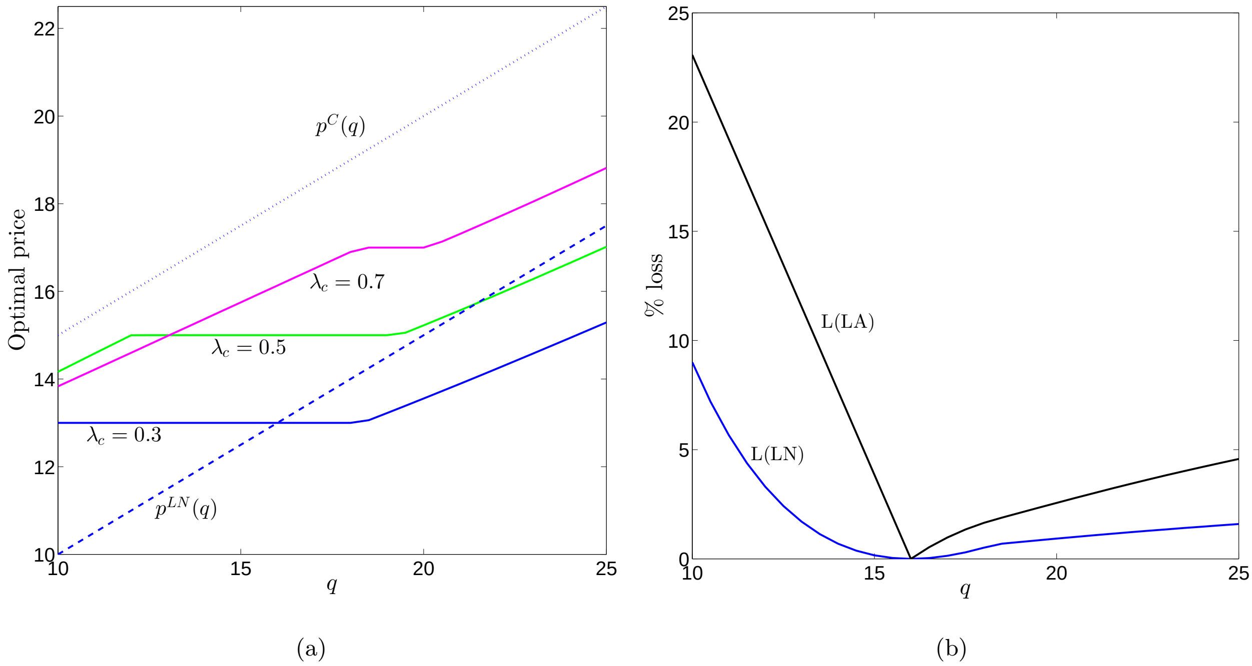


Figure 3 **In both panels we have $\lambda_p = 0.6$, $v_0 = 10$, and $\theta \sim U[0,1]$. Panel (a) plots the optimal price as a function of q for different values of λ_c. In Panel (b) the top curve $L(LA)$ plots the percentage profit loss from wrongly assuming that the consumer is loss neutral when she is in fact loss averse (with $\lambda_c = 0.3$) and the lower curve $L(LN)$ plots the same loss from wrongly assuming that the consumer is loss averse when she is in fact loss neutral.**

(Loss aversion may also decrease the responsiveness of price to quality more generally, which we refer to as price compression.) A second property, (P2), is that $p_q^{\mathrm{LA}}(q) < p_q^{\mathrm{i}}(q)$ for $i \in \{\mathrm{LN}, \mathrm{C}\}$. This property holds everywhere in Figure 3(a): for a given q, the both the loss-neutral and no-uncertainty price schedules (the dashed and dotted lines, respectively) are steeper than any loss-averse price schedule.

Here we address the case corresponding to Assumptions 1–4 (the flat portion of the pricing schedules); additional results can be derived when these conditions do not hold (see Appendix C). We next state the paper's main result on price compression and price uniformity.

Proposition 3. *Suppose Assumptions 1–4 hold. (a) If $\theta_0 = 0$, then (P1) and (P2) hold. (b) If $\theta_0 > 0$, then (P2) holds provided* $\min(\mathbb{E}\theta, \theta^{\mathrm{LN}}) > \frac{1+\lambda_c}{1+\beta_p}\theta_0$.

Proposition 3(a) explains why vertically differentiated products with a wide range of quality may sell at the same price: we have $p_q^{\mathrm{LA}}(q) = 0$ and so (P1) holds; because $p_q^{\mathrm{LN}}(q) > 0$ and $p_q^{\mathrm{C}}(q) > 0$, we conclude that (P2) holds as well. Thus price is less responsive to quality under loss aversion than under loss neutrality. We now illustrate the relevance of Proposition 3(a) when the consumer's taste is uniformly distributed.

Corollary 1. *Assume that taste is distributed* Uniform$[0,1]$ *and that* $\lambda_c < 3$. *If* $\frac{6\lambda_c}{3-\lambda_c} \leq \frac{q}{v_0} < 1+\lambda_p+\frac{\lambda_c}{1+\lambda_c}$ *then* $p^{\mathrm{LA}}(q) = (1+\lambda_c)v_0$ *and consumption plan* $\theta = 0$ *maximizes the firm's profits.*

The conditions stated in Corollary 1 are tight bounds for $p_q^{\mathrm{LA}} = 0$ (the flat segments on Figure 3(a)). Uniform pricing can be optimal with consumption loss aversion alone or with monetary loss aversion alone. Even so, the two sources of loss aversion play asymmetric roles. *Monetary loss aversion* can only make it more likely that uniform pricing is optimal, and for high quality products ($q > 2v_0$) only monetary loss aversion can make uniform pricing optimal. *Consumption loss aversion* cannot be too strong if uniform pricing is to be optimal. Not only does an increase in consumption loss aversion lower expected utility, but also PC is violated for high enough λ_c. When $\frac{6\lambda_c}{3-\lambda_c} > \frac{q}{v_0}$, the full-consumption price must be set below p_0^{LA} in order for PC to hold.

Proposition 3(b) shows that (P2) holds for $\theta_0 > 0$ when both θ^{LN} and $\mathbb{E}\theta$ are sufficiently large relative to θ_0. What matters is any taste uncertainty that creates a gap between the lowest possible taste draw and the rest of the distribution. The intuition here is that, under loss aversion, $p_q^{\mathrm{LA}}(q) = \theta^{\mathrm{LA}}(q)L(\theta^{\mathrm{LA}}(q)) = \frac{1+\lambda_c}{1+\beta_p}\theta_0$ because $\theta_q^{\mathrm{LA}}(q) = 0$. With taste certainty we have $p_q^{\mathrm{C}}(q) = \mathbb{E}\theta$ whereas under loss neutrality, a lower bound for $p_q^{\mathrm{LN}}(q)$ is $\theta^{\mathrm{LN}}(q)$.[9]

If the consumer is loss averse, then a firm that deviates from uniform pricing will see its profits decline substantially. The reason is that the firm's optimal profits are achieved at the full-consumption corner and the firm's profits are not flat at the optimal price. Any deviation from the optimal price imposes a first-order loss as illustrated by Figure 3(b). For $\lambda_c = 0.3$, the top curve (L(LA)) plots the percentage profit loss from wrongly assuming that the consumer is loss neutral (when she is in fact loss averse) and hence charging $p^{\mathrm{LN}}(q)$ instead of $p^{\mathrm{LA}}(q)$; the bottom curve (L(LN)) plots the loss from making the opposite mistake. When $\lambda_c = 0.3$ we have $p^{\mathrm{LN}}(16) = p^{\mathrm{LA}}(16)$ and so $L(\mathrm{LA}) = L(\mathrm{LN}) = 0$ for $q = 16$. This establishes a benchmark case for which there is no cost of not knowing whether the consumer is loss neutral or loss averse. We see that L(LN) stays close to zero for small deviations from $q = 16$ because small pricing mistakes have only a second-order effect on profits under loss neutrality. Yet this is not the case for L(LA): when the consumer is loss averse, even small mistakes can have a large negative effect on profits.

We remark that it is essential to assume the complementarity between the taste draw and product quality in the consumer valuation. To see why, assume to the contrary that the taste draw and product quality are additive; that is, the consumer valuation is $\tilde{q} + \theta$ (instead of $v_0 + q\theta$), where $\tilde{q}$ is an additive quality component. The analysis proceeds as before once we put $v_0 = \tilde{q}$ and $q = 1$. The optimal price under loss aversion is $p^{\mathrm{LA}}(\tilde{q}) = (\tilde{q} + \theta_0)\frac{1+\lambda_c}{1+\beta_p}$, and so $p_{\tilde{q}}^{\mathrm{LA}}(\tilde{q}) = \frac{1+\lambda_c}{1+\beta_p}$. Under loss

[9] We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$, where $q\theta_q^{\mathrm{LN}}(q) > 0$ is the indirect effect due to re-optimizing price.

neutrality, the price schedule is such that $p_{\tilde{q}}^{\text{LN}}(\tilde{q}) < 1$.[10] The price schedule is steeper under loss aversion than under loss neutrality: $p_{\tilde{q}}^{\text{LA}}(\tilde{q}) > p_{\tilde{q}}^{\text{LN}}(\tilde{q})$. In both the additive and multiplicative cases, loss aversion increases consumption; that is, the consumer buys the product for all taste draws. But this alone is not sufficient to make the price schedule flatter; in addition, product quality and the taste draw must be complements. If that is the case then the firm's price, which is equal to the lower bound of the valuation support, is relatively unresponsive to a change in quality.

Under general conditions, the consumer's expected utility increases with product quality. We use $\widetilde{\text{EU}}(q) = \text{EU}(\theta_0, p_0^{\text{LA}}(q))$ to denote the consumer's expected utility from product q when the price is $p_0^{\text{LA}}(q)$. Then

$$\frac{\partial \widetilde{\text{EU}}(q)}{\partial q} = \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

LEMMA 7. *Let $\lambda_c \geq \beta_p$ and assume that Assumption 3 holds. Then $\frac{\partial \widetilde{\text{EU}}(q)}{\partial q} \geq (\mathbb{E}\theta - \theta_0)(1 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)) - \frac{\lambda_c - \beta_p}{1+\beta_p}\theta_0 \geq 0$.*

Lemma 7 states that although consumers suffer a greater consumption gain–loss utility from higher-quality products, the overall effect (after accounting for direct consumption utility) is that utility increases with quality. This establishes a key feature of the uniform pricing puzzle: price compression is observed even though consumers receive a strictly larger surplus from better products.

6. Conclusions

The uniform pricing puzzle has not been resolved for the case of vertically differentiated products. We present a model of monopoly pricing with vertically differentiated products, consumer loss aversion, and taste uncertainty; in this model the consumer compares current purchases using a lagged expectation of transactions involving products of the same quality. Thus loss aversion applies within a class of products of the same quality but *not* across quality classes. We show that uniform pricing can be optimal across quality classes up to a quality threshold. This will be the case if the consumer is sufficiently loss averse in monetary utility (but not too loss averse in consumption utility), if taste is sufficiently uncertain, and if product quality and the consumer's idiosyncratic taste draw are complements. If these conditions are satisfied, then the consumer consumes for all taste draws and the price is affected by quality only through the latter's effect on the consumer's lowest possible valuation. Price compression occurs because that lowest valuation responds little to quality, and price uniformity is optimal when the lowest valuation does not depend on quality. In both cases, the price differences across quality classes is smaller under loss aversion than under loss neutrality, and consumer surplus increases with quality (which is consistent with

[10] We have $p^{\text{LN}}(\tilde{q}) = \tilde{q} + \theta^{\text{LN}}(\tilde{q})$ and because $\theta_{\tilde{q}}^{\text{LN}}(\tilde{q}) < 0$, it follows that $p_{\tilde{q}}^{\text{LN}}(\tilde{q}) = 1 + \theta_{\tilde{q}}^{\text{LN}}(\tilde{q}) < 1$.

casual observation). Finally, the loss from mistakenly assuming that the consumer is loss neutral dominates the cost of mistakenly assuming she is loss averse. This finding is of relevance to the empirical literature that compares the profitability of price uniformity and variable pricing (e.g., Chu et al. 2011, Shiller and Waldfogel 2011).

Our aim in this paper is to demonstrate that loss aversion together with uncertain taste for quality can explain price compression and price uniformity for vertically differentiated products. Although our approach is plausible in the contexts described here, we do not rule out other explanations based on menu cost, contractual constraints, or other rationales in the various contexts where uniform pricing applies. The main message delivered by our research is that even small values of loss aversion can have significant effect on the firm's optimal price when consumers' valuation is uncertain.

References

Abeler, J., A. Falk, L. Goette, and D. Huffman. 2011. Reference points and effort provision. *American Economic Review* 101 (2): 470–492.

Anderson, E. T., and J. D. Dana, Jr. 2009. When is price discrimination profitable? *Management Science* 55 (6): 980–989.

Ariely, D., J. Huber, and K. Wertenbroch. 2005. When do losses loom larger than gains? *Journal of Marketing Research* 42 (2): 134–138.

ArtsJournal 2013. Why is dynamic pricing so rarely used? (February 28).

Baron, O., M. Hu, S. Najafi-Asadolahi, and Q. Qian. 2015. Newsvendor selling to loss-averse consumers with stochastic reference points. *Manufacturing and Service Operations Management*. forthcoming.

Bolton, P., and M. Dewatripont. 2005. *Contract theory.* MIT press.

Carbajal, J. C., and J. C. Ely. 2012. Optimal contracts for loss averse consumers. Working paper.

Chen, Y., and T. H. Cui. 2013. The benefit of uniform price for branded variants. *Marketing Science* 32 (1): 36–50.

Chu, C. S., P. Leslie, and A. Sorensen. 2011. Bundle-size pricing as an approximation to mixed bundling. *American Economic Review* 101 (1): 263–303.

Courty, P., and M. Pagliero. 2014. The pricing of art and the art of pricing: Pricing styles in the concert industry. *Handbook of the Economics of Art and Culture* 2:299–356.

Deneckere, R. J., and P. R. McAfee. 1996. Damaged goods. *Journal of Economics & Management Strategy* 5 (2): 149–174.

Eckert, A., and D. S. West. 2013. Proliferation of brewers' brands and price uniformity in canadian beer markets. *Review of Industrial Organization* 42 (1): 63–83.

Eliaz, K., and R. Spiegler. 2015. X-games. *Games and Economic Behavior* 89 (93): 93–100.

Ericson, K. M. M., and A. Fuster. 2011. Expectations as endowments: Evidence on reference-dependent preferences from exchange and valuation experiments. *Quarterly Journal of Economics* 126 (4): 1879–1907.

Hahn, J.-H., J. Kim, S.-H. Kim, and J. Lee. 2015. Price discrimination with loss averse consumers. *Working Paper*.

Heidhues, P., and B. Kőszegi. 2005. The impact on consumer loss aversion on pricing. Mimeo. University of California, Berkeley.

Heidhues, P., and B. Kőszegi. 2008. Competition and price variation when consumers are loss averse. *American Economic Review* 98 (4): 1245–1268.

Heidhues, P., and B. Kőszegi. 2014. Regular prices and sales. *Theoretical Economics* 9 (1): 217–251.

Herweg, F. 2013. The expectation-based loss-averse newsvendor. *Economic Letters* 120 (3): 429–432.

Herweg, F., and K. Mierendorff. 2013. Uncertain demand, consumer loss aversion, and flat-rate tariffs. *Journal of the European Economic Association* 11 (2): 399–432.

Iyer, G., and P. B. Seetharaman. 2003. To price discriminate or not: Product choice and the selection bias problem. *Quantitative Marketing and Economics* 1 (2): 155–178.

Kahneman, D., J. L. Knetsch, and R. Thaler. 1986. Fairness as a constraint on profit seeking: Entitlements in the market. *American Economic Review* 76 (4): 728–741.

Karle, H., and M. Peitz. 2014. Competition under consumer loss aversion. *RAND Journal of Economics* 45 (1): 1–31.

Kőszegi, B., and M. Rabin. 2006. A model of reference-dependent preferences. *Quarterly Journal of Economics* 121 (4): 1133–1165.

Kőszegi, B., and M. Rabin. 2009. Reference-dependent consumption plans. *American Economic Review* 99 (3): 909–936.

Leslie, P., and A. Sorensen. 2014. Resale and rent-seeking: An application to ticket markets. *Review of Economic Studies* 81 (1): 266–300.

Lindsey, R. 2011. State-dependent congestion pricing with reference-dependent preferences. *Transportation Research Part B: Methodological* 45 (10): 1501–1526.

Liu, Q., and D. Zhang. 2013. Dynamic pricing competition with strategic customers under vertical product differentiation. *Management Science* 59 (1): 84–101.

McMillan, R. S. 2007. Different flavor, same price: The puzzle of uniform pricing for differentiated products. *Same Price: The Puzzle of Uniform Pricing for Differentiated Products (January 17, 2007)*.

Moorthy, K. S. 1984. Market segmentation, self-selection and product line design. *Marketing Science* 3 (4): 288–305.

Mussa, M., and S. Rosen. 1978. Monopoly and product quality. *Journal of Economic Theory* 18 (2): 301–317.

Nasiry, J., and I. Popescu. 2011. Dynamic pricing with loss-averse consumers and peak-end anchoring. *Operations Research* 59 (6): 1361–1368.

Novemsky, N., and D. Kahneman. 2005. The boundaries of loss aversion. *Journal of Marketing research* 42 (2): 119–128.

Orbach, B. Y., and L. Einav. 2007. Uniform prices for differentiated goods: The case of the movie-theater industry. *International Review of Law and Economics* 27 (2): 129–153.

Pope, D. G., and M. E. Schweitzer. 2011. Is tiger woods loss averse? persistent bias in the face of experience, competition, and high stakes. *American Economic Review* 101 (1): 129–157.

Popescu, I., and Y. Wu. 2007. Dynamic pricing strategies with reference effects. *Operations Research* 55 (3): 413–429.

Shiller, B., and J. Waldfogel. 2011. Music for a song: An empirical look at uniform pricing and its alternatives. *Journal of Industrial Economics* 59 (4): 630–660.

Talluri, K., and G. van Ryzin. 2004. Revenue management under a general discrete choice model of consumer behavior. *Management Science* 50 (1): 15–33.

The Atlantic 2012. Why do all movie tickets cost the same? (January 3).

The Guardian 2006. Rent rise brings down curtain on easycinema. (May 29).

Tversky, A., and D. Kahneman. 1991. Loss aversion in riskless choice: A reference-dependent model. *Quarterly Journal of Economics* 106 (4): 1039–1061.

Verboven, F. 1999. Product line rivalry and market segmentationwith an application to automobile optional engine pricing. *Journal of Industrial Economics* 47 (4): 399–425.

Yang, L., P. Guo, and Y. Wang. 2014. Service pricing with loss averse customers. SSRN working paper.

Zhou, J. 2011. Reference dependence and market competition. *Journal of Economics & Management Strategy* 20 (4): 1073–1097.

Appendix A: Notation

Table 2 summarizes the notation used in the paper.

Table 2 Notation

q	product quality
$\theta \in \boldsymbol{\Theta} = [\theta_0, \theta_1]$	consumer's taste draw
$g(\theta)$, $G(\theta)$	p.d.f and c.d.f. of θ
$\hat{\theta}(\theta)$	symmetric value of $\theta \in [\theta_0, \mathbb{E}\theta]$ relative to $\mathbb{E}\theta$
λ_c, β_c	loss–gain parameters in consumption utility
λ_p, β_p	loss–gain parameters in monetary utility
$\pi(\theta)$	consumption probability in state θ
$\bar{\pi} = \{\pi(\theta)\}_{\theta \in \boldsymbol{\Theta}}$	consumption plan
$u(\bar{\pi}\|\bar{\pi}, \theta)$	utility from consumption plan $\bar{\pi}$ in state θ; equation (17)
$\mathrm{EU}(\bar{\pi})$	expected utility of consumption plan $\bar{\pi}$; equation (2)
$u^0(\theta^*)$	utility of not consuming for threshold θ^*; equation (6)
$u^1(\theta, \theta^*)$	utility of consuming for taste draw θ and threshold θ^*; equation (7)
$\mathrm{EU}(\theta, p)$	expected utility from threshold θ; equations (12) and (13)
$L(\theta)$	ratio of attachment effect to comparison effect; equation (9)
$\tilde{\theta}$	state θ at which $L(\theta) = 1$
$\boldsymbol{\Theta}^{\mathrm{PE}}(p)$	set of interior PEs associated with price p
$\boldsymbol{\Theta}^{\mathrm{PPE}}(p)$	PPE associated with price p
$R^{\mathrm{LA}}(\theta; q)$	firm's revenue when consumers are loss averse; equation (15)
$\theta^{\mathrm{LA}}(q)$, $p^{\mathrm{LA}}(q)$	consumption threshold and price-maximizing revenue under loss aversion
p_0^{LA}	highest price such that full consumption is a PE
$\theta^{\mathrm{LN}}(q)$, $p^{\mathrm{LN}}(q)$	consumption threshold and price-maximizing revenue under loss neutrality

Appendix B: Proofs

Proof of Lemma 1: We first establish the ex post utility $u(\pi(\theta)|\bar{\pi}, \theta)$ in its most general form and then identify some of its derivatives. Table 3 presents the utility from following a random consumption plan $\pi(\theta)$ in

Table 3 Utility $u(\pi(\theta)|\bar{\pi}, \theta)$

Consumption Utility	$\pi(\theta)(v_0 + q\theta - p)$	
	Gain–Loss from: Consumption Utility	Gain–Loss from: Monetary Utility
Case 1: Consume (probability $\pi(\theta)$)	$-\lambda_c q \int_\theta^{\theta_1} \pi(\theta')(\theta' - \theta)\, dG(\theta')$ $+\beta_c(v_0 + q\theta) \int_{\boldsymbol{\Theta}} (1 - \pi(\theta'))\, dG(\theta')$ $+\beta_c q \int_{\theta_0}^{\theta} \pi(\theta')(\theta - \theta')\, dG(\theta')$	$-\lambda_p p \int_{\boldsymbol{\Theta}} (1 - \pi(\theta'))\, dG(\theta')$
Case 2: Not Consume (probability $1 - \pi(\theta)$)	$-\lambda_c \int_{\boldsymbol{\Theta}} \pi(\theta')(v_0 + q\theta')\, dG(\theta')$	$\beta_p p \int_{\boldsymbol{\Theta}} \pi(\theta')\, dG(\theta')$

state θ when the the reference consumption plan is $\bar{\pi}$. The first line corresponds to the standard consumption utility, and the other terms correspond to the consumption and monetary gain–loss utilities. These gain–loss terms compare what actually happens in state θ (consume with probability $\pi(\theta)$) with what the consumer expects to happen in her reference transaction (consume with probability $\pi(\theta')$ in state θ', which occurs with density $g(\theta')$). The ex post utility simplifies to

$$u(\pi(\theta)|\bar{\pi},\theta) = \pi(\theta)(v_0+q\theta-p) - (1-\pi(\theta))\int_{\Theta}\pi(\theta')\big(\lambda_c(v_0+q\theta')-p\beta_p\big)dG(\theta')$$
$$+\pi(\theta)\int_{\Theta}\big(\pi(\theta')\big(\beta_c(\theta-\theta')^+ - \lambda_c(\theta'-\theta)^+\big)q + (1-\pi(\theta'))\big(\beta_c(v_0+q\theta)-\lambda_p p\big)\big)\,dG(\theta'). \quad (17)$$

The derivative with respect to $\pi(\theta)$ is

$$\frac{\partial u(\pi(\theta)|\bar{\pi},\theta)}{\partial\pi(\theta)} = v_0+q\theta-p+\int_{\Theta}\pi(\theta')\big(\lambda_c(v_0+q\theta')-p\beta_p\big)\,dG(\theta')$$
$$+\int_{\Theta}\big(\pi(\theta')\big(\beta_c(\theta-\theta')^+ - \lambda_c(\theta'-\theta)^+\big)q + (1-\pi(\theta'))\big(\beta_c(v_0+q\theta)-\lambda_p p\big)\big)\,dG(\theta'),$$

so the cross partial derivative with respect to $\pi(\theta)$ and θ is

$$\frac{\partial^2 u(\pi(\theta)|\bar{\pi},\theta)}{\partial\theta\partial\pi(\theta)} = q+\lambda_c q\int_{\theta}^{\theta_1}\pi(\theta')\,dG(\theta') + \beta_c q\int_{\theta_0}^{\theta}\pi(\theta')\,dG(\theta') + \beta_c q\int_{\Theta}(1-\pi(\theta'))\,dG(\theta') > 0. \quad (18)$$

Next we show that $\pi(\theta)$ is nondecreasing in θ. The proof proceeds by way of contradiction. Assume there exist $\theta_i<\theta_j$ such that $\pi(\theta_i)>\pi(\theta_j)$. Then

$$u(\pi(\theta_i)|\bar{\pi},\theta_i) \ge u(\pi(\theta_j)|\bar{\pi},\theta_i),$$
$$u(\pi(\theta_j)|\bar{\pi},\theta_j) \ge u(\pi(\theta_i)|\bar{\pi},\theta_j).$$

Summing up these two inequalities yields

$$(u(\pi(\theta_i)|\bar{\pi},\theta_j) - u(\pi(\theta_i)|\bar{\pi},\theta_i)) - (u(\pi(\theta_j)|\bar{\pi},\theta_j) - u(\pi(\theta_j)|\bar{\pi},\theta_i)) \le 0,$$

which contradicts (18).

Finally, we show that $\pi(\theta)\in\{0,1\}$ almost everywhere. Assume by contradiction that this is not the case. Then there exists an interval $[\theta_a,\theta_b]$ such that $\pi(\theta)\in(0,1)$ for $\theta\in[\theta_a,\theta_b]$ and so $u(1|\bar{\pi},\theta)=u(0|\bar{\pi},\theta)$ for $\theta\in[\theta_a,\theta_b]$. However,

$$u(0|\bar{\pi},\theta) = -\lambda_c\int_{\Theta}\pi(\theta')(v_0+q\theta')\,dG(\theta') + \beta_p p\int_{\Theta}\pi(\theta')\,dG(\theta'),$$

and $\frac{\partial u(0|\bar{\pi},\theta)}{\partial\theta}=0$ for $\theta\in[\theta_a,\theta_b]$ whereas

$$\frac{\partial u(1|\bar{\pi},\theta)}{\partial\theta} = q+\lambda_c q\int_{\theta}^{\theta_1}\pi(\theta')\,dG(\theta') + \beta_c q\int_{\theta_0}^{\theta}\pi(\theta')\,dG(\theta') + \beta_c q\int_{\Theta}(1-\pi(\theta'))\,dG(\theta') > 0,$$

which is a contradiction. □

Proof of Lemma 2: If $u^1(\theta_0,\theta_0)-u^0(\theta_0)>0$ then $\theta=\theta_0$ is a corner PE, and if $u^1(\theta_1,\theta_1)-u^0(\theta_1)<0$ then $\theta=\theta_1$ is a corner PE. If neither inequality holds then, by continuity of the function $u^1(x,x)-u^0(x)$, there exists an interior PE $\theta\in(\theta_0,\theta_1)$ such that $u^1(\theta,\theta)=u^0(\theta)$. Thus a PE always exists, the existence of a PPE follows as a necessary consequence. □

Proof of Lemma 3: Recall from (6) and (7) that

$$u^0(\theta) = -\lambda_c\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta') + \beta_p p\bar{G}(\theta) \quad \text{and}$$

$$u^1(\theta',\theta) = v_0 + q\theta' - p - \lambda_c q\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'') + \beta_c\big((v_0+q\theta')G(\theta) + q\int_{\theta}^{\theta'}(\theta'-\theta'')\,dG(\theta'')\big) - \lambda_p pG(\theta),$$

for $\theta' > \theta$. Plugging these terms into (12), we obtain

$$\begin{aligned}\mathrm{EU}(\theta,p) &= (1-(\lambda_c-\beta_c)G(\theta))\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta') - (\lambda_c-\beta_c)q\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta')\\ &\quad - p\bar{G}(\theta)(1+(\lambda_p-\beta_p)G(\theta)).\end{aligned} \tag{19}$$

Observe that

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta') = \int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta''\,dG(\theta'')\,dG(\theta') - \int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta'\,dG(\theta'')\,dG(\theta').$$

Applying integration by parts to the first term yields $-G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta') + \int_{\theta}^{\theta_1}G(\theta')\theta'\,dG(\theta')$. Collecting terms, we obtain

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')\,dG(\theta'')\,dG(\theta') = -\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta') - G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta').$$

Plugging this expression into (19), we obtain

$$\begin{aligned}\mathrm{EU}(\theta,p) &= (1-(\lambda_c-\beta_c)G(\theta))\int_{\theta}^{\theta_1}(v_0+q\theta')\,dG(\theta')\\ &\quad + (\lambda_c-\beta_c)q\big(\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta') + G(\theta)\int_{\theta}^{\theta_1}\theta'\,dG(\theta')\big) - p\bar{G}(\theta)(1+(\lambda_p-\beta_p)G(\theta))\\ &= \int_{\theta}^{\theta_1}(v_0+q\theta'-p)\,dG(\theta') - (\lambda_c-\beta_c)v_0G(\theta)\bar{G}(\theta) + (\lambda_c-\beta_c)q\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'\,dG(\theta')\\ &\quad - (\lambda_p-\beta_p)pG(\theta)\bar{G}(\theta).\end{aligned}$$

Because $G(\theta)\bar{G}(\theta) = -\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\,dG(\theta')$, we can write $\mathrm{EU}(\theta,p)$ as

$$\mathrm{EU}(\theta,p) = \int_{\theta}^{\theta_1}(v_0+q\theta'-p)\,dG(\theta') - (\lambda_c-\beta_c)\int_{\theta}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(v_0+q\theta')\,dG(\theta') - (\lambda_p-\beta_p)pG(\theta)\bar{G}(\theta).$$

□

Proof of Lemma 4: We state and prove two claims that together prove Lemma 4.

CLAIM 1. *Suppose θ^* is a PE in $[\theta_0,\mathbb{E}\theta]$ such that $\frac{dEU(\theta)}{d\theta}\big|_{\theta=\theta^*} < 0$ and $\mathrm{EU}(\theta^*) \geq 0$. Then $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$ for $\theta \in [\theta^*,\mathbb{E}\theta)$.*

Proof of Claim 1: From Lemma 3 we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta) = g(\theta)(M(\theta)-F(\theta)), \tag{20}$$

where $M(\theta) = p-(v_0+q\theta)$ and $F(\theta) = (\bar{G}(\theta)-G(\theta))((\lambda_c-\beta_c)(v_0+q\theta)+(\lambda_p-\beta_p)p))$. Since by assumption $\frac{d\mathrm{EU}(\theta)}{d\theta}\big|_{\theta=\theta^*} < 0$, it follows that $F(\theta^*) > M(\theta^*)$. Furthermore, $v_0+q\mathbb{E}\theta \geq p$ implies that $F(\mathbb{E}\theta) = 0 \geq p-(v_0+q\mathbb{E}\theta) = M(\mathbb{E}\theta)$.

We also have $\frac{dF(\theta)}{d\theta} = -2((\lambda_p-\beta_p)p+(\lambda_c-\beta_c)(v_0+q\theta))g(\theta) + (\lambda_c-\beta_c)q(\bar{G}(\theta)-G(\theta))$ and $\frac{d^2F(\theta)}{d\theta^2} = -4(\lambda_c-\beta_c)qg(\theta) - 2((\lambda_p-\beta_p)p+(\lambda_c-\beta_c)(v_0+q\theta))g'(\theta)$. Both terms are negative and so we obtain $\frac{d^2F(\theta)}{d\theta^2} < 0$ for $\theta < \mathbb{E}\theta$.

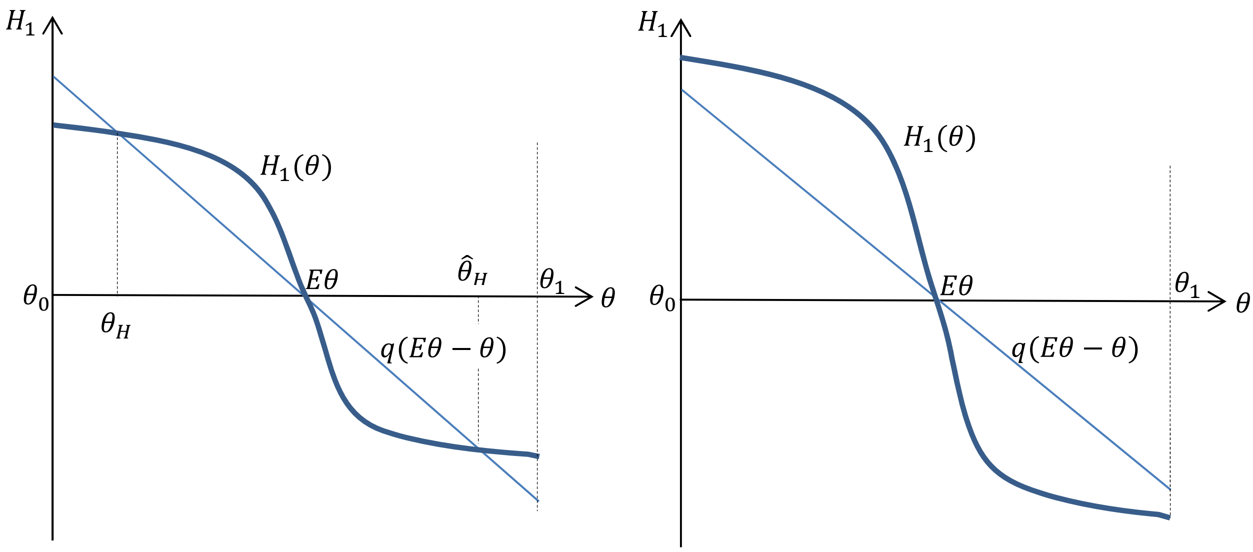
To sum up, we have $F(\theta^*) > M(\theta^*)$, $F(\cdot)$ is concave over $[\theta^*,\mathbb{E}\theta]$, and $F(\mathbb{E}\theta) \geq M(\mathbb{E}\theta)$. Hence we conclude that $F(\theta) > M(\theta)$ and $\frac{d\mathrm{EU}(\theta)}{d\theta} = g(\theta)(M(\theta)-F(\theta)) < 0$ for $\theta \in [\theta^*,\mathbb{E}\theta)$. □

Claim 2. *Suppose Assumption 1 holds, and θ^* is a PE in $[\theta_0, \mathbb{E}\theta]$ such that $\frac{dEU(\theta)}{d\theta}\big|_{\theta=\theta^*} < 0$, and $\mathrm{EU}(\theta^*) \geq 0$. Then $\mathrm{EU}(\theta) < \mathrm{EU}(\theta^*)$ for $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.*

Proof of Claim 2: By Lemma 3,

$$\frac{d\mathrm{EU}(\theta)}{d\theta} = g(\theta)(H(\theta) + K(\theta)), \tag{21}$$

with $H(\theta) = q(\mathbb{E}\theta - \theta) - (\bar{G}(\theta) - G(\theta))((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + (\lambda_p - \beta_p)p)$ and $K(\theta) = (\lambda_c - \beta_c)q(\mathbb{E}\theta - \theta)(\bar{G}(\theta) - G(\theta)) - (v_0 + q\mathbb{E}\theta - p)$. We can also write $H(\theta) = q(\mathbb{E}\theta - \theta) - H_1(\theta)$ and $K(\theta) = K_1(\theta) - (v_0 + q\mathbb{E}\theta - p)$. Given these definitions, the following properties hold (see Figures 4 and 5 for schematic representations)



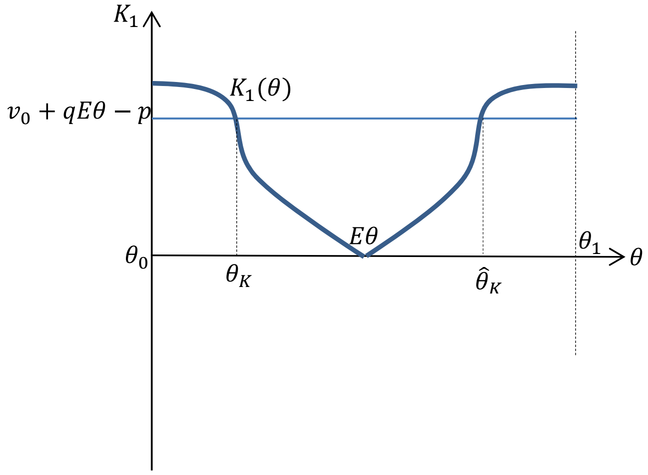
(a) $H(\theta_0) \geq 0$ (b) $H(\theta_0) < 0$

Figure 4 $H(\theta) = q(\mathbb{E}\theta - \theta) - H_1(\theta)$.

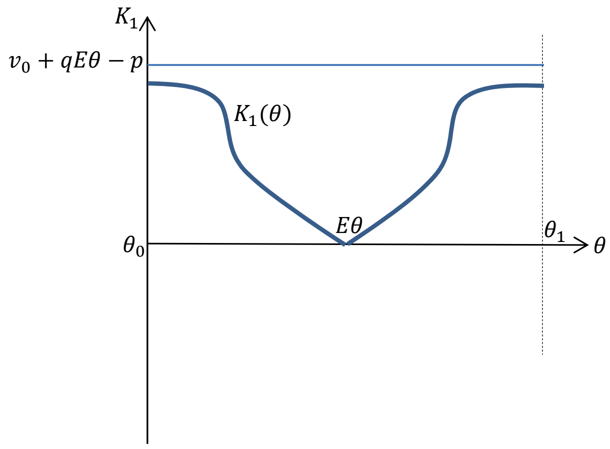
($\triangle$1): $H_1(\theta)$ is positive over $[\theta_0, \mathbb{E}\theta]$, $\frac{dH_1(\theta)}{d\theta} = -2g(\theta)((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + p(\lambda_p - \beta_p)) \leq 0$ for $\theta \in [\theta_0, \theta_1]$, $\frac{d^2H_1(\theta)}{d\theta^2} = -2g'(\theta)((\lambda_c - \beta_c)(v_0 + q\mathbb{E}\theta) + p(\lambda_p - \beta_p)) \leq 0$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, and $H_1(\mathbb{E}\theta - x) = -H_1(\mathbb{E}\theta + x)$. Next, we claim that (i) when $H(\theta_0) < 0$, $H_1(\theta)$ never crosses $q(\theta - \mathbb{E}\theta)$ over $[\theta_0, \mathbb{E}\theta)$ (see Figure 4(b)); and (ii) when $H(\theta_0) \geq 0$, $H_1(\theta)$ crosses $q(\theta - \mathbb{E}\theta)$ exactly once over $[\theta_0, \mathbb{E}\theta)$ at a point that we denote by θ_H (see Figure 4(a)). Take the latter statement. If $H(\theta_0) \geq 0$, then $H_1(\cdot)$ is weakly lower than $q(\theta - \mathbb{E}\theta)$ at $\theta = \theta_0$ ($H_1(\theta_0) \leq q\mathbb{E}\theta$), the two are equal at $\theta = \mathbb{E}\theta$ ($H_1(\mathbb{E}\theta) = 0$), and H_1 is decreasing and concave while $q(\theta - \mathbb{E}\theta)$ is linear. Thus the two curves cross exactly once.

($\triangle$2): $K_1(\theta) \geq 0$, $\frac{dK_1(\theta)}{d\theta} = -(\lambda_c - \beta_c)q(\bar{G}(\theta) - G(\theta) + 2g(\theta)(\mathbb{E}\theta - \theta)) \leq 0$ for $\theta \in [\theta_0, \mathbb{E}\theta]$, and $K_1(\mathbb{E}\theta - x) = K_1(\mathbb{E}\theta + x)$. If $K(\theta_0) \geq 0$ (Figure 5(a)), then $K_1(\theta)$ intercepts $v_0 + p\mathbb{E}\theta - p$ exactly once in $[\theta_0, \mathbb{E}\theta]$ at a point that we denote by θ_K. When $K(\theta_0) < 0$ (Figure 5(b)), $K(\theta) < 0$ for $\theta \in [\theta_0, \theta_1]$. Because $\frac{d\mathrm{EU}(\theta_0)}{d\theta} < 0$, we have $H(\theta_0) + K(\theta_0) < 0$. Using this fact, we distinguish three cases.

Case 1: $K(\theta_0) < 0$,, $H(\theta_0) \geq 0$, and $\theta^* \leq \theta^H$; (see Figures 5(b) and 4(b)). We distinguish three intervals as follows. (1) $\theta \in [\theta^*, \theta_H]$. By Claim 1, $\mathrm{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_H, 1 - \theta_H]$. We have the following



(a) $K(0) \geq 0$



(b) $K(0) < 0$

Figure 5 $K(\theta) = K_1(\theta) - (v_0 + q\mathbb{E}\theta - p)$.

properties: $H(\theta_H) = H(1-\theta_H) = 0$; and $(\triangle 2)$ implies that $\int_{\theta_H}^{\theta} H(\theta)\, dG(\theta) \leq 0$. We can now use the inequality $K(\theta) < 0$ to conclude that $\int_{\theta_H}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$. (3) $\theta \in [1-\theta_H, \hat{\theta}^*]$. As a result, $H(\theta) \leq 0$ and $K(\theta) < 0$; hence $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$. Combining the conclusions drawn for each of these three intervals, we conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

Case 2: $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* \leq \theta^K$ (see Figures 4(a) and 5(a)). Again we distinguish three intervals. (1) $\theta \in [\theta^*, \theta_K]$. By Claim 1, $\mathrm{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_K, 1-\theta_K]$. We have the following properties: because $\theta_K \leq \mathbb{E}\theta$, $(\triangle 2)$ implies that $\int_{\theta_K}^{\theta} H(\theta)\, dG(\theta) \leq 0$ for any $\theta \in [\theta_K, 1-\theta_K]$; since $K(\theta) \leq 0$, we conclude that $\int_{\theta_K}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' \leq 0$. (3) $\theta \in [1-\hat{\theta}_K, \hat{\theta}^*]$. We now have $H(\theta) \geq 0$ and $K(\theta) \geq 0$ for $\theta \in [1-\theta_K, \theta_1]$. Therefore, $\mathrm{EU}(\theta)$ increases over $[1-\theta_K, \theta_1]$ and reaches its maximum $\mathrm{EU}(\theta_1) = 0$ at θ_1. Over that interval, we have $\mathrm{EU}(\theta) \leq \mathrm{EU}(\theta_1) = 0 \leq \mathrm{EU}(\theta^*)$. We combine the conclusions drawn for each of the intervals intervals (1)–(3) to conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' < 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

Case 3: This includes all remaining cases: (3a) $K(\theta_0) < 0$, $H(\theta_0) \geq 0$, and $\theta^* > \theta^H$ (Figures 5(b) and 4(b)); (3b) $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* > \theta^K$ (Figures 4(a) and 5(a)); (3c) $H(\theta_0) < 0$ and $K(\theta_0) < 0$ (Figures 4(a) and 5(b)). The argument in each case is the same: $K(\theta) < 0$ and $\int_{\theta^*}^{\theta} H(\theta')\, dG(\theta') \leq 0$ for any $\theta \in [\theta^*, \hat{\theta}^*]$. Again $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d\mathrm{EU}(\theta')}{d\theta}\, d\theta' \leq 0$ for any $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$.

In each of Cases 1–3, we obtain $U(\theta^*) > U(\theta)$ for $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$. Thus the PE θ^* dominates any other candidate PE $\theta \in [\mathbb{E}\theta, \hat{\theta}^*]$. $\square$

Proof of Lemma 5: Differentiating (13) with respect to θ, we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta) = -g(\theta)\big(v_0 + q\theta - p + (\bar{G}(\theta) - G(\theta))((\lambda_c - \beta_c)(v_0 + q\theta) + p(\lambda_p - \beta_p))\big).$$

Therefore, $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$ is equivalent to

$$1 - (\lambda_p - \beta_p)(\bar{G}(\theta) - G(\theta)) < \frac{v_0 + q\theta}{p}\big(1 + (\lambda_c - \beta_c)(\bar{G}(\theta) - G(\theta))\big).$$

By (8), at an interior PE we have $\frac{v_0+q\theta^*}{p} = \frac{1+\beta_p+(\lambda_p-\beta_p)G(\theta^*)}{1+\beta_c+(\lambda_c-\beta_c)\bar{G}(\theta^*)}$. Evaluating $\frac{d\mathrm{EU}(\theta)}{d\theta} < 0$ at $\theta = \theta^*$ now yields

$$1-(\lambda_p-\beta_p)(\bar{G}(\theta^*)-G(\theta^*)) < \frac{1+\beta_p+(\lambda_p-\beta_p)G(\theta^*)}{1+\beta_c+(\lambda_c-\beta_c)\bar{G}(\theta^*)}\big(1+(\lambda_c-\beta_c)(\bar{G}(\theta^*)-G(\theta^*))\big).$$

After some simplification, we obtain the inequality claimed in the lemma. $\square$

Proof of Proposition 1: The expected utility under full consumption is given by (14). Observe that $\int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))\,dG(\theta') = 0$ and $\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta')-\bar{G}(\theta'))(\theta'-\mathbb{E}\theta)\,dG(\theta') = \int_{\mathbb{E}\theta}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(\theta'-\mathbb{E}\theta)\,dG(\theta')$. We can use these identities to rewrite the loss aversion component in (14) as

$$\int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))\theta'\,dG(\theta') = \int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(\theta'-\mathbb{E}\theta)\,dG(\theta') = 2\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta')-\bar{G}(\theta'))(\theta'-\mathbb{E}\theta)\,dG(\theta').$$

In addition, for $\theta \le \mathbb{E}\theta$ we have $-1 \le G(\theta')-\bar{G}(\theta') \le 0$. Hence

$$\int_{\theta_0}^{\mathbb{E}\theta}(G(\theta')-\bar{G}(\theta'))(\theta'-\mathbb{E}\theta)\,dG(\theta') \le \int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\,dG(\theta').$$

We next establish that $\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\,dG(\theta') \le (\mathbb{E}\theta-\theta_0)/4$. The first step in proving this relation is to show that there must exist a $\theta_k \in [\theta_0,\mathbb{E}\theta]$ such that $g(\theta) \le \frac{1}{\theta_1-\theta_0}$ for $\theta \le \theta_k$ and $g(\theta) \ge \frac{1}{\theta_1-\theta_0}$ for $\theta \ge \theta_k$. Assume this is not the case. Then, because $g(\cdot)$ is positive and increasing on $[\theta_0,\mathbb{E}\theta]$, we must have either that $g(\theta) > \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0,\mathbb{E}\theta]$, which leads to the contradiction $G(\mathbb{E}\theta) > \frac{1}{2}$, or that $g(\theta) < \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0,\mathbb{E}\theta]$, which leads to the contradiction $G(\mathbb{E}\theta) < \frac{1}{2}$.

The second step is to show that

$$\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' \le 0.$$

Yet this inequality holds since

$$\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' = \int_{\theta_0}^{\theta_k}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' + \int_{\theta_k}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta'$$

and since, moreover, $\int_{\theta_0}^{\theta_k}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' \le (\mathbb{E}\theta-\theta_k)\int_{\theta_0}^{\theta_k}\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta'$ (because $g(\theta') \le \frac{1}{\theta_1-\theta_0}$ for $\theta \le \theta_k$) and $\int_{\theta_k}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' \le (\mathbb{E}\theta-\theta_k)\int_{\theta_k}^{\mathbb{E}\theta}\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta'$ (because $g(\theta') \ge \frac{1}{\theta_1-\theta_0}$ for $\theta \ge \theta_k$). Taking these inequalities into account, we arrive at

$$\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' \le (\mathbb{E}\theta-\theta_k)\int_{\theta_0}^{\mathbb{E}\theta}\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' = (\mathbb{E}\theta-\theta_k)(\frac{1}{2}-\frac{1}{2}) = 0.$$

To conclude, we remark that $\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')\big(g(\theta')-\frac{1}{\theta_1-\theta_0}\big)\,d\theta' \le 0$ can be rewritten as

$$\int_{\theta_0}^{\mathbb{E}\theta}(\mathbb{E}\theta-\theta')g(\theta')\,d\theta' \le \int_{\theta_0}^{\mathbb{E}\theta}\frac{\mathbb{E}\theta-\theta'}{\theta_1-\theta_0}\,d\theta' = \frac{\theta_1-\theta_0}{8} = \frac{\mathbb{E}\theta-\theta_0}{4}.$$

Putting everything together yields an upper bound for the loss aversion component:

$$\int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))\theta'\,dG(\theta') \le \frac{\mathbb{E}\theta-\theta_0}{2}.$$

After replacing the loss aversion component in equation (14), we obtain

$$\mathrm{EU}\big(\theta_0,p_0^{\mathrm{LA}}\big) \ge v_0+q\mathbb{E}\theta-p_0^{\mathrm{LA}}-(\lambda_c-\beta_c)q\frac{\mathbb{E}\theta-\theta_0}{2}.$$

Thus $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$ for any $G(\cdot)$ if

$$v_0 + q\mathbb{E}\theta - p_0^{\mathrm{LA}} - (\lambda_c - \beta_c)q\frac{\mathbb{E}\theta - \theta_0}{2} \geq 0.$$

This expression simplifies to Assumption 3 once we plug in the value for p_0^{LA}. $\square$

Proof of Lemma 6: We have

$$\frac{\frac{d}{d\theta}[(v_0 + q\theta)\bar{G}(\theta)]}{(v_0 + q\theta)\bar{G}(\theta)} = \frac{q}{v_0 + q\theta} - \frac{g(\theta)}{\bar{G}(\theta)}$$

and

$$\frac{L_\theta(\theta)}{L(\theta)} = \frac{-(\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p)g(\theta)}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))}.$$

Therefore, $\frac{\frac{d}{d\theta}[(v_0+q\theta)\bar{G}(\theta)]}{(v_0+q\theta)\bar{G}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)}$ is equivalent to

$$1 + \bar{G}(\theta)\frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))} \geq \frac{q\bar{G}(\theta)}{(v_0 + q\theta)g(\theta)}.$$

A bound for the first fraction in this inequality is

$$\frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \beta_p + (\lambda_p - \beta_p)G(\theta))(1 + \beta_c + (\lambda_c - \beta_c)\bar{G}(\theta))} \geq \frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)} = 1 - \frac{1 + \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)}.$$

It follows that $R^{\mathrm{LA}}(\theta)$ is decreasing as long as

$$1 + \bar{G}(\theta)\left(1 - \frac{1 + \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)}\right) \geq \frac{q\bar{G}(\theta)}{(v_0 + q\theta)g(\theta)}.$$

For $\theta \in [\theta_0, \theta^{\mathrm{LN}})$, we have

$$1 + \bar{G}(\theta)\left(1 - \frac{1 + \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)}\right) \geq 1 + \bar{G}(\theta^{\mathrm{LN}})\left(1 - \frac{1 + \beta_c\beta_p}{(1 + \lambda_p)(1 + \lambda_c)}\right) \geq \varepsilon_0^{-1} \geq \frac{q\bar{G}(\theta)}{(v_0 + q\theta)g(\theta)}$$

where the middle inequality follows from Assumption 4.

In the uniform case with $\theta_0 = 0$, $\theta_1 = 1$, and $\beta_c = \beta_p = 0$, equation (15) states that the firm's revenue as a function of θ is

$$R^{\mathrm{LA}}(\theta) = \frac{(1 - \theta)(q\theta + v_0)(1 + \lambda_c(1 - \theta))}{\theta\,\lambda_p + 1}. \tag{22}$$

Differentiating $R^{\mathrm{LA}}(\theta)$ now gives

$$\frac{dR^{\mathrm{LA}}}{d\theta} = \frac{2\lambda_c\lambda_p q\theta^3 - (2\lambda_c\lambda_p q - \lambda_c\lambda_p v_0 - 3\lambda_c q + \lambda_p q)\theta^2 - 2(2\lambda_c q - \lambda_c v_0 + q)\theta - (\lambda_c\lambda_p + 2\lambda_c + \lambda_p + 1)v_0 + (\lambda_c + 1)q}{(1 + \lambda_p\theta)^2}.$$

We have $\frac{dR^{\mathrm{LA}}}{d\theta}\big|_{\theta=1} = -\frac{q\lambda_p + \lambda_p v_0 + q + v_0}{(\lambda_p+1)^2} < 0$ as well as $\frac{dR^{\mathrm{LA}}}{d\theta}\big|_{\theta=0} = -((1 + \lambda_c)(1 + \lambda_p) + \lambda_c)v_0 + q(1 + \lambda_c)$. The condition specified in the lemma, $\frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$, is necessary for the revenue function to be decreasing at θ_0. Given that condition, it suffices to show that the derivative is negative for all $\theta \in (0, 1)$. We begin by noting some properties exhibited by the derivative of the numerator of $\frac{dR^{\mathrm{LA}}}{d\theta}$: (i) it is a quadratic and convex function of θ; (ii) at $\theta = 0$ it takes the value $2(q(-2\lambda_c - 1) + \lambda_c v_0)$ and at $\theta = 1$ the value $2(1 + \lambda_p)(q(\lambda_c - 1) + \lambda_c v_0)$, (iii) $2\ (q(-2\lambda_c - 1) + \lambda_c v_0) < 2(1 + \lambda_p)(q(\lambda_c - 1) + \lambda_c v_0)$.

Next we distinguish three cases. (1) $2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0) \leq 0$. In this case, the derivative is decreasing in $\theta \in (0, 1)$ and, because the derivative is negative at $\theta = 0$, it is negative for all $\theta \in [0, 1]$. (2) $(q(-2\lambda_c -$

$1)+\lambda_c v_0) \geq 0$. In this case, the derivative is increasing in $\theta \in (0,1)$ and, because the derivative is negative at $\theta = 1$, it is negative for all $\theta \in [0,1]$. (3) $(q(-2\lambda_c - 1) + \lambda_c v_0) < 0 < 2(1+\lambda_p)(q(\lambda_c - 1) + \lambda_c v_0)$. In this case, the derivative first decreases and then increases; because the derivative is negative at both $\theta = 0$ and $\theta = 1$, we conclude that it is negative over $\theta \in [0,1]$. So in all cases, the R^{LA} is decreasing over $[0,1]$. $\square$

Proof of Proposition 3: (a) $p_0^{\mathrm{LA}} = \frac{1+\lambda_c}{1+\beta_p} v_0$ and $p_q = 0$. (b) We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$ and $\theta_q^{\mathrm{LN}}(q) = -\frac{g(\theta^{\mathrm{LN}})v_0}{qR_{\theta\theta}^{\mathrm{LN}}(\theta^{\mathrm{LN}})} > 0$. The inequality follows because $R^{\mathrm{LN}}(\theta)$ is concave at θ^{LN}.

Now, if $\theta_0 = 0$, then $p_q^{\mathrm{LN}}(q) > 0 = p_q^{\mathrm{LA}}(q)$ and if $\theta_0 > 0$ then $\theta^{\mathrm{LN}}(q) \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0$. Putting these statements together, we conclude that

$$p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q) \geq \theta^{\mathrm{LN}}(q) \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0 = p_q^{\mathrm{LA}}(q). \qquad \square$$

Proof of Corollary 1: According to Lemma 6, $R^{\mathrm{LA}}(\theta)$ is decreasing in θ when $\frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$. We need to check that the participation constraint holds at PE θ_0 for $p = (1+\lambda_c)v_0$. If it does then, by Proposition 1, θ_0 is a PPE for $p = (1+\lambda_c)v_0$. We have $\mathrm{EU}(\theta = 0) = \frac{q}{2} - p + v_0 - \frac{\lambda_c q}{6}$. Plugging in the price $p = (1+\lambda_c)v_0$ gives $\mathrm{EU}(\theta = 0, p = (1+\lambda_c)v_0) = \frac{q}{2} - (1+\lambda_c)v_0 + v_0 - \frac{\lambda_c q}{6}$, in which case the PC is equivalent to $\frac{6\lambda_c}{3-\lambda_c} \leq \frac{q}{v_0}$. Since $R^{\mathrm{LA}}(\theta)$ is decreasing in θ, there can be no other PE or PPE that yields more revenue than $(1+\lambda_c)v_0$. $\square$

Proof of Lemma 7: We have

$$\frac{d\widetilde{\mathrm{EU}}(q)}{dq} = \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

Assumption 3 implies that the PC holds. That is,

$$\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) = v_0 + q\mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0) - (\lambda_c - \beta_c)q\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta') \geq 0,$$

or, equivalently,

$$q\mathbb{E}\theta \geq v_0(\frac{1+\lambda_c}{1+\beta_p} - 1) + \frac{1+\lambda_c}{1+\beta_p}q\theta_0 - (\lambda_c - \beta_c)q\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta'),$$

and, if $\lambda_c \geq \beta_p$, the above inequality implies that

$$\mathbb{E}\theta \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta').$$

We can use our derivations from the proof of Proposition 1 to write

$$\mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta'\, dG(\theta') \geq \mathbb{E}\theta - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)^2 \geq 0.$$

We conclude that $\frac{d\widetilde{\mathrm{EU}}(q)}{dq} \geq (\mathbb{E}\theta - \theta_0)(1 - (\lambda_c - \beta_c)(\mathbb{E}\theta - \theta_0)) - \frac{\lambda_c - \beta_p}{1+\beta_p}\theta_0 \geq 0.$ $\square$

Appendix C: Binding PC; Interior Equilibrium

As mentioned in Section 4.3, it is not possible in general to characterize fully the firms's optimal consumption threshold θ^{LA}. We can, however, make specific statements. Recall that the firm maximizes $R^{\mathrm{LA}}(\theta)$ subject to $\theta \in \mathbf{\Theta}^U \cap \bigcup_{p>0} \mathbf{\Theta}^{\mathrm{PPE}}(p)$. The optimal threshold may be an interior value or a corner at the boundary of the set $\mathbf{\Theta}^U \cap \bigcup_{p>0} \mathbf{\Theta}^{\mathrm{PPE}}(p)$. Consider the case where PC binds: θ^{LA} is a corner located on the boundary of $\mathbf{\Theta}^U$. In Figure 3(a), this corresponds to the section of the curves to the left of the flat segments. An increase in consumption loss aversion reduces the slope of the price schedule.

PROPOSITION 4. $p^{\mathrm{LA}}_{q,\lambda_c}(q) < 0$ *when PC binds.*

Proof of Proposition 4: The participation constraint, equation (3), is binding

$$\int_{\theta}^{\theta_1}(v_0+q\theta'-p^{\mathrm{LA}}(q,\lambda_c))\,dG(\theta')-(\lambda_c-\beta_c)\int_{\theta}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(v_0+q\theta')\,dG(\theta')-(\lambda_p-\beta_p)p^{\mathrm{LA}}(q,\lambda_c)G(\theta)\bar{G}(\theta)=0$$

We have $p^{\mathrm{LA}}_{q,\lambda_c}(q) < 0$ and $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$. □

In Figure 3(a), the curves get flatter to the left of the first kink. This response is characteristic of low-quality products and occurs also when the consumption loss aversion coefficient is large enough so that EU from equation (14) is negative. Monetary loss aversion has no effect on the slope of the price schedule, $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$.

Next we consider the case of interior consumption thresholds. In Figure 3(a), these correspond to the section of the curves to the right of the flat segments. Interior PPEs are also always chosen for small enough values of loss aversion. Consumption loss aversion and monetary loss aversion increase consumption.

PROPOSITION 5. $\frac{\partial\theta^{\mathrm{LA}}}{\partial\lambda_c} < 0$ *and* $\frac{\partial\theta^{\mathrm{LA}}}{\partial\lambda_p} < 0$.

Proof of Proposition 5: Assume that the first-order approach holds. Then the derivative of the firm's revenue, equation (15), with respect to θ is

$$R^{\mathrm{LA}}_{\theta} = \frac{d}{d\theta}[(v_0+q\theta)\bar{G}(\theta)]L + (v_0+q\theta)\bar{G}(\theta)L_{\theta}.$$

An interior solution is characterized by $R^{\mathrm{LA}}_{\theta} = 0$ or

$$\frac{\frac{d}{d\theta}[(v_0+q\theta)\bar{G}(\theta)]}{(v_0+q\theta)\bar{G}(\theta)} = -\frac{L_{\theta}}{L}.$$

An increase in either λ_c or λ_p does not change the expression's left-hand side. However, $-\frac{d}{d\lambda_p}\frac{L_\theta}{L} > 0$ (similarly $-\frac{d}{d\lambda_c}\frac{L_\theta}{L} > 0$); that is, an increase in loss aversion increases the RHS. Since $\frac{d}{d\theta}[(v_0+q\theta)\bar{G}(\theta)]$ is decreasing in θ, it must be that θ decreases. □

Although the consumption threshold decreases with consumption and price loss aversion, the effect on the slope of the price schedule is not possible to sign. For interior equilibria, we have

$$p^{\mathrm{LA}}_q(q) = \theta^{\mathrm{LA}}(q)L(\theta^{\mathrm{LA}}(q)) + \big(qL(\theta^{\mathrm{LA}}(q)) + (v_0+q\theta^{\mathrm{LA}}(q))L_{\theta}(\theta^{\mathrm{LA}}(q))\big)\theta^{\mathrm{LA}}_q(q).$$

An increase in either λ_c or λ_p reduces $\theta^{\mathrm{LA}}(q)$, but its effect on the other terms cannot be signed.