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LOSS AVERSION IN A CONSUMPTION/SAVINGS MODEL

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Abstract

Psychological evidence indicates that a person's well-being depends not only on his current consumption of goods, but on a reference level determined by his past consumption. According to Kahneman and Tversky's (1979) prospect theory, people care much more about losses relative to their reference points than about gains, are risk-averse over gains, and risk-loving over losses. We define these characteristics as loss aversion. We incorporate an extended form of loss aversion into a simple two-period savings model. Our main conclusion is that, when there is sufficient income uncertainty, a person resists lowering consumption in response to bad news about future income, and this resistance is greater than the resistance to increasing consumption in response to good news. We discuss some recent empirical research that confirms this predicted asymmetry in behavior, which seems inconsistent with other models of consumption.

Loss Aversion in a Consumption/Savings Model

David Bowman, Deborah Minehart, Matthew Rabin[1]

1. Introduction

Psychological evidence indicates that a person's well-being depends not only on his current consumption of goods, but also on how his current consumption compares to his past consumption.[2] As documented by Kahneman and Tversky (1979,1991,1992), underlying many instances of such reference-dependent preferences is a fundamental asymmetry in how increases and decreases in consumption are evaluated: People care much more about losses relative to their reference point than about gains. Moreover, while people are risk averse in gains, they are *risk loving* in losses. We refer to these combined characteristics as *loss aversion*.[3]

In this paper, we incorporate an extended form of loss aversion into a two-period consumption/savings model where a consumer faces uncertainty over his second-period income. To capture the idea that preferences depend on past consumption, we follow the recent literature on *habit persistence*[4] by assuming that a person's second-period

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[1]International Finance Division of the Board of Governors of the Federal Reserve System, Boston University, and the University of California at Berkeley respectively. This paper represents the views of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. We thank Danny Kahneman and Richard Thaler for helpful conversations, and especially Eddie Dekel-Tabak and Halsey Rogers for helpful comments. We are also grateful to Berkeley's Institute for Business and Economic Research for funding of research assistance. Rabin thanks the National Science Foundation for financial support, through research grant SES-9210323.

[2]For an excellent survey of the psychological evidence along these lines, see Kahneman, Knetsch, and Thaler (1991). Other recent papers considering effects of the status quo on behavior are Samuelson and Zeckhauser (1988) and Thaler (1980). For empirical evidence regarding habit persistence, see Ferson and Constantinides (1991).

[3]We are abusing terminology here; Kahneman and Tversky (1979) coined the term "loss aversion" to mean the basic gain-loss asymmetry alone.

[4]See Ryder and Heal (1973), and Becker and Murphy (1988), Boyer (1983), Constantinides

reference point is a weighted average of his initial reference point and his first-period consumption.

Our main conclusion is that, when there is sufficient uncertainty, a person resists consuming below his reference point in the first period even when his expected average per-period income is below his reference point. This resistance to changing consumption is asymmetric. Formally, though a consumer will never consume below his reference level when he anticipates an average per-period income above his reference level, he might consume *above* his reference level when he anticipates an average per-period income below his reference level. This asymmetry is due largely to the risk-loving attitude towards losses, which most clearly differentiates our model from both the classical utility function and recent theoretical literature on habit persistence.

Our model conforms with the general intuition--posited at least as far back as Duesenberry (1952)--that people resist decreasing their standard of living in response to bad news about income. Our results also conform with recent empirical research by Shea (1993a, 1993b), who provides evidence of asymmetric behavior consistent with our model. Using information on union contracts to construct a measure of expected wage growth for each household in his sample, Shea (1993a) finds that the response of consumption to predictable declines in wages is greater than to predictable increases. He also shows that, while Hall's (1978) test of the Permanent Income Hypothesis is rejected for predicted wage declines, it cannot be rejected for predicted wage increases. Shea (1993b) tests for and finds the same asymmetry in aggregate U.S. data. These findings appear to be robust, and are inconsistent with other standard explanations of apparent violations of the Permanent Income Hypothesis, such as liquidity constraints or Campbell and Mankiw's (1989) rule-of-thumb behavior.[5]

(1990), Sundaresan (1989), and Detemple and Zapatero (1991).

[5]Another alternative hypothesis that has received attention recently is *myopia*. Because he associates it with Campbell and Mankiw's rule-of-thumb behavior, Shea (1993a,1993b)

Shea's empirical findings also help differentiate our model from the recent literature on habit persistence. While this literature posits reference-dependent behavior, they assume that utility is concave both in consumption and in the reference point, thus omitting loss aversion. In these models, as in ours, a person who receives bad news about future income may be slow to adjust consumption downward, because breaking with habit is painful. But, in these models, he should be equally slow to adjust consumption *upward* in response to good news. The asymmetry predicted by our model relies on the person's risk-loving attitude towards losses, resistance to lowering consumption in response to bad news means that a person is willing to risk an even more dramatic future drop in consumption so as to avoid consuming below his reference point today.

Because loss aversion encapsulates important psychological facts about how preferences and attitudes towards risk are influenced by reference points, we feel it deserves significant attention by mainstream economists. Beyond our results on savings, therefore, we see this paper as an attempt to grapple with some more general issues involved in incorporating loss aversion into formal economic analysis. Our research indicates, for instance, that loss aversion may have little predictive power in formal models of dynamic decision-making unless ancillary assumptions are made about how a person's welfare is affected by *changes* in his reference point. We assume that preferences exhibit *acclimation*: Fixing a person's consumption, he is happier if his reference point matches his consumption than if it is either higher or lower. As we show in Section 3, this assumption guarantees that if a person is certain that his average income will be just sufficient to consume each period at his reference level, he will do

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argues that myopia does not explain his evidence of asymmetry. More generally, it is hard to imagine that any natural definition of myopia would cause the asymmetry found by Shea (1993a,1993b). Myopia could, however, *magnify* the asymmetry; Benartzi and Thaler (1992) in fact argue that loss aversion and myopia *together* explain the phenomenon of underinvestment in risky assets.

so rather than vary his consumption across periods.

In developing our formal model, we have made some compromises with behavioral evidence. For instance, while we have adopted loss aversion from Kahneman and Tversky (1979,1992), we have ignored other aspects of their more general *prospect theory*. They demonstrate that people are subject to *framing effects*--a person may behave differently if a logically equivalent decision is framed to him in two different ways.[6] They also show that the *decision weights* people use in evaluating the riskiness of their choices may differ from the true probabilities. We ignore these and other issues because we feel they are simply not as tractable nor as generalizable as loss aversion, and, in taking the pragmatic strategy of incorporating new assumptions one at a time, loss aversion seems an ideal candidate to start with. Moreover, it does not appear that adding other behavioral assumptions would substantially change the qualitative differences between our model and the standard life-cycle model of savings.

We begin in Section 2 by presenting Kahneman and Tversky's (1979) original formulation of loss aversion, our extensions of loss aversion, and two simple propositions comparing loss aversion to the standard economic model of risk aversion. In Section 3, we present our savings model and our formal results. We conclude in Section 4 by discussing possible extensions of our savings model, and briefly considering other economic applications of loss aversion.

2. Loss Aversion and Acclimation

In this section, we review the basic model of preferences developed by Kahneman and Tversky (1979) and outline our extensions. To formally incorporate the idea of

[6]Thaler's (1980,1985,1990) notion of mental accounting is somewhat related to the idea of framing effects, and Thaler (1990) demonstrates that mental accounting may play an important role in savings behavior, implying that people treat different sources of income differently even when standard theory predicts that they are interchangible.

reference dependence, we assume that a person's utility is a function of both his level of consumption, c, and his reference level for consumption, r. We assume for simplicity that utility is separable in the reference level and the deviation of actual consumption from the reference level:

$$U(r,c) \equiv w(r) + v(c-r),$$

where $w(\cdot)$ is the "reference utility," $v(\cdot)$ is the "gain-loss utility," and $U(\cdot,\cdot)$ is overall utility. We shall assume that the function is defined for all non-negative values of r and c. For convenience, we shall also assume that both components of the utility function are continuous and, except when $c = r$, are twice differentiable.

2.1 Loss Aversion

We begin with the assumptions on the utility function incorporated into Kahneman and Tversky's (1979,1992) prospect theory. Because their analysis concerned the case where the reference point is fixed, tastes are determined entirely by $v(\cdot)$. We normalize $v(\cdot)$ to be equal to zero if $c = r$, so that $v(0) = 0$ and $U(r,r) = w(r)$. Kahneman and Tversky posit the following three characteristics of $v(\cdot)$:

<u>Assumptions</u> <u>V1</u>-<u>V3</u>:

V1. $v(x)$ is strictly increasing in x.

V2. $v(x)$ is strictly concave for $x > 0$ and strictly convex for $x < 0$.

V3. If $y > x > 0$, then $v(y) + v(-y) < v(x) + v(-x) < 0$.

Assumption V1 says that "more is better"--$v(\cdot)$ is an increasing function, which implies that $U(\cdot,\cdot)$ is increasing in c. Assumption V2 implies that people are risk averse in situations involving a sure gain, but also implies that people are risk *loving* in situations involving a sure loss. Assumption V3 is that the marginal utility of a loss is strictly greater than the marginal utility of a comparable gain.

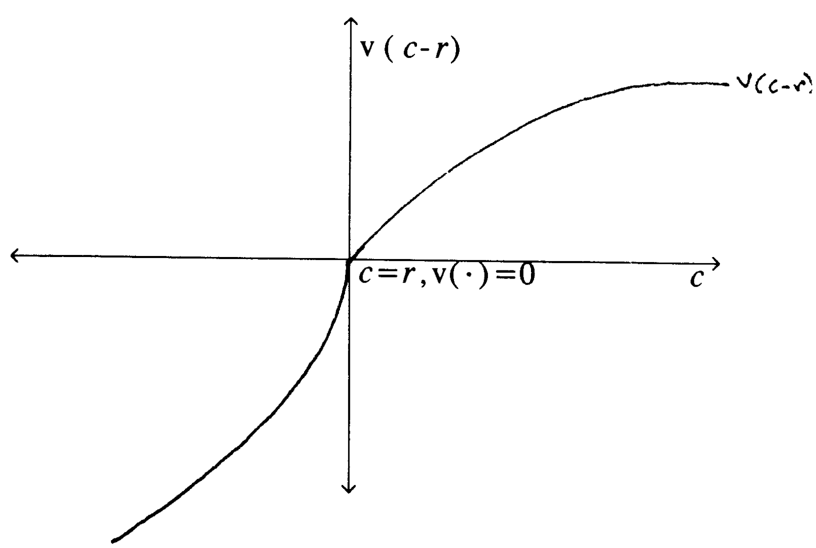
The experimental evidence (see Tversky and Kahneman (1992)) indicates that the ratio of loss aversion--the marginal utility of losses divided by the marginal utility of gains--is often approximately 2. That is, a loss of one unit is two times more unpleasant than a gain of one unit is pleasant. Moreover, evidence indicates that the relative distaste for losses exists even for very small changes, and the algebraic examples Tversky and Kahneman (1991,1992) estimate have a "kink" such that the ratio of loss aversion is strictly greater than 1 for even infinitesimal gains and losses. We formalize this idea in Assumption V4:

Assumption V4:

$$\text{Limit}_{x \to 0} \; v'(-x)/v'(x) \equiv L > 1.$$

Assumptions V1-V4 can be graphically represented by the following classical diagram:

Figure 1



Standard theory predicts that people are very close to risk neutral for small bets, and will accept any slightly-better-than-fair bet if it is small enough (Arrow (1974), Pratt (1964), and Samuelson (1961)). Because of Assumption V4, loss aversion predicts the opposite result--that people will reject any slightly-better-than-fair bet if it is small enough.

Formally, consider a bet as a distribution f over gains and losses, and say that we are scaling it by $k > 0$ when we consider the bet where all values of f are multiplied by k. Then standard theory predicts that for every better-than-fair bet f, there exists $\varepsilon > 0$ such that for all $k < \varepsilon$, a person will *accept* the bet f scaled by k.

In contrast, loss aversion predicts:

Proposition 2.1:[7]

Let f be any better-than-fair bet with a lower bound on losses, and with a positive probability of a loss; let g be any fair bet with a lower bound on losses; and let $h(p) \equiv pf + (1-p)g$ for $p \in (0,1]$. Assumptions V1-V4 imply that there exists an $\varepsilon > 0$ and $p^* \in (0,1]$ such that for all $k < \varepsilon$ and $p < p^*$, a person will *reject* $h(p)$ when scaled by k.

Locally, then, loss aversion implies a stronger form of risk aversion than standard theory.[8] Globally, however, the situation is different. As is well known, the standard assumption of a concave utility implies that a person will reject all fair bets. While loss aversion implies risk aversion for both small bets and symmetric bets of any size, the more general risk aversion predicted by the concave utility function is

[7]All proofs are in the Appendix.
[8]Recently, papers such as Segal and Spivak (1990) consider ways to modify utility theory to get "first-order risk aversion" (where slightly-better-than-fair bets are turned down no matter how small) of the type we get, rather than the classical "second-order risk aversion." This literature, however, attains first-order risk aversion by relaxing the assumption of expected-utility maximization. By contrast, Proposition 2.1 shows that first-order risk aversion can be a consequence of loss aversion, even if one works within the expected-utility framework.

not guaranteed. In fact, empirical evidence suggests that some systematic deviations from risk aversion do exist.[9] However, we do not consider these deviations to be a central issue in savings behavior, and so on this issue we conform to the standard theory. Specifically, Assumption V5 states that a person will turn down any fair bet centered around his reference point--including asymmetric ones of any size:

<u>Assumption</u> <u>V5</u>:

For all $1 \geq p \geq 0$, $x > 0$ and $y < 0$ such that $p \cdot x + (1-p) \cdot y = 0$, $p \cdot v(x) + (1-p) \cdot v(y) < 0$.

Assumption V5 is equivalent to the condition that the marginal disutility of a loss is everywhere greater than the marginal utility of a gain. Formally:

<u>Proposition</u> <u>2.2</u>:

If $v(\cdot)$ satisfies Assumptions V1-V4, then it satisfies Assumption V5 *if and only if* for all $x \geq 0$ and $y < 0$, $v'(y) > v'(x)$.

If preferences meet Assumptions V1-V5, we say they exhibit *general loss aversion.*

2.2 Acclimation

In the type of intertemporal setting we consider in this paper, future reference points may be affected by current choices. Individuals may realize this, and be influenced by reference-point effects when making consumption decisions. For instance, a

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[9] While Kahneman and Tversky do not assume that people refuse all fair bets, their discussion of violations of this assumption focuses more on the manner in which people use probabilities in making decisions (for instance, ignoring small, but non-zero, probability outcomes) than on the properties of $v(\cdot)$. Tversky and Kahneman (1992) estimate a parameterization of $v(\cdot)$ that implies that the marginal disutility of a loss is greater than the marginal utility of a gain whenever the loss is less than 860 times the size of the gain; only extremely asymmetric fair bets will be accepted. Especially because we do not in any event conceive of our results as being relevant in the case of very large losses in income, we feel that the addition of Assumption V5 does relatively little damage to the realism of our results.

consumer may be cautious in developing a luxurious lifestyle because he knows that doing so will make him less happy if he later becomes impoverished. We must therefore make assumptions about how a person's welfare depends on his reference point. We have little empirical guidance on this issue, but make what we feel are sensible choices for the problem at hand.

The gain-loss utility $v(\cdot)$ represents the amount of pleasure or displeasure the consumer experiences from gains or losses relative to his reference point $w(\cdot)$, whereas $w(\cdot)$ corresponds to that portion of a person's well-being that is determined by his reference point. Assuming that reference points tend eventually to adjust to levels of consumption, $w(\cdot)$ can be interpreted a kind of "long-term" utility function, representing how much material pleasure a consumer gets from consuming amount r after he becomes accustomed to that consumption level. We shall make the conventional assumptions that $w(\cdot)$ exhibits non-satiation and diminishing marginal utility:

Assumption W1:

$w(\cdot)$ is increasing and weakly concave everywhere.

While our assumptions about $w(\cdot)$ are relatively straightforward, we must also make assumptions about the relationship between $w(\cdot)$ and $v(\cdot)$. For reasons which will be explained shortly, we assume the following:

Assumption VW1:

We assume $U(r,c)$ exhibits *acclimation*: When $r > c$, $\partial U(r,c)/\partial r < 0$, and when $r < c$, $\partial U(r,c)/\partial r > 0$. For preferences meeting Assumptions V1-V5 and W1, this is true if and only if for all $r \geq 0$, $x \leq 0$, and $y \geq 0$, $v'(x) > w'(r) > v'(y)$.

This assumption says that, *fixing his consumption level*, an individual is happier the smaller his loss or the smaller his gain. The first part of this assumption seems uncontroversial, but the second part may seem less natural--it says that somebody who is consuming a lot now will be happier if he is accustomed to it than if he had previously been poor. We show in the next section, however, that this assumption is needed to guarantee reasonable intertemporal behavior. In particular, acclimation guarantees that a person will not purposely consume below his reference point in the present solely to consume above his reference point in the future.[10]

On a technical level, acclimation is simply an upper envelope condition. To see this, suppose for the moment that we relax the separability assumption and set $v(c - r) = w(c) - w(r)$ for $c \geq r$. Then $U(r, c) = w(c)$ for $c \geq r$. This captures the idea that consumers adjust to gains immediately, evaluating them according to their "long-term" utility function. Then as long as $U(r,c)$ satisfies general loss aversion, W1, and the more intuitive part of acclimation, all of our results in section 3 still obtain. Although in what follows we assume separability for expositional simplicity, more generally the second part of acclimation can be expressed as $U(r,c) \leq w(c)$ for $c \geq r$.

[10]Some empirical and experimental research (see, for instance, Frank and Hutchens (1990), Loewenstein and Sicherman (1991), and Loewenstein and Prelec (1992)) has found that, fixing average lifetime income, people prefer their income to be increasing over time, rather than either flat or decreasing. The assumption of acllimation is consistent with this behavior, as long as a person is not willing to consume *below his current reference point* in order to consume above his reference level in the future.

3. A Two-Period Model of Savings

We now consider the implications of general loss aversion and acclimation in a simple two-period model of a consumer's savings decision. We assume that there are two periods remaining in a consumer's life, and any uncertainty in his total remaining income will be resolved only in the second period. We do not allow a consumer to consume less than zero in either period, and assume that he cannot borrow against *uncertain* future income (which would risk default); but we assume that there are no liquidity constraints which prevent him from consuming any *guaranteed* second-period income in the first period. For simplicity, we assume no discounting and that savings earn no interest. We let r_t be the reference level in period t, and let c_t be consumption level in period t. The consumer's total two-period utility is given by:

$$U(r_1,c_1;r_2,c_2) \equiv [w(r_1) + v(c_1-r_1)] + E\ \{[w(r_2) + v(c_2-r_2)]\}.$$

The consumer chooses his first-period consumption after he knows his first-period income, but with probabilistic beliefs about his second-period income. The nature of the maximization problem is clearly determined by the way in which reference points are formed. We will model reference-point formation in a manner consistent with recent literature on habit persistance in consumption. We take the first-period reference point, r_1, as exogenously determined. The second-period reference point, r_2, will be determined in part by r_1, and in part by the first-period consumption level, c_1:

$$r_2 \equiv (1-\alpha)r_1 + \alpha c_1, \text{ where } \alpha \in [0,1].$$

The parameter α represents the speed at which the reference point changes in response to recent consumption. If $\alpha = 0$, then first-period consumption has no effect on the consumer's second-period reference level, so that utility is time-separable; if $\alpha = 1$, then the second-period reference level adjusts fully to first-period consumption.

3.1 Consumption Behavior When Income is Certain

We begin our analysis by examining the consumer's behavior when he faces no uncertainty. We denote by $c_1(Y)$ and $c_2(Y)$ the consumer's choice of first- and second-period consumption if faced with a sure total income of Y. Our results are summarized by Theorem 1:

Theorem 1:

If the constraints $c_1 \geq 0$ and $c_2 \geq 0$ are ignored, then for all $\alpha > 0$, $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing, with $c_1(Y) = c_2(Y) = r_1$ when $Y = 2r_1$.

If the constraints $c_1 \geq 0$ and $c_2 \geq 0$ are imposed, then for all $\alpha > 0$, $c_1(Y)$ and $c_2(Y)$ are continuous and non-decreasing, and for $Y \geq (1-\alpha)r_1$, $c_1(Y)$ and $c_2(Y)$ are strictly increasing, with $c_1(Y) = c_2(Y) = r_1$ when $Y = 2r_1$.

Theorem 1 says that--except for cases where the constraint that consumption cannot be negative in either period is binding--a consumer will increase both his first and his second period consumption in response to an increase in his lifetime income. In other words, both first and second period consumption are normal goods. It also says that he will consume below his current reference level if and only if his lifetime income will not be able to support continued consumption at or above his current reference level. We consider both these features of behavior to be natural when there is no uncertainty.

In Section 2, we introduced the assumption of acclimation (Assumption VW1), which describes a consumer's preferences over his reference point for a *fixed* consumption level. We discussed how one aspect of acclimation--that a person with a high standard of living is happier if he is acclimated to that standard of living than if he is accustomed to being poor--may strike some readers as counterintuitive. In particular, this assumption may seem to go against the intuition that people like improvements rather than staying where they are. Proposition 3.1 establishes, however, that this aspect of acclimation is in fact necessary in order to obtain the behavior implied by Theorem 1.

Proposition 3.1:

If preferences meet Assumptions V1-V5 and W1, then Theorem 1 holds for all $r_1 \geq 0$ and all Y only if for all $r \geq 0$ and $c > r$, $w'(r) > v'(c-r)$. Because $v(\cdot)$ is concave for positive values, this last condition holds if and only if for all $r \geq 0$ and $c > r$, $w(r) = U(c,c) > U(r,c)$.

Proposition 3.1 suggests an indirect empirical test for the validity of acclimation, which is hard to test directly: If the behavior implied by Theorem 1 captures reasonable intemporal behavior when a person's lifetime income is certain, Proposition 3.1 indicates that acclimation may be a realistic restriction on preferences.

The more intuitive aspect of acclimation--that, when a person's reference level is above his current consumption, he is happier the lower is his reference point--is sufficient but not necessary for Theorem 1 to hold. Proposition 3.2 establishes a weaker upper envelope condition on preferences which also conforms with this intuition and is in fact necessary for Theorem 1:

Proposition 3.2:

If preferences meet Assumptions V1-V5 and W1, then Theorem 1 holds for all $r_1 \geq 0$ and all Y only if for all $c \geq 0$ and $r > c$, $w(r) = U(c,c) > U(r,c)$.

3.2 Consumption Behavior When Income is Uncertain

In the conventional concave-utility framework, the monotonicity obtained in Theorem 1 extends to cases of uncertainty: If lottery $g(\cdot)$ first-order stochastically dominates lottery h(·), then first-period consumption will be higher if faced with income stream g(·) than if faced with h(·). Example 1 shows that this feature does not hold in our model. The convexity of v(·) over losses can cause a stochastic rise in income to yield a decline in first-period consumption.

<u>Example 1</u>:

A consumer solves the problem

$$\max_{c_1, c_2} \ U(r_1, c_1) + E\{ U(r_2, c_2) \}$$

subject to $c_1 + c_2 = Y_1 + Y_2$

$r_1 = 1$, $r_2 = c_1$ (i.e., $\alpha = 1$)

$Y_1 = 2.75$, $\text{prob}(Y_2 = 0) = 3/8$, $\text{prob}(Y_2 = 1) = 5/8$

$$\text{where } U(r,c) \equiv \begin{cases} 1.25r + (c-r) - .25(c-r)^2 & \text{if } c \geq r \\ 1.25r + 2.5(c-r) + .5(c-r)^2 & \text{if } c \leq r \end{cases}$$

In this example, the optimal choice of c_1 is locally approximated by the equation $c_1 \approx 3.25 - .59 \cdot Y_1$, so that increases in Y_1 lead to decreases in c_1. The intuition for this non-monotonicity is as follows. When the consumer chooses optimally in this example, he will choose to risk the possibility of consuming below his reference point in period 2. A rise in income gives the consumer the ability to erase some of his possible loss in the second period; since the marginal utility of consumption is rising in the loss region, the consumer may even reduce first-period consumption in order to take advantage of this new opportunity. In this example, first-period consumption is an inferior good.[11]

Strict non-monotonicities as exhibited in Example 1 appear to require that preferences are highly risk loving in losses and therefore are, in a loose sense, unlikely. Example 2 illustrates what seems, however, to be a very common outcome in our

[11]It is also possible to construct examples in which *second*-period consumption is an inferior good, so that a rise in expected income persuades the consumer to raise first-period consumption by more than the expected increase in income.

model: when expected average per-period income is near the reference level of consumption, a consumer may rigidly set $c_1 = r_1$, and be totally unresponsive to modest changes in expected income.

Example 2:

A consumer solves the problem

$$\text{Max}_{c_1,c_2} \ U(r_1,c_1) + E\{ U(r_2,c_2) \}$$

subject to $c_1 + c_2 = Y_1 + Y_2$

$r_1 = 1,\ r_2 = c_1$

$1.5 \leq Y_1 \leq 2.5$ and Y_2 uniformly distributed over $[-.5,.5]$

$$\text{where } U(r,c) \equiv \begin{cases} 3r + 2(c-r) - .25(c-r)^2 & \text{if } c \geq r \\ 3r + 4(c-r) + .25(c-r)^2 & \text{if } c \leq r \end{cases}$$

The solution is:

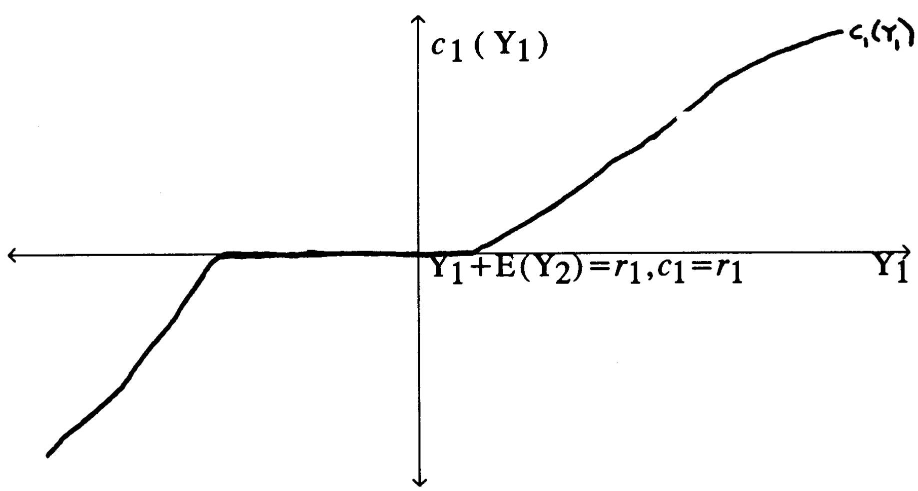
$$c_1 = 2.0625 + .5\cdot Y_1 - .25\cdot(73.0625 + Y_1)^{1/2} \qquad \text{if } Y_1 \geq 2.131$$

$$c_1 = r_1 = 1 \text{ if } 1.7081 \leq Y_1 \leq 2.131$$

$$c_1 = 1.9375 + .5\cdot Y_1 - .25\cdot(53.0625 - Y_1)^{1/2} \qquad \text{if } Y_1 \leq 1.7081$$

The results from Example 2 can be illustrated as follows:

Figure 2



The flat part of c_1(Y) in Example 2 reflects the non-differentiability in the value function $v(\cdot)$. But the more general fact that consumption changes little around the reference point demonstrates the key feature of our model: When there is enough uncertainty, people resist lowering consumption in response to the news that they will probably not be able to maintain the standard of living to which they are accustomed. If a person learns that his average per-period income is likely to be lower than his current reference point, he will not immediately consume below his reference level unless he is *very* sure that he will never recover his former income level. Theorem 2 states that as long as future per-period income exceeds the current reference level with at least probability $\alpha/(1+\alpha)$, a person will consume at least at his reference level in the first period:

<u>Theorem 2</u>:

Suppose that $P[Y/2 \geq r_1] \geq \alpha/(1+\alpha)$. Then $c_1 \geq r_1$ if $P[Y \geq r_1] = 1$.

The intuition for this result depends on precisely the two aspects of loss aversion that are most non-standard to economists--the dependence on reference points, and risk-loving preferences in losses relative to the reference point.[12] Consumers want to avoid losses, but they are willing to risk rather large losses in the future to avoid even small losses in their current standard of living. This "risk-loving" behavior is therefore crucial in yielding the prediction that people will resist decreasing their consumption.

Theorem 2 also provides some insight into how changes in the speed at which a person's reference point adjusts to recent consumption affects his behavior. The higher is α, the more willing is a person to decrease his current consumption in response to bad news about future income. This is because a higher α means that lowering consumption

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[12]In fact, our proof of Theorem 2 shows that only a weaker form of Assumption VW1 is required. So long as Assumptions V1-V5 and W1 hold, then Theorem 2 holds if $w'(r) \geq v_g'(c-r)$ for all $c \geq r$.

will more dramatically lower the future reference point; if future consumption is likely to be lower, then such a lowering of the reference point is a positive feature. Note that if a person's reference point is completely unaffected by current changes in consumption--if $\alpha = 0$--then Theorem 2 says that the person will never consume below his reference level unless his income is so low that consuming above his reference level in the first period risks default.

While Example 2 illustrates that consumption can be sticky both when expected income is below the reference level and when it is above, there is more generally an asymmetry in our model regarding such stickiness. Theorem 2 implies that consumer behavior under loss aversion is asymmetric with respect to the reference level, because there *are* examples where a consumer will consume above his reference level in response to bad news. Example 3 describes a situation in which, even though the consumer expects income in the second period to be below his reference level, he consumes above his reference point in the first period.

Example 3:

A consumer solves the problem

$$\max_{c_1,c_2} U(r_1,c_1) + E\{U(r_2,c_2)\}$$

subject to $c_1 + c_2 = Y_1 + Y_2$

$r_1 = 1;\ r_2 = .5r_1 + .5c_1$

$1 \le Y_1 \le 2$

Y_2 uniformly distributed over $[0,.5]$

$$\text{where } U(r,c) \equiv \begin{cases} r + (c-r) - .25(c-r)^2 & \text{if } c \ge r \\ r + 1.05(c-r) + .0625(c-r)^2 & \text{if } c \le r \end{cases}$$

The solution is:

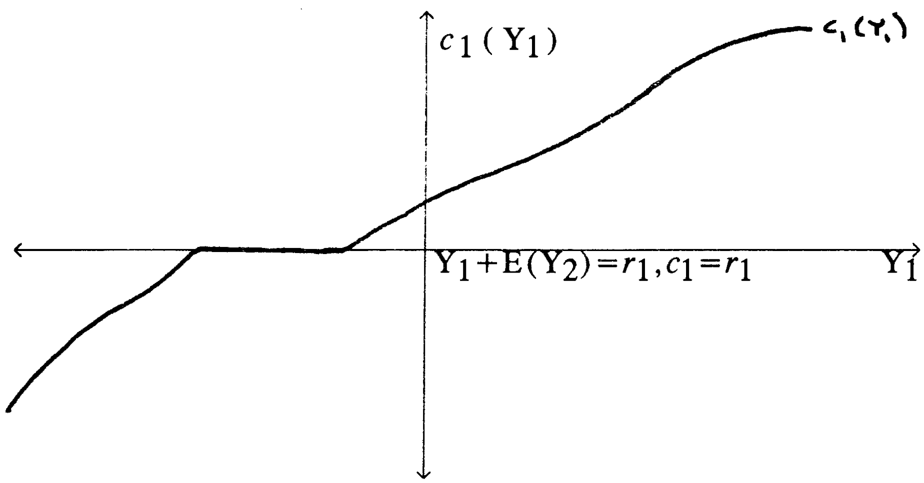
$$c_1 = .1052 + .6667Y_1 - (-.2126 + .1580Y_1)^{1/2} \quad \text{if } 1.6944 \le Y_1 \le 2$$

$$c_1 = 1 \quad \text{if } 1 \le Y_1 \le 1.6944$$

For $1.6944 < Y_1 < 1.75$, $Y_1 + E(Y_2) < 2 = 2r_1$, but $c_1 > 1 = r_1$.

Example 3 can be illustrated graphically as follows:

Figure 3



It is this asymmetry implied by Theorem 2 and Example 3 together that we feel is the main feature differentiating loss aversion from other models of preferences.

3.3 Consumption Behavior when Income Uncertainty is Increased : Precautionary Savings

It is a commonly held belief that people will increase savings in response to an increase in uncertainty. Leland (1968) shows that--in a conventional, time-separable, concave-utility model--this intuition is correct only when the third derivative of the utility function is positive (see also Kimball (1990a,1990b)).

In our model, an increase in uncertainty can either increase consumption or decrease it. Combining Theorems 1 and 2 show that an increase in uncertainty can decrease savings. Theorem 1 shows that if a consumer knows for sure that his future income is below his current reference level, he will consume below his reference level. Theorem 2 shows that if the consumer faces expected per-period income below his reference level, but believes that there is a high probability that his per-period income will be above his reference level, he will consume at or above his reference level.

Thus, when expected income is below the reference level by a small amount, then an increase in the probability that a consumer will be able to consume above his reference level each period will lead him to decrease savings. Part (i) of Proposition 3.3 illustrates this idea. Part (ii), on the other hand, suggests a set of circumstances where the classical intuition is likely to hold: If expected per-period income is above the reference level by a small amount, then an increase in uncertainty is likely to increase savings.

Proposition 3.3:

Let $c_1(Y,k)$ be the consumer's first-period consumption when faced with probabilistic total income $(.5\,(1-k)Y,\ .5\,(1+k)Y)$, where $k \in [0,1]$.

i) There exists $Y_* < 2 \cdot r_1$ such that for all $Y \in (Y_*, 2r_1)$ and all k for which $(1-k) \cdot Y > r_1$, $c_1(Y,k) > c_1(Y,0)$.

ii) There exists $k^* > 0$ and $Y^* > 2 \cdot r_1$ such that for all $Y \in (2r_1, Y^*)$ and all $k < k^*$, $c_1(Y,0) > c_1(Y,k)$;

While part (ii) of Proposition 3.3 need not generalize to non-binary symmetric distributions, the idea driving the proof is that when expected average per-period income exceeds your reference point, an increase in uncertainty *that raises the odds that you will not be able to maintain your current consumption level* will increase your savings.[13] Roughly, this holds whenever the effect of increasing the odds of being forced to consume below one's reference level is not outweighed by behavioral changes relating to third-derivative features of the various components of the utility function.

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[13]Theorems 1 and 2 make clear, however, that part (i) generalizes to any symmetric distribution. The fact that part (i) of Proposition 3.3 holds more generally than part (ii) seems to be related to the asymmetry in Theorem 2 and Example 3.

4. Discussion and Conclusion

Our exploration of the savings problem calls for a few natural extensions. We would like to extend the analysis to a multiple-period or infinite-horizon model. To do this, we would need to add discounting and positive interest rates to the model. Given our analysis, one can make an informed guess about how results would be altered under these extensions. No matter how many periods there are, for instance, it is staightforward to show that, if $\alpha = 0$, an agent will never consume below his reference point unless his income absolutely forces him to. As α rises, a consumer will be more willing to consume below his reference point, just as in the two-period model.

Extending the analysis to many periods would allow us to explore some issues that cannot be completely addressed within the two-period framework. As we show in Proposition 3.3, consumers may respond to an increase in income uncertainty either by saving more or by saving less, depending on whether expected per-period income is above or below the reference level. Given that losses are so costly, it is natural to suppose that over a longer time frame consumers will plan their path of consumption in order to avoid losses. In this case, the probability of a consumer having to consume below his reference level will be greatly diminished. As in standard models of precautionary savings, the consumer's typical response to small increases in uncertainty might therefore be to increase savings.[14]

As we stated in the introduction, we feel that loss aversion can usefully be incorporated more generally into economics. Benartzi and Thaler (1992) invoke loss aversion in explaining under-investment in risky assets. And Sheffrin and Statman (1985) find that trading volume on the stock market falls when stock values fall, which they interpret as due to loss aversion--investors are unwilling to realize the loss on their

[14]For discussions of precautionary savings in the standard framework, see Sibley (1975), Miller (1976), Zeldes (1989), and Caballero (1990).

investment and so hold on to the stocks.[15] Fershtman (1993) considers the effect of loss aversion on the willingness of incumbents in an industry to fight potential entrants. We feel that a formal model along the lines developed in this paper will help researchers begin to systematically investigate the implications of loss aversion in a wider array of economic situations.

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[15]These examples, and indeed the savings example in this paper, all have a similar flavor to the results found in some studies in non-economic settings. McGlothlin (1956) documents the tendency of race track bettors to bet increasingly long shots as the betting day goes on, behavior that he interprets as an attempt to end the day without a loss. Thaler and Johnson (1990) discover a similar pattern in a different gambling context.

Appendix: Proofs

Note: In all proofs, we denote the function v(x) by $v_l(x)$ when $x \le 0$, and by $v_g(x)$ when $x \ge 0$.

Proof of Proposition 2.1:

Assumption V4 implies that there exists $\delta > 0$, $\gamma > 0$ such that

$$v_l'(y) - v_g'(0) > \gamma > 0 \text{ for all } y \in (-\delta,0].$$

Let G and H denote the c.d.f.'s of g and h, and let $y_i \equiv$ be the lowest y realized with positive probability by either f or g.

Set $\varepsilon \equiv -\delta/y_i$. Then $k < \varepsilon$ implies that

i)
$$\begin{aligned} E\{v(k\cdot g)\} &= \int_{y_i}^{0} v_l(kx)dG(x) + \int_0^\infty v_g(kx)dG(x) \\ &= \int_{y_i}^{0} [\int_0^x v_l'(ks)ds]\cdot dG(x) + \int_0^\infty [\int_0^x v_g'(ks)ds]\cdot dG(x) \\ &< \int_{y_i}^{0}[v_g'(0)+\gamma]xdG(x) + \int_0^\infty v_g'(0)\cdot xdG(x), \\ &= v_g'(0)\cdot\int_{y_i}^{\infty}xdG(x) + \gamma\int_{y_i}^{0} xdG(x) \\ &= \gamma \int_{y_i}^{0} [v_g'(0)+\gamma]\cdot xdG(x) < 0. \end{aligned}$$

ii)
$$\begin{aligned} E\{v(k\cdot f)\} &= \int_{y_i}^{0} v_l(kx)dF(x) + \int_0^\infty v_g(kx)dF(x) \\ &< v_g'(0)\cdot[\int_{y_i}^{0} x\cdot dF(x) + \int_0^\infty x\cdot dF(x)] \\ &= v_g'(0)\cdot\int_{y_i}^{\infty}xdF(x). \end{aligned}$$

Set $a(p) \equiv p\cdot v_g'(0)\cdot\int_{y_i}^{\infty}xdF(x) + (1-p)\cdot\gamma\cdot\int_{y_i}^{0} x\cdot dG(x)$.

Then $a(1) > 0$, since f is better-than-fair;

$a(0) < 0$.

Thus, there exists $p^* > 0$ such that $a(p^*) = 0$ and $a(p) < 0$ for all $p < p^*$.

So $p < p^*$ and $k < \varepsilon$ imply that

$$E\{v(kh(p)\} = p\cdot E\{v(kf)\} + (1-p)\cdot E\{v(kg)\}.$$

But this is less than $a(p)$, which is negative, so you reject $h(p)$ scaled by $k < \varepsilon$.

Q.E.D.

Remark on Proposition 2.1:

This proposition would also be true even if the supports of f and g contained infinitely negative outcomes, so long as the expected value of losses were finite for each of the bets.

Proof of Proposition 2.2:

Proof of "only if" direction:

Assume $v(\cdot)$ satisfies V1-V5.

Suppose there exists $y_0 < 0$ such that $v'(y_0) \leq v_g'(0)$. Then $v'(y_1) < v_g'(0)$ for all $y_1 < y_0$. Therefore, the continuity of $v_g'(\cdot)$ implies that $v'(y_1) \leq v_g'(\varepsilon)$ for some $\varepsilon > 0$. Thus, by Assumption V2, $v'(y) < v'(\varepsilon)$ for all $y < y_1$ and $v'(\varepsilon) - v'(y)$ increases as y decreases.

Then $v(y) - v'(\varepsilon)\cdot y = \int_y^0 [v'(\varepsilon)-v'(s)]ds$

$$= \int_{y_0}^0 [v'(\varepsilon)-v'(s)]ds + \int_y^{y_0} [v'(\varepsilon)-v'(s)]ds$$

and $\lim_{y \to -\infty} v(y) - v'(\varepsilon)\cdot y = \infty$.

In particular, we can choose $\tilde{y} < 0$ sufficiently negative such that $v(\tilde{y}) - v'(\varepsilon)\cdot\tilde{y} > 2v(\varepsilon)$.

Set $\tilde{p} \equiv \varepsilon/(\varepsilon-\tilde{y})$. Then i) $\tilde{p}\cdot\tilde{y} + (1-\tilde{p})\cdot\varepsilon = 0$, and

ii) $\tilde{p}\cdot v(\tilde{y}) + (1-\tilde{p})\cdot v(\varepsilon)$

$$> \tilde{p}\cdot[v'(\varepsilon)\cdot\tilde{y} + 2v(\varepsilon)] + (1-\tilde{p})\cdot v(\varepsilon)$$

$$> -\varepsilon\cdot v'(\varepsilon) + v(\varepsilon)(\tilde{p} + 1), \text{ (since } 0 > \tilde{p}\cdot\tilde{y} = -(1-\tilde{p})\cdot\varepsilon > -\varepsilon.)$$

$$= v'(\varepsilon) - \varepsilon\cdot v'(\varepsilon) + \tilde{p}\cdot v(\varepsilon)$$

$$> 0 \quad \text{(by Assumption V2).}$$

Proof of "if" direction:

Suppose $v'(y) > v_g'(0)$ for all $y < 0$.

Choose any y,z such that $p\cdot y + (1-p)\cdot z = 0$. Thus, $z = -p\cdot y/(1-p)$.

Then $p\cdot v(y) + (1-p)\cdot v(z)$

$$< p\cdot v_g'(0)\cdot y + (1-p)\cdot v(z) \quad \text{(since } v'(y) > v_g'(0))$$

$$< p\cdot v_g'(0)\cdot y - [(1-p)\cdot p/(1-p)]\cdot v_g'(0)\cdot y$$

$$= p\cdot v_g'(0)\cdot y - p\cdot v_g'(0)\cdot y$$

$$= 0$$

Q.E.D.

Proof of Theorem 1:

For any given r_1 and Y, substituting Y - c_1 for c_2 and $\alpha c_1 + (1-\alpha)r_1$ for r_2, we can write $U(c_1,r_1;c_2,r_2)$ as a function of c_1 alone:

$$u(c_1) \equiv w(r_1) + v(c_1-r_1) + w(\alpha c_1+(1-\alpha)r_1) + v(Y - (1+\alpha)c_1 - (1-\alpha)r_1)$$

We will let c_1^* denote the c_1 which maximizes $u(c_1)$, and let $c_2^* \equiv Y - c_1^*$. If we impose the non-negativity constraint, we say that c_1 is feasible if $c_1 \in [0,Y]$; if we do not impose the non-negativity constraint, then any c_1 is feasible.

(1) Claim: $c_1(Y) = c_2(Y) = r_1$ when $Y = 2r_1$.

Proof: When $Y = 2r_1$, if $c_1 > r_1$ then $c_2 < r_2$, and if $c_1 < r_1$ then $c_2 > r_2$. If $c_1 > r_1$ and $c_2 < r_2$, then $u'(c_1) = v_g'(c_1-r_1) + \alpha w'(r_2) - (1+\alpha)v_1'(Y-c_1-r_2)$. By VW1 this is negative, so that the consumer will decrease c_1. If $c_1 < r_1$ and $c_2 > r_2$, then $u'(c_1) = v_1'(c_1-r_1) + \alpha w'(r_2) - (1+\alpha)v_g'(Y-c_1-r_2)$. By VW1 this is positive, so that the consumer will increase c_1.

(2) Claim: $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing when $Y > 2r_1$, and both converge to r_1 when Y approaches $2r_1$ from above.

Proof: Assume $Y > 2r_1$. We know from the above argument that at the optimum, $c_1^* \geq r_1$ and $c_2^* \geq r_2$; this implies $r_1 \leq c_1^* \leq (Y-(1-\alpha)r_1)/(1+\alpha)$. Note that $u(c_1)$ is strictly concave in this region. If $c_1 = r_1$, then, by VW1,

$$u'(c_1) = v_1'(0) + \alpha w'(r_1) - (1+\alpha)v_g'(Y-2r_1) > 0.$$

Therefore, $c_1 > r_1$ if $Y > 2r_1$. Now consider $c_2 = r_2$; this implies $c_1 = (Y-(1-\alpha)r_1)/(1+\alpha)$. Then

$$u'(c_1) = v_g'((Y-2r_1)/(1+\alpha)) + \alpha w'((\alpha Y + (1-\alpha)r_1)/(1+\alpha)) - (1+\alpha)v_g'(0).$$

If $u'(c_1)$ is greater than zero when evaluated at this point, then $c_1^* = (Y-(1-\alpha)r_1)/(1+\alpha)$ and $c_2^* = (\alpha Y+(1-\alpha)r_1)/(1+\alpha)$. As Y increases, $u'((Y-(1-\alpha)r_1)/(1+\alpha))$ decreases continuously by the concavity of v_g and w, and by the assumption that v_g' and w' are continuous.

If $u'((Y-(1-\alpha)r_1)/(1+\alpha) \leq 0$, then there is a $\bar{c}_1 \in (r_1,(Y-(1-\alpha)r_1)/(1+\alpha)]$ such that $u'(\bar{c}_1) = 0$. Then $c_1^* = \bar{c}_1$ and $c_2^* = Y - \bar{c}_1$. We know $\bar{c}_1$ is a continuous function of Y since v_g' and w' are continuous. Totally differentiating the equation $u'(\bar{c}_1) = 0$ yields $1/(1+\alpha) > d\bar{c}_1/dY > 0$.

Therefore, $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing, and since (Y-

$(1-\alpha)r_1)/(1+\alpha) \to r_1$ as $Y \to 2r_1$, we have $c_1(Y)$ and $c_2(Y) \to r_1$ as $Y \to 2r_1$ from above.

(3) Claim: If $Y < 2r_1$ and negative values of c_1 and c_2 are allowed, then $c_1(Y) = (Y - (1-\alpha)r_1)/(1+\alpha)$. If $Y < 2r_1$ and the constraints $c_1 \geq 0$ and $c_2 \geq 0$ are imposed, then $c_1(Y) = \max[0,(Y-(1-\alpha)r_1)/(1+\alpha)]$.

Proof: From the proof of (1), we know that $c_1^* \leq r_1$ and $c_2^* \leq r_2$, so that $(Y-(1-\alpha)r_1)/(1+\alpha) \leq c_1^* \leq r_1$. Consider $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$. In this region $c_1-r_1 \leq c_2-r_2 \leq 0$, and so we have

$$u'(c_1) = v_1'(c_1-r_1) + \alpha w'(\alpha c_1+(1-\alpha)r_1) - (1+\alpha)v_1'(c_2-r_2) < 0$$

by VW1 and V2. Now consider the region $c_1 \in ((Y+\alpha r_1)/(2+\alpha), r_1]$. In this region we have $0 \geq c_1 - r_1 > c_2 - r_2$. We can rewrite this range of possible choices for c_1 as $(Y-2r_1)/(2+\alpha) < c_1 - r_1 \leq 0$. Consider any choice of $c_1-r_1 \equiv z$ in this range, then $u(r_1+z) = v_1(z) + w(\alpha z+r_1) + v_1(Y-(1+\alpha)z-2r_1)$ is the utility from this choice. Compare this to the choice of $c_2-r_2 = z$ (This choice implies a c_1 which lies in the range $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$). If $c_1 = r_1+z$ is feasible, then $c_2 = r_2+z$ is also feasible. The choice $c_2 = r_2+z$ results in utility

$$u(Y-r_2-z) = v_1(Y-(1+\alpha)z-2r_1+k) + w(\alpha z+r_1-k) + v_1(z),$$

where $k \equiv \alpha[2r_1-Y+(2+\alpha)z]/(1-\alpha) > 0$. Then

$$u(Y-r_2-z) - u(r_1+z) = [v_1(Y-(1+\alpha)z-2r_1+k) - v_1(Y-(1+\alpha)z-2r_1)] + [w(\alpha z+r_1-k) - w(\alpha z+r_1)].$$

VW1 implies $v_1(x+q) - v_1(x) > w(s+q) - w(s)$ for any $q > 0$, $x \leq -q$ and $s \geq 0$. Therefore $u(Y-r_2-z) > u(r_1+z)$ for any z such that $(Y-2r_1)/(2+\alpha) < z \leq 0$. This implies that for any feasible choice of $c_1 \in ((Y+\alpha r_1)/(2+\alpha), r_1]$ there is a feasible choice of $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$ which is preferred. Therefore

$$c_1^* \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)],$$

and therefore if negative values of c_1 and c_2 are allowed, $c_1(Y) = (Y-(1-\alpha)r_1)/(1+\alpha)$; if $c_1 < 0$ and $c_2 < 0$ are not allowed, then $c_1(Y) = \text{Max}[0,(Y-(1-\alpha)r_1)/(1+\alpha)]$.

This establishes Claim (3); Claims (1), (2), and (3) together establish the theorem. Q.E.D.

<u>Remark on Theorem 1</u>:

The proof above shows that $c_1(Y)$ is never steeper for $Y/2 > r_1$ than it is for $Y/2 < r_1$.

<u>Proof</u> <u>of</u> <u>Proposition</u> <u>3.1</u>:

For any given r_1 and Y, substituting Y - c_1 for c_2 and $\alpha c_1 + (1-\alpha)r_1$ for r_2, we can write $U(c_1,r_1;c_2,r_2)$ as a function of c_1 alone:

$$u(c_1) \equiv w(r_1) + v(c_1-r_1) + w(\alpha c_1+(1-\alpha)r_1) + v(Y - (1+\alpha)c_1 - (1-\alpha)r_1)$$

We will let c_1^* denote the c_1 which maximizes $u(c_1)$, and let $c_2^* \equiv Y - c_1^*$.

Suppose that there is some $\bar{r} \geq 0$, $\bar{c} \geq \bar{r}$, such that $v'(\bar{c}-\bar{r}) \geq w'(\bar{r})$. Then by W1 for any $r > \bar{r}$ and $\bar{c} \geq c > r$ we have $v'(c-r) > w'(r)$. Let $r_1 = r$ and take $Y \geq 2r_1$. Theorem 1 implies that $c_1 \geq r$ and $c_2 \geq r$. If $c_1 = r$ then $u'(r) = v_g'(0) + \alpha w'(r) - (1+\alpha)v_g'(Y-2r)$.

Then $\mathrm{Lim}_{Y\to 2r_1} u'(r) = \alpha\cdot[w'(r) - v_g'(0)] < 0$. Because v_g' and w' are continuous, this implies that there is some $x > 0$ such that for all $0 < y \leq x$,

$$u'(r) = v_g'(0) + \alpha w'(r) - (1+\alpha)v_g'(y) < 0.$$

$u(c_1)$ is concave in the region $r \leq c_1 \leq (Y-(1-\alpha)r)/(1+\alpha)$. Therefore if c_1^* lies in this region, $c_1^* = r$ for $Y \in [2r,2r+x]$. This means $c_1(Y)$ is not strictly increasing in this region, which contradicts Theorem 1.

To see that $c_1 \in [r,(Y-(1-\alpha)r)/(1+\alpha)]$ note that by supposition $r \leq c_1^* \leq Y-r$. Then

$$u(c_1 = Y-(1-\alpha)r)/(1+\alpha)) = w(\bar{r}) + v_g((Y-2r)/(1+\alpha)) + w((\alpha Y+(1-\alpha)r)/(1+\alpha)).$$

Compare this to $u(c_1 = (Y-(1-\alpha)r)/(1+\alpha) + z)$, where $z > 0$:

$$u(Y-(1-\alpha)r)/(1+\alpha)+z) = w(\bar{r}) +$$
$$v_g(((Y-2r)/(1+\alpha))+z) + w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) + v_l(-(1+\alpha)z).$$

Let $X \equiv u((Y-(1-\alpha)r)/(1+\alpha))+z) - u(Y-(1-\alpha)r)/(1+\alpha))$

$$= [v_g(((Y-2r)/(1+\alpha))+z) - v_g((Y-2r)/(1+\alpha))] + v_l(-(1+\alpha)z) + [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))].$$

Then $X \leq [v_g(z) - v_g(0)] + v_l(-(1+\alpha)z)$
$+ [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))].$
$= v_g(z) + v_l(-(1+\alpha)z)$
$+ [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))].$

By supposition, $c_1(2r) = c_2(2r) = r$. Therefore when $Y = 2r$ we must have $u(r+k) < u(r)$ for any $z > 0$.

$$u(r+z) - u(r) = v_g(z) + w(r+\alpha z) + v_l(-(1+\alpha)z) - w(r)$$
$$= v_g(z) + v_l(-(1+\alpha)z) + [w(r+\alpha z) - w(r)].$$

Therefore, $u(r+z) - u(r) < 0$ for all $r \geq 0$, $z > 0$, and $Y = 2r$ implies $X < 0$. Therefore $u(((Y-(1-\alpha)r)/(1+\alpha))+z) < u((Y-(1-\alpha)r)/(1+\alpha))$ for any $z > 0$, which implies $r \leq c_1^* \leq (Y-(1-\alpha)r)/(1+\alpha)$.

This proves that theorem 1 and V1-V5, W1 imply $w'(r) > v'(c-r)$ for any $r \geq 0$ and c will more dramatically lower the future reference point; if future consumption is likely to be lower, then such a lowering of the reference point is a positive feature. Note that if a person's reference point is completely unaffected by current changes in consumption--if $\alpha = 0$--then Theorem 2 says that the person will never consume below his reference level unless his income is so low that consuming above his reference level in the first period risks default.

While Example 2 illustrates that consumption can be sticky both when expected income is below the reference level and when it is above, there is more generally an asymmetry in our model regarding such stickiness. Theorem 2 implies that consumer behavior under loss aversion is asymmetric with respect to the reference level, because there *are* examples where a consumer will consume above his reference level in response to bad news. Example 3 describes a situation in which, even though the consumer expects income in the second period to be below his reference level, he consumes above his reference point in the first period.

Example 3:

A consumer solves the problem

$$\max_{c_1,c_2} U(r_1,c_1) + E\{U(r_2,c_2)\}$$

subject to $c_1 + c_2 = Y_1 + Y_2$

$r_1 = 1;\ r_2 = .5r_1 + .5c_1$

$1 \leq Y_1 \leq 2$

Y_2 uniformly distributed over $[0,.5]$

$$\text{where } U(r,c) \equiv \begin{cases} r + (c-r) - .25(c-r)^2 & \text{if } c \geq r \\ r + 1.05(c-r) + .0625(c-r)^2 & \text{if } c \leq r \end{cases}$$

The solution is:

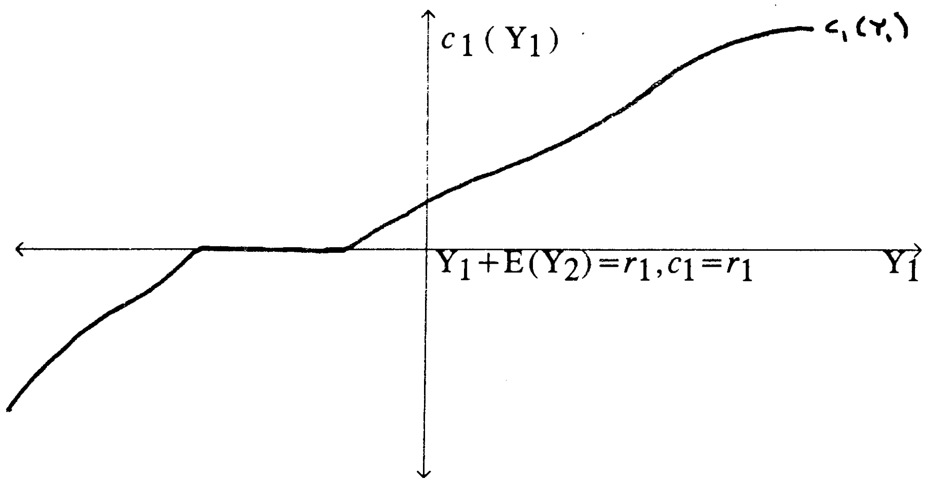
$$c_1 = .1052 + .6667Y_1 - (-.2126 + .1580Y_1)^{1/2} \quad \text{if } 1.6944 \leq Y_1 \leq 2$$

$$c_1 = 1 \quad \text{if } 1 \leq Y_1 \leq 1.6944$$

For $1.6944 < Y_1 < 1.75$, $Y_1 + E(Y_2) < 2 = 2r_1$, but $c_1 > 1 = r_1$.

Example 3 can be illustrated graphically as follows:

Figure 3



It is this asymmetry implied by Theorem 2 and Example 3 together that we feel is the main feature differentiating loss aversion from other models of preferences.

3.3 Consumption Behavior when Income Uncertainty is Increased : Precautionary Savings

It is a commonly held belief that people will increase savings in response to an increase in uncertainty. Leland (1968) shows that--in a conventional, time-separable, concave-utility model--this intuition is correct only when the third derivative of the utility function is positive (see also Kimball (1990a, 1990b)).

In our model, an increase in uncertainty can either increase consumption or decrease it. Combining Theorems 1 and 2 show that an increase in uncertainty can decrease savings. Theorem 1 shows that if a consumer knows for sure that his future income is below his current reference level, he will consume below his reference level. Theorem 2 shows that if the consumer faces expected per-period income below his reference level, but believes that there is a high probability that his per-period income will be above his reference level, he will consume at or above his reference level.

Thus, when expected income is below the reference level by a small amount, then an increase in the probability that a consumer will be able to consume above his reference level each period will lead him to decrease savings. Part (i) of Proposition 3.3 illustrates this idea. Part (ii), on the other hand, suggests a set of circumstances where the classical intuition is likely to hold: If expected per-period income is above the reference level by a small amount, then an increase in uncertainty is likely to increase savings.

Proposition 3.3:

Let $c_1(Y,k)$ be the consumer's first-period consumption when faced with probabilistic total income $(.5\ (1\text{-}k)Y,\ .5\ (1+k)Y)$, where $k \in [0,1]$.

i) There exists $Y_* < 2 \cdot r_1$ such that for all $Y \in (Y_*, 2r_1)$ and all k for which $(1\text{-}k) \cdot Y > r_1$, $c_1(Y,k) > c_1(Y,0)$.

ii) There exists $k^* > 0$ and $Y^* > 2 \cdot r_1$ such that for all $Y \in (2r_1, Y^*)$ and all $k < k^*$, $c_1(Y,0) > c_1(Y,k)$;

While part (ii) of Proposition 3.3 need not generalize to non-binary symmetric distributions, the idea driving the proof is that when expected average per-period income exceeds your reference point, an increase in uncertainty *that raises the odds that you will not be able to maintain your current consumption level* will increase your savings.[13] Roughly, this holds whenever the effect of increasing the odds of being forced to consume below one's reference level is not outweighed by behavioral changes relating to third-derivative features of the various components of the utility function.

[13]Theorems 1 and 2 make clear, however, that part (i) generalizes to any symmetric distribution. The fact that part (i) of Proposition 3.3 holds more generally than part (ii) seems to be related to the asymmetry in Theorem 2 and Example 3.

4. Discussion and Conclusion

Our exploration of the savings problem calls for a few natural extensions. We would like to extend the analysis to a multiple-period or infinite-horizon model. To do this, we would need to add discounting and positive interest rates to the model. Given our analysis, one can make an informed guess about how results would be altered under these extensions. No matter how many periods there are, for instance, it is staightforward to show that, if $\alpha = 0$, an agent will never consume below his reference point unless his income absolutely forces him to. As α rises, a consumer will be more willing to consume below his reference point, just as in the two-period model.

Extending the analysis to many periods would allow us to explore some issues that cannot be completely addressed within the two-period framework. As we show in Proposition 3.3, consumers may respond to an increase in income uncertainty either by saving more or by saving less, depending on whether expected per-period income is above or below the reference level. Given that losses are so costly, it is natural to suppose that over a longer time frame consumers will plan their path of consumption in order to avoid losses. In this case, the probability of a consumer having to consume below his reference level will be greatly diminished. As in standard models of precautionary savings, the consumer's typical response to small increases in uncertainty might therefore be to increase savings.[14]

As we stated in the introduction, we feel that loss aversion can usefully be incorporated more generally into economics. Benartzi and Thaler (1992) invoke loss aversion in explaining under-investment in risky assets. And Sheffrin and Statman (1985) find that trading volume on the stock market falls when stock values fall, which they interpret as due to loss aversion--investors are unwilling to realize the loss on their

[14]For discussions of precautionary savings in the standard framework, see Sibley (1975), Miller (1976), Zeldes (1989), and Caballero (1990).

investment and so hold on to the stocks.[15] Fershtman (1993) considers the effect of loss aversion on the willingness of incumbents in an industry to fight potential entrants. We feel that a formal model along the lines developed in this paper will help researchers begin to systematically investigate the implications of loss aversion in a wider array of economic situations.

[15]These examples, and indeed the savings example in this paper, all have a similar flavor to the results found in some studies in non-economic settings. McGlothlin (1956) documents the tendency of race track bettors to bet increasingly long shots as the betting day goes on, behavior that he interprets as an attempt to end the day without a loss. Thaler and Johnson (1990) discover a similar pattern in a different gambling context.

Appendix: Proofs

Note: In all proofs, we denote the function v(x) by $v_l(x)$ when $x \le 0$, and by $v_g(x)$ when $x \ge 0$.

Proof of Proposition 2.1:

Assumption V4 implies that there exists $\delta > 0$, $\gamma > 0$ such that

$$v_l'(y) - v_g'(0) > \gamma > 0 \text{ for all } y \in (-\delta, 0].$$

Let G and H denote the c.d.f.'s of *g* and *h*, and let $y_i \equiv$ be the lowest y realized with positive probability by either *f* or *g*.

Set $\varepsilon \equiv -\delta/y_i$. Then $k < \varepsilon$ implies that

i)
$$\begin{aligned} E\{v(k\cdot g)\} &= \int_{y_i}^{0} v_l(kx)dG(x) + \int_0^\infty v_g(kx)dG(x) \\ &= \int_{y_i}^{0} [\int_0^x v_l'(ks)ds]\cdot dG(x) + \int_0^\infty [\int_0^x v_g'(ks)ds]\cdot dG(x) \\ &< \int_{y_i}^{0}[v_g'(0)+\gamma]xdG(x) + \int_0^\infty v_g'(0)\cdot xdG(x), \\ &= v_g'(0)\cdot\int_{y_i}^{\infty}xdG(x) + \gamma\int_{y_i}^{0} xdG(x) \\ &= \gamma \int_{y_i}^{0} [v_g'(0)+\gamma]\cdot xdG(x) < 0. \end{aligned}$$

ii)
$$\begin{aligned} E\{v(k\cdot f)\} &= \int_{y_i}^{0} v_l(kx)dF(x) + \int_0^\infty v_g(kx)dF(x) \\ &< v_g'(0)\cdot[\int_{y_i}^{0} x\cdot dF(x) + \int_0^\infty x\cdot dF(x)] \\ &= v_g'(0)\cdot\int_{y_i}^{\infty}xdF(x). \end{aligned}$$

Set $a(p) \equiv p\cdot v_g'(0)\cdot\int_{y_i}^{\infty}xdF(x) + (1-p)\cdot\gamma\cdot\int_{y_i}^{0} x\cdot dG(x)$.

Then $a(1) > 0$, since *f* is better-than-fair;

$a(0) < 0$.

Thus, there exists $p^* > 0$ such that $a(p^*) = 0$ and $a(p) < 0$ for all $p < p^*$.

So $p < p^*$ and $k < \varepsilon$ imply that

$$E\{v(kh(p)\} = p\cdot E\{v(kf)\} + (1-p)\cdot E\{v(kg)\}.$$

But this is less than $a(p)$, which is negative, so you reject $h(p)$ scaled by $k < \varepsilon$.

Q.E.D.

Remark on Proposition 2.1:

This proposition would also be true even if the supports of *f* and *g* contained infinitely negative outcomes, so long as the expected value of losses were finite for each of the bets.

Proof of Proposition 2.2:

Proof of "only if" direction:

Assume $v(\cdot)$ satisfies V1-V5.

Suppose there exists $y_0 < 0$ such that $v'(y_0) \leq v_g'(0)$. Then $v'(y_1) < v_g'(0)$ for all $y_1 < y_0$. Therefore, the continuity of $v_g'(\cdot)$ implies that $v'(y_1) \leq v_g'(\varepsilon)$ for some $\varepsilon > 0$. Thus, by Assumption V2, $v'(y) < v'(\varepsilon)$ for all $y < y_1$ and $v'(\varepsilon) - v'(y)$ increases as y decreases.

Then $v(y) - v'(\varepsilon)\cdot y = \int_y^0 [v'(\varepsilon)-v'(s)]ds$

$$= \int_{y_0}^0 [v'(\varepsilon)-v'(s)]ds + \int_y^{y_0} [v'(\varepsilon)-v'(s)]ds$$

and $\lim_{y \to -\infty} v(y) - v'(\varepsilon)\cdot y = \infty$.

In particular, we can choose $\tilde{y} < 0$ sufficiently negative such that $v(\tilde{y}) - v'(\varepsilon)\cdot\tilde{y} > 2v(\varepsilon)$.

Set $\tilde{p} \equiv \varepsilon/(\varepsilon-\tilde{y})$. Then i) $\tilde{p}\cdot\tilde{y} + (1-\tilde{p})\cdot\varepsilon = 0$, and

ii) $\tilde{p}\cdot v(\tilde{y}) + (1-\tilde{p})\cdot v(\varepsilon)$

$$> \tilde{p}\cdot[v'(\varepsilon)\cdot\tilde{y} + 2v(\varepsilon)] + (1-\tilde{p})\cdot v(\varepsilon)$$

$$> -\varepsilon\cdot v'(\varepsilon) + v(\varepsilon)(\tilde{p} + 1), \text{ (since } 0 > \tilde{p}\cdot\tilde{y} = -(1-\tilde{p})\cdot\varepsilon > -\varepsilon.)$$

$$= v'(\varepsilon) - \varepsilon\cdot v'(\varepsilon) + \tilde{p}\cdot v(\varepsilon)$$

$$> 0 \quad \text{(by Assumption V2).}$$

Proof of "if" direction:

Suppose $v'(y) > v_g'(0)$ for all $y < 0$.

Choose any y,z such that $p\cdot y + (1-p)\cdot z = 0$. Thus, $z = -p\cdot y/(1-p)$.

Then $p\cdot v(y) + (1-p)\cdot v(z)$

$$< p\cdot v_g'(0)\cdot y + (1-p)\cdot v(z) \quad \text{(since } v'(y) > v_g'(0))$$

$$< p\cdot v_g'(0)\cdot y - [(1-p)\cdot p/(1-p)]\cdot v_g'(0)\cdot y$$

$$= p\cdot v_g'(0)\cdot y - p\cdot v_g'(0)\cdot y$$

$$= 0$$

Q.E.D.

Proof of Theorem 1:

For any given r_1 and Y, substituting Y - c_1 for c_2 and $\alpha c_1 + (1-\alpha)r_1$ for r_2, we can write $U(c_1,r_1;c_2,r_2)$ as a function of c_1 alone:

$$u(c_1) \equiv w(r_1) + v(c_1-r_1) + w(\alpha c_1+(1-\alpha)r_1) + v(Y - (1+\alpha)c_1 - (1-\alpha)r_1)$$

We will let c_1^* denote the c_1 which maximizes $u(c_1)$, and let $c_2^* \equiv Y - c_1^*$. If we impose the non-negativity constraint, we say that c_1 is feasible if $c_1 \in [0,Y]$; if we do not impose the non-negativity constraint, then any c_1 is feasible.

(1) Claim: $c_1(Y) = c_2(Y) = r_1$ when $Y = 2r_1$.

Proof: When $Y = 2r_1$, if $c_1 > r_1$ then $c_2 < r_2$, and if $c_1 < r_1$ then $c_2 > r_2$. If $c_1 > r_1$ and $c_2 < r_2$, then $u'(c_1) = v_g'(c_1-r_1) + \alpha w'(r_2) - (1+\alpha)v_l'(Y-c_1-r_2)$. By VW1 this is negative, so that the consumer will decrease c_1. If $c_1 < r_1$ and $c_2 > r_2$, then $u'(c_1) = v_l'(c_1-r_1) + \alpha w'(r_2) - (1+\alpha)v_g'(Y-c_1-r_2)$. By VW1 this is positive, so that the consumer will increase c_1.

(2) Claim: $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing when $Y > 2r_1$, and both converge to r_1 when Y approaches $2r_1$ from above.

Proof: Assume $Y > 2r_1$. We know from the above argument that at the optimum, $c_1^* \geq r_1$ and $c_2^* \geq r_2$; this implies $r_1 \leq c_1^* \leq (Y-(1-\alpha)r_1)/(1+\alpha)$. Note that $u(c_1)$ is strictly concave in this region. If $c_1 = r_1$, then, by VW1,

$$u'(c_1) = v_l'(0) + \alpha w'(r_1) - (1+\alpha)v_g'(Y-2r_1) > 0.$$

Therefore, $c_1 > r_1$ if $Y > 2r_1$. Now consider $c_2 = r_2$; this implies $c_1 = (Y-(1-\alpha)r_1)/(1+\alpha)$. Then

$$u'(c_1) = v_g'((Y-2r_1)/(1+\alpha)) + \alpha w'((\alpha Y + (1-\alpha)r_1)/(1+\alpha)) - (1+\alpha)v_g'(0).$$

If $u'(c_1)$ is greater than zero when evaluated at this point, then $c_1^* = (Y-(1-\alpha)r_1)/(1+\alpha)$ and $c_2^* = (\alpha Y+(1-\alpha)r_1)/(1+\alpha)$. As Y increases, $u'((Y-(1-\alpha)r_1)/(1+\alpha))$ decreases continuously by the concavity of v_g and w, and by the assumption that v_g' and w' are continuous.

If $u'((Y-(1-\alpha)r_1)/(1+\alpha) \leq 0$, then there is a $\bar{c}_1 \in (r_1,(Y-(1-\alpha)r_1)/(1+\alpha)]$ such that $u'(\bar{c}_1) = 0$. Then $c_1^* = \bar{c}_1$ and $c_2^* = Y - \bar{c}_1$. We know $\bar{c}_1$ is a continuous function of Y since v_g' and w' are continuous. Totally differentiating the equation $u'(\bar{c}_1) = 0$ yields $1/(1+\alpha) > d\bar{c}_1/dY > 0$.

Therefore, $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing, and since (Y-

$(1-\alpha)r_1)/(1+\alpha) \to r_1$ as $Y \to 2r_1$, we have $c_1(Y)$ and $c_2(Y) \to r_1$ as $Y \to 2r_1$ from above.

(3) Claim: If $Y < 2r_1$ and negative values of c_1 and c_2 are allowed, then $c_1(Y) = (Y - (1-\alpha)r_1)/(1+\alpha)$. If $Y < 2r_1$ and the constraints $c_1 \geq 0$ and $c_2 \geq 0$ are imposed, then $c_1(Y) = \max[0,(Y-(1-\alpha)r_1)/(1+\alpha)]$.

Proof: From the proof of (1), we know that $c_1^* \leq r_1$ and $c_2^* \leq r_2$, so that $(Y-(1-\alpha)r_1)/(1+\alpha) \leq c_1^* \leq r_1$. Consider $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$. In this region $c_1-r_1 \leq c_2-r_2 \leq 0$, and so we have

$$u'(c_1) = v_1'(c_1-r_1) + \alpha w'(\alpha c_1+(1-\alpha)r_1) - (1+\alpha)v_1'(c_2-r_2) < 0$$

by VW1 and V2. Now consider the region $c_1 \in ((Y+\alpha r_1)/(2+\alpha), r_1]$. In this region we have $0 \geq c_1 - r_1 > c_2 - r_2$. We can rewrite this range of possible choices for c_1 as $(Y-2r_1)/(2+\alpha) < c_1 - r_1 \leq 0$. Consider any choice of $c_1-r_1 \equiv z$ in this range, then $u(r_1+z) = v_1(z) + w(\alpha z+r_1) + v_1(Y-(1+\alpha)z-2r_1)$ is the utility from this choice. Compare this to the choice of $c_2-r_2 = z$ (This choice implies a c_1 which lies in the range $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$). If $c_1 = r_1+z$ is feasible, then $c_2 = r_2+z$ is also feasible. The choice $c_2 = r_2+z$ results in utility

$$u(Y-r_2-z) = v_1(Y-(1+\alpha)z-2r_1+k) + w(\alpha z+r_1-k) + v_1(z),$$

where $k \equiv \alpha[2r_1-Y+(2+\alpha)z]/(1-\alpha) > 0$. Then

$$u(Y-r_2-z) - u(r_1+z) =$$
$$[v_1(Y-(1+\alpha)z-2r_1+k) - v_1(Y-(1+\alpha)z-2r_1)] + [w(\alpha z+r_1-k) - w(\alpha z+r_1)].$$

VW1 implies $v_1(x+q) - v_1(x) > w(s+q) - w(s)$ for any $q > 0$, $x \leq -q$ and $s \geq 0$. Therefore $u(Y-r_2-z) > u(r_1+z)$ for any z such that $(Y-2r_1)/(2+\alpha) < z \leq 0$. This implies that for any feasible choice of $c_1 \in ((Y+\alpha r_1)/(2+\alpha), r_1]$ there is a feasible choice of $c_1 \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)]$ which is preferred. Therefore

$$c_1^* \in [(Y-(1-\alpha)r_1)/(1+\alpha),(Y+\alpha r_1)/(2+\alpha)],$$

and therefore if negative values of c_1 and c_2 are allowed, $c_1(Y) = (Y-(1-\alpha)r_1)/(1+\alpha)$; if $c_1 < 0$ and $c_2 < 0$ are not allowed, then $c_1(Y) = \text{Max}[0,(Y-(1-\alpha)r_1)/(1+\alpha)]$.

This establishes Claim (3); Claims (1), (2), and (3) together establish the theorem. Q.E.D.

<u>Remark on Theorem 1</u>:

The proof above shows that $c_1(Y)$ is never steeper for $Y/2 > r_1$ than it is for $Y/2 < r_1$

Proof of Proposition 3.1:

For any given r_1 and Y, substituting Y - c_1 for c_2 and $\alpha c_1 + (1-\alpha)r_1$ for r_2, we can write $U(c_1,r_1;c_2,r_2)$ as a function of c_1 alone:

$$u(c_1) \equiv w(r_1) + v(c_1-r_1) + w(\alpha c_1+(1-\alpha)r_1) + v(Y - (1+\alpha)c_1 - (1-\alpha)r_1)$$

We will let c_1^* denote the c_1 which maximizes $u(c_1)$, and let $c_2^* \equiv Y - c_1^*$.

Suppose that there is some $\bar{r} \geq 0$, $\bar{c} \geq \bar{r}$, such that $v'(\bar{c}-\bar{r}) \geq w'(\bar{r})$. Then by W1 for any $r > \bar{r}$ and $\bar{c} \geq c > r$ we have $v'(c-r) > w'(r)$. Let $r_1 = r$ and take $Y \geq 2r_1$. Theorem 1 implies that $c_1 \geq r$ and $c_2 \geq r$. If $c_1 = r$ then $u'(r) = v_g'(0) + \alpha w'(r) - (1+\alpha)v_g'(Y-2r)$.

Then $\text{Lim}_{Y\to 2r_1} u'(r) = \alpha\cdot[w'(r) - v_g'(0)] < 0$. Because v_g' and w' are continuous, this implies that there is some $x > 0$ such that for all $0 < y \leq x$,

$$u'(r) = v_g'(0) + \alpha w'(r) - (1+\alpha)v_g'(y) < 0.$$

$u(c_1)$ is concave in the region $r \leq c_1 \leq (Y-(1-\alpha)r)/(1+\alpha)$. Therefore if c_1^* lies in this region, $c_1^* = r$ for $Y \in [2r,2r+x]$. This means $c_1(Y)$ is not strictly increasing in this region, which contradicts Theorem 1.

To see that $c_1 \in [r,(Y-(1-\alpha)r)/(1+\alpha)]$ note that by supposition $r \leq c_1^* \leq Y-r$. Then

$$u(c_1 = Y-(1-\alpha)r)/(1+\alpha)) = w(\bar{r}) + v_g((Y-2r)/(1+\alpha)) + w((\alpha Y+(1-\alpha)r)/(1+\alpha)).$$

Compare this to $u(c_1 = (Y-(1-\alpha)r)/(1+\alpha) + z)$, where $z > 0$:

$$u(Y-(1-\alpha)r)/(1+\alpha)+z) = w(\bar{r}) + v_g(((Y-2r)/(1+\alpha))+z) + w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) + v_1(-(1+\alpha)z).$$

$$\begin{aligned} \text{Let } X &\equiv u((Y-(1-\alpha)r)/(1+\alpha))+z) - u(Y-(1-\alpha)r)/(1+\alpha)) \\ &= [v_g(((Y-2r)/(1+\alpha))+z) - v_g((Y-2r)/(1+\alpha))] + v_1(-(1+\alpha)z) + \\ &\quad [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))]. \end{aligned}$$

$$\begin{aligned} \text{Then } X &\leq [v_g(z) - v_g(0)] + v_1(-(1+\alpha)z) \\ &\quad + [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))]. \\ &= v_g(z) + v_1(-(1+\alpha)z) \\ &\quad + [w(((\alpha Y+(1-\alpha)r)/(1+\alpha))+\alpha z) - w((\alpha Y+(1-\alpha)r)/(1+\alpha))]. \end{aligned}$$

By supposition, $c_1(2r) = c_2(2r) = r$. Therefore when $Y = 2r$ we must have $u(r+k) < u(r)$ for any $z > 0$.

$$\begin{aligned} u(r+z) - u(r) &= v_g(z) + w(r+\alpha z) + v_1(-(1+\alpha)z) - w(r) \\ &= v_g(z) + v_1(-(1+\alpha)z) + [w(r+\alpha z) - w(r)]. \end{aligned}$$

Therefore, $u(r+z) - u(r) < 0$ for all $r \geq 0$, $z > 0$, and $Y = 2r$ implies $X < 0$. Therefore $u(((Y-(1-\alpha)r)/(1+\alpha))+z) < u((Y-(1-\alpha)r)/(1+\alpha))$ for any $z > 0$, which implies $r \leq c_1^* \leq (Y-(1-\alpha)r)/(1+\alpha)$.

This proves that theorem 1 and V1-V5, W1 imply $w'(r) > v'(c-r)$ for any $r \geq 0$ and c

$\geq r$. Given that w and v_g are strictly concave, this can be true if and only if $U(c,c) > U(r,c)$ for any $r \geq 0$ and $c > r$. Q.E.D.

Proof of Proposition 3.2:

For any given r_1 and Y, substituting $Y - c_1$ for c_2 and $\alpha c_1 + (1-\alpha)r_1$ for r_2, we can write $U(c_1,r_1;c_2,r_2)$ as a function of c_1 alone:

$$u(c_1) \equiv w(r_1) + v(c_1-r_1) + w(\alpha c_1+(1-\alpha)r_1) + v(Y - (1+\alpha)c_1 - (1-\alpha)r_1)$$

We will let c_1^* denote the c_1 which maximizes $u(c_1)$, and let $c_2^* \equiv Y - c_1^*$.

Consider the case in which negative values of c_1 and c_2 are allowed, so that theorem 1 implies that $c_1(Y)$ and $c_2(Y)$ are continuous and strictly increasing for all Y. Consider $Y \leq 2r_1$. By supposition $c_1^* \leq r_1$ and $c_2^* \leq r_1$. We can show that $(Y-(1-\alpha)r_1)/(1+\alpha) \leq c_1^* \leq r_1$. If $c_1 < (Y-(1-\alpha)r_1)/(1+\alpha)$ then $c_2 > r_2$, so that

$$u'(c_1) = v_1'(c_1-r_1) + \alpha w'(\alpha c_1+(1-\alpha)r_1) -(1+\alpha)v_g'(Y-(1-\alpha)r_1-(1+\alpha)c_1) > 0$$

by assumption V5 and the proof of Proposition 3.1.

Over the range $(Y-(1-\alpha)r_1)/(1+\alpha) \leq c_1 \leq r_1$

$$u(c_1) = v_1'(c_1-r_1) + w'(\alpha c_1+(1-\alpha)r_1) + v_1'(Y-(1-\alpha)r_1-(1+\alpha)c_1)$$

In this region $u(c_1)$ need be neither concave or convex. Suppose c_1^* is in the interior of this region. For this to be so $u(c_1)$ must be concave in some neighborhood of c_1^*. At such an optimum we must have $u'(c_1^*) = 0$. Totally differentiating this equation with respect to c_1^* and Y implies that $dc_1^*/dY = [(1+\alpha)v_1''(Y-(1-\alpha)r_1-(1+\alpha)c_1^*)]/u''(c_1^*) < 0$, which contradicts the claim that $c_1(Y)$ is strictly increasing. Therefore there can be no interior optimum, which implies that either $c_1^* = r_1$ or $c_1^* = (Y-(1-\alpha)r_1)/(1+\alpha)$. Since $c_1^* = r_1$ also violates the claim that $c_1(Y)$ is strictly increasing, we must have

$$c_1^* = (Y-(1-\alpha)r_1)/(1+\alpha).$$

$c_1^* = (Y-(1-\alpha)r_1)/(1+\alpha)$ implies that $u((Y-(1-\alpha)r_1)/(1+\alpha)) \geq u(c_1)$ for any c_1. In particular we must have $X \equiv u((Y-(1-\alpha)r_1)/(1+\alpha)) - u(r_1) > 0$ if $Y < 2r_1$. Writing out the terms of X we have:

$$X \equiv v_1((Y-2r_1)/(1+\alpha)) + w(z) - w(z + k) - v_1(((Y-2r_1)/(1+\alpha)) -k) > 0,$$

where $k = \alpha(2r_1-Y)/(1+\alpha) > 0$, and $z = r_1 - k$. Since

$[v_1(0) - v_1(-k)] > [v_1((Y-2r_1)/(1+\alpha)) - v_1(((Y-2r_1)/(1+\alpha)) - k)],$

$X > 0$ for any $r_1 \geq 0$ and $Y < 2r_1$ implies $-v_1(-k) > w(z + k) - w(z)$ for any $k > 0$, $z \geq 0$. This last inequality implies $U(c,c) > U(r,c)$ for any $c \geq 0$ and $r > c$. Q.E.D.

Proof of Theorem 2:

We prove the proposition for lotteries that are represented by probability measures of the form $\mathbf{m} = \mathbf{f}\lambda + \mathbf{d}$, where λ is Lebesgue measure on $\mathbb{R}$, $\mathbf{f}: \mathbb{R} \longrightarrow \mathbb{R}$ is a measurable function, and $\mathbf{d}$ is a measure which has support consisting of a countable collection of point masses with no point of accumulation.

Recall the consumer's problem:

$$\text{(P)} \quad \max_{c_1} U(r_1,c_1;r_2,c_2)$$

$$\text{s.t. } c_1 + c_2 \leq Y \equiv Y_1 + Y_2,$$

where Y_1 is a fixed amount representing first-period income, and Y_2 is a lottery over second-period income.

The proof has two parts. First we show the theorem for binomial bets in which Y_2 takes on only two values, Y_H and Y_L. Then we show that bets of this type can be combined, to yield the result for all Y in the class described above.

Part I of proof:

Suppose Y_2 is such that $P[Y_2=Y_H] = p$, $P[Y_2=Y_L] = 1-p$, where $(Y_1+Y_H)/2 \geq r_1$ and $p \geq (\alpha/(1+\alpha))$. For binomial bets Y_2, this is equivalent to the hypothesis of the proposition. We show that $c_1 \geq r_1$. To prove this, we consider two cases:

Case 1 of Part I:

$(Y_1+Y_L)/2 \geq r_1$. Substituting $Y_1 + Y_2 - c_1$ for c_2 and $(1-\alpha)r_1 + c_1$ for r_2, we can rewrite (P) as a function of c_1:

$$\text{(P2)} \quad \max_{c_1} U(c_1) = w(r_1) + v(c_1-r_1) + w((1-\alpha)r_1+\alpha c_1) +$$

$$p\cdot v(Y_1+Y_H-((1-\alpha)r_1+(1+\alpha)c_1)) + (1-p)\cdot v(Y_1+Y_L-((1-\alpha)r_1+(1+\alpha)c_1)).$$

Then $c_1 < r_1$ implies, by Assumption VW1, that

$$\frac{\partial U(c_1)}{\partial c_1} = v_1'(c_1-r_1) + \alpha w'((1-\alpha)r_1+\alpha c_1) - p(1+\alpha)v_g'(Y_1+Y_H-((1-\alpha)r_1+(1+\alpha)c_1)$$

$$- (1-p)(1+\alpha)v_g'(Y_1+Y_L-((1-\alpha)r_1+ (1+\alpha)c_1))$$

is greater than zero. Thus maximization in c_1 implies $c_1 \geq r_1$.

Case 2 of Part I:

$(Y_1+Y_H)/2 \geq r_1 > (Y_1+Y_L)/2$

i) $c_1 < \min\{r_1,(Y_1+Y_L-(1-\alpha)r_1)/(1+\alpha)\}$:

$$\frac{\partial U(c_1)}{\partial c_1} = v_1'(c_1-r_1)+\alpha w'((1-\alpha)r_1+\alpha c_1) - p(1+\alpha)v_g'(Y_1+Y_H-((1-\alpha)r_1+(1+\alpha)c_1)$$

$$-(1-p)(1+\alpha)v_g'(Y_1+Y_L-((1-\alpha)r_1+(1+\alpha)c_1)) > 0$$

by assumption VW1.

ii) $r_1 \geq c_1 \geq (Y_1+Y_L-(1-\alpha)r_1)/(1+\alpha)$

Let $U(c_1) = x_1 + x_2$, where

$$x_1 = (1-p)(1+\alpha)v_1(c_1-r_1)+(1-p)v_1(Y_1+Y_L-((1-\alpha)r_1+(1+\alpha)c_1))$$

$$x_2 = [1-(1-p)(1+\alpha)]v_1(c_1-r_1)+w((1-\alpha)r_1+\alpha c_1) + pv_g(Y_1+Y_H-((1-\alpha)r_1+(1+\alpha)c_1))$$

Then if $c_1 < r_1$

$$\frac{\partial x_2}{\partial c_1} = [(1+\alpha)p-\alpha]v_1'(c_1r_1)+\alpha w'((1-\alpha)r_1+\alpha c_1) - (1+\alpha)pv_g'(Y_1+Y_H-(1-\alpha)r_1+(1+\alpha)c_1)$$

$$> [(1+\alpha)p-\alpha]v_1'(c_1-r_1)-[(1+\alpha)p-\alpha]v_g'(Y_1+Y_H-((1-\alpha)r_1+(1+\alpha)c_1)) \geq 0$$

since $p \geq (\alpha/(1+\alpha))$. So x_2 is maximized in this region at $c_1 = r_1$.

$$\frac{\partial x_1}{\partial c_1} = (1-p)(1+\alpha)v_1'(c_1-r_1)-(1-p)(1+\alpha)v_1'(Y_1+Y_L-((1-\alpha)r_1+(1+\alpha)c_1))$$

$$\frac{\partial^2 x_1}{\partial c_1 \partial c_1} = (1-p)(1+\alpha)v_1''(c_1-r_1) + (1-p)(1+\alpha)^2 v_1''(Y_1+Y_L-((1-\alpha)r_1+(1+\alpha)c_1)) > 0$$

Since x_1 is convex in this region, it has exactly two local maxima, at $c_1 = r_1$ and at $c_1 = (Y_1+Y_L-(1-\alpha)r_1)/(1+\alpha)$ respectively. But by V2

$$x_1(Y_1+Y_L-(1-\alpha)r_1)/(1+\alpha)) = (1-p)(1+\alpha)v_1((Y_1+Y_L-2r_1)/(1+\alpha))$$

$$< (1-p)v_1(Y_1+Y_L-2r_1) = x_1(r_1)$$

Thus x_1 is strictly maximized over the region at $c_1 = r_1$. Since x_1 and x_2 are both maximized at $c_1 = r_1$, $U(c_1)$ is strictly maximized over $[Y_1+Y_L-(1-\alpha)r_1)/(1+\alpha), r_1]$ at $c_1 = r_1$.

Case 1 and Case 2 imply that (P2) is maximized at $c_1 \geq r_1$.

Part II of the Proof:

Case 1 of Part II:

Suppose Y_2 is a countable sum of point masses with no accumulation point:

$$Y_2 = \sum_{i=1}^{i=N_+} p_i \delta_{x_i} + \sum_{j=1}^{j=N_-} q_j \delta_{y_j}$$

$p_i, q_j > 0$, $x_i \geq 0$, $y_j < 0$;

$i > i'$, $j > j' \Rightarrow x_i > x_{i'}$, $y_j < y_{j'}$;

$N_+, N_- \in \mathbb{N} \cup \{\infty\}$.

Claim: Because $P[Y \geq 2r_1] \geq \alpha/(1+\alpha)$, we can rewrite Y_2 in the form

$$Y_2 = \sum_{i=1}^{i=M} \tilde{p}_i \delta_{\tilde{x}_i} + \sum_{i=1}^{i=M} \left(\frac{\tilde{p}_i}{\alpha}\right) \delta_{\tilde{y}_i} + \sum_{j=1}^{j=Q} \tilde{s}_j \delta_{\tilde{z}_j},$$

$\tilde{z}_i, \tilde{x}_i \geq 0, \tilde{y}_i < 0$

$\tilde{z}_1 \geq \tilde{x}_M, \; i > j \Rightarrow \tilde{x}_i > \tilde{x}_j, \tilde{z}_i > \tilde{z}_j, \tilde{y}_i < \tilde{y}_j$.

Proof of Claim: The proof of this is tedious, but straightforward. Match successively lower outcomes with successively higher outcomes in the proportion of $1/\alpha$ to 1. Because the support of Y_2 has no point of accumulation, this countable process eventually exhausts all of the negative weight of Y_2.

Then $Y_2 = \sum_{i=1}^{i=M} (1+1/\alpha)\, \tilde{p}_i[\alpha/(1+\alpha)\delta_{\tilde{x}_i} + 1/(1+\alpha)\delta_{\tilde{y}_i}] + \sum_{j=1}^{j=Q} \tilde{s}_j \delta_{\tilde{z}_j}$,

Each term in brackets [] is a binomial bet (the $\delta_{\tilde{z}_j}$ are trivially so) satisfying the hypothesis of Proposition 3.2.

(P2) $\qquad \max_{c_1} U(c_1)$

$$U(c_1) = \sum_{i=1}^{M} \left((1+1/\alpha)\tilde{p}_i\right)[w(r_1) + v(c_1-r_1) + w(r_2) +$$
$$(1/(1+\alpha))v(Y_1 + \tilde{y}_i - ((1-\alpha)r_1 + (1+\alpha)c_1)) + \alpha/(1+\alpha)v(Y_1 + \tilde{x}_i - ((1-\alpha)r_1 + (1+\alpha)c_1))]$$
$$+ \sum_{j=1}^{Q} \tilde{s}_j[w(r_1) + v(c_1-r_1) + w(r_2) + v(Y_1 + z_j - ((1-\alpha)r_1 + (1+\alpha)c_1))].$$

From Part 1, we know that each square bracketed term in (P2) takes on a larger value at $c_1 = r_1$ than at $c_1 < r_1$. But this implies that $U(r_1) > U(c_1)$ for $c_1 < r_1$, and hence that (P2) is maximized at $c_1 \geq r_1$.

Case 2 of Part II:

Suppose Y_2 has a density with respect to Lebesgue measure in the sense that there is a measurable function **h** such that $P[Y_2 \geq x] = \int_x^\infty \mathbf{h}(x)dx$. Further suppose $\int_{(2r_1-Y_1)}^\infty \mathbf{h}(x)dx \geq \alpha/(1+\alpha)$. Let $H(a) = \int_{-\infty}^{a} \mathbf{h}(x)dx$ be the cumulative distribution function for **h**. Define $q(a) \equiv \inf H^{-1}[(\alpha+1)H((2r_1-Y_1)) - \alpha H(a)]$. Then

$$\alpha \int_a^{(2r_1-Y_1)} \mathbf{h}(x)dx = \int_{(2r_1-Y_1)}^{q(a)} \mathbf{h}(x)dx. \qquad (\%)$$

Then q is increasing and hence measurable.

Claim: For measurable $f: \mathbb{R} \longrightarrow \mathbb{R}$, $\alpha \int_a^0 f(q(x))\mathbf{h}(x)dx = \int_{[0,q(a)]} f(x)\mathbf{h}(x)dx$.

Proof of Claim: The statement holds for simple functions f (those taking a finite number of values), since we can then break the integrals down into the sum of integrals over regions $[a_i,b_i]$ so that f is constant on both $[a_i,b_i]$ and $[q(b_i),q(a_i)]$. On these regions the equalities follow from (%). Approximate f by simple functions f_n, $0 \leq f_n \leq f$, so that $f_n(x) \longrightarrow f(x)$ for all $x \in \mathbb{R}$. Then we have:

$$\alpha\int_a^0 f_n(q(x))\mathbf{h}(x)dx = \int_{[0,q(a)]} f_n(x)\mathbf{h}(x)dx. \qquad (\%\%)$$

Since q is increasing, $f_n(q(x))$ is a simple function defined on $(-\infty,0]$. Clearly $0 \leq f_n(q(x)) \leq f(q(x))$, and $f_n(q(x)) \longrightarrow f(q(x))$ for all $x \in (-\infty,0]$. Applying Lebesgue's theorem of monotone convergence to both sides of the equation (%%), we are done.

Finally,using the claim, we can rewrite the consumer's problem:

$$\text{(P2)} \max_{c_1} (1+\alpha)\int_{-\infty}^{(2r_1-Y_1)}[w(r_1) + v(c_1- r_1) + w(r_2) + 1/(1+\alpha)v(Y_1+x-c_1-r_2)$$
$$+ \alpha/(1+\alpha)v(Y_1+ q(x)-c_1-r_2)]\mathbf{h}(x)dx$$
$$+ \int_{q(-\infty)} [w(r_1) + v(c_1- r_1) + w(r_2) + v(Y_1+x-c_1-r_2)]\mathbf{h}(x)dx.$$

In the first integral, $Y_1 + q(x) \geq 2r_1$, while in the second integral, $Y_1 + x \geq 2r_1$. So each term in brackets [] represents the utility from a binomial lottery on income satisfying the hypothesis of Proposition 3.2, and hence is larger at $c_1=r_1$ than at any $c_1 < r_1$. That is, (P2) is maximized at $c_1 \geq r_1$.

Q.E.D.

Proof of Proposition 3.3:

Part (i) is a straightforward implication of Theorems 1 and 2. By the proof of Theorem 1, we know that $c_1(Y,0) < r_1$ if $Y < 2\cdot r_1$. But Theorem 2 says that if we choose Y_* such that $Y_*(1+k) = 2\cdot r_1$, then $c_1(Y,k) \geq r_1$ for all $Y \geq Y_*$, as long as $(1-k)Y \geq r_1$.

Proof of part (ii):

The consumer will maximize with respect to c_1

$$U \equiv w(\alpha\cdot c_1 + (1-\alpha)\cdot r_1) + v(c_1-r_1) + .5\cdot v(Y(1+k) - (1+\alpha)\cdot c_1 - (1-\alpha)\cdot r_1)$$
$$+ .5\cdot v(Y(1-k) - (1+\alpha)\cdot c_1 - (1-\alpha)\cdot r_1).$$

By VW1, we know that we can choose $Y^* > 2\cdot r_1$ sufficiently close to $2\cdot r_1$ such that, for all $Y \in (2\cdot r_1,Y^*)$,

$$(1+\alpha)v_g'(0) \leq v_g'((Y-2r_1)/(1+\alpha)) + \alpha\cdot w'((\alpha\cdot Y+(1-\alpha)\cdot r_1)/(1+\alpha)).$$

As in Theorem 1, this implies that $c_1(Y,0) = (Y-(1-\alpha)\cdot r_1)/(1+\alpha)$. By VW1, we can take k^* sufficiently small such that, for all $k \in [0,k^*)$,

$$((1+\alpha)/2)\cdot(v_1'(-2kY) + v_g'(2kY)) > v_g'(0) + \alpha\cdot w'(r_1).$$

Claim: $c_1(Y,k) \leq ((1-k)Y-(1-\alpha)\cdot r_1)/(1+\alpha) < (Y-(1-\alpha)\cdot r_1)/(1+\alpha) = c_1(Y,0)$.

Proof of claim:

Note first that, since $Y > 2r_1$, we have $(1+k)Y > 2r_1$. Thefore, $c_1 \geq r_1$ by Theorem 2 provided $k \leq 1/2$.

a) Suppose that $((1-k)Y-(1-\alpha)\cdot r_1)/(1+\alpha) < c_1 < ((1+k)Y-(1-\alpha)\cdot r_1)/(1+\alpha)$. Then

$$\begin{aligned}\partial U/\partial c_1 &= v_g'(c_1-r_1) + \alpha\cdot w'(r_2) - ((1+\alpha)/2)\cdot[v_1'((1-k)Y-(1+\alpha)\cdot c_1-(1-\alpha)r_1) \\ &\quad + v_g'((1+k)Y-(1+\alpha)c_1-(1-\alpha)r_1)] \\ &< v_g'(0) + \alpha\cdot w'(r_1) - ((1+\alpha)/2)\cdot(v_1'(-2kY) + v_g'(2kY)) \\ &< 0.\end{aligned}$$

(The second-to-last inequality follows from the fact that, for c_1 in this region, $\partial[v_g'(c_1-r_1) + \alpha\cdot w'(r_2)]/\partial c_1 < 0$, $[(1-k)Y-(1+\alpha)\cdot c_1-(1-\alpha)r_1] \in (-2kY,0)$ and $[(1+k)Y-(1+\alpha)\cdot c_1-(1-\alpha)r_1] \in (0,2kY)$.)

b) Suppose that $c_1 > [(1+k)Y - (1-\alpha)r_1]/(1+\alpha)$.

Then

$$\begin{aligned}\partial U/\partial c_1 &= v_g'(c_1-r_1) + \alpha\cdot w'(r_2) - ((1+\alpha)/2)\cdot[v_1'((1-k)Y-(1+\alpha)\cdot c_1-(1-\alpha)r_1) + \\ &\quad v_1'((1+k)Y-(1+\alpha)c_1-(1-\alpha)r_1)],\end{aligned}$$

which, by VW1, is negative. (a) and (b) together establish the claim, which proves Proposition 3.2.

Q.E.D.

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