sustainability

Article

Low-Carbon Supply Chain Coordination Based on Carbon Tax and Government Subsidy Policy

Wenxue Ran * and Teng Xu

School of Logistics and Management Engineering, Yunnan University of Finance and Economics, Kunming 650221, China
* Correspondence: ranwxa@ynufe.edu.cn

Abstract: To meet the demands of society's transition to a low-carbon economy, this study analyzes and designs a coordination contract that is suitable for a low-carbon supply chain, under the circumstances of a carbon tax policy and government subsidies; this is to achieve a reduction in emissions and a growth in the total profits of the supply chain, while simultaneously improving the sustainable competitiveness and coordination of the supply chain. Manufacturers and retailers make up the two levels of the supply chain that are the focus of this study. Both centralized and decentralized decision-making models are created using the Stackelberg game method. By analyzing the supply chain decision-making and emission-reduction strategies in both cases, the revenue-sharing contract is designed to achieve the sustainable coordination of the y chain. The results of the numerical analysis show the following: first, that more orders are placed and emissions are reduced under centralized decision-making than under decentralized decision-making; second, that the total supply chain's profits are higher when all parties comply with the revenue-sharing contract than when there are no contracts; third, that the revenue-sharing contract allows for the free allocation of supply chain gross margins in the enterprise for supply chain coordination.

Keywords: carbon tax; government subsidies; revenue-sharing contract; supply chain coordination

Citation: Ran, W.; Xu, T. Low-Carbon Supply Chain Coordination Based on Carbon Tax and Government Subsidy Policy. *Sustainability* **2023**, *15*, 1135. https://doi.org/10.3390/su15021135

Academic Editors: Marc A. Rosen and Wantao Yu

Received: 13 October 2022
Revised: 30 December 2022
Accepted: 4 January 2023
Published: 6 January 2023

1. Introduction

Climate change is an urgent global issue, which requires all parties to make efforts to reduce CO_2 emissions and explore more sustainable ways of development. This study mainly analyzes and solves the problem of enterprise emission reduction and coordination in the specific context that sees the government collecting carbon tax and subsidizing enterprises. The goal of this study is to design a revenue-sharing contract that can be applied in the low-carbon supply chain, so as to help enterprises reduce emissions, obtain more revenue, and achieve supply chain coordination.

Recently, as the international community has become more aware of environmental protection, governments of various countries have gradually advocated for a low-carbon economy and for reduced carbon emissions [1]. The protection of the environment has become an issue of great concern for the international community [2,3], and more than 100 countries and government agencies (such as the European Union, China, Japan and South Africa) have formulated, or are considering, a net zero emission target [4]. In this context, a reduction in carbon emissions has become a necessary measure for global sustainable development, and countries are actively adopting a series of policies [5–7]. For example, the 2005-instituted European Emissions Trading System (EU-ETS) has made a huge contribution to global carbon reduction; through the mandatory regulation of corporate carbon emissions, combined with market regulation [8,9], the EU-ETS reduced its carbon emissions by approximately 21% in 2020 (compared with the level in 2005), and the EU has set a challenging goal to cut its emissions by 40% by 2030 (compared with the level in 1990). However, EU government agencies are working hard to increase their goal to 55%, and fully achieve carbon neutrality by 2050 [10]. In September 2010, the federal government of Germany suggested lowering CO2 emissions from their 1990 levels by 40% by 2020, 55% by 2030, and 80–95% by 2050. However, by the end of 2011, emissions had only fallen approximately 27 percent from their 1990 levels [11]. According to the data of the German Federal Environment Agency (UBA), Germany's carbon emissions in 2021 were 762 million tons, an increase of 4.5%. This indicates that one year later, the actual goal of cutting greenhouse gas emissions by 40%, compared to 1990, has not been met. In the long-term comparison, the emissions decreased by only 38.7%. At the 2015 Paris Climate Change Conference, Chinese authorities clearly established a goal of reducing carbon dioxide emissions, per unit of GDP, by 40%, compared with 2005 [12]. Additionally, the Chinese government explicitly states in the Outline of the 14th Five-Year Plan for National Economic and Social Development of the People's Republic of China that, during the plan's duration, energy consumption, per unit of GDP, and carbon dioxide emissions, per unit of GDP, will be reduced by 13.5 percent and 18 percent, respectively. These two targets have been listed as legally binding targets [13]. The Paris Agreement's primary objective is to keep the rise in the average global temperature this century to under 2 °C. To meet the objectives of the Paris Agreement, all participating countries should significantly improve the emission-reduction target, increase the emission-reduction efforts, and achieve the global emission peak as soon as possible [14].

In order to reduce carbon emissions in the supply chain, research on low-carbon supply chain management (LCSCM) has been stimulated by increasing public awareness of climate change [15]. For instance, to help managers make the most effective inventory decisions while taking into account logistical costs and carbon emissions, Astanti et al. suggested a supply chain inventory model [16]. Manufacturers are under pressure to achieve a competitive edge through low-carbon production, due to intense market competition and customers' awareness of environmental protection. Research shows that low-carbon production cannot mitigate the detrimental effects of growing competition intensity on company earnings in chain-to-chain competition, and fierce competition weakens manufacturers' efforts to reduce emissions [17]. Reduced carbon emissions in the supply chain, without compromising long-term economic interests, is the ultimate objective of low-carbon supply chain management (LCSCM). Therefore, there is a trade-off between economic objectives and environmental objectives in the low-carbon supply chain. Environmental sustainability is significantly affected by supply chain practices [18], and sustainable innovation and enterprise competitiveness have a strong and positive association; in addition, enterprise competitiveness has a significant relationship with financial, environmental, and operational performance [19]. To be able to improve the competitiveness of the supply chain and improve the level of emission reduction without damaging the interests of the supply chain, to develop new forms of collaboration, enterprises need to engage in sustainable innovation.

Based on the above background, to enhance sustainable competitiveness of the supply chain, and reduce carbon emissions so that enterprises can achieve sustainable operation coordination, this study considers the coordination problem of the secondary supply chain; this is composed of manufacturers and retailers, whose decisions include order quantity and emission reduction. Under the influence of carbon tax and government subsidy, what order quantity should enterprises maintain? Viewed through the lens of the supply chain, what is the order quantity, profit gap and emission-reduction gap between centralized and decentralized decisions? Is there a contract that can increase the willingness of supply chain participants to minimize carbon emissions, while producing the same amount of orders and profits as the centralized decision-making? All of these issues appeal to us.

The remainder of this essay is structured as follows: Section 2 summarizes the literature relevant to this field; Section 3 describes the assumptions of the model and the related symbol meaning, and the focus is on building models and relevant inferences; Section 4 offers a numerical validation analysis of the government subsidy and tax policy on emission reduction, and the impact of the supply chain profit; and Section 5 summarizes the relevant conclusions, the study's shortcomings and the next directions for research.

2. Literature Review

As environmental issues are becoming increasingly serious, carbon emission reduction has been paid increasing attention. Many researchers have conducted studies on low-carbon development. For example, Wangsa et al. proposed a mixed-integer linear programming system for the fresh food supply chain; this was used to optimize carbon emission costs by improving product inventory and delivery [20]. Zhang et al. propounded a new production profit model based on risk constraints, in order to explore the decision-making methods of carbon emissions and the production of factories [21]. Climate change and global warming are exacerbated by continued anthropogenic greenhouse gas emissions. An expansion in industrialization, population growth, and supply networks are responsible for a large portion of these human greenhouse gas emissions. Controlling carbon emissions in the supply chain can, therefore, both address the issue of climate change and comply with legal requirements. Therefore, any organization must reduce carbon emissions from the supply chain [22]. Academics and businesses are interested in low-carbon supply chain management (LCSCM). At present, the study of the low-carbon supply chain focuses on three primary areas: low-carbon operation, low-carbon supply chain design and carbon management [23]. Low-carbon supply chain operation includes inventory management and transportation issues, low-carbon supply chain design includes network design and supply-chain coordination design, and carbon management includes various carbon policies.

Influenced by carbon tax and government carbon subsidy, to improve the sustainable competitiveness and reduce carbon emissions, this study analyzes and designs a contractual cooperation to enable supply chain enterprises to achieve sustainable coordination. Therefore, this study focuses primarily on three aspects of the literature: carbon tax, government carbon subsidies and supply chain coordination.

2.1. Carbon Tax Policy

An important issue in the realm of environmental protection is carbon emission. At present, there are three main policies implemented to reduce a company's carbon emissions in the supply chain, namely, carbon restriction [24], carbon trade [25] and carbon tax [26]. Carbon tax policy, also known as environmental tax, is a way for the government to control the carbon emissions of enterprises through taxation. Meng et al. simulated the impact of the carbon tax proposed by the government on the environment and the economy, including and excluding compensation policies. As a result, carbon tax can effectively reduce emissions, but it will lead to moderate economic contraction [27]. Meng et al. studied how manufacturers should choose their own manufacturing or outsourcing to a third party according to the carbon tax policy. The production inventory, wholesale price and carbon tax rate were taken as decision variables, and the cost and profit under different tax rates were analyzed and studied. The results show that the government's tax rate should not be fixed, but should adjust the carbon tax strategy in stages, according to the actual situation [28]. Zhou et al. focused on the effect of carbon tax on the decision-making of the whole supply chain and the effect of carbon emission reduction; they considered two situations with and without retailer competition, and presented the research results [29]. Manufacturers, retailers, and governments, with or without a government carbon tax, were analyzed using a Stackelberg game model, developed by Wang et al. They discovered that the retail prices and government carbon taxes were higher in the decentralized choice model than in the centralized decision model [30].

2.2. Government Carbon Subsidies

Government carbon subsidy is implemented to subsidize the cost of emission reduction by enterprises, aiming to increase business enthusiasm for reductions in emissions. In the low-carbon supply chain, to analyze the effect of government subsidies on supply chain business, Li et al. modeled and analyzed the effect of government subsidies on supply chain business. According to the study, government incentives, depending on the degree of carbon emission reduction, can effectively encourage businesses to cut carbon emissions even more [31]. Sheu et al. also studied the profitability of enterprises through government subsidy policies. The research results show that the profitability of enterprises, with or without government subsidy, is different, and the profitability of enterprises with government subsidy is significantly higher than those without government subsidy [32]. Li et al. examined the effects of emission-reduction subsidies and green technology investment subsidies on the overall amount of supply chain carbon emission reduction [33]. The above research focuses on the effect of different carbon policies on the decision-making of supply chain members, but ignores the willingness of business to actively reduce carbon emissions and difficulties they face; consequently, there is a dearth of research on the supply chain's internal incentive system.

Some academics have developed a model based on the composite aim of emission reduction and economic gains, in order to properly apply carbon tax limits and subsidies [34]. After weighing the benefits and drawbacks of carbon emissions, it is possible to conclude that collaborative emission reduction not only enhances business initiatives to cut emissions, but also contributes to environmental protection [35]. The supply chain's collaboration and rivalry between upstream and downstream businesses will have a favorable effect on the carbon emissions of the entire supply chain, hence lowering the system's overall carbon emissions [36]. By constructing a two-level supply chain, composed of manufacturers and retailers, Gao et al. examined the cooperative emission-reduction decision and cost-sharing decision among supply chain members [37].

2.3. Supply Chain Coordination

Supply chain coordination is an interdependent mechanism for enterprises to make decisions. The goal of low-carbon supply chain coordination is to maximize the total profits of the supply chain, retailers and manufacturers, within strict carbon emission constraints [38]. The implementation of carbon emission reduction by manufacturing businesses is conducive to enhancing their performance as a whole, but the high cost of low-carbon production, "free riding" and other problems, reduce their enthusiasm for emission reduction. At present, in order to enlist the support of businesses to decrease emissions and achieve supply chain coordination, academia mainly studies and designs various contracts to achieve the above goals. For example, there are two-part pricing contracts [39], quantity discount contracts [40], revenue-sharing contracts [20] and cost-sharing contracts [41], among which revenue-sharing contracts are directly related to this study. Over the past 20 years, revenue-sharing contracts have become increasingly prevalent, notably among well-known platform distributors such as the Apple App Store, Google Games, and Amazon. As a result, supply chain management research has focused heavily on these contracts [42]. The performance of channel members, with and without revenue-sharing contracts, is examined by Cai et al., who find that revenue-sharing can result in a Pareto improvement in channel members [43]. Considering the reliability of the members, Feng et al. studied the revenue-sharing with reliability (RSR) contract in the N-stage supply chain, which can coordinate the supply chain and arbitrarily distribute the total profit [44]. The above study only considered the static case, and Krishnan et al. combined the dynamic case by allowing inventory carryover in discrete time to lay the foundation for adjusting the revenue-sharing contract in the incentive mechanism [45].

The above studies mainly focus on coordination in conventional supply chains, and few studies attempt to study revenue-sharing contracts in low-carbon supply chains. With the improvement of carbon emission-reduction requirements, customers are increasingly favoring low-carbon products, and consumers' environmental awareness has an impact on the ecological performance of manufacturers and retailers [46]. The demand and profit functions of the customer, provider, and the global supply chain have been reformed with new carbon emission limitations in the case of low-carbon supply chain coordination [47].

Under the restrictions of carbon emissions, the aim of low-carbon supply chain coordination is to optimize the combined earnings of the supply chain, retailers, and producers [48]. Different solutions, such as total limit, carbon subsidy, and carbon tax, have been included in the supply chain coordination strategy [49,50]. In their supply chain design choice model, Tseng et al. added the social costs associated with carbon emissions [51]. Based on the above discussion, we find that, with the increasing consumer preference for low-carbon products, the traditional supply chain coordination strategy needs to be redesigned, and the decision variables, such as production quantity and order quantity, also need to be redefined.

The literature assessment reveals that research either concentrates on government subsidies or carbon price policy, but fails to consider the two issues that supply chain enterprises must face together. Meanwhile, under the comprehensive influence of carbon tax policy and government subsidies, how to realize supply chain coordination, while reducing carbon emission, has not been discussed; a supply chain coordination contract, adapted to the low-carbon background, needs more research. According to the literature, the Stackelberg method has been widely used in this research field, and its effectiveness has been proven. Therefore, this research utilizes the Stackelberg game method to build a centralized and decentralized decision-making model, based on the government's carbon tax and carbon tax subsidies, in order to enhance the sustainable coordination ability of the supply chain and reduce carbon emissions. By designing a coordination contract, the ordering decisions and optimal emission-reduction strategies of each enterprise in the supply chain are analyzed to realize the coordination.

3. Model Establishment and Analysis

3.1. Model Description and Assumptions

This study mainly analyzes and solves the problem of enterprise emission reduction and coordination in the specific context that sees the government collecting carbon tax and subsidizing enterprises. Combined with the current situation, regarding the development of a low-carbon supply chain coordination mechanism, aimed at solving the problem of supply chain coordination under carbon tax and carbon subsidy, this paper established a decision-making model. This model used a supply chain, based on manufacturers and retailers, and the Stackelberg game method; this was in order to solve the model, further coordinate the contract design and finally achieve the overall coordination. The method and process of this research are shown in Figure 1 below:

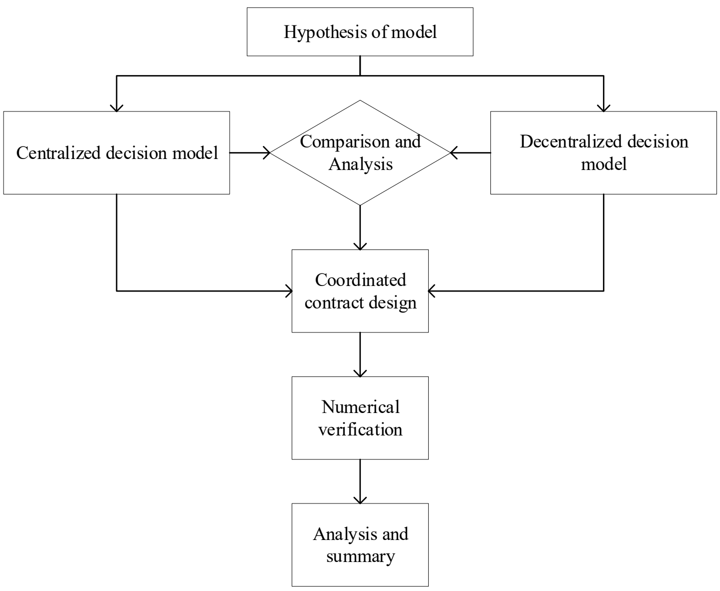

Figure 1. Research Methods and Process.

The following assumptions are propounded for this purpose, and the following symbols are defined:

Assumption 1. *There is no backorder cost for the manufacturer*.

Assumption 2. *The government-mandated carbon tax rate is an exogenous variable*.

Assumption 3. *Production costs per unit of product are not affected by abatement input costs*.

Assumption 4. *The interests of manufacturers and retailers are both prioritized when making decisions*.

Assumption 5. *In the case of a single sales cycle, the manufacturer only provides one ordering opportunity to the retailer in one cycle*.

Assumption 6. *The penalty costs of exceeding or failing to implement low-carbon policies are infinite, that is, manufacturers and retailers are strictly adhering to low-carbon policies*.

Assumption 7. *Production and operation costs include the production cost of unit product, fixed cost of starting production (including order processing, production preparation and other costs), carbon emission-reduction cost and transportation cost*.

Assumption 8. The manufacturer's production (Q) *is equal to the retailer's order (q) and there is no inventory cost*.

Assumption 9. *Consumers have a preference for low carbon, and the market demand for products is affected by both price and carbon emissions. The market demand for the product falls if the price rises and rises if the amount of emissions is reduced*.

Assumption 10. *Suppose the inverse demand function is*

$$p = N - \alpha q + \beta(\Delta e_m + \Delta e_r) \tag{1}$$

where,$\alpha > 0, \beta > 0, p > 0$. *represents the market capacity,*α*represents the sensitive coefficient of price on demand, and*β*represents the sensitive coefficient of price on carbon emission reduction. When*$\Delta e_i = 0, i = m, r$*, it means that there is no carbon tax constraint, and it is assumed that the customer demand can be fully met, that is,*q *is the quantity ordered by the retailer.*

Assumption 11. *Assume that the total emission-reduction cost of enterprise iis*

$$C(\Delta e_i) = \frac{1}{2}\xi_i \Delta e_i{}^2 \tag{2}$$

where, $i = m, r$, ξ_i *represents the degree of emission-reduction effort, also known as the difficulty coefficient of carbon emission reduction. The larger the* ξ_i *is, the more carbon emission-reduction investment funds are required under the same emission reduction, and vice versa. In addition,* $C(\Delta e_i)$ *satisfies* $C(0) = 0, C(1) = +\infty, C'(\Delta e_i) > 0\ C''(\Delta e_i) > 0$.

The parameters and variables used in our model are as follows (i represents the enterprise, where m is used by the manufacturer and r is used by the retailer):

c: Manufacturer's production cost per unit of product (related only to the quantity produced);

C_{0m}: The manufacturer's fixed cost of producing a product (related only to the number of times it is produced);

C_{0r}: Fixed subscription fee for products ordered by retailers (related to number of production only);

C_t: The transportation cost per unit of product in the process of product transportation shall be borne by the manufacturer;

w: Wholesale price per unit of product;

p: Retail price per unit of product;

q: Market demand for the product;

e_i: Initial carbon emissions per unit of product produced or sold by enterprise i (before emission reduction);

Δe_i: Carbon emission reduction per unit product of enterprise $i(0 < \Delta e_i < e_i)$;

λ_i: The rate at which the government subsidizes the emission reduction cost of enterprise i;

t: A government-set tax rate per unit of carbon emissions;

ξ_i: Emission-reduction effort coefficient of enterprise i;

$C(\Delta e_i)$: Carbon emission-reduction cost of enterprise i;

3.2. Model Construction and Analysis

The supply chain decision models—the centralized decision model and the decentralized decision model—are established in this section, based on the carbon tax policy.

3.2.1. Centralized Decision Model

In the centralized decision-making model, the manufacturer and the retailer can be regarded as a whole and reduce emissions as a whole. Maximizing the supply chain's overall profit is the primary objective when choosing the most effective strategy. Firstly, the government determines the carbon tax rate after understanding the market environment and enterprises' estimated carbon emissions. Secondly, enterprises determine the emission-reduction rate and the best selling price of products, according to the government subsidy coefficient. Let $\prod_c^N$ represent the total profit of the centralized decision-making supply chain, under the constraint of carbon tax policy. Therefore, the total profit function can be expressed as follows:

$$\begin{aligned}\prod\nolimits_c^N = \;& (p-c)q - [(1-\lambda_m)C(\Delta e_m) + (1-\lambda_r)C(\Delta e_r)] \\ & -[t(e_m-\Delta e_m) + t(e_r - \Delta e_r)]q - C_{0m} - C_{0r}\end{aligned} \tag{3}$$

In the above equation, $t(e_m - \Delta e_m)$ and $t(e_r - \Delta e_r)$, respectively, represent the effect of carbon tax paid by manufacturers and retailers on the carbon emissions of unit products. In addition,

$(1 - \lambda_m)C(\Delta e_m)$ and $(1 - \lambda_r)C(\Delta e_r)$, respectively, represent the emission-reduction costs consumed by them. In Equation (3), the inverse demand function and emission reduction-cost function are substituted to obtain:

$$\begin{aligned}\prod\nolimits_c^N = \;& [N - \alpha q + \beta(\Delta e_m + \Delta e_r) - c - t(e_m - \Delta e_m + e_r - \Delta e_r)]q \\ & -\tfrac{1}{2}[(1-\lambda_m)\xi_m(\Delta e_m)^2 + (1-\lambda_r)\xi_r(\Delta e_r)^2] - C_{0m} - C_{0r}\end{aligned} \tag{4}$$

In the above equation, market demand q and emission reduction $\Delta e_m, \Delta e_r$ are the decision variables of the model, and the optimal decision variable $(q_c^{N^*}, \Delta e_{mc}^{N^*}, \Delta e_{rc}^{N^*})$ is obtained by solving the model to make $\prod_c^N$ reach the maximum value. First, the derivative of Equation (4), with respect to q, can be obtained as follows:

$$\frac{\partial \prod_c^N}{\partial q} = N - 2\alpha q + \beta(\Delta e_m + \Delta e_r) - c - t(e_m - \Delta e_m + e_r - \Delta e_r) \tag{5}$$

It can be verified that $\prod_c^{N''}(q) = -2\alpha < 0$ through Equation (5), that is, $\prod_c^N$ is a convex function with respect to q. After the first-order linear condition $\prod_c^{N'}(q) = 0$, as can be observed, in order to increase the supply chain's overall profit, the unique optimal production volume (ordered quantity) is determined as $q_c^{N^*}$:

$$q_c^{N^*} = \frac{N + \beta(\Delta e_m + \Delta e_r) - c - t(e_m - \Delta e_m + e_r - \Delta e_r)}{2\alpha} \tag{6}$$

Once Equation (6) has been entered into Equation (4), the following can be obtained:

$$\begin{aligned}\prod_c^N = \; & \frac{[N+\beta(\Delta e_m+\Delta e_r)-c-t(e_m-\Delta e_m+e_r-\Delta e_r)]^2}{4\alpha} \\ & -\frac{1}{2}[(1-\lambda_m)\xi_m(\Delta e_m)^2+(1-\lambda_r)\xi_r(\Delta e_r)^2]-C_{0m}-C_{0r}\end{aligned} \tag{7}$$

The Hessian matrix H of Equation (7), with respect to emission reduction Δe_m and Δe_r, can be calculated as follows:

$$\begin{aligned} H & = \begin{bmatrix} \frac{\partial^2 \prod_c^N}{\partial \Delta e_m^2} & \frac{\partial^2 \prod_c^N}{\partial \Delta e_m \cdot \partial \Delta e_r} \\ \frac{\partial^2 \prod_c^N}{\partial \Delta e_m \cdot \partial \Delta e_r} & \frac{\partial^2 \prod_c^N}{\partial \Delta e_r^2} \end{bmatrix} \\ & = \begin{bmatrix} \frac{(\beta+t)^2}{2\alpha}-(1-\lambda_m)\xi_m & \frac{(\beta+t)^2}{2\alpha} \\ \frac{(\beta+t)^2}{2\alpha} & \frac{(\beta+t)^2}{2\alpha}-(1-\lambda_r)\xi_r \end{bmatrix}\end{aligned}$$

When the Hessian matrix is negative, there is an optimal solution to Equation (4), then the following two equations can be obtained:

$$|H_1| = \frac{(\beta+t)^2}{2\alpha} - (1-\lambda_m)\xi_m < 0$$

$$|H_2| = (1-\lambda_m)\xi_m(1-\lambda_r)\xi_r - [(1-\lambda_m)\xi_m + (1-\lambda_r)\xi_r]\frac{(\beta+t)^2}{2\alpha} > 0$$

There is an optimal solution when $2\alpha(1-\lambda_m)\xi_m > (\beta+t)^2$ and $2\alpha(1-\lambda_m)\xi_m(1-\lambda_r)\xi_r > [(1-\lambda_m)\xi_m + (1-\lambda_r)\xi_r](\beta+t)^2$, and there is an optimal value for variables Δe_m and Δe_r. By taking the first partial derivatives of emission reduction Δe_m and Δe_r, and setting them equal to 0, the optimal emission reduction $\Delta e_{mc}^{N^*}$ and $\Delta e_{rc}^{N^*}$ can be solved. The equations are as follows:

$$\begin{cases} \Delta e_{mc}^{N^*} = \frac{(1-\lambda_r)\xi_r(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r} \\ \Delta e_{rc}^{N^*} = \frac{(1-\lambda_m)\xi_m(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m} \end{cases} \tag{8}$$

By substituting (8) into Equation (6), the optimal order quantity of the retailer, using the centralized decision mode, can be obtained as follows:

$$q_c^{N^*} = \frac{(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m} \tag{9}$$

after summarizing Equations (8) and (9), we can obtain:

$$\begin{cases} q_c^{N^*} = \frac{(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m} \\ \Delta e_{mc}^{N^*} = \frac{(1-\lambda_r)\xi_r(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r} \\ \Delta e_{rc}^{N^*} = \frac{(1-\lambda_m)\xi_m(\beta+t)[N-c-t(e_r+e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m} \end{cases} \tag{10}$$

3.2.2. Decentralized Decision Model

Under the decentralized decision-making mode, the maximization of self-interest is the starting point of enterprise decision-making. This section considers the case that the government provides emission-reduction subsidies and assumes that the manufacturer is the dominant player in the supply chain. The profit functions of the manufacturer and retailer, under the decentralized decision-making mode, are given as follows:

Under the car-bon tax policy, manufacturers' decentralized decision-making is based on the following business model:

$$\prod_{m}^{N} = (w-c)q - (1-\lambda_m)C(\Delta e_m) - t(e_m - \Delta e_m)q - C_{0m} \tag{11}$$

The profit model of the decentralized decision-making of retailers, under the carbon tax policy, is as follows:

$$\prod_{r}^{N} = (p-w)q - (1-\lambda_r)C(\Delta e_r) - t(e_r - \Delta e_r)q - C_{0r} \tag{12}$$

After substituting the inverse demand function and emission-reduction cost function into Equations (11) and (12), we can obtain:

$$\prod_{m}^{N} = [w - c - t(e_m - \Delta e_m)]q - \frac{1}{2}(1-\lambda_m)\xi_m(\Delta e_m)^2 - C_{0m} \tag{13}$$

$$\prod_{r}^{N} = [N - \alpha q + \beta(\Delta e_m + \Delta e_r) - w - t(e_r - \Delta e_r)]q - \frac{1}{2}(1-\lambda_r)\xi_r(\Delta e_r)^2 - C_{0r} \tag{14}$$

To solve the model, partial derivatives of Equation (14), with respect to q and e, are calculated respectively, and the following equations can be obtained:

$$\begin{cases} \frac{\partial \prod_r^N}{\partial q} = N + \beta(\Delta e_m + \Delta e_m) - 2\alpha p - w - t(e_r - \Delta e_r) = 0 \\ \frac{\partial \prod_r^N}{\partial \Delta e_r} = (\beta + t)q - (1-\lambda_r)\xi_r \Delta e_r = 0 \end{cases} \tag{15}$$

Solving the above equation leads to:

$$\begin{cases} q = \frac{(1-\lambda_r)\xi_r(N+\beta\Delta e_m - w - te_r)}{2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2} \\ \Delta e_r = \frac{(\beta+t)[N - w - te_r + \beta\Delta e_m]}{2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2} \end{cases} \tag{16}$$

Then, the Hessian matrix H of Equation (13), regarding the ordered quantity q and emission reduction Δe_r, is obtained:

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 \prod_c^N}{\partial \Delta q^2} & \frac{\partial^2 \prod_c^N}{\partial q \cdot \partial \Delta e_r} \\ \frac{\partial^2 \prod_c^N}{\partial q \cdot \partial \Delta e_r} & \frac{\partial^2 \prod_c^N}{\partial \Delta e^2{}_r} \end{bmatrix} \\ &= \begin{bmatrix} -2\alpha & \beta + t \\ \beta + t & -(1-\lambda_r)\xi_r \end{bmatrix} \end{aligned}$$

We know, from the Hessian matrix, that

$$-2\alpha < 0, |H| = 2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2$$

The negative definite condition of Hessian matrix is

$$2\alpha(1-\lambda_m)\xi_m > (\beta+t)^2$$

Therefore, q, Δe_r, satisfying Equation (15), is the optimal emission reduction and ordering decision $(q^{N^*}, \Delta e_r^{N^*})$ of the retailer's decentralized decision under the constraint of the carbon tax policy. Next, the obtained q^{N^*} and $\Delta e_r^{N^*}$ are substituted into Equation (12), and after sorting, the following equation can be obtained:

$$\begin{aligned}\prod_m^N = \quad & [w - c - t(e_m - \Delta e_m)]\frac{(1-\lambda_r)\xi_r(N+\beta\Delta e_m - w - te_r)}{2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2} \\ & -\frac{1}{2}(1-\lambda_m)\xi_m(\Delta e_m)^2 - C_{0m}\end{aligned} \tag{17}$$

It can be easily verified by Equation (17) that

$$\prod_m^{N''}(\Delta e_m) = -(1-\lambda_m)\xi_m < 0$$

Then, $\prod_m^N$ is a convex function with respect to Δe_m. The derivative of Equation (17), with respect to Δe_m, can be obtained as follows.

$$\begin{aligned}\frac{\partial \prod_m^N}{\partial \Delta e_m} = \quad & t\frac{(1-\lambda_r)\xi_r(N+\beta\Delta e_m - w - te_r)}{2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2} - (1-\lambda_m)\xi_m\Delta e_m \\ & + \frac{(1-\lambda_r)\xi_r\beta}{2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2}[w - c - t(e_m - \Delta e_m)]\end{aligned} \tag{18}$$

When the derivative is equal to 0, the manufacturer's optimal emission reduction $\Delta e_m^{N^*}$, under decentralized decision mode, can be found:

$$\Delta e_m^{N^*} = \frac{(1-\lambda_r)\xi_r[t(N - w - te_r) + \beta(w - c - te_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_m)\xi_m - 2\beta t(1-\lambda_r)\xi_r} \tag{19}$$

Next, substituting $\Delta e_m^{N^*}$ into Equation (16) can find the optimal emission reduction and ordering decision $(q^{N^*}, \Delta e_r^{N^*})$ of the retailer, under the decentralized decision mode:

$$\begin{cases} q^{N^*} = \frac{(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m[N - w - t(e_r + e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - 2\beta t(1-\lambda_m)\xi_m} \\ \Delta e_r^{N^*} = \frac{(1-\lambda_m)\xi_m[t(N - w - te_m) + \beta(w - c - te_r)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - 2\beta t(1-\lambda_m)\xi_m} \end{cases} \tag{20}$$

After summarizing Equations (19) and (20), we can obtain:

$$\begin{cases} q^{N^*} = \frac{(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m[N - w - t(e_r + e_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - 2\beta t(1-\lambda_m)\xi_m} \\ \Delta e_m^{N^*} = \frac{(1-\lambda_r)\xi_r[t(N - w - te_r) + \beta(w - c - te_m)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_m)\xi_m - 2\beta t(1-\lambda_r)\xi_r} \\ \Delta e_r^{N^*} = \frac{(1-\lambda_m)\xi_m[t(N - w - te_m) + \beta(w - c - te_r)]}{2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - 2\beta t(1-\lambda_m)\xi_m} \end{cases} \tag{21}$$

3.2.3. Model Comparison and Analysis

By comparing these two decision models, we can draw two inferences, as follows:

Inference 1. $q^{N^*} < q_c^{N^*}$ *(The order quantity under decentralized decision is less than that of the centralized decision). The derivation is as follows:*

To compare $q_c^{N^*}$ and q^{N^*} in Equations (10) and (21), the denominators in fractions $q_c^{N^*}$ and q^{N^*} are first compared; this follows on from the property of the inequality that $(\beta + t)^2 \geq 2\beta t$. Then, the following equation can be obtained:

$$\begin{aligned}& 2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - (\beta+t)^2(1-\lambda_m)\xi_m \\ & < 2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m - (\beta+t)^2(1-\lambda_r)\xi_r - 2\beta t(1-\lambda_m)\xi_m.\end{aligned}$$

Second, the molecules in fractions $q_c^{N^*}$ and q^{N^*} are compared, because $w > c$; therefore, we can obtain

$$N - c - t(e_r + e_m) > N - w - t(e_r + e_m).$$

Finally, according to the properties of the fraction, we know that $q^{N^*} < q_c^{N^*}$.

Inference 2. $\Delta e_{mc}^{N^*} > \Delta e_m^{N^*}$ *(The manufacturer's emission reduction in centralized decision-making mode is greater than that in the decentralized decision-making mode). The derivation is as follows:*

By comparing $\Delta e_{mc}^{N^*}$ and $\Delta e_m^{N^*}$ in Equations (10) and (21), firstly, through numerator comparison, we can obtain:

$$(\beta + t)[N - c - t(e_r + e_m)] - [t(N - w - te_r) + \beta(w - c - te_m)] \\ = \beta(N - w - te_r) + t(w - c - te_m)$$

From the assumptions, we know that $N - w - te_r > 0, w - c - te_m > 0$, and $(\beta + t)^2 \geq 2\beta t$, so the above conclusion can be drawn after a comprehensive comparison.

Below, according to Equation (21), the derivatives of q^{N^*},$\Delta e_m^{N^*}$and$\Delta e_r^{N^*}$, with respect toα, can be obtained easily:

$$\frac{\partial q^{N^*}}{\partial \alpha} < 0, \frac{\partial \Delta e_m^{N^*}}{\partial \alpha} < 0, \frac{\partial \Delta e_r^{N^*}}{\partial \alpha} < 0$$

α is the sensitive coefficient of price to market demand. From the above equation, it can be known that q^{N^*}, $\Delta e_m^{N^*}$ and $\Delta e_r^{N^*}$ are the subtraction functions of α, which are negatively correlated with α, and decrease with the rise of α under the constraint of the carbon tax policy.

Then, according to Equation (21), the derivatives of q^{N^*}, $\Delta e_m^{N^*}$ and $\Delta e_r^{N^*}$, with respect to β, can be obtained easily:

$$\frac{\partial q^{N^*}}{\partial \beta} < 0, \frac{\partial \Delta e_m^{N^*}}{\partial \beta} < 0, \frac{\partial \Delta e_r^{N^*}}{\partial \beta} < 0$$

β is the sensitive coefficient of price to carbon emission reduction. From the above equation, it can be known that q^{N^*}, $\Delta e_m^{N^*}$ and $\Delta e_r^{N^*}$ are the subtraction functions of β, which are negatively correlated with β, and decrease with the rise of α under the constraint of the carbon tax policy.

From the results of the previous section, it can be inferred that, to realize supply chain coordination, only the optimal decisions in centralized and decentralized modes need to be equalized. Since orders will be reduced under decentralized decision-making, the manufacturer can give the retailer a certain wholesale discount, which means that the manufacturer will lose some profits. In this situation, the retailer can share the earnings with the manufacturer to achieve a win–win situation. This win–win situation needs to be realized by a revenue-sharing contract (w, φ). The details are as follows: the manufacturer sells the product to the retailer at a wholesale price w below its production cost $(w < c)$. While the retailer wants to compensate the supplier by means of profit sharing, namely the retailer's income minus the carbon tax and emission-reduction cost, including the proportion of retailers for φ and the manufacturer's share ratio of $1 - \varphi(0 \leq \varphi \leq 1)$, the innovation of the design contract is that the manufacturer shares the retailer's profits and also that some of the costs are shared.

3.2.4. Retailer's Optimal Decision under Contract

The model in this section is solved by backward induction. Firstly, the total profit function of the retailer under the contract is calculated as follows:

$$\prod_{r1}^{N} = \varphi[pq - (1 - \lambda_r)C(\Delta e_r) - t(e_r - \Delta e_r)q] - wq \tag{22}$$

After substituting the inverse demand function and emission-reduction cost function into the arrangement, the results shown here can be attained:

$$\prod_{r1}^{N} = \varphi[N - \alpha q + \beta(\Delta e_m + \Delta e_r) - t(e_r - \Delta e_r)]q - \frac{1}{2}\varphi(1 - \lambda_r)\xi_r(\Delta e_r)^2 - wq \tag{23}$$

To find the solution of the model, we first need to find the partial derivatives of Equation (21), with respect to q and a, and find the solution of the following system of equations at the same time:

$$\begin{cases} \frac{\partial \prod_{r1}^{N}}{\partial q} = \varphi[N + \beta(\Delta e_m + \Delta e_r) - t(e_r - \Delta e_r)] - 2\alpha\varphi p - w = 0 \\ \frac{\partial \prod_{r1}^{N}}{\partial \Delta e_r} = \varphi(\beta + t)q - \varphi(1 - \lambda_r)\xi_r \Delta e_r = 0 \end{cases}$$

By solving the above equations, the following can be obtained:

$$\begin{cases} q_1^N = \frac{\varphi(1-\lambda_r)\xi_r(N+\beta\Delta e_m - te_r) - (1-\lambda_r)\xi_r w}{\varphi[2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2]} \\ \Delta e_{r1}^N = \frac{\varphi(\beta+t)(N+\beta\Delta e_m - te_r) - (\beta+t)w}{\varphi[2\alpha(1-\lambda_r)\xi_r - (\beta+t)^2]} \end{cases} \tag{24}$$

Then it can be found that the $q, \Delta e_r$ Hessian matrix of Equation (23) is:

$$\begin{aligned} H &= \begin{bmatrix} \frac{\partial^2 \prod_c^N}{\partial \Delta q^2} & \frac{\partial^2 \prod_c^N}{\partial q \cdot \partial \Delta e_r} \\ \frac{\partial^2 \prod_c^N}{\partial q \cdot \partial \Delta e_r} & \frac{\partial^2 \prod_c^N}{\partial \Delta {e_r}^2} \end{bmatrix} \\ &= \begin{bmatrix} -2\alpha\varphi & (\beta + t)\varphi \\ (\beta + t)\varphi & -(1 - \lambda_r)\xi_r\varphi \end{bmatrix} \end{aligned}$$

According to the Hessian matrix, the following can be obtained:

$$-2\alpha\varphi < 0, |H| = [2\alpha(1 - \lambda_r)\xi_r - (\beta + t)^2]\varphi^2$$

When $2\alpha(1 - \lambda_r)\xi_r > (\beta + t)^2$, the Hessian matrix is negative definite; therefore, q and Δe_r, satisfying Equation (24), are the optimal emission reduction and ordering decision $(q_1^{N^*}, \Delta e_{r1}^{N^*})$ of the retailer in the revenue-sharing contract.

3.2.5. Manufacturer's Optimal Decision under Revenue-Sharing Contract

For the manufacturer, the total expected profit function of the manufacturer, under the revenue-sharing contract (w, φ), is:

$$\begin{aligned} \prod_{m1}^{N} = \ & (w - c)q - (1 - \lambda_m)C(\Delta e_m) - t(e_m - \Delta e_m)q \\ & + (1 - \varphi)[pq - (1 - \lambda_r)C(\Delta e_r) - t(e_r - \Delta e_r)q] \end{aligned} \tag{25}$$

after substituting the inverse demand function and emission-reduction cost function into the arrangement, the following results can be obtained:

$$\begin{aligned} \prod_{m1}^{N} = \ & [w - c - t(e_m - \Delta e_m)]q - \tfrac{1}{2}(1 - \lambda_m)\xi_m(\Delta e_m)^2 \\ & + (1 - \varphi)[N - \alpha q + \beta(\Delta e_m + \Delta e_r)] \\ & - t(e_r - \Delta e_r)q - (1 - \varphi)\tfrac{1}{2}(1 - \lambda_r)\xi_r(\Delta e_r)^2 \end{aligned} \tag{26}$$

Substituting $q_1^{N^*}$ and $\Delta e_{r1}^{N^*}$ into Equation (26) to find their derivatives, with respect to Δe_m, and setting the derivative equal to 0, we can obtain:

$$\Delta e_m^{N^*} = \frac{\varphi(1 - \lambda_r)\xi_r(\beta + t)[N - t(e_r + e_m)] - (1 - \lambda_r)\xi_r(\beta + t)w}{\varphi[2\alpha(1 - \lambda_r)\xi_r(1 - \lambda_m)\xi_m - (\beta + t)^2(1 - \lambda_m)\xi_m - (\beta + t)^2(1 - \lambda_r)\xi_r]} \tag{27}$$

From the above equation, it can be obtained that $\prod_{m1}^{N''}(\Delta e_m) < 0$, and then that $\prod_{m1}^{N}$ is a convex function with respect to Δe_m. Substituting Equation (27) into Equation (24), $(q_1^{N^*}, \Delta e_{r1}^{N^*})$.

Can be obtained as follows:

$$\begin{cases} q_1^{N^*} = \frac{\varphi(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m[N-t(e_r+e_m)]-(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m w}{\varphi[2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m]} \\ \Delta e_{r1}^{N^*} = \frac{\varphi(1-\lambda_m)\xi_m(\beta+t)[N-t(e_r+e_m)]-(1-\lambda_m)\xi_m(\beta+t)w}{\varphi[2\alpha(1-\lambda_r)\xi_r(1-\lambda_m)\xi_m-(\beta+t)^2(1-\lambda_r)\xi_r-(\beta+t)^2(1-\lambda_m)\xi_m]} \end{cases} \tag{28}$$

For supply chain coordination to be realized, it is necessary to set

$$q_1^{N^*} = q_c^{N^*}, \Delta e_{m1}^{N^*} = \Delta e_{mc}^{N^*}, \Delta e_{r1}^{N^*} = \Delta e_{rc}^{N^*}$$

So that the parameters of the revenue-sharing contract (w^{N^*}, φ^{N^*}) should meet the following conditions:

$$w^{N^*} = \varphi^{N^*} c \tag{29}$$

Then it can be concluded that, when the parameters of the revenue-sharing contract (w^{N^*}, φ^{N^*}) meet the condition of $w^{N^*} = \varphi^{N^*} c$, the decentralized decision model is consistent with the enterprise decision under the centralized decision.

After comparing Equations (4) and (23) and (26), we can obtain:

$$\prod_c^N = \prod_m^N (w^{N^*}, \varphi^{N^*}) + \prod_r^N (w^{N^*}, \varphi^{N^*}) \tag{30}$$

$$\prod_m^N (w^{N^*}, \varphi^{N^*}) = (1 - \varphi^{N^*}) \prod_c^N \tag{31}$$

$$\prod_r^N (w^{N^*}, \varphi^{N^*}) = \varphi^{N^*} \prod_c^N \tag{32}$$

It can be seen from Equations (30)–(32) that the revenue-sharing contract (w^{N^*}, φ^{N^*}) can realize the perfect coordination of the supply chain. At this time, $1 - \varphi^{N^*}$ of the total profit of the supply chain is the manufacturer's optimal profit, and φ^{N^*} of the retailer's optimal profit is the total profit of the supply chain. The size of φ^{N^*} is determined by the specific circumstances of their bargaining, and the total profit of the supply chain is shared by both parties in proportion. When $\varphi^{N^*} \to 0$, all profit winners of the supply chain are retailers, which is similar to the situation before the contract is signed. When $\varphi^{N^*} \to 1$, the manufacturer obtains all the profits of the supply chain, that is, the manufacturer is the absolute leader of the supply chain. However, in this case, the retailer's profit is not positive, so this is an extreme situation that cannot happen in real life.

However, to make both parties willing to abide by the revenue-sharing contract, their respective profits under decentralized decision-making must be greater than their profits without the contract, that is, $(\varphi^{N^*} = 1)$. Therefore, this value should have a lower limit φ_a^N and an upper limit φ_b^N, satisfying $\prod_{r1}^{N^*}(w^{N^*}, \varphi_a^N) = \prod_r^{N^*}(w^{N^*}, \varphi = 1)$ and $\prod_{m1}^{N^*}(w^{N^*}, \varphi_b^N) = \prod_m^{N^*}(w^{N^*}, \varphi = 1)$. Therefore, it can be concluded that the optimal revenue-sharing contract (w^{N^*}, φ^{N^*}) satisfies $w^{N^*} = \varphi^{N^*} c$, and $\varphi_a^N < \varphi^N < \varphi_b^N$; this makes both parties willing to make the same ordering and production decisions in decentralized decision-making mode as in centralized decision-making mode. To maximize the total profit of the supply chain, it can be known that:

$$\varphi_a^N = \frac{\prod_{r1}^{N^*}(w, \varphi = 1)}{\prod_c^N}, \varphi_b^N = \frac{\prod_{m1}^{N^*}(w, \varphi = 1)}{\prod_c^N} \tag{33}$$

The following inferences can be drawn from this:

Inference 3. *If the contract parameters meet the range $\varphi_a^N < \varphi^N < \varphi_b^N$, both parties can freely distribute the total profit of the supply chain.*

Inference 4. *Under the optimal revenue-sharing contract parameters* (w^{N^*}, φ^{N^*}) *(satisfies Equation (29)), the manufacturer's wholesale price is directly proportional to* φ^{N^*}. *Therefore, if the manufacturer wants to realize the independent control of revenue-sharing proportion, it can achieve this purpose by mediating the wholesale price.*

Inference 5. *The manufacturer's optimal wholesale price is lower than the cost, indicating that the manufacturer realizes its own revenue increase by sharing the retailer's profit.*

It can be seen that the revenue-sharing contract can not only realize the perfect coordination of the supply chain, but also has the advantages of flexibility, practicality and high feasibility, compared with other contracts.

4. Numerical Analysis

Numerical analysis is carried out in this section to explore the effects of starting carbon emission levels, subsidies, and tax policies on carbon emission levels, and the profitability of manufacturers and retailers; it also clarifies and validates the findings of this article.

To make the calculation simpler, the fixed production and ordering costs are not considered. Through the model constructed in the previous section, the value range of φ^{N^*} can be obtained as [0.78, 0.93]. Therefore, if $\varphi^{N^*} = 0.85$, the manufacturer's optimal wholesale price $w^{N^*} = 17$ can be obtained from Equation (29). Then, to simplify the calculation without a loss of generality, we suppose that $N = 50, c = 20, \alpha = 0.5, \beta = 0.3, w = 50, \lambda_m = \lambda_r = 0.2$, $\xi_m = 100, \xi_r = 90, e_m = e_r = 20$ and $C_{0m} = C_{0r} = 0$. Knowing that all of the model's constraint requirements have been met is simple.

According to the given parameter values, the following Table and Figures can be obtained via a calculation.

According to Table 1, Figure 2 can be concluded as follows:

Table 1. Impact of carbon tax rate on supply chain enterprises.

Carbon Tax Rate	Retailer's Optimal Order Quantity	Manufacturer's Optimal Emission Reduction	Retailer Optimal Emission Reduction	Retailer's Profit (10^4)	Manufacturer 's Profit (10^4)
0	482	1.8	2.0	10.73	1.45
0.1	480	2.3	2.6	10.61	1.36
0.2	478	2.8	3.2	10.48	1.27
0.3	476	3.3	3.8	10.35	1.18
0.4	474	3.8	4.4	10.22	1.09
0.5	472	4.3	5.0	10.09	1.00
0.6	470	4.8	5.6	9.95	0.91
0.7	468	5.3	6.2	9.82	0.82
0.8	466	5.8	6.8	9.70	0.73
0.9	464	6.3	7.4	9.57	0.64
1	462	6.8	8	9.43	0.55

Figure 2 demonstrates that, when all supply chain related enterprises comply with the revenue-sharing contract, no matter what the carbon tax rate is, the retailer's optimal order quantity is greater than that without the contract. Therefore, the production and sales problems of enterprises can be effectively improved through the revenue-sharing contract, so as to improve the enterprise's own revenue. Regardless of whether there is a revenue-sharing contract, when the carbon tax rate rises, the retailer's optimal order quantity will decrease. From the actual situation, this is mainly due to the continuous increase in the tax rate per unit of product. The more products enterprises produce and sell, the higher the carbon tax will be. As a result, the revenue of enterprises will decrease continuously, and then the enterprises will be forced to cut output to reduce excessive carbon dioxide emissions.

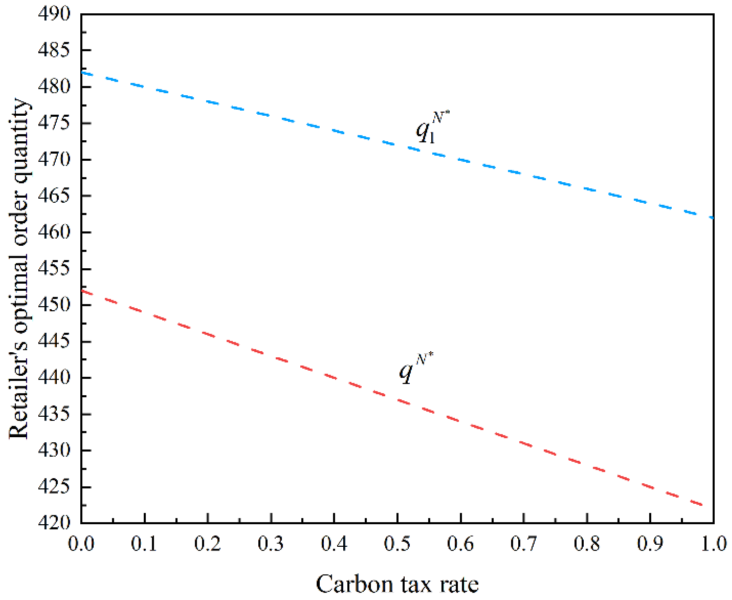

Figure 2. The effect of carbon tax rate on the optimal order quantity of retailers, under the two conditions of revenue-sharing contract coordination.

Meanwhile, the influence of tax rate on the optimal emission reduction in supply chain enterprises can be obtained under the two conditions of whether there is contract coordination or not, as shown in Figure 3 below:

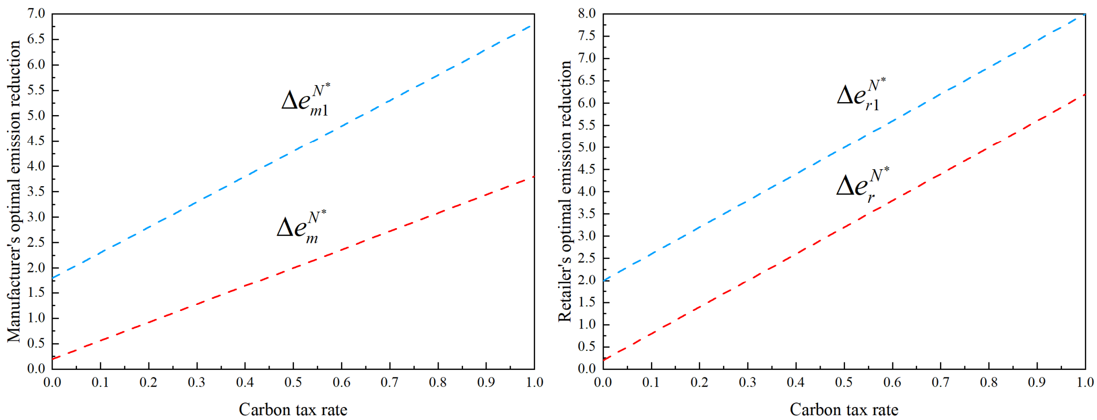

Figure 3. Effect of carbon tax rate on optimal emission reduction in supply chain enterprises, under the two conditions of revenue-sharing contract coordination.

Figure 3 demonstrates that, when all supply chain related enterprises comply with the revenue-sharing contract, the results with the coordinated contract are better than those without the coordinated contract, regardless of the value of the carbon tax rate; the reduction in carbon emissions by manufacturers is positively correlated with the carbon tax rate. Combined with real life analysis, the reason is that, after the carbon tax rate increases, manufacturers need to pay more expensive costs for each unit of carbon emissions. Therefore, manufacturers must reduce their own output and increase their emission reduction to reduce the cost burden of carbon tax. When all the relevant enterprises in the supply chain comply with the revenue-sharing contract, the results with the coordinated contract are better than those without the coordinated contract, regardless of the value of the carbon tax rate; the optimal emission reduction by the retailer is positively correlated with the carbon tax rate. The reason is that, after the carbon tax rate increases, retailers need to pay more expensive costs for each unit of carbon emissions. Retailers must lower their order volume and sales volume to guarantee that they pay less carbon tax, in order to lessen the financial strain brought on by the carbon tax.

Then, the effect of carbon tax rate on the earnings of supply chain enterprises is shown in Figure 4:

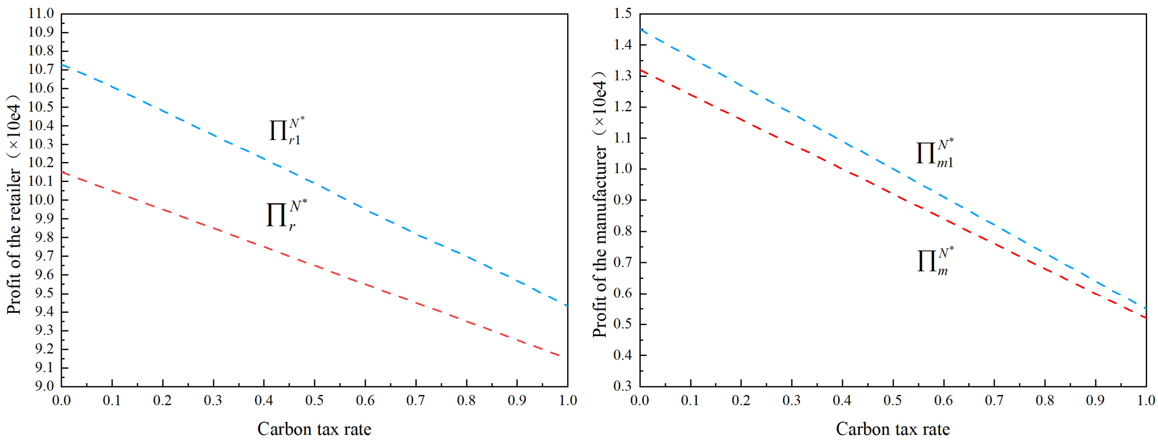

Figure 4. Effect of carbon tax rate on profit of supply chain enterprises, under the two conditions of revenue-sharing contract coordination.

Figure 4 shows that, when the supply chain-related businesses comply with the revenue-sharing contract, regardless of the value of the carbon tax rate, retailers profit more than without a contract; this suggests that retailers' profits can be effectively increased through revenue-sharing contract implementation, and, under the coordination of the contract, that the retailer can accomplish the task and improve their income. When the relevant enterprises in the supply chain comply with the revenue-sharing contract, the manufacturer's profit is greater than that without the contract, regardless of the value of the carbon tax rate; this indicates that the manufacturer's profit can also be effectively increased through the revenue-sharing contract. At the same time, the influence of the supply chain's total profit is shown in Figure 5 below:

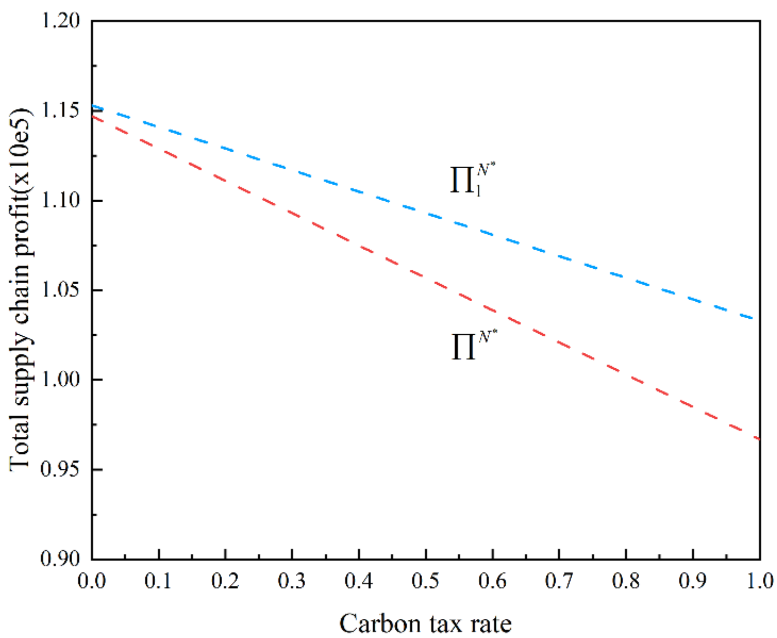

Figure 5. Effect of carbon tax rate on total profit of supply chain, under the two conditions of revenue-sharing contract coordination.

Figure 5 shows that, when all supply chain-related firms comply with the revenue-sharing contract, the results of the coordinated contract are better than those of the uncoordinated contract, regardless of the carbon tax rate; this indicates that the contract has a practical effect on improving the total earnings of the supply chain. However, when the carbon tax rate increases, the total earnings of the supply chain will continue to decline, which further illustrates the importance of the government to formulate a scientific and reasonable carbon tax rate in the process of economic development.

5. Conclusions

In order to address the major environmental issues that are becoming more prevalent, for the coordination of and reduction in carbon emissions in the supply chain business, there must be an increase in the supply chain's sustainable competitiveness and a reduction in the emissions. This study considers the coordination problem of the secondary supply chain, composed of manufacturers and retailers; it constructs and solves the centralized decision-making and decentralized decision-making models, respectively, according to the Stackelberg game method, and further designs the coordination contract. The supply chain decision-making and emission-reduction strategies, under the constraints of a carbon policy, are analyzed, and the overall coordination of the supply chain is finally achieved. The numerical analysis verifies the design of this study.

The findings indicate the following: first, that under the constraint of a low-carbon policy, manufacturers and retailers are more suited to adopting a centralized decision-making mode to maximize the profits of the supply chain and maximize a reduction in carbon emission. Second, with the increase in the carbon tax rate, the manufacturers and retailers' reduction in emissions will increase, but the order quantity of retailers, the total profit of the supply chain and the respective profits of enterprises will decrease. Third, the supply chain coordination contract is designed reasonably in this paper; this can increase the reduction in the supply chain's carbon emissions and mobilize the enterprise's enthusiasm for emission reduction. Meanwhile, the total profits can be distributed, according to the adjustment of the coordination contract, so as to achieve the overall coordination. Fourth, the government needs to implement reasonable low-carbon subsidy policies and tax rates, in order to effectively stimulate the enthusiasm of enterprises to reduce emissions and promote the healthy and coordinated development of supply chain enterprises.

The main contributions of this study are as follows: First, we comprehensively consider the effect of carbon tax, government subsidies and supply chain coordination, and supplemented relevant theories. Second, we prove that the total supply chain profit and emission reduction, under centralized decision-making, are higher than those under decentralized decision-making. Thirdly, we analyze a revenue-sharing contract to coordinate the order quantity, so that the total profit and emission reduction are higher than without a coordination contract; this can be used by supply chain enterprises for reference. Fourth, we find that, when both parties abide by the revenue-sharing contract, they can achieve coordination and can freely distribute the gross profits of the supply chain.

There are two main limitations to this paper. First, this study only considers the constant government subsidy rate, and does not discuss the impact of dynamic changes in tax rates and subsidy rates on supply chain enterprises. Second, the established model only considers manufacturers and retailers, and does not include more participating enterprises in the supply chain into the model. In future research, other contracts can be further improved to adapt to the low-carbon development of the supply chain, such as cost-sharing contracts. At the same time, we can study the changing carbon tax rate and government subsidy rate, in order to find the optimal combination; this would stimulate enterprises to reduce emissions. Finally, more supply chain enterprises should take this into consideration, establish an overall analysis model, and systematically analyze the differences in profits and the emission-reduction decisions of different enterprises in the supply chain.

Author Contributions: Conceptualization, W.R. and T.X. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zhou, Y.; Ye, X. Differential game model of joint emission reduction strategies and contract design in a dual-channel supply chain. *J. Clean. Prod.* **2018**, *190*, 592–607. [CrossRef]
2. Friedler, F. Process integration, modelling and optimisation for energy saving and pollution reduction. *Appl. Therm. Eng.* **2010**, *30*, 2270–2280. [CrossRef]
3. Hariga, M.; As'ad, R.; Shamayleh, A. Integrated economic and environmental models for a multi stage cold supply chain under carbon tax regulation. *J. Clean. Prod.* **2017**, *166*, 1357–1371. [CrossRef]
4. van Soest, H.L.; den Elzen, M.; van Vuuren, D. Net-zero emission targets for major emitting countries consistent with the Paris Agreement. *Nat. Commun.* **2021**, *12*, 2140. [CrossRef]
5. De, A.; Kumar, S.K.; Gunasekaran, A.; Tiwari, M.K. Sustainable maritime inventory routing problem with time window constraints. *Eng. Appl. Artif. Intell.* **2017**, *61*, 77–95. [CrossRef]
6. Varbanov, P.S.; Manenti, F.; Klemeš, J.J.; Lund, H. Special section: Process Integration, Modelling and Optimisation for Energy Saving and Pollution Reduction—PRES 2014. *Energy* **2015**, *90*, 1–4. [CrossRef]
7. Wu, P.; Jin, Y.; Shi, Y.; Shyu, H. The impact of carbon emission costs on manufacturers' production and location decision. *Int. J. Prod. Econ.* **2017**, *193*, 193–206. [CrossRef]
8. Li, J.; Su, Q.; Ma, L. Production and transportation outsourcing decisions in the supply chain under single and multiple carbon policies. *J. Clean. Prod.* **2017**, *141*, 1109–1122. [CrossRef]
9. Fukui, H.; Miyoshi, C. The impact of aviation fuel tax on fuel consumption and carbon emissions: The case of the US airline industry. *Transp. Res. Part D Transp. Environ.* **2017**, *50*, 234–253. [CrossRef]
10. Mihalakas, A.P.; Hyde, E. Implementation of Nationally Determined Contributions under the Paris Agreement—Comparing the Approach of China and the EU. *Athens J. Law* **2020**, *6*, 407–430. [CrossRef]
11. Gullberg, A.T.; Ohlhorst, D.; Schreurs, M. Towards a low carbon energy future—Renewable energy cooperation between Germany and Norway. *Renew. Energy* **2014**, *68*, 216–222. [CrossRef]
12. Du, S.; Hu, L.; Song, M. Production optimization considering environmental performance and preference in the cap-and-trade system. *J. Clean. Prod.* **2016**, *112*, 1600–1607. [CrossRef]
13. Hepburn, C.; Qi, Y.; Stern, N.; Ward, B.; Xie, C.; Zenghelis, D. Towards carbon neutrality and China's 14th Five-Year Plan: Clean energy transition, sustainable urban development, and investment priorities. *Environ. Sci. Ecotechnol.* **2021**, *8*, 100130. [CrossRef]
14. Zhou, S.; Tong, Q.; Pan, X.; Cao, M.; Wang, H.; Gao, J.; Ou, X. Research on low-carbon energy transformation of China necessary to achieve the Paris agreement goals: A global perspective. *Energy Econ.* **2021**, *95*, 105137. [CrossRef]
15. Damert, M.; Feng, Y.; Zhu, Q.; Baumgartner, R.J. Motivating low-carbon initiatives among suppliers: The role of risk and opportunity perception. *Resour. Conserv. Recycl.* **2018**, *136*, 276–286. [CrossRef]
16. Astanti, R.D.; Daryanto, Y.; Dewa, P.K. Low-Carbon Supply Chain Model under a Vendor-Managed Inventory Partnership and Carbon Cap-and-Trade Policy. *J. Open Innov. Technol. Mark. Complex.* **2022**, *8*, 30. [CrossRef]
17. Xia, T.; Wang, Y.; Lv, L.; Shen, L.; Cheng, T.C.E. Financing decisions of low-carbon supply Chain under Chain-to-Chain competition. *Int. J. Prod. Res.* **2022**, *2022*, 1–24. [CrossRef]
18. Jum'a, L.; Zimon, D.; Ikram, M. A Relationship between Supply Chain Practices, Environmental Sustainability and Financial Performance: Evidence from Manufacturing Companies in Jordan. *Sustainability* **2021**, *13*, 2152. [CrossRef]
19. Le, T.T.; Ikram, M. Do sustainability innovation and firm competitiveness help improve firm performance? Evidence from the SME sector in vietnam. *Sustain. Prod. Consum.* **2022**, *29*, 588–599. [CrossRef]
20. Wangsa, I.D.; Vanany, I.; Siswanto, N. An optimization model for fresh-food electronic commerce supply chain with carbon emissions and food waste. *J. Ind. Prod. Eng.* **2023**, *40*, 1–21. [CrossRef]
21. Zhang, X.; Dou, Y.; Zhang, C.; Ding, L.; Lv, H. Carbon emission management of coal power plant from the perspective of production planning in China. *J. Ind. Prod. Eng.* **2023**, *40*, 22–34. [CrossRef]
22. Jabbour, C.J.C.; Neto, A.S.; Gobbo Jr, J.A.; de Souza Ribeiro, M.; de Sousa Jabbour, A.B.L. Eco-innovations in more sustainable supply chains for a low-carbon economy: A multiple case study of human critical success factors in Brazilian leading companies. *Int. J. Prod. Econ.* **2015**, *164*, 245–257. [CrossRef]
23. Shaharudin, M.S.; Fernando, Y.; Jabbour, C.J.C.; Sroufe, R.; Jasmi, M.F.A. Past, present, and future low carbon supply chain management: A content review using social network analysis. *J. Clean. Prod.* **2019**, *218*, 629–643. [CrossRef]
24. Chen, X.; Benjaafar, S.; Elomri, A. The carbon-constrained EOQ. *Oper. Res. Lett.* **2013**, *41*, 172–179. [CrossRef]
25. Du, S.; Zhu, L.; Liang, L.; Ma, F. Emission-dependent supply chain and environment-policy-making in the 'cap-and-trade' system. *Energy Policy* **2013**, *57*, 61–67. [CrossRef]

26. Marron, D.B.; Toder, E.J. Tax Policy Issues in Designing a Carbon Tax. *Am. Econ. Rev.* **2014**, *104*, 563–568. [CrossRef]
27. Meng, S.; Siriwardana, M.; McNeill, J. The Environmental and Economic Impact of the Carbon Tax in Australia. *Environ. Resour. Econ.* **2012**, *54*, 313–332. [CrossRef]
28. Meng, X.; Yao, Z.; Nie, J.; Zhao, Y. Make or buy? It is the question: A study in the presence of carbon tax. *Int. J. Prod. Econ.* **2018**, *195*, 328–337. [CrossRef]
29. Zhou, Y.; Hu, F.; Zhou, Z. Pricing decisions and social welfare in a supply chain with multiple competing retailers and carbon tax policy. *J. Clean. Prod.* **2018**, *190*, 752–777. [CrossRef]
30. Wang, C.; Wang, W.; Huang, R. Supply chain enterprise operations and government carbon tax decisions considering carbon emissions. *J. Clean. Prod.* **2017**, *152*, 271–280. [CrossRef]
31. Li, B.; Geng, Y.; Xia, X.; Qiao, D. The Impact of Government Subsidies on the Low-Carbon Supply Chain Based on Carbon Emission Reduction Level. *Int. J. Environ. Res. Public Health* **2021**, *18*, 7603. [CrossRef]
32. Sheu, J.-B.; Chen, Y. Impact of government financial intervention on competition among green supply chains. *Int. J. Prod. Econ.* **2012**, *138*, 201–213. [CrossRef]
33. Li, Z.; Pan, Y.; Yang, W.; Ma, J.; Zhou, M. Effects of government subsidies on green technology investment and green marketing coordination of supply chain under the cap-and-trade mechanism. *Energy Econ.* **2021**, *101*, 105426. [CrossRef]
34. Liu, Z.; Hu, B.; Zhao, Y.; Lang, L.; Guo, H.; Florence, K.; Zhang, S. Research on Intelligent Decision of Low Carbon Supply Chain Based on Carbon Tax Constraints in Human-Driven Edge Computing. *IEEE Access* **2020**, *8*, 48264–48273. [CrossRef]
35. Gao, H.; Liu, S.; Xing, D.; Cao, G. Optimization strategy of cooperation and emission reduction in supply chain under carbon tax policy. *J. Discret. Math. Sci. Cryptogr.* **2018**, *21*, 825–835. [CrossRef]
36. Liu, Z.; Lang, L.; Hu, B.; Shi, L.; Huang, B.; Zhao, Y. Emission reduction decision of agricultural supply chain considering carbon tax and investment cooperation. *J. Clean. Prod.* **2021**, *294*, 126305. [CrossRef]
37. Liu, Z.; Hu, B.; Huang, B.; Lang, L.; Guo, H.; Zhao, Y. Decision Optimization of Low-Carbon Dual-Channel Supply Chain of Auto Parts Based on Smart City Architecture. *Complexity* **2020**, *2020*, 1–14. [CrossRef]
38. Zhou, Y.; Bao, M.; Chen, X.; Xu, X. Co-op advertising and emission reduction cost sharing contracts and coordination in low-carbon supply chain based on fairness concerns. *J. Clean. Prod.* **2016**, *133*, 402–413. [CrossRef]
39. Du, W.; Fan, Y.; Tang, X. Two-part pricing contracts under competition: The South-to-North Water Transfer Project supply chain system in China. *Int. J. Water Resour. Dev.* **2016**, *32*, 895–911. [CrossRef]
40. Heydari, J.; Momeni, B. Retailers' coalition and quantity discounts under demand uncertainty. *J. Retail. Consum. Serv.* **2021**, *61*, 102557. [CrossRef]
41. Li, T.; Zhang, R.; Zhao, S.; Liu, B. Low carbon strategy analysis under revenue-sharing and cost-sharing contracts. *J. Clean. Prod.* **2019**, *212*, 1462–1477. [CrossRef]
42. Bart, N.; Chernonog, T.; Avinadav, T. Revenue-sharing contracts in supply chains: A comprehensive literature review. *Int. J. Prod. Res.* **2020**, *59*, 6633–6658. [CrossRef]
43. Cai, G. Channel Selection and Coordination in Dual-Channel Supply Chains. *J. Retail.* **2010**, *86*, 22–36. [CrossRef]
44. Feng, X.; Moon, I.; Ryu, K. Revenue-sharing contracts in an N-stage supply chain with reliability considerations. *Int. J. Prod. Econ.* **2014**, *147*, 20–29. [CrossRef]
45. Krishnan, H.; Winter, R. On the role of revenue-sharing contracts in supply chains. *Oper. Res. Lett.* **2011**, *39*, 28–31. [CrossRef]
46. Liu, Z.; Anderson, T.; Cruz, J. Consumer environmental awareness and competition in two-stage supply chains. *Eur. J. Oper. Res.* **2012**, *218*, 602–613. [CrossRef]
47. Du, S.; Hu, L.; Wang, L. Low-carbon supply policies and supply chain performance with carbon concerned demand. *Ann. Oper. Res.* **2015**, *255*, 569–590. [CrossRef]
48. Ji, J.; Zhang, Z.; Yang, L. Carbon emission reduction decisions in the retail-/dual-channel supply chain with consumers' preference. *J. Clean. Prod.* **2017**, *141*, 852–867. [CrossRef]
49. Bai, Q.; Chen, M.; Xu, L. Revenue and promotional cost-sharing contract versus two-part tariff contract in coordinating sustainable supply chain systems with deteriorating items. *Int. J. Prod. Econ.* **2017**, *187*, 85–101. [CrossRef]
50. Toptal, A.; Çetinkaya, B. How supply chain coordination affects the environment: A carbon footprint perspective. *Ann. Oper. Res.* **2015**, *250*, 487–519. [CrossRef]
51. Tseng, S.C.; Hung, S. A strategic decision-making model considering the social costs of carbon dioxide emissions for sustainable supply chain management. *J. Environ. Manag.* **2014**, *133*, 315–322. [CrossRef] [PubMed]