Low-Complexity Cooperative Coding for Sensor Networks using Rateless and LDGM Codes

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Abstract—Given limitations with current technology, nodes in a sensor network have stringent energy and complexity constraints. This paper presents a scheme for cooperative error-control coding, using rateless and low-density generator-matrix codes, for sensor networks. Assuming knowledge of the source-relay channel quality, we show that the proposed scheme achieves good performance and a good energy tradeoff despite low computational complexity. The scheme exploits the flexibility of rateless and LDGM codes to permit, depending on the channel conditions, independent, relay and cooperative modes of operation. As a motivating example, we analyze networks of two cooperating nodes communicating with a more sophisticated receiver. We also discuss the generalization of our framework to a multi-node system.

I. Introduction

A sensor network is a system in which distributed sensors take local measurements of a phenomenon and form a network to share their information, or to transmit it to some central authority. Such networks have a wide variety of potential applications, from wildlife monitoring [1] to load monitoring in structures [2]. Many of these applications require the network to be unobtrusive and ubiquitous, and to function with little or no maintenance. Nodes, therefore, must be as small, inexpensive, and efficient as possible, though the data sink (receiver) may be quite sophisticated.

In the literature, an important strategy for efficient communication in a network uses the relay channel [3], in which a transmitter is assisted by intermediate transceiver in sending a message, where the transceiver has no message of its own to send. This idea can be generalized to cooperative diversity [4], in which two (or more) transmitters assist each other in sending their messages to a common receiver. This idea has been developed for wireless ad-hoc and sensor networks. In [5], cooperative diversity was combined with error-control coding as a more flexible strategy than merely repetition by the partner while in [6], the two component codes of a Turbo code were split up between two relaying nodes, and used to implement a distributed Turbo code.

It is notable that much current research in sensor network communication, including research cited above, ignores the computational limitations of the deployed sensor nodes. For instance, many proposed schemes for sensor networking, such as [7], rely on the capability of the sensor node to decode complicated error-control codes, such as LDPC codes. Even the encoding of such codes requires relatively high complexity,

large amounts of memory, or both. If the true gains of error-control coding are to be achieved in practical sensor networks, it will be necessary to find powerful codes that are simple to encode, and relay schemes that can operate without intermediate decoding. In this regard, the proposal in [8], though using a simplistic code, is interesting for its simplicity of encoding.

Another challenge for sensor nodes is to ensure reliable communication in channels with widely varying qualities. In order for a sensor's signal to be discerned successfully, a sufficient amount of energy must arrive at a receiver. With a traditional error-correcting code, since the block length (and hence the transmission time, for fixed symbol rate) is fixed, this may be done by changing the transmitted power. However, there has recently been much interest in rateless codes, such as Luby transform (LT) codes [9], which can vary their block length to adapt to any channel condition. Rateless codes also have the appealing property the encoding process is extremely simple. Rateless codes have recently been proposed for relay channels [7], though in a manner that greatly increases the complexity and requires intermediate decoding at the relay. These codes are very similar to low-density generator-matrix (LDGM) codes, which are easy to encode, at the expense of some performance as compared to LDPC codes [10]. In fact, one may think of an LDGM code as a rateless code where the rate is fixed in advance.

The main contribution of this paper is a low-complexity error-control coding and cooperation scheme based on rateless and LDGM codes. The proposed system is unique in the literature in that it has been expressly designed with flexibility and simplicity in mind, and should be usable on contemporary sensor networking hardware. We are most interested in using rateless codes in the relay as a tool to improve efficiency that is, using the extra information provided by the code and the relay to reduce the computational or energy burden at each of the sensors. Of work in the literature, our approach is most similar to [6]-[8]. However, unlike [6], [8], we provide the flexibility of an inherently variable rate to match a wide range of possible channel conditions, and the use of the relay is not mandatory to gain the benefit of the full code. Furthermore, our approach has a much lower computational burden than the one proposed in [7]. We point out that Turbo encoding, such as suggested in [6], is relatively simple, and that much work has been done on reducing the complexity of LDPC encoding

(such as [11]), but LDGM encoding is extremely simple, and neither of these existing codes can be used ratelessly.

The remainder of the paper is organized as follows. In Section II, we introduce our system model and motivating example. In Section III, we discuss the use of rateless and low-density generator-matrix codes, and show the modifications required to operate in our framework. In Section IV we show how our framework can be generalized beyond two nodes. Finally, in Section V, we present some preliminary results found using density evolution.

II. SYSTEM MODEL

We are concerned with sensor networks that transmit their measurements to a central authority, i.e., the sensor network is composed of several sensors and one information sink. The sensors are equipped with simple two-way digital radios, and are capable of computational tasks of limited complexity. The energy resources of the sensors are also limited, and the most energy-intensive task of the sensor is assumed to be wireless transmission; all other tasks are assumed to have negligible energy cost. On the other hand, the information sink is considered to have effectively unlimited power and computational resources. The task of the sensors is to communicate their measurements as accurately as possible to the sink.

Since reception is far more energy efficient than transmission, it makes sense for the sensors to cooperate with each other in transmitting their measurements. However, we assume that the constrained computing resources of the sensors will be fully occupied with *encoding* an error-correcting code. Decoding the relayed transmission is assumed to be beyond the sensor's abilities.

As the motivating example of this paper, consider the system in Fig. 1, with two sensors, numbered 1 and 2. Let $\mathbf{x}_1 \in \{0,1\}^k$ represent the k-bit binary information sequence observed by sensor 1, and $\mathbf{x}_2 \in \{0,1\}^k$ the k-bit information sequence observed by sensor 2. Because it observes part of sensor 1's transmission, sensor 2 may use part of its transmission to act as a relay for channel 2.

The vectors of information bits observed by sensors 1 and 2 are \mathbf{x}_1 and \mathbf{x}_2 , respectively. Node 1 encodes \mathbf{x}_1 (in a manner to be described in the next section) to generate \mathbf{w}_1 , and transmits it across the channel. Node 2 observes \mathbf{y}_c , which is the noisy version of \mathbf{w}_1 , and selects hard decisions from \mathbf{y}_c and its own data stream \mathbf{x}_2 to encode \mathbf{w}_2 , which is also transmitted. The receiver observes \mathbf{y}_1 and \mathbf{y}_2 from nodes 1 and 2, respectively, where

$$\mathbf{y}_1 = A_1 \mathbf{w}_1 + \mathbf{n}_1$$

$$\mathbf{y}_2 = A_2 \mathbf{w}_2 + \mathbf{n}_2,$$

where \mathbf{n}_1 and \mathbf{n}_2 are vectors of unit-variance additive white Gaussian noise, and A_1 and A_2 are the (possibly random) channel amplitudes for nodes 1 and 2, respectively. It is assumed that \mathbf{w}_1 and \mathbf{w}_2 are generated with BPSK modulation. Meanwhile, although the cross link between nodes 1 and 2 is a Gaussian channel, it becomes an equivalent binary symmetric

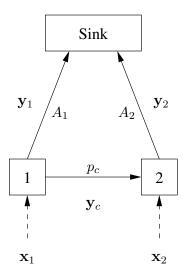


Fig. 1. Schematic of the system model. The cross link is a BSC with crossover probability $p_{\rm c}$, while the direct links are additive Gaussian noise channels with the given amplitudes.

channel (BSC) with crossover probability p_c due to the hard decisions made by the relay.

For ease of analysis, we assume that only sensor 2 acts as a relay, but this assumption may be relaxed. The assumption that the relay uses hard decisions of y_c is a reasonable one, since we have assumed that the relay makes no attempt to decode the code, and since it would be impractical for a relay to quantize a continuous Gaussian value beyond the single bit of a hard decision.

As illustrated in Fig. 2, three possible system architectures are considered in this paper: *independent transmission*, in which relays are not used; *relay transmission*, in which sensor 2 has no information of its own, and only acts as a relay for sensor 1; and *cooperative transmission*, in which sensor 2 splits its transmission between relaying for sensor 1 and transmitting its own information.

III. LDGM AND RATELESS CODES

A. Single rateless and LDGM codes

A rateless code is a code in which an information sequence x is mapped into a semi-infinite sequence w, so that any prefix of w is a codeword of a good error-correcting code. Such codes are called rateless because they have no pre-determined rate, and the prefix property implies that the code can be terminated at will (for example, when an acknowledgment is received). An LDGM code is a linear code in which the generator matrix, G, is sparse. The block length, and hence the rate, of an LDGM code is fixed. In this paper, we restrict ourselves to systematic LDGM codes, so that the generator matrix has the form

$$G = [I P],$$

where P is called the *parity generator matrix*, and P must be sparse.

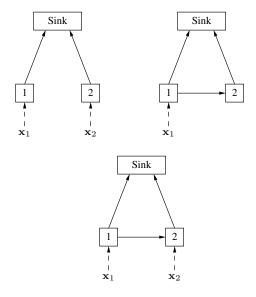


Fig. 2. System modes of operation. Clockwise from top left: independent; relay; cooperative.

The rateless code that we use is related to the LT code [9]; codes of this type are also related to LDGM codes. In a LT code, for $1 \leq i \leq \infty$, the encoded bit w_i is obtained by forming an even parity of a random subset of \mathbf{x} (of size u_i), chosen uniformly from all possibilities. More formally, let $\mathcal{N} = \{1, 2, \ldots, k\}$ represent an index set, let $\mathcal{N}_i \subseteq \mathcal{N}$ represent a subset of \mathcal{N} such that $|\mathcal{N}_i| = u_i$, and let σ_i : $\{1, 2, \ldots, u_i\} \to \mathcal{N}_i$ be an arbitrary bijective mapping. Let \mathbf{p}_i be a binary row vector of length k such that $p_{i,j} = 1$ if $j \in \mathcal{N}_i$ and $p_{i,j} = 0$ otherwise. Then

$$w_i = \mathbf{x}\mathbf{p}_i^T$$

where the superscript T represents transposition. For any fixed length n, the rateless codeword may thus be represented as $\mathbf{w}^{(n)} = \mathbf{x}\mathbf{P}^{(n)}$, where $\mathbf{w}^{(n)}$ represents the first n bits of \mathbf{w} , and the parity generator matrix $\mathbf{P}^{(n)}$ is given by

$$\mathbf{P}^{(n)} = \left[egin{array}{c} \mathbf{p}_1 \ \mathbf{p}_2 \ dots \ \mathbf{p}_n \end{array}
ight]^T$$
 .

If u_i is usually small, then $\mathbf{P}^{(i)}$ is sparse. Thus, we define a systematic rateless code as a sequence of systematic LDGM codes with parity generator matrices $\{\mathbf{P}^{(1)}, \mathbf{P}^{(2)}, \ldots\}$, whose codewords (for fixed parity length n) are of the form

$$\mathbf{w} = [\mathbf{x} \ \mathbf{w}^{(n)}].$$

With simplicity of the encoder in mind, we propose using a single value for every check degree, i.e., $u_i = u$ a constant, $\forall i$. It is known that LT codes achieve the Shannon capacity of every erasure channel, though no degree distribution exists to achieve the capacity of every channel with symmetric noise (such as the AWGN channel) [12].

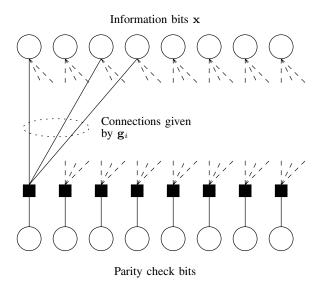


Fig. 3. Factor graph representation of a rateless code.

Consider some properties of this LT code. First, the parity check matrix is given by

$$\mathbf{H}^{(n)} = \left[(\mathbf{P}^{(n)})^T \mathbf{I} \right],\tag{1}$$

which is a low-density matrix since $\mathbf{P}^{(n)}$ is sparse. As with a standard low-density parity-check matrix, this code can be represented on a factor graph, as in Fig. 3. Second, consider the column Hamming weight of $\mathbf{P}^{(n)}$, which gives the degrees of the information variable nodes in the factor graph. Since the subsets \mathcal{N}_i are selected uniformly at random from all possible subsets of size m, the variable node degrees are also random. If k and k are asymptotically large, it is easy to see that these degrees have a Poisson distribution with parameter k [9], where

$$\nu = u\left(\frac{n-k}{k}\right).$$

As a result of the Poisson distribution, some of the rows of the generator matrix must have low weight, so such a code has poor minimum distance properties. Thus, it is known that LDGM codes suffer from high error floors, which can be mitigated by concatenation of two such codes [10]. Further, it is known that unmodified rateless codes in noisy channels suffer from error floors [13]. However, the concatenation strategy introduces extra complexity, and makes it difficult to use the codes without a fixed rate. In Section V, we will present results to show that, for practical code rates and channels, the error floors might not present a problem.

Since the parity check matrix $\mathbf{H}^{(n)}$ from (1) is sparse, decoding of the rateless code may be accomplished using the sum-product algorithm over the factor graph [14]. This algorithm is now well established; we omit the details and direct the reader to the reference.

B. Rateless codes in relaying and cooperation

We now discuss the impact of relaying on the rateless coding scheme. As mentioned previously, we do not wish the relaying node to make any attempt to decode the code. Instead, we wish the relay to directly incorporate the noisy observations of its neighbor into its information string.

In our proposed scheme, the first sensor encodes its symbols in a systematic code and transmits them. The second sensor observes the first sensor's transmission, or a fraction thereof. Let \mathbf{w}_1 represent the transmitted sequence from sensor 1, and let \mathbf{y}_c represent the received sequence at sensor 2, so that

$$\mathbf{y}_c = \mathbf{w}_1 \oplus \mathbf{z},\tag{2}$$

where \oplus represents componentwise mod-2 addition, and **z** represents a binary noise sequence.

In relay mode, where sensor 2 has no information of its own to send, its information sequence x_2 is formed by selecting k elements from y_c , including both systematic and parity bits. In the cooperative, joint transmission mode, we more generally define ϵ as the fraction of \mathbf{x}_2 devoted to relaying as opposed to sending independent information. Thus, x_2 is composed of $k(1-\epsilon)$ bits of independent information and $k\epsilon$ bits selected from y_c (again, either all systematic observations, or a mixture). A codeword is formed from the information vector \mathbf{x}_2 using a rateless or LDGM code, in the same manner as for the first sensor. Notice that setting $\epsilon = 0$ corresponds to independent transmission, and $\epsilon = 1$ corresponds to the case of relayed transmission, with all settings in between corresponding to cooperative transmission. Empirically, we have found that the relayed bits should be weighted towards relaying more information bits than parity bits, though the parity bits should not be excluded altogether; in our simulation results, we use a 70/30 mixture of information bits to parity bits.

Decoding is slightly more complicated in the relay case, but still relies on passing messages with the sum-product algorithm. The encoded sequences \mathbf{w}_1 and \mathbf{w}_2 are encoded as described in the previous section, so these variables are connected to factor graph structures similar to Fig. 3. However, where node 2 acts as a relay for node 1, these sequences are themselves correlated random variables, and are therefore connected with a factor graph structure. For example, sensor 1 transmits $w_{1,i}$ (which includes \mathbf{x}_1 since the code is systematic), and sensor 2 observes $y_{c,i}$. This symbol is then included in \mathbf{w}_2 as $w_{2,j}$. From (2), $w_{2,j} = w_{1,i} \oplus z_i$. Since we can write $\Pr(w_{2,j} = w_{1,i}) = \Pr(z_i = 0)$, $w_{2,j}$ and $w_{1,i}$ are correlated random variables. Thus, the two rateless code factor graphs are connected with structures as depicted in Fig. 4.

Messages passed through the connecting node are obtained simply from the sum-product algorithm. The messages that arrive at the connecting node represent the *a posteriori* probabilities of each symbol connected to it. The *a posteriori* probability of $w_{2,j}$ given $w_{1,i}$ is

$$p_{W_{2,j}}(w_{2,j}) \propto \sum_{w_{1,i} \in \{0,1\}} p_{W_{2,j},W_{1,i}}(w_{2,j},w_{1,i}) p_{W_{1,i}}(w_{1,i})$$

where $p_{W_{1,i}}(w_{1,i})$ is the *a posteriori* probability of $w_{1,i}$, provided by the sum-product algorithm, and the constant of proportionality is independent of $w_{2,j}$. Then the message from

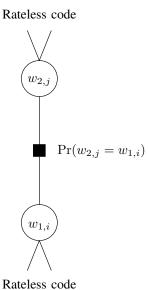


Fig. 4. Linkage between information symbols in the relay factor graph. These nodes connect the two rateless code factor graphs.

the connecting node to the node representing $w_{2,j}$ in Fig. 4 would be given as a log-likelihood ratio by

would be given as a log-likelihood ratio by
$$\ell = \log \frac{\sum_{w_{1,i} \in \{0,1\}} p_{W_{2,j},W_{1,i}}(w_{2,j} = 0, w_{1,i}) p_{W_{1,i}}(w_{1,i})}{\sum_{w_{1,i} \in \{0,1\}} p_{W_{2,j},W_{1,i}}(w_{2,j} = 1, w_{1,i}) p_{W_{1,i}}(w_{1,i})},$$
(3)

which contains all the summarized information concerning $w_{2,j}$ based on $w_{1,i}$. A similar calculation is performed for the message in the reverse direction. Note that this factor graph connection is only used for relayed nodes.

C. A note on encoder complexity

Our primary motivation in proposing these codes is the low complexity with which they can be encoded. In this section, we justify our assertion that the encoder is computationally simple.

There are two computational tasks involved in encoding a linear code:

- 1) Storing the information string; and
- 2) For each column of the generator matrix \mathbf{G} , calculating $w_i = \mathbf{x}\mathbf{g}_i^T$.

The first task requires random-access memory equal to the length of the information string. For the second task, we deliberately formulated the sequence of parity generator matrices $\{\mathbf{P}^{(1)},\mathbf{P}^{(2)},\ldots\}$ so that each parity check was generated by selecting variables at random. As a result, instead of storing a generator matrix, we can implement a pseudo-random number generator with a known seed, selecting elements of \mathbf{x} pseudorandomly and taking their mod-2 sum. A pseudo-random number generator can be implemented simply using a finite-state machine. To reduce the complexity further, we maintain the same check degree u for all parity checks, so the length of every mod-2 sum is the same. Furthermore, note that the bits w_i need not be stored beyond time i, as they can transmitted immediately.

Clearly, these hardware requirements are not particularly strenuous; or alternatively, on a general-purpose microcontroller, these tasks could be implemented in a straightforward manner in assembly language.

IV. GENERALIZATION TO LARGER NETWORKS

It is quite easy to extend the method proposed here to a large sensor network, so long as the sensors were constrained to operate through at most one relay, and so long as all nodes in the network are able to communicate directly with the receiver. In that case, the complication would be to find a useful assignment of sensors and relays.

In a large network that disposed of these constraints, there are examples of interesting features that could be found. Here we describe two of them and indicate how they could be included in our framework, which involve minor changes to the factor graph at the receiver:

- Multiple relays. In a multiple relay, more than one relay node is used to convey information from source to destination. The multiple relays, numbered $r=\{2,3,\ldots,r_{\max}\}$, would be represented by multiple variables $w_{r,j}$ correlated with $w_{1,i}$, obeying the relation (2). If each of the relays were protected by an LDGM or rateless code, the factor graphs for each code would be connected by structures such as in Fig. 4, with extra edges and variables representing the multiple relay symbols, appearing more like a "star" radiating from $w_{1,i}$ than a simple connection.
- Compound relays. In a compound relay, the path from source to destination includes more than one relay. We can represent a series of consecutive relays with a factor graph structure similar to Fig. 4, extending the line with more nodes and variables corresponding to each new relay in the path.

Since these behaviors can be described on a factor graph, the familiar sum-product algorithm may be used to decode the codes in a system containing them. A further challenge is the formulation of a protocol to allow them to be exploited optimally, but such a protocol is beyond the scope of this work.

V. RESULTS AND DISCUSSION

This section presents some proof-of-concept results on the proposed cooperation scheme, focusing mostly on LDGM codes in relay mode. It is straightforward to extend these results and show that similar performance can be expected for rateless codes, and in co-operative mode.

Since the codes we use have error floors, and since we are making no effort to mitigate these error floors, it is inevitable that there will be some nonzero average probability of error for every possible rate. This may be disappointing to some rateless coding researchers, who are accustomed to rateless codes always being decoded correctly, if the rate is large enough. On the other hand, rateless codes exhibit *thresholds*: for channels worse than the threshold, the probability of symbol error is very poor (on the order of 10^{-1}), but for channels better than the threshold, the probability of symbol error is quite good (on

the order of 10^{-5}). As we investigate the rateless properties of this coding system, we will emphasize the strong thresholds of the codes, rather than trying to determine where the probability of error vanishes.

A useful definition in this section is the *redundancy* of a relay coding system, ρ , which is the total number of channel uses per information bit. For a single code of rate R, clearly $\rho=1/R$. However, in relay mode, it is easier to use redundancy since it is additive. For example, if $\rho=2$ on the direct link and $\rho=1$ on the relay link, then overall we have $\rho=3$. In all of our results, we used codes with 2000 information bits, so a redundancy of ρ implies that the total code length was 2000ρ .

Our first simulation result, in Fig. 5, shows the improvement in performance of the system when operated in relay mode. Each node has a systematic LDGM code with a fixed rate, and with fixed check degree u=10 in both codes. The redundancy on each link is 3, so the overall redundancy in relay mode is $\rho=6$, and in independent mode is $\rho=3$. The SNR of both the cross link (i.e., node 1 to node 2) and the link from node 2 to the sink is 3 dB, which implies that $A_2=1.4125$ and $p_c=0.0786$. We compare the performance when the relay is included to the case when the relay is excluded, and see that a gain of nearly 2 dB can be expected. The simulation results indicate that the noise floor of the code is below 10^{-4} , and most likely much lower.

Notice that the "without relay" case (i.e., $\epsilon=0$) in Fig. 5 corresponds to the independent mode of operation, and the "with relay" case (i.e., $\epsilon=1$) corresponds to the full relay mode of operation. These are the two extremes of system operation, and for any ϵ between 0 and 1, the performance will fall between the two curves.

Our second simulation result, in Fig. 6, demonstrates the energy efficiency of the scheme for reasonable values of SNR. Using rateless codes, there are two possible ways in which a node can surpass the code threshold and achieve low error rates: it can lower the rate and generate more parity bits, or it can use its neighbor to act as a relay. Assuming that each bit has the same energy cost, whether transmitted by the original node or the relay, the redundancy ρ is directly proportional to the amount of energy expended per information bit. Thus, the more energy-efficient scheme is that which has better performance for the same redundancy. For the relay case we have a redundancy of $\rho = 3$ on each link of the relay, and for the single transmitter case, we have a redundancy of $\rho = 6$, so the redundancies of the two schemes are the same. In this case, we have that the cross link and the link from node 2 to the sink have SNRs of 4 dB, which implies that $A_2 = 1.5849$ and $p_c = 0.0565$. The result demonstrates an energy advantage to using the relay, even when the relay SNRs are relatively low and not enormously different from the direct channel SNR.

Our final result, in Fig. 7, indicates the performance of the system in fading. We assume quasi-static Rayleigh fading, so that A_1 and A_2 are Rayleigh-distributed random variables that remain constant over a codeword, and p_c is a random variable

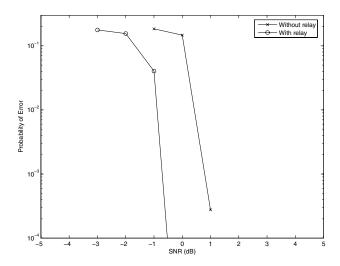


Fig. 5. Result for channels with fixed strength: source-to-relay and relay-to-sink SNRs are fixed at 3 dB

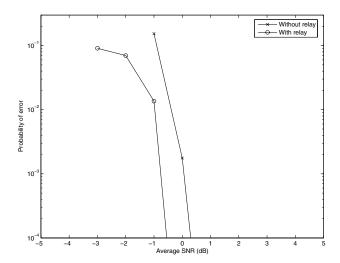


Fig. 6. Result for channels with fixed strength, where the single-user and relay cases have the same redundancy: source-to-relay and relay-to-sink SNRs are fixed at 4 dB

such that

$$p_c = \frac{1}{2}\operatorname{erfc}\left(\frac{A_c}{\sqrt{2}}\right),$$

where A_c (i.e., the amplitude of the cross channel) is a Rayleigh-distributed random variable. The codes have u=10, and are LDGM codes with a total redundancy of $\rho=6$, operated in full relay mode. Each of the three random amplitudes $(A_1, A_2, \text{ and } A_c)$ is assumed to be independent, and have the same average SNR. We clearly observe that, as expected, the channel with the relay achieves clear diversity gain at the higher values of SNR in the figure.

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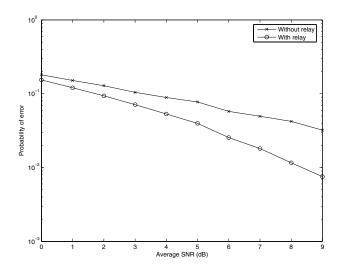


Fig. 7. Result for Rayleigh fading. All channels have the same average SNR.

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