Low-Complexity Lattice Reduction-Aided Regularized Block Diagonalization for MU-MIMO Systems

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***Abstract*—By employing the regularized block diagonalization (RBD) preprocessing technique, the MU-MIMO broadcast channel is decomposed into multiple parallel independent SU-MIMO channels and achieves the maximum diversity order at high data rates. The computational complexity of RBD, however, is relatively high due to two singular value decomposition (SVD) operations. In this letter, a low-complexity lattice reduction-aided RBD is proposed. The first SVD is replaced by a QR decomposition, and the orthogonalization procedure provided by the second SVD is substituted by a lattice-reduction whose complexity is mainly contributed by a QR decomposition. Simulation results show that the proposed algorithm can achieve almost the same sum-rate as RBD, substantial bit error rate (BER) performance gains and a simplified receiver structure, while requiring a lower complexity.**

***Index Terms*—MU-MIMO, regularized block diagonalization (RBD), low-complexity.**

I. Introduction

UNLIKE the received signal in single user multi-input multi-output (SU-MIMO) systems, the received signals of different users in multiuser multi-input multi-output (MU-MIMO) systems not only suffer from the noise and inter-antenna interference but are also affected by the multiuser interference (MUI). Channel inversion based strategies such as zero forcing (ZF) and minimum mean squared error (MMSE) precoding [1], [2], [3] can be still used to cancel the MUI, but they result in a reduced throughput or require higher power at the transmitter [4]. Block diagonalization (BD) precoding has been proposed in [4] to improve the sum-rate or reduce the transmitted power. However, BD precoding only takes the MUI into account and suffers a performance loss at low signal to noise ratios (SNRs) when the noise is the dominant factor. Therefore, the regularized block diagonalization (RBD) precoding which introduces a regularization to take the noise term into account has been proposed in [5].

The main steps for BD or RBD are two SVD operations. The first SVD is implemented to transform the MU-MIMO channel into a set of parallel equivalent SU-MIMO channels, where each user channel has the same properties as a conventional SU-MIMO channel [6]. The second SVD is used to orthogonalize the equivalent SU-MIMO channels and obtain a power loading matrix. For the BD or RBD algorithm we still need a unitary matrix for decoding, which is obtained by the second SVD, to orthogonalize each user's stream. The second SVD can be either computed at the transmit side or the receive side. If the second SVD is implemented at the transmit side, the corresponding decoding matrix needs to be informed to each distributed receiver, which requires an extra control overhead [7]. If the second SVD is implemented at the receive side, not only the complexity of the receiver will be increased but also the corresponding equivalent channel state information (CSI) after the first SVD precoding must be known or estimated by each receiver. In addition, due to these two SVD operations, the computational complexity of BD or RBD is relatively high compared with the channel inversion schemes.

In order to reduce the complexity of RBD, the first SVD of RBD is replaced with a less complex QR decomposition [8] in this work. We term the RBD in [8] as QR/SVD RBD and adopt it to get the first precoding filters due to its lower computational complexity and equivalent performance to the original RBD in [5]. In order to reduce the complexity further and to obtain a better bit error rate (BER) performance, the second SVD is replaced by a complex lattice reduction (CLR) whose complexity is mainly due to a QR decomposition. The aim of the CLR algorithm is to find a new basis which is shorter and nearly orthogonal as compared to the original matrix. Therefore, if the second precoding filters for the equivalent SU-MIMO channels after the first SVD were designed based on the lattice reduced channel matrix, a better BER performance can be achieved. Then, a CLR-aided RBD precoding algorithm is proposed, which not only has a lower complexity but also achieves a better BER performance than the RBD or QR/SVD RBD.

It is worth noting that the two SVDs are no longer required in the proposed algorithm which only needs the CSI at the transmitter and a quantization procedure at the receiver. Hence, the required computational effort for each user's receiver is reduced and a significant amount of transmit power can be saved which is very important considering the mobility of the users. For convenience, the proposed algorithm is termed as LC-RBD-LR in this letter.

II. System Model

We consider an uncoded MU-MIMO broadcast channel, with N_T transmit antennas at the base station (BS) and N_i receive antennas at the ith user equipment (UE). With K users in the system, the total number of receive antennas is $N_R = \sum_{i=1}^{K} N_i$. We assume a flat fading MIMO channel and the received signal at the ith user is given by

$$\boldsymbol{y}_i = \beta^{-1}(\boldsymbol{H}_i\boldsymbol{P}_i\boldsymbol{s}_i + \boldsymbol{H}_i\sum_{j=1,j\neq i}^{K}\boldsymbol{P}_j\boldsymbol{s}_j + \boldsymbol{n}_i), \qquad (1)$$

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where $\boldsymbol{H}_i \in \mathbb{C}^{N_i \times N_T}$ is the ith user's channel matrix. The quantity $\boldsymbol{P}_i \in \mathbb{C}^{N_T \times N_i}$ is the precoding matrix, β is a scalar chosen to make sure the energy of the precoded signal is not greater than the average transmit power E_s. The quantity $\boldsymbol{s}_i \in \mathbb{C}^{N_i}$ is the ith user's transmit signal, and $\boldsymbol{n}_i \in \mathbb{C}^{N_i}$ is the ith user's Gaussian noise with independent and identically distributed (i.i.d.) entries of zero mean and variance σ_n^2.

III. Proposed LC-RBD-LR Algorithm

From the system model, the combined channel matrix is given by $\boldsymbol{H} = [\boldsymbol{H}_1^T\ \boldsymbol{H}_2^T\ \ldots\ \boldsymbol{H}_K^T]^T$. We exclude the ith user's channel matrix and define $\overline{\boldsymbol{H}}_i = [\boldsymbol{H}_1^T\ \boldsymbol{H}_{i-1}^T\ \boldsymbol{H}_{i+1}^T\ \ldots\ \boldsymbol{H}_K^T]^T$, so that $\overline{\boldsymbol{H}}_i \in \mathbb{C}^{\overline{N}_i \times N_T}$, where $\overline{N}_i = N_R - N_i$. The proposed precoder design is performed in two steps. Correspondingly, the precoding matrix in (1) can be rewritten as $\boldsymbol{P}_i = \beta \boldsymbol{P}_i^a \boldsymbol{P}_i^b$.

Step 1: Obtain the first precoding matrix $\boldsymbol{P}_i^a$ by a QR decomposition of an extension of the matrix $\overline{\boldsymbol{H}}_i$.

For user i, the channel extension of $\overline{\boldsymbol{H}}_i$ is defined as

$$\overline{\overline{\boldsymbol{H}}}_i = \{\rho \boldsymbol{I}_{\overline{N}_i}, \overline{\boldsymbol{H}}_i\}, \tag{2}$$

where $\rho = \sqrt{\frac{N_R \sigma_n^2}{E_s}}$ and $\boldsymbol{I}_{\overline{N}_i}$ is a $\overline{N}_i \times \overline{N}_i$ identity matrix. The QR decomposition of $\overline{\overline{\boldsymbol{H}}}_i^H$ is given by

$$\overline{\overline{\boldsymbol{H}}}_i^H = \boldsymbol{Q}_i \boldsymbol{R}_i, \tag{3}$$

where $\boldsymbol{Q}_i$ is an $(\overline{N}_i + N_T) \times (\overline{N}_i + N_T)$ unitary matrix and $\boldsymbol{R}_i$ is an $(\overline{N}_i + N_T) \times \overline{N}_i$ upper triangular matrix. Then the first precoding matrix $\boldsymbol{P}_i^a$ for the ith user is obtained as

$$\boldsymbol{P}_i^a = \boldsymbol{Q}_i(\overline{N}_i + 1 : \overline{N}_i + N_T,\ \overline{N}_i + 1 : \overline{N}_i + N_T), \tag{4}$$

and the first combined precoding matrix for all users is

$$\boldsymbol{P}^a = [\boldsymbol{P}_1^a,\ \boldsymbol{P}_2^a,\ \ldots,\ \boldsymbol{P}_K^a]. \tag{5}$$

It is proved in [8] that the first precoding matrix $\boldsymbol{P}^a$ is equivalent to the one obtained by the first SVD in the conventional RBD in [5].

Step 2: Employ the CLR algorithm instead of the second SVD to implement the size-reduction, and obtain the second precoding matrix $\boldsymbol{P}^b$ by implementing channel inversion.

The aim of the CLR transformation is to find a new basis $\tilde{\boldsymbol{H}}$ which is nearly orthogonal compared to the original matrix $\boldsymbol{H}$ for a given lattice $L(\boldsymbol{H})$. After the first precoding, the effective channel matrix for the ith user is

$$\boldsymbol{H}_{\text{eff}_i} = \boldsymbol{H}_i \boldsymbol{P}_i^a. \tag{6}$$

We perform the CLR transformation on $\boldsymbol{H}_{\text{eff}_i}^T$ in the precoding scenario [11], that is

$$\tilde{\boldsymbol{H}}_{\text{eff}_i} = \boldsymbol{U}_i \boldsymbol{H}_{\text{eff}_i}, \tag{7}$$

where $\boldsymbol{U}_i$ is a unimodular matrix ($\det|\boldsymbol{U}_{\text{i}}| = 1$) and all elements of $\boldsymbol{U}_i$ are complex integers, i.e. $u_{l,k} \in \mathbb{Z} + j\mathbb{Z}$.

By using the ZF precoding, the second precoding matrix for user i is given as

$$\tilde{\boldsymbol{P}}_{\text{ZF}_i}^b = \tilde{\boldsymbol{H}}_{\text{eff}_i}^H (\tilde{\boldsymbol{H}}_{\text{eff}_i} \tilde{\boldsymbol{H}}_{\text{eff}_i}^H)^{-1}. \tag{8}$$

The MMSE precoding is equivalent to ZF with respect to an extended system model [12], [13]. The extended channel matrix $\underline{\boldsymbol{H}}$ for the precoding scheme is defined as

$$\underline{\boldsymbol{H}} = [\boldsymbol{H},\ \sigma_n \boldsymbol{I}_{N_R}]. \tag{9}$$

The MMSE precoding filter can be rewritten as $\boldsymbol{P}_{\text{MMSE}} = \boldsymbol{A}\underline{\boldsymbol{H}}^H(\underline{\boldsymbol{H}}\underline{\boldsymbol{H}}^H)^{-1}$, where $\boldsymbol{A} = [\boldsymbol{I}_{N_T},\ \boldsymbol{0}_{N_T,N_R}]$. The rows of $\underline{\boldsymbol{H}}$ determine the effective transmit power amplification. Correspondingly, the CLR transformation should be applied to the transpose of the extended channel matrix $\underline{\boldsymbol{H}}_{\text{eff}_i}^T = [\boldsymbol{H}_{\text{eff}_i},\ \sigma_n \boldsymbol{I}_{N_i}]^T$ for the MMSE precoding, and thus the CLR transformed channel matrix $\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}$ is obtained. Then, the CLR-aided MMSE precoding filter is given by

$$\tilde{\boldsymbol{P}}_{\text{MMSE}_i}^b = \boldsymbol{A}\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}^H (\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i} \underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}^H)^{-1}. \tag{10}$$

Finally, the second precoding matrix $\tilde{\boldsymbol{P}}^b$ for all users is

$$\tilde{\boldsymbol{P}}^b = \begin{bmatrix} \tilde{\boldsymbol{P}}_1^b & 0 & \ldots & 0 \\ 0 & \tilde{\boldsymbol{P}}_2^b & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \tilde{\boldsymbol{P}}_K^b \end{bmatrix}. \tag{11}$$

The resulting precoding matrix is $\tilde{\boldsymbol{P}} = \beta \boldsymbol{P}^a \tilde{\boldsymbol{P}}^b$, where the gain factor $\beta = \sqrt{E_s/(\|\boldsymbol{P}^a \tilde{\boldsymbol{P}}^b\|^2)}$. Since the lattice reduced precoding matrix $\tilde{\boldsymbol{P}}^b$ has near-orthogonal columns, it is able to reduce the interference to a lower level than the precoder $\boldsymbol{P}^b$ obtained from the linear or BD designs. The required transmit power will be reduced and a better BER performance can be achieved by the proposed LC-RBD-LR algorithm.

The received signal is finally obtained as

$$\boldsymbol{y} = \beta^{-1}(\boldsymbol{H}\tilde{\boldsymbol{P}}\boldsymbol{s} + \boldsymbol{n}). \tag{12}$$

The mainly processing work left for the receiver is to quantize the received signal $\boldsymbol{y}$ to the nearest transmitted symbols.

IV. Computational Complexity Analysis

In this section we use the total number of FLOPs to measure the computational complexity of the proposed and existing algorithms. According to [10], the average complexity of the CLR algorithm is almost 1.6 times of the QR decomposition. FLOPs for real QR, SVD and complex QR decomposition are given in [9]. In real arithmetic, a multiply followed by an addition needs 2 FLOPs. With complex-valued quantities, a multiplication followed by an addition needs 8 FLOPs. Thus, the complexity of a complex matrix multiplication is nearly 4 times its real counterpart. For a complex $m \times n$ matrix $\boldsymbol{A}$, its SVD is given by $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^H$, where $\boldsymbol{U}$ and $\boldsymbol{V}$ are unitary matrices and $\boldsymbol{\Sigma}$ is a diagonal matrix containing the singular values of matrix $\boldsymbol{A}$. Rewriting this formulation, we have

$$\begin{bmatrix} \boldsymbol{A}_r & \boldsymbol{A}_i \\ -\boldsymbol{A}_i & \boldsymbol{A}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{U}_r & \boldsymbol{U}_i \\ \boldsymbol{U}_i & -\boldsymbol{U}_r \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma} & 0 \\ 0 & \boldsymbol{\Sigma} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_r^T & \boldsymbol{V}_i^T \\ \boldsymbol{V}_i^T & -\boldsymbol{V}_r^T \end{bmatrix}. \tag{13}$$

From (13), the number of FLOPs required by a $m \times n$ complex SVD is equivalent to the complexity required by its extended $2m \times 2n$ real matrix. We summarize the total FLOPs needed for the matrix operations below:

TABLE I
COMPUTATIONAL COMPLEXITY OF PROPOSED LC-RBD-LR ALGORITHM

Steps	Operations	Flops	Case $(2,2,2)\times 6$
1	QR($\overline{\overline{\boldsymbol{H}}}_i^H$)	$16K(N_T^2\overline{N}_i + N_T\overline{N}_i^2 + \frac{1}{3}\overline{N}_i^3)$	12544
2	$\boldsymbol{H}_i\boldsymbol{P}_i^a$	$8N_RN_T^2$	1728
3_{ZF}	CLR$(\boldsymbol{H}_{\text{eff}_i}^T)^T$	$25.6K(N_T^2N_i - N_TN_i^2 + \frac{1}{3}N_i^3)$	3891
3_{MMSE}	CLR$(\underline{\boldsymbol{H}}_{\text{eff}_i}^T)^T$	$25.6K(N_T^2N_i + N_TN_i^2 + \frac{1}{3}N_i^3)$	7578
4_{ZF}	$\tilde{\boldsymbol{H}}_{\text{eff}_i}^H(\tilde{\boldsymbol{H}}_{\text{eff}_i}\tilde{\boldsymbol{H}}_{\text{eff}_i}^H)^{-1}$	$K(2N_i^3 - 2N_i^2 + N_i + 16N_TN_i^2)$	1182 Total 19345
4_{MMSE}	$\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}^H(\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}\underline{\tilde{\boldsymbol{H}}}_{\text{eff}_i}^H)^{-1}$	$K(18N_i^3 - 2N_i^2 + N_i + 16N_TN_i^2)$	1566 Total 23416

TABLE II
COMPUTATIONAL COMPLEXITY OF CONVENTIONAL RBD

Steps	Operations	Flops	Case $(2,2,2)\times 6$
1	$\boldsymbol{U}_i^a\boldsymbol{\Sigma}_i^a\boldsymbol{V}_i^{a\,H}$	$32K(N_T\overline{N}_i^2 + 2\overline{N}_i^3)$	21504
2	$(\boldsymbol{\Sigma}_i^{a\,T}\boldsymbol{\Sigma}_i^a + \rho^2\boldsymbol{I}_T)^{-\frac{1}{2}}$	$K(18N_T + \overline{N}_i)$	336
3	$\boldsymbol{V}_i^a\boldsymbol{D}_i^a, (\boldsymbol{D}_i^a \leftarrow 2)$	$8KN_T^3$	5184
4	$\boldsymbol{H}_i\boldsymbol{P}_i^a$	$8N_RN_T^2$	1728
5	$\boldsymbol{U}_i^b\boldsymbol{\Sigma}_i^b\boldsymbol{V}_i^{b\,H}$	$64K(\frac{9}{8}N_i^3 + N_TN_i^2 + \frac{1}{2}N_T^2N_i)$	13248 Total 42000

TABLE III
COMPUTATIONAL COMPLEXITY OF QR/SVD RBD [8]

Steps	Operations	Flops	Case $(2,2,2)\times 6$
1	$\overline{\overline{\boldsymbol{H}}}_i^H = \boldsymbol{Q}_i\boldsymbol{R}_i$	$16K(N_T^2\overline{N}_i + N_T\overline{N}_i^2 + \frac{1}{3}\overline{N}_i^3)$	12544
2	$\boldsymbol{H}_{\text{eff}_i} = \boldsymbol{H}_i\boldsymbol{P}_i^a$	$8N_RN_T^2$	1728
3	$\boldsymbol{H}_{\text{eff}_i} = \boldsymbol{U}_i^b\boldsymbol{\Sigma}_i^b\boldsymbol{V}_i^{b\,H}$	$64K(\frac{9}{8}N_i^3 + N_TN_i^2 + \frac{1}{2}N_T^2N_i)$	13248 Total 27520

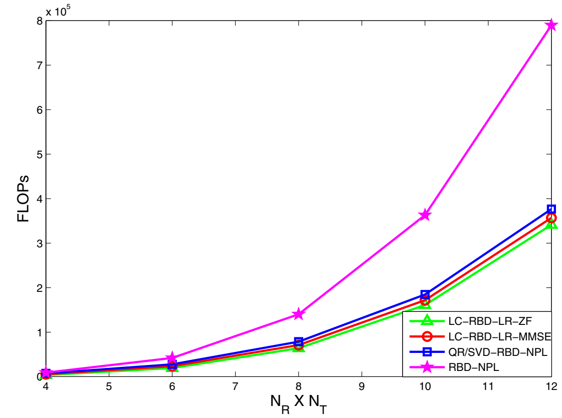

Fig. 1. Computational complexity in FLOPs for MU-MIMO systems.

- Multiplication of $m \times n$ and $n \times p$ complex matrices: $8mnp$;
- QR decomposition of an $m\times n$ $(m \le n)$ complex matrix: $16(n^2m - nm^2 + \frac{1}{3}m^3)$;
- SVD of an $m \times n$ $(m \le n)$ complex matrix where only $\boldsymbol{\Sigma}$ and $\boldsymbol{V}$ are obtained: $32(nm^2 + 2m^3)$;
- SVD of an $m \times n$ $(m \le n)$ complex matrix where $\boldsymbol{U}$, $\boldsymbol{\Sigma}$ and $\boldsymbol{V}$ are obtained: $8(4n^2m + 8nm^2 + 9m^3)$;
- Inversion of an $m \times m$ real matrix: $2m^3 - 2m^2 + m$.

For the case shown in Table I, Table II and Table III, the complexity of the proposed LC-RBD-LR-ZF is about 46.1% of RBD and 70.3% of the QR/SVD RBD, while the complexity of the proposed LC-RBD-LR-MMSE is about 55.8% of RBD and 85.1% of the QR/SVD RBD. Clearly, the proposed algorithm requires the lowest complexity.

The required number of FLOPs of the proposed and existing algorithms are simulated for different system dimensions and the results are displayed in Fig.1. The simulations are implemented in Matlab and we average the curves over 100 independent trials. Moreover, we assume that each user is equipped with $N_i = 2$ antennas and the number of users K is set to make the total number of receive antennas N_R equal to the number of transmit antennas N_T. From Fig.1., it is clear that the proposed LC-RBD-LR algorithms show a lower computational complexity than that of the RBD and QR/SVD RBD algorithms. It is worth noting that with the increase of the system dimension, the complexity reduction becomes more considerable.

V. SIMULATION RESULTS

A system with $N_T = 6$ transmit antennas and $K = 3$ users each equipped with $N_i = 2$ receive antennas is considered; this scenario is denoted as $(2,2,2)\times 6$ case.

The transmitted signal $\boldsymbol{s}_i$ of the ith user employs QPSK modulation. The channel matrix $\boldsymbol{H}_i$ of the ith user is modeled as a complex Gaussian channel matrix with zero mean and unit variance. We assume an uncorrelated block fading channel, that is, the channel is static during each transmit packet and there is no correlation between the antennas. We also assume that the channel estimation is perfect at the receive side and the feedback channel is error free. For simplicity we do not consider the power loading between users and streams and we term this strategy as no power loading (NPL). The number of simulation trials is 10^6 and the packet length is 10^2 symbols. The E_b/N_0 is defined as $E_b/N_0 = \frac{N_R E_s}{N_T M N_0}$ with M being the number of transmitted information bits per channel symbol.

Fig. 2. shows the BER performance of the proposed and existing algorithms. It is clear that the proposed algorithm has a better performance compared to the BD, RBD and QR/SVD RBD algorithms. The QR/SVD RBD has the same BER performance as the RBD. At the BER of 10^{-2}, LC-RBD-LR-ZF has a gain of more than 6 dB compared to the RBD, whereas LC-RBD-LR-MMSE has a gain of more than 7 dB over RBD. It is worth noting that the BER performance of RBD is outperformed by the proposed LC-RBD-LR-MMSE in the whole E_b/N_0 range and the improved BER gains become more significant with the increase of E_b/N_0.

Fig. 3. illustrates the sum-rate of the proposed and existing algorithms. The information rate is calculated using [14]:

$$C = \log(\det(\boldsymbol{I} + \sigma_n^{-2}\boldsymbol{H}\boldsymbol{P}\boldsymbol{P}^H\boldsymbol{H}^H)) \quad \text{(bits/Hz)}. \qquad (14)$$

From Fig. 3., the proposed LC-RBD-LR-MMSE has the same sum-rate as RBD at low E_b/N_0s. At high E_b/N_0s, it is

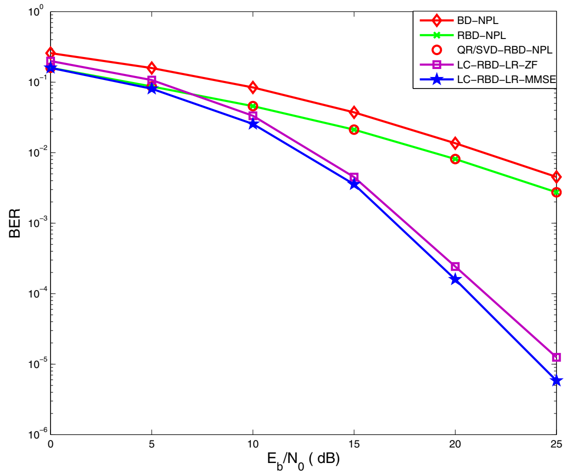

Fig. 2. BER performance, $(2, 2, 2) \times 6$ MU-MIMO.

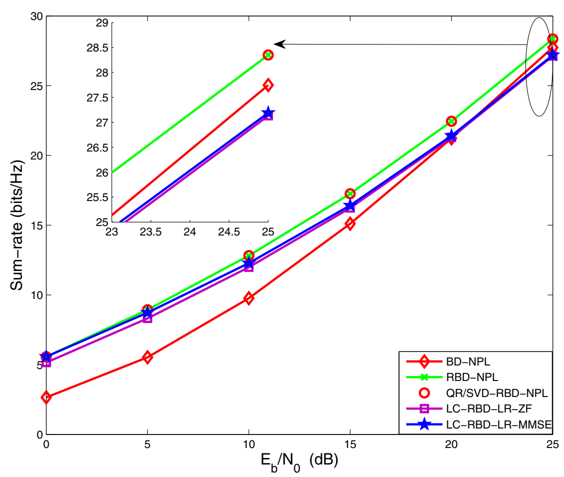

Fig. 3. Sum-rate performance, $(2, 2, 2) \times 6$ MU-MIMO.

slightly inferior to the RBD but requires a lower computational complexity.

VI. CONCLUSION

In this paper, a low-complexity precoding algorithm for MU-MIMO systems has been proposed. The complexity of the precoding process is reduced and a considerable BER gain is achieved by the proposed LC-RBD-LR algorithm at a cost of a slight sum-rate loss at high SNRs. The computational complexity of the proposed LC-RBD-LR algorithm is analyzed and compared to existing algorithms. It is worth noting that the receiver is simplified by employing the proposed LC-RBD-LR algorithm at the transmit side as there is no need for an SVD at the receiver.

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