Low-Complexity OFDMA Channel Allocation With Nash Bargaining Solution Fairness

Zhu Han, Zhu Ji, and K. J. Ray Liu
Electrical and Computer Engineering Department , University of Maryland, College Park

***Abstract*— A fair and simple scheme to allocate subcarrier, rate, and power for multiuser OFDMA systems is considered. The problem is to maximize the overall system rate, under each user's maximal power and minimal rate constraints, while considering the fairness among users. The approach proposes the fairness and low complexity implementation based on Nash Bargaining Solutions and coalitions. First, a two-user algorithm is developed to bargain subcarrier usage between both users. Based on this algorithm, we develop a multiuser bargaining algorithm where optimal coalition pairs among users are constructed. Simulation results show that the proposed algorithms not only provide fair resource allocation among users, but also have comparable overall system rate with the scheme maximizing the total rate without considering fairness. They also have much higher rates than the scheme with max-min fairness. The proposed algorithms have complexity $O(N \log N)$, where N is the number of subcarriers.**

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is a promising multi-access technique for high data rate transmissions over wireless radio channels. Efficient resource allocation, which involves bit loading, transmission power control, and subcarrier assignment, can greatly improve system performance and so draw a great attention in recent researches.

Most of previous resource allocation approaches [1]- [7] study how to efficiently maximize the total transmission rate or minimize the total transmitted power under some constraints. The formulated problem and their solutions are focused on the efficiency issue. But these approaches benefit the users closer to the base station or with a higher power capability. The fairness issue has been mostly ignored. On the other hand, as for the fairness among users, max-min criterion has been considered for channel allocation in multiuser OFDM systems [4]. However, by using this criterion, it is not easy to take into account of the notions that users might have different requirements. Moreover, since the max-min approach deals with the worst case scenario, it penalizes users with better channels and reduces the system efficiency. In addition, most of the existing solutions have high complexities, which prohibit them from practical implementation. Therefore, it is necessary to develop an approach that considers altogether the fairness of resource allocation, system efficiency, and complexity.

In daily life, a market is served as a central gathering point, where people can exchange goods and negotiate transactions, so that people will be satisfied through bargaining. Similarly, in single cell multiuser OFDMA systems, there is a base station that can serve as a function of the market. The distributed users can negotiate via base station to cooperate in making the decisions on the subcarrier usage, such that each of them will operate at its optimum and joint agreements are made about their operating points. Such a fact motivates us to apply the cooperative game theory [8], [9], [11], [16], which can achieve the crucial notion of fairness and maximize the overall system rate. The concepts of Nash Bargaining Solution (NBS) and coalitions are taken into consideration, because they provide a fair operation point in a distributed implementation.

Motivated by the above reasons, we apply the cooperative game theory for resource allocation in OFDMA systems. The goal is to maximize the overall system rate, under the constraints of each user's minimal rate requirement and maximal transmitted power. First we develop a fast two-user bargaining algorithm to negotiate the usage of subcarriers. The approach is based on NBS which maximizes the system performance while keeping the NBS fairness. Then we group the users into groups of size two, which is defined as a coalition. Within each coalition, we use two-user algorithm to improve the performance. In the next iteration, new coalitions are formed and subcarrier allocation is optimized until no improvement can be obtained. By using Hungarian method, optimal coalitions are formed and the number of iterations can be greatly reduced. A significant point for the proposed algorithm is that the complexity is only $O(N \log N)$. Moreover, this approach can also be applied to other formulated problems dealing with multiuser communications with different optimization goals and constraints. From the simulation results, the proposed algorithms allocate resources fairly and efficiently compared to the other two schemes: maximal rate and max-min fairness.

This paper is organized as follows: In Section II, the system model is given. In Section III, basics for NBS of cooperative game theory are presented. In Section IV, the optimization problem is formulated. A two-user algorithm and a multiuser algorithm are constructed. In Section V, simulations are developed. In Section VI, conclusions are drawn.

II. SYSTEM MODEL AND DESCRIPTION

Consider an uplink scenario of a single cell multiuser OFDMA system. There are totally K users randomly located within the cell. The users want to share their transmissions among N different subcarriers. Each subcarrier has a bandwidth of W. The i^{th} user's transmission rate is R_i and is allocated to different subcarriers as $R_i = \sum_{j=1}^{N} r_{ij}$, where r_{ij} is the i^{th} user's transmission rate in the j^{th} subcarrier. Define the rate allocation matrix $\mathbf{r}$ with $[\mathbf{r}]_{ij} = r_{ij}$. Define the subcarrier assignment matrix $[\mathbf{A}]_{ij} = a_{ij}$, where $a_{ij} = 1, \text{if } r_{ij} > 0; a_{ij} = 0, \text{otherwise.}$. For single cell OFDMA, no subcarrier can support the transmissions for more than one user, i.e., $\sum_{i=1}^{K} a_{ij} = 1, \forall j$.

Adaptive modulation provides each user with the ability to match each subcarrier's transmission rate r_{ij}, according to its channel condition. MQAM is a modulation method with a high spectrum efficiency, which is adopted in our system without loss of generality. In [13], bit error rate (BER) of

 0-7803-8794-5/04/$20.00 © 2004 IEEE

MQAM as a function of rate and signal to noise ratio (SNR) is approximated by $\text{BER}_{ij} \approx c_1 e^{-c_2 \frac{\Gamma_{ij}}{2^{r_{ij}}-1}}$, where $c_1 \approx 0.2$, $c_2 \approx 1.5$, and Γ_{ij} is the i^{th} user's SNR at the j^{th} subcarrier:

$$\Gamma_{ij} = \frac{P_{ij} G_{ij}}{\sigma^2} \tag{1}$$

where G_{ij} is the subcarrier channel gain and P_{ij} is the transmitted power for the i^{th} user in the j^{th} subcarrier. The thermal noise power for each subcarrier is assumed to be the same and equal to σ^2. Define power allocation matrix $[\mathbf{P}]_{ij} = P_{ij}$. Without loss of generality, we assume a fixed and the same BER for all users in all subcarriers. We have

$$r_{ij} = W \log_2 \left(1 + \frac{P_{ij} G_{ij} c_3}{\sigma^2}\right) \tag{2}$$

where $c_3 = c_2 / \ln(c_1/\text{BER})$ with $\text{BER} = \text{BER}_{ij}, \forall i, j$.

We assume the slow fading channel such that the channel is stable within each OFDM frame. The channel conditions of different subcarriers for each user are assumed perfectly estimated. There exist reliable feedback channels from base station to mobile users without any delay. Moreover for a practical system, the OFDM frequency offset between the mobile user and the base station is around several tenth Hz. The inter-carrier-interference caused by the frequency offset may cause some error floor increase. However this is not the bottleneck limiting the system performance and this offset can be feed back to the mobile for adjustment. In [12], guard subcarrier is put at the edge of each subcarrier such that multiple access interference can be minimized and synchronized algorithm is applicable for each subcarrier. So in this paper, we assume mobiles and base station are synchronized.

III. Basics for Nash Bargaining Solution

The bargaining problem of cooperative game theory can be described as follows [8], [9], [11]: Let $\mathbf{K} = \{1, 2, \ldots, K\}$ be the set of players. Let $\mathbf{S}$ be a closed and convex subset of $\Re^K$ to represent the set of feasible payoff allocations that the players can get if they all work together. Let R^i_{min} be the minimal payoff that the i^{th} player would expect; otherwise, he will not cooperate. Suppose $\{R_i \in \mathbf{S} | R_i \geq R^i_{min}, \forall i \in \mathbf{K}\}$ is a nonempty bounded set. Define $\mathbf{R}_{min} = (R^1_{min}, \ldots, R^K_{min})$, the pair $(\mathbf{S}, \mathbf{R}_{min})$ is called a K-person bargaining problem.

Within the feasible set $\mathbf{S}$, we define the notion of Pareto optimal as a selection criterion for the bargaining solutions.

Definition 1: The point $(R_1, \ldots R_K)$ is said to be **Pareto optimal**, if and only if there is no other allocation R'_i such that $R'_i \geq R_i, \forall i$, and $R'_i > R_i, \exists i$, i.e. there exists no other allocation that leads to superior performance for some users without inferior performance for some other users.

There might be infinite number of Pareto optimal points. We need further criteria to select a bargaining result. A possible criterion is the fairness. One commonly used fairness criterion is max-min [4], where the performance of the user with worst channel conditions is maximized. This criterion penalizes the users with good channels and as a result generates inferior overall system performance. In this paper, we use the criterion of fairness NBS. The intuitive idea is that, after the minimal requirements are satisfied for all users, the rest resources are allocated proportionally to users according to their conditions. We will discuss the *proportional fairness* concept which is a special case of NBS fairness in the next section and show the fair results in the simulation section. There exist many kinds of cooperative game solutions [11]. Among them, NBS provides a unique and fair Pareto optimal operation point under the following conditions. NBS is briefly explained as follows:

Definition 2: $\bar{\mathbf{r}}$ is said to be a **Nash Bargaining Solution** [1] in $\mathbf{S}$ for $\mathbf{R}_{min}$, i.e., $\bar{\mathbf{r}} = \phi(\mathbf{S}, \mathbf{R}_{min})$, if the following Axioms are satisfied: *Individual Rationality*, *Feasibility*, *Pareto Optimality*, *Independence of Irrelevant Alternatives*, *Independence of Linear Transformations*, and *Symmetry* [11]

Theorem 1: **Existence and Uniqueness of NBS**: There is a unique solution function $\phi(\mathbf{S}, \mathbf{R}_{min})$ that satisfies all six axioms in Definition 1. And this solution satisfies [11]

$$\phi(\mathbf{S}, \mathbf{R}_{min}) \in \arg \max_{\bar{\mathbf{r}} \in \mathbf{S}, \bar{R}_i \geq R^i_{min}, \forall i} \prod_{i=1}^{K} \left(\bar{R}_i - R^i_{min}\right). \tag{3}$$

As discussed above, the cooperative game in the multiuser OFDMA system can be defined as follows: Each user has R_i as its objective function, where R_i is bounded above and have a nonempty, closed, and convex support. The goal is to maximize all R_i simultaneously. $\mathbf{R}_{min}$ represents the minimal performance and is called the initial agreement point. Define $\mathbf{S}$ as the feasible set of rate allocation matrix $\mathbf{r}$ that satisfies $R_i \geq R^i_{min}, \forall i$. The problem, then, is to find a simple way to choose the operating point in $\mathbf{S}$ for all users, such that this point is optimal and fair.

IV. Cooperative Game Approaches

A. Problem Formulation

Since a channel for a specific subcarrier may be good for more than one user, there is a competition among users for their transmissions over the subcarriers with large G_{ij}. Moreover the maximal transmitted power for each user is bounded by the maximal transmitted power P_{max} and each user has a minimal rate requirement R^i_{min} if it is admitted to the system. In this paper, the optimization goal is to determine different users' channel assignment matrix $\mathbf{A}$ and power matrix $\mathbf{P}$ such that the cost function will be maximized, i.e.,

$$\max_{\mathbf{A},\mathbf{P}} U \tag{4}$$

$$\text{subject to} \begin{cases} \sum_{i=1}^{K} a_{ij} = 1, \forall j; \\ R_i \geq R^i_{min}, \forall i; \\ \sum_{j=1}^{N} P_{ij} \leq P_{max}, \forall i. \end{cases}$$

where U can have three definitions in terms of the objectives:

$$\text{Maximal Rate: } U = \sum_{i=1}^{N} R_i, \tag{5}$$

$$\text{Max-min Fairness: } U = \min R_i, \tag{6}$$

$$\text{Nash Bargaining Solutions: } U = \prod_{i=1}^{K} \left(R_i - R^i_{min}\right). \tag{7}$$

For maximal rate optimization, the overall system rate is maximized. For max-min fairness optimization, the worst case

[1]Because of length limitation of the paper, the detailed descriptions of the definition are omitted.

situation is optimized with strict fairness. In this paper, we proposed the Nash Bargaining Solutions with the following two reasons. First, it will be shown later that this form will ensure fairness of allocation in the sense that NBS fairness is a generalized proportional fairness. Second, cooperative game theories prove that there exists a unique and efficient solution under the six axioms. The difficulty to solve (4) by traditional methods lies in the fact that the problem itself is a constrained combinatorial problem and the constraints are nonlinear. Thus the complexities of the traditional schemes are high especially with large number of users. Moreover, distributed algorithms are desired for uplink OFDMA systems, while centralized schemes are dominant in literature. In addition, most of the existing work doesn't discuss the issue of fairness. We will use the bargaining concept to develop simple and distributed algorithms that can achieve an efficient and fair resource allocation in the rest of this section.

We will show in the following definition and theorem that proportional fairness [10], which is widely used in wired networks, is a special case of the fairness provided by NBS.

Definition 3: We call the rate distribution is proportionally fair, when any change in the distribution of rates will result in the sum of the proportional changes of the utilities to be non-positive, i.e.,

$$\sum_i \frac{R_i - \tilde{R}_i}{\tilde{R}_i} \leq 0, \ \forall R_i \in \mathbf{S}. \tag{8}$$

where $\tilde{R}_i$ and R_i are the proportionally fair rate distribution and any other feasible rate distribution for the i^{th} user, respectively.

Theorem 2: When $R_{min}^i = 0, \forall i$, the NBS fairness is the same as the proportional fairness.

Proof: Since the function of $\ln$ is concave and monotonic, the NBS in (3) is equivalent to

$$\max_{\mathbf{r} \in \mathbf{S}} \sum_{i=1}^{K} \ln\left(R_i\right), \tag{9}$$

when $R_{min}^i = 0, \forall i$. Define $\hat{U}_i = \ln(R_i)$. As shown in [15], the following optimality condition holds for all feasible $R_i \in \mathbf{S}, \forall i$:

$$\sum_i \frac{\partial \hat{U}_i}{\partial R_i}|_{\tilde{R}_i}(R_i - \tilde{R}_i) \leq 0. \tag{10}$$

The above equation is the same as the proportional fairness definition in (8). So the proportionally fair is a special case of NBS fairness when $R_{min}^i = 0, \forall i$. Since minimal rate requirement is desired in practice, we apply NBS fairness in this paper. **QED**

Next, we want to demonstrate that there exists a unique and optimal solution in (4), when the feasible set satisfying the constraints is not empty. We show the uniqueness and optimality in two steps. First, we prove the uniqueness and optimality with fixed channel assignment matrix $\mathbf{A}$. Then, we prove that the probability that there exists more than one optimum is zero for different channel assignment matrix $\mathbf{A}$.

First, under fixed channel assignment matrix $\mathbf{A}$, each user tries to maximize its own rate under the power constraint independently, because any subcarrier is not shared by more than one users. This is similar to the single user case. Since we assume the feasible set that satisfies the constraints in (4) is not empty. Within the feasible set, each user can get its minimal rate requirement R_{min}^i by allocating its power to the assigned channel set. For all three cost functions in (5), (6), and (7), the problem in (4) is reduced to the following problem:

$$\max_{\mathbf{P}} U_i = R_i \tag{11}$$

$$\text{subject to } \sum_{j=1}^{N} P_{ij} a_{ij} \leq P_{max}, \forall i.$$

Obviously, the above problem is a water filling problem [17] and has a unique optimal solution. Define

$$I_{ij} = \begin{cases} \frac{\sigma^2}{c_3 G_{ij}}, & \text{if } a_{ij} = 1; \\ \infty, & \text{otherwise.} \end{cases} \tag{12}$$

The unique optimal solution is

$$P_{ij} = (\mu_i - I_{ij})^+ \text{ and } r_{ij} = W \log_2(1 + \frac{P_{ij} a_{ij}}{I_{ij}}), \tag{13}$$

where $y^+ = \max(y, 0)$. Here μ_i is the water level and can be solved by bisection search of $\sum_{j=1}^{N} P_{ij} a_{ij} = P_{max}$.

We have proved the optimality and uniqueness with fixed channel assignment. The channel assignment is a combinatorial problem with finite number of combinations. For example, the total number of combinations for the system with K users and N subcarriers is K^N. So, we can obtain the optimal solution by solving the following problem

$$\arg\max_{\mathbf{A}} U, \tag{14}$$

where U is obtained by solving (11) with respect to each $\mathbf{A}$.

The above problem can be solved by means of full search to get the optimal channel assignment and power allocation. Because the optimization goal U, the channel gains, and the rates are continuous random values, it has zero probability to have two channel assignment matrices that generate the same optimization goal. So there exists a unique channel assignment matrix that generates the optimal solution in (4).

B. Bargaining Algorithm for Two-user Case

In this subsection, we consider the case when $K = 2$ and we will develop a fast two-user bargaining algorithm. Similar to bargaining in a real market, the intuitive idea to solve the two-user problem is to allow two users to negotiate and exchange their subcarriers such that mutual benefits will be obtained. The difficulty is to determine how to optimally exchange subcarriers, which is a complex integer programming problem. An interesting low complexity algorithm was given in [3]. The idea is to sort the order of subcarriers first and then to use a simple two-band partition for the subcarrier assignment. When SNR is high, the two-band partition for two-user subcarrier assignment is near optimal for the optimization goal of maximizing the weighted sum of both users' rates.

We propose a fast algorithm between two users for the optimization goals by exchanging their subcarriers as shown

TABLE I: Two-user Algorithm

1. Initialization: Initialize subcarrier assignment with minimal rate requirements. For *Maximal Rate*, $\varrho_1 = \varrho_2 = 1$; For *NBS*, calculate $\varrho_i = 1/(R_i - R^i_{min})$. If $R_i = R^i_{min}$, assign a big number to ϱ_i.
2. Sort the subcarriers: Arrange index from largest to smallest $\frac{g_{1j}^{\varrho_1}}{g_{2j}^{\varrho_2}}$.
3. For j=1,. . . , N-1 User 1 occupies and water-fills subcarrier 1 to j; User 2 occupies and water-fills subcarrier j+1 to N. Waterfill both users to assigned subcarrier sets. Calculate U. **End**
4. Choose the two-band partition (the corresponding j) that generates the largest U satisfing the constraints. Calculate **A**, **P**, R_1, and R_2.
5. Update channel assignment -*Maximal Rate*: Return -*NBS*: If U can not be increased by updating ϱ_i, exit; otherwise, update ϱ_i; go to Step 2.

in Table I. First all subcarriers are initially assigned. Any heuristical or greedy approaches can be applied to have subcarriers assigned while the system is still feasible. Then, two users' subcarriers are sorted and a two-band partition algorithm is applied for them to negotiate the subcarriers. For maximal rate optimization goal, only one iteration is necessary. For NBS optimization goal, an intermediate parameter needs to be updated for every iteration. From the simulations, the iterations between Step 2 and Step 5 is converged within 2 to 3 rounds. The algorithm has the complexity of $O(N^2)$ and can be further improved by using a binary search algorithm with a complexity of only $O(N \log N)$.

Theorem 3: The algorithm in Table 1 is near optimal for both the problem of maximal rate and NBS goals in (4) with the number of users equal to two, when SNR of each subcarrier for all users in (1) is much greater than 1 [20].

C. Multiple-User Algorithm Using Coalitions

For the case where the number of users is larger than two, most work in literature concentrates on solving the OFDMA resource allocation problem for all users together in a centralized way [1]- [7]. Because the problem itself is combinatorial and nonlinear, the computational complexity is very high with respect to the number of subcarriers by the existing methods [1]- [7]. In this paper, we propose a two-step iterative scheme: First, users are grouped into pairs, which are called coalitions. Then for each coalition, the algorithm in Table 1 is applied for two users to negotiate and improve their performances by exchanging subcarrier sets. Further, the users are regrouped and renegotiates again and again until convergence. By using this scheme, the computational cost can be greatly reduced. First, we give the strict definition of coalition as follows.

Definition 4: For a K-person game, any nonempty subset of the set of players is called a **coalition**.

The question now is how to group users into coalitions with size 2. A straightforward algorithm is to form the coalition randomly and let the users bargain arbitrarily. We call this algorithm *random method*, which can be described by the steps in Table II. During the initialization, the goal is to assign all subcarriers to users and try to satisfy the minimal rate and maximal power constraint. We develop a fast algorithm. Starting from the user with the best channel conditions, if the user has rate larger than or equal to R^i_{min}, it is removed from the assignment list. After every user has enough rate, the rest of subcarriers are greedily assigned to the users according to their channel gains. Note that there is no need for the initial assignment to satisfy all constraints. The constraints will be satisfied during the iterations of negotiations.

We quantify the convergence speed by the round of negotiations. The convergence speed of the random method becomes slow with the number of users increasing. This is because the negotiations within arbitrarily grouped coalitions are less effective and most negotiations turn out to be the same as or little improvement than the performance of the channel allocation before the negotiations. So the optimal cooperation grouping among subsets of the users should be taken into consideration. In order to speed up convergence, each user needs to carefully select who it should negotiate with.

Each user's channel gains are varying over different subcarriers. A user may be preferred by many users to form coalitions, while only two-user coalition is allowed. Thus, the problem to decide the coalition pairs can be stated as an assignment problem [14]: "a special structured linear programming which is concerned with optimally assigning individuals to activities, assuming that each individual has an associated value describing its suitability to execute that specific activity".

Now, we formulate the assignment problem in details. Define a $K \times K$ assignment table **X**. Each component represents whether or not there is a coalition between two users. $X_{ij} = 1,$ if user i negotiates with user j; $X_{ij} = 0,$ otherwise. Obviously matrix **X** is symmetric, $\sum_{i=1}^{K} X_{ij} = 1, \forall j$, and $\sum_{j=1}^{K} X_{ij} = 1, \forall i$.

Define the benefit for the i^{th} user to negotiate with the j^{th} user as b_{ij}. Obviously $b_{ii} = 0, \forall i$. For the other cases, from (4), each element of the cost table **b** can be expressed as:

$$b_{ij} = \max(U(\tilde{R}_i, \tilde{R}_j) - U(\hat{R}_i, \hat{R}_j), 0), \tag{15}$$

where $\tilde{R}_i$ and $\tilde{R}_j$ are the rates if the negotiation happens, and $\hat{R}_i$ and $\hat{R}_j$ are the original rates, respectively. Obviously **b** is also symmetric.

So the assignment problem is how to select the pairs of negotiations such that the overall benefit will be maximized, which is stated as:

$$\max_{\mathbf{X}} \sum_{i=1}^{K} \sum_{j=1}^{K} X_{ij} b_{ij} \tag{16}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{K} X_{ij} = 1 & j = 1 \ldots K, \forall i; \\ \sum_{j=1}^{K} X_{ij} = 1 & i = 1 \ldots K, \forall j; \\ X_{ij} \in \{0, 1\} & \forall i, j. \end{cases}$$

One of the solutions for (16) is Hungarian method [14] which can always find the optimal coalition pairs. In each round, the optimal coalition pairs are determined by Hungarian method and then the users are set to bargain together using the two-user algorithm in Table I. The whole algorithm stops when no bargaining can further improve the performance, i.e., **b** is equal to a zero matrix. Based on the above explanations,

TABLE II: Multiuser Algorithm

1. Initialize the channel assignment: Assign all subcarriers to users.
2. Coalition Grouping: If the number of users is even, the users are grouped into coalitions; otherwise, a dummy user is to make the total number of users even. No user can exchange its resource with this dummy user. - *Random Method*: Randomly form 2-user coalition. - *Hungarian Method*: Form user coalitions by Hungarian algorithm.
3. Bargain within Each Coalition: Negotiate between two users in all coalitions to exchange subcarriers using the two-user algorithm in Table I.
4. Continue: Repeat Step 2 and Step 3, until no improvement can be achieved.

we develop the multi-user resource allocation in the multiuser OFDMA systems in Table II.

In each iteration, the optimization function U is nondecreasing in Step 2 and Step 3 and the optimal solution is upper bounded. Consequently the proposed multiuser algorithm is convergent. The complexity of Hungarian method is $O(K^4)$. Since the number of users is much less than the number of subcarriers, the complexity of the proposed algorithm is much lower than the schemes that apply Hungarian method directly to the subcarrier domain [5], [6]. Since the channel responses for each user over different subcarriers are known in the base station, the random method can be implemented in a distributed manner, while the Hungarian method needs some limited centralized control within base station. The signaling overhead for such schemes are negligible because the channel responses are obtained by the base station anyway and the information for channel assignment matrix only cost N bit per user.

V. Simulation Results

In order to evaluate the performance of the proposed schemes, we consider a two-user and a multiple user simulation setups. Three different optimization goals (maximal rate, max-min, and NBS) are compared.

First, a two-user OFDMA system is taken into consideration. We simulate the OFDMA system with 128 subcarriers over 3.2 MHz band. To make the tones orthogonal to each other, the symbol duration is $40\mu s$. An additional $10\mu s$ guard interval is used to avoid inter-symbol-interference due to channel delay spread. This results in a total block length as $50\mu s$ and a block rate as 20k. The maximal power is $P_{max} = 50$mWatts, and the desired BER is 10^{-2} (without channel coding). The thermal noise level is $\sigma^2 = 10^{-11}$Watts. The propagation loss factor is 3. The distance between user 1 and base station is fixed at $D_1 = 50$m, while D_2 is varying from 10m to 200m. $R^i_{min} = 100\text{Kbps}, \forall i$. Doppler frequency is 100Hz.

To evaluate the performances, we have tested 10^5 sets of frequency selective fading channels, which is simulated using four-ray Rayleigh model [19] with the exponential power profile and $100ns$ root-mean-square (RMS) delay spread.

In Fig. 1, the rates of both users for the NBS, maximal rate, and max-min schemes are shown vs. D_2. For the maximal rate scheme, the user closer to the base station has higher rate and the rate difference is very large when D_1 and D_2 are different. For the max-min scheme, both users have the same rate which is reduced when D_2 is increasing. This is because

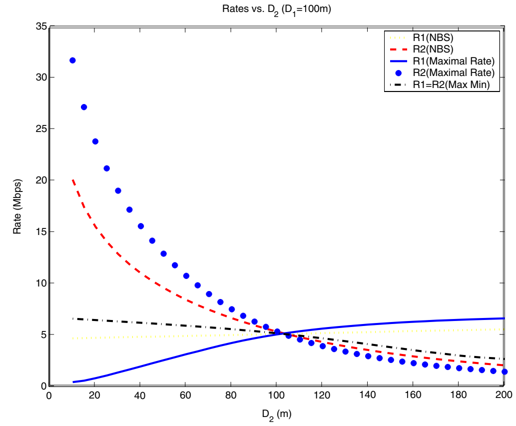

Fig. 1: Each User's Rate (Mbps) vs. D_2: Fairness

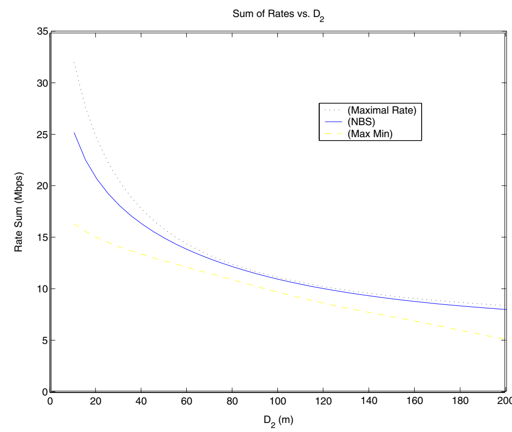

Fig. 2: Total Rate (Mbps) of the Two Users: Efficiency

the system has to accommodate the user with the worst channel condition. While for the NBS scheme, user 1's rate is almost the same regardless D_2 and user 2's rate is reduced when D_2 is increasing. This shows that the NBS algorithm is fair in the sense that the user's rate is determined only by its channel condition and not by other interfering users' conditions.

In Fig. 2, we show the overall rate of two users for three schemes vs D_2. Because the max-min algorithm is for the worst case scenario, it has the worst performance, especially when the two users have the very different channel conditions, because the user with worse channel conditions will limit the usage of the system resources. The NBS scheme has the performance between the maximal rate scheme and max-min scheme, while the maximal rate scheme is extremely unfair. Moreover, the performance loss of NBS scheme to that of the maximal rate scheme is small. As we mentioned before, the NBS scheme maintains the fairness in a way that one user's performance is unchanged to the other user's channel conditions. So the proposed algorithm is a good tradeoff between the fairness and the overall system performance.

We setup the simulations with more users to test the proposed algorithms. All the users are randomly located within the cell of radius 200m. One base station is located in the middle of the cell. Each user has the minimal rate $R^i_{min} =$ 25kbps. The other settings are the same as those of the two-user case simulations.

In Fig. 3, we show the sum of all users' rates vs. the number

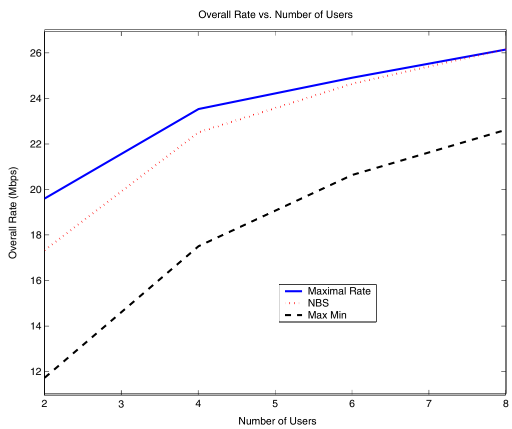

Fig. 3: Overall Rate (Mbps) vs. No. of Users

of users in the system for three schemes. We can see that all three schemes have better performances when the number of users increases. This is because of multiuser diversity, provided by the independent varying channels across the different users. The performance improvement satiates gradually. The NBS scheme has a similar performance to that of the maximal rate scheme and has a much better performance than that of the max-min scheme. The performance gap between the maximal rate scheme and the NBS scheme reduces when the number of users is large. This is because more bargain pair choices are available to increase the system performance.

In Fig. 4, we show the histogram of the number of rounds that is necessary for convergence of the random method and Hungarian method with eight users. Hungarian method converges in about 1 to 6 rounds, while the random method may converge very slowly. The average converge rounds for the random method is 4.25 times to that of Hungarian method. By using Hungarian method, the best negotiation pairs can be found. Consequently, the convergence rate is much quicker and the computation cost is reduced.

VI. Conclusions

In this paper, we use cooperative game theory including NBS and coalitions to develop a fast and fair algorithm for adaptive subcarrier, rate, and power allocation in the uplink multiuser OFDMA systems. The optimization problem takes consideration of fairness and the practical implementation constraints. The proposed algorithm consists of two steps. First a Hungarian method is constructed to determine optimal bargaining pairs among users. Then a fast two-user bargaining algorithm is developed for two users to exchange their resources. The above two steps are taken iteratively for users to negotiate the optimal resource allocation. The approach can also be applied to other optimization goals. The computation complexity can be greatly reduced.

From the simulation results, the proposed algorithm shows similar overall rate to that of the maximal rate scheme and much better performance than that of the max-min scheme. The NBS fairness is demonstrated by the fact that a user's rate is not determined by the interfering users. The proposed algorithm provides a near optimal fast solution and finds a good tradeoff between the overall rate and fairness. The significance of the proposed algorithm is the bargaining and NBS fairness that result in the low complexity of $O(N \log N)$ with fair individual performance and good overall system performance.

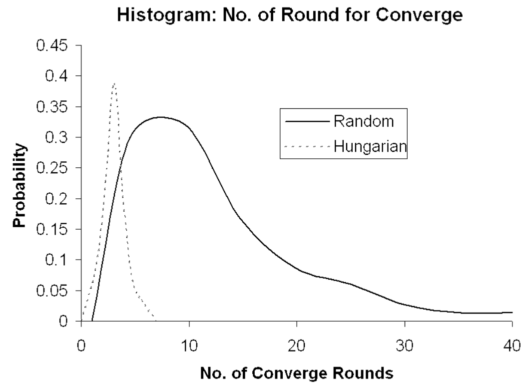

Fig. 4: Histogram for Convergence

References

[1] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE Jour. on Sel. Areas in Comm.*, vol.17, no.10, pp.1747-1758, Oct. 1999.

[2] H. Yin and H. Liu, "An efficient multiuser loading algorithm for OFDM-based broadband wireless systems," *IEEE Globecom Conference*, vol.1, pp. 103-107, 2000.

[3] W. Yu and J. M. Cioffi, "FDMA capacity of Gaussian multi-access channels with ISI," *IEEE Trans. on Comm.*, vol. 50, pp. 102-111, Jan. 2002.

[4] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," *Proc. of IEEE VTC'00*, pp. 1085-1089, 2000.

[5] C. Y. Wong, C. Y. Tsui, R. S. Cheng, and K. B. Letaief, "A real-time sub-carrier allocation scheme for multiple access downlink OFDM transmission," *Proc. of IEEE VTC'99*, pp. 1124-1128, 1999.

[6] S. Pietrzyk and G. J. M. Janssen, "Multiuser subcarrier allocation for QoS provision in the OFDMA systems," *IEEE VTC fall*, Sep. 2002.

[7] T. Keller and L. Hanzo, "Adaptive modulation techniques for duplex OFDM transmission", *IEEE Trans. on Veh. Techn.*, vol.49, no.5, pp.1893-1906, Sep. 2000.

[8] H. Yaiche, R.R. Mazumdar, and C. Rosenberg, "A game theoretic framework for bandwidth allocation and pricing in broadband networks", *IEEE/ACM Trans. on Networking*, vol. 8, no. 5, pp. 667-678, Oct. 2000.

[9] D. Grosu, A. T. Chronopoulos, M.Y. Leung, "Load balancing in distributed systems: an approach using cooperative games", *Proc. of IPDPS 2002*, pp. 52-61, 2002.

[10] F. Kelly, "Charging and rate control for elastic traffic", *European Transactions on Telecommunications*, vol.28, no.1, pp.33-37, 1997.

[11] G. Owen, *Game theory, 3rd ed.* Academic Press, 2001.

[12] J.J. Beek, etc., "A time and frequency synchronization scheme for multiuser OFDM", *IEEE J. Select. Areas Commun.* vol.17, no.11, pp.1900-1914, Nov. 1999.

[13] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view", *IEEE Trans. Commun.* , vol.49, pp.1561-1571, Sep. 2001.

[14] H.W. Kuhn, "The Hungarian method for the assignment problem", *Naval Research Logistics*, Quarterly 2, 1955.

[15] F. P. Kelly, A.K. Maulloo and D.K.H. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability", *Journal of the Operational Research Society*, vol.49, p.p. 237-252, 1998.

[16] R. Mazumdar, L.G. Mason, and C. Douligeris, "Fairness in network optimal flow control: optimality of product forms", *IEEE Trans. on Comm.*, vol.39, pp.775-782, 1991.

[17] T.M. Cover and J.A. Thomas, *Elements of information theory*, Wiley-interscience, 1990.

[18] D. P. Bertsekas, *Nonlinear programming, 2nd edition*, Athena Scienific, 1999.

[19] T. S. Rappaport, *Wireless communications*, Prentice Hall, 1996.

[20] Z. Han, Z. Ji, and K. J. Ray Liu, "Fair low-complexity multiuser channel allocation for OFDMA networks using Nash Bargaining Solutions and coalitions", submitted to *IEEE Transactions on Communications*, 2004.