

# Low energy incomplete fusion: Observation of a significant incomplete fusion fraction at $\ell < \ell_{crit}$

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**Abstract.** To study the low energy incomplete fusion, excitation functions have been measured in the  $^{16}O + ^{165}Ho$  system using a well-established activation technique. The analysis of the present work has been carried out in the frame work of the statistical model code PACE4. The results show that the yields of complete fusion channels agree well with the theoretical predictions of the model code PACE4. However,  $\alpha$ -emitting channels have a significant incomplete fusion fraction even at  $\ell < \ell_{crit}$

## 1 Introduction

At moderate excitation energies the dominating fusion processes are (i) Complete Fusion (CF) and Incomplete Fusion (ICF) [1, 2]. However, in recent years at low projectile energies i.e., near and/or above the Coulomb barrier (CB), the ICF sets in, where the CF is supposed to play a key role to the total fusion cross-section. The distinction between CF and ICF can be made on the basis of driving input angular momenta [3, 4] as suggested by Wilczynski et al. [3] in their sumrule model hypothesis. In this hypothesis they assumed a sharp cut-off approximation, according to which the probability of CF is assumed to be unity for  $\ell = \ell_{crit}$  and expected to be zero for  $\ell > \ell_{crit}$ . However, in recent time several studies have observed the occurrence of ICF processes in the vicinity of Coulomb barrier [5–8]. Yadav et al. [5] has observed the existence of ICF reactions at  $\ell < \ell_{crit}$ . Moreover, in our recent observation [5], we found a diffused boundary in the  $\ell$ -window that may penetrate near the barrier. Different theoretical models [9–12] have been presented for the ICF, but anyhow those are not able to explain the underlying reaction dynamics at energies below 10 MeV/nucleon. Hence, in order to study the ICF reaction dynamics at low incident energies, we have measured the excitation functions (EFs) of evaporated residues in  $^{16}O + ^{165}Ho$  system at energies  $\approx 4$ -7 MeV/nucleon. The present work also evidences the occurrence of a diffused boundary in the  $\ell$ -window for the fusion processes.

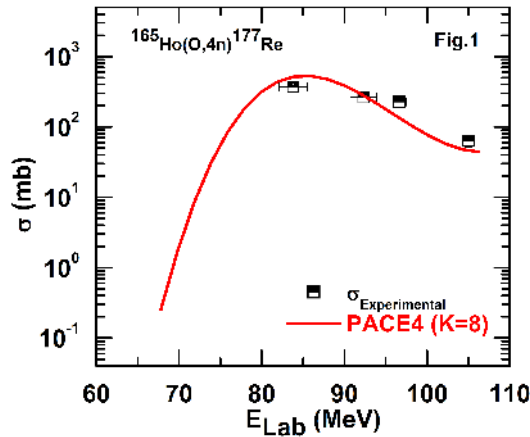
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## 2 Experimental Details

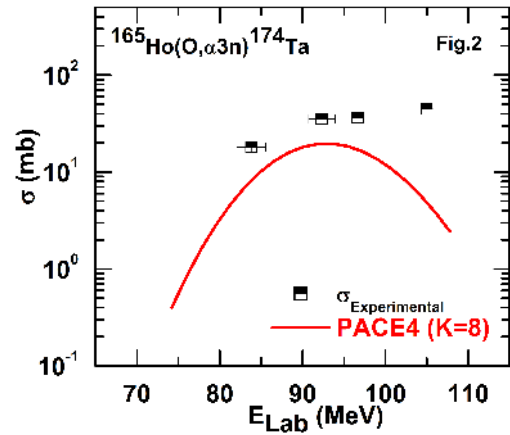
The experiment was done at the Inter University Accelerator Centre (IUAC), New Delhi, India. Stack consisting of five Self supporting  $^{165}Ho$  targets of thickness  $\approx 1.3$ -1.4 mg/cm<sup>2</sup> were prepared by rolling natural holmium. In stack each target was backed with aluminum foils of thicknesses  $\approx 2.1$ -2.15 mg/cm<sup>2</sup>. These foils are used to catch the evaporated residues. The irradiation has been carried out for  $\approx 10$  hours in the General Purpose Scattering Chamber (GPSC) using  $^{16}O$ -beam of 105 MeV energy. The beam current was  $\approx 4$  pA and a Faraday cup has been used to collect the charge. Post irradiation analysis was done using a high resolution, large active volume 100 c.c. pre-calibrated High Purity Germanium (HPGe) detector coupled with a data acquisition system. A  $^{152}Eu$  standard source of known strength was used to determine the efficiency and calibration of the detector. The evaporation residues (ERs) were identified on the basis of measured half-lives and characteristic gamma-ray energies. The spectroscopic properties of evaporated residues are given in Table.1. Further, details of the experimental arrangement, formulation used and error analysis etc. are discussed in detail in Ref. [5]. The overall errors in the present work were estimated to be lying between 12-16% including the statistical errors.

## 3 Results and Discussion

In order to study the low energy ICF reaction dynamics, the EFs of twelve evaporation residues have been measured in the  $^{16}O + ^{165}Ho$  interaction at energies  $\approx 4$ -7 MeV/nucleon. The present analysis has been carried out in the frame work of the statistical model code PACE4



**Figure 1.** Excitation functions of  $^{177}\text{Re}$  ( $4n$ ) reaction expected to be formed via complete fusion.

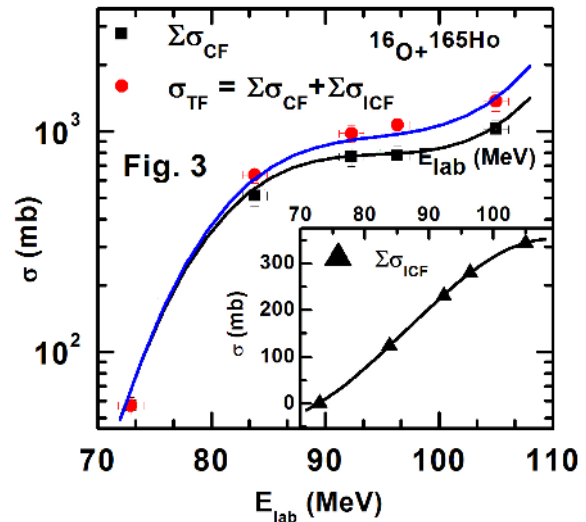


**Figure 2.** Excitation functions of  $^{174}\text{Ta}$  ( $\alpha 3n$ ) reaction expected to be formed via complete as well as incomplete fusion.

**Table 1.** List of reactions with residues and spectroscopic properties produced in  $^{16}\text{O} + ^{165}\text{Ho}$  system via complete and/or incomplete fusion.

Residues	$T_{1/2}$	$J_{\pi}$	$E_{\gamma}$ (keV)	$I^{\gamma}$
$^{178}\text{Re}$ ( $3n$ )	13.2 min	$3^{+}$	237.0 939.1	45.0 8.9
$^{177}\text{Re}$ ( $4n$ )	14.0 min	$5/2^{-}$	197.0	8.4
$^{176}\text{Re}$ ( $5n$ )	5.2 min	$3^{+}$	240.0 109.08	48.0 25.0
$^{175}\text{Re}$ ( $6n$ )	6.0 min	$5/2^{-}$	184.5	4.8
$^{177}\text{W}$ ( $p3n$ )	2.25 h	$1/2^{-}$	186.3 115.2	8.9 51.0
$^{176}\text{W}$ ( $p4n$ )	2.3 h	$0^{+}$	426.8 100.2	13.2 73.01
$^{175}\text{W}$ ( $p5n$ )	1.81 h	$1/2^{+}$	166.7 270.25	9.0 12.6
$^{176}\text{Ta}$ ( $\alpha n$ )	8.09 h	$1^{-}$	201.87 1159.2	5.5 24.6
$^{175}\text{Ta}$ ( $\alpha 2n$ )	10.5 h	$7/2^{+}$	1225.03 349.0 125.9	6.0 11.4 5.8
$^{174}\text{Ta}$ ( $\alpha 3n$ )	62.6 min	$3^{+}$	206.5 266.9	57.0 10.8
$^{173}\text{Ta}$ ( $\alpha 4n$ )	3.56 h	$5/2^{-}$	172.2 160.4	17.5 4.9
$^{166}\text{Tm}$ ( $3\alpha 3n$ )	7.7 h	$2^{+}$	778.8 785.9	18.1 9.9

[13]. This model follows the correct procedure for angular momentum coupling at each stage of de-excitation. The angular momentum conservation is explicitly taken into account at each step. In this code the input fusion cross section was calculated using the Bass formula [14]. The details of this model are given in our earlier work [1]. In this model the most important parameter is Level Density Parameter (LDP). The LDP ( $a = A/K$ ), where  $A$  is the atomic mass of the compound nucleus (CN) and  $K$  is a free parameter. The prescription of Kataria et al. [15]



**Figure 3.** Total fusion cross section ( $\sigma_{TF}$ ) along with the sum of complete ( $\Sigma\sigma_{CF}$ ) and incomplete fusion contributions ( $\Sigma\sigma_{ICF}$ ) at different energies in  $^{16}\text{O} + ^{165}\text{Ho}$  system.

for the level density is employed for this purpose, which takes into account the excitation energy dependence of the level density parameter. For  $^{16}\text{O} + ^{165}\text{Ho}$  system at present studied energies the most suitable value found to be  $K = 8$ . As a representative case the measured EFs of  $^{177}\text{Re}$  ( $4n$ ) has been plotted in Fig. 1. From this figure we can see that the experimentally measured excitation function agrees well with the theoretical predictions of the statistical model code PACE4. However, the EFs for  $\alpha$ -emitting channels show the enhancement over the theoretical predictions of model code PACE4. This enhancement in EFs may be due to ICF contribution. As can be seen from Fig. 2, where  $^{174}\text{Ta}$  ( $\alpha 3n$ ) shows enhancement over the theoretical predictions of code PACE4. The experimentally measured production cross sections for these channels may be attributed to both CF and/or ICF. Although, it

**Table 2.** Experimentally measured cross-section  $\Sigma\sigma_{CF}$ ,  $\Sigma\sigma_{ICF}$ ,  $\sigma_{TF}$  and  $F_{ICF}$  in mb.

$E_{lab}$ (MeV)	$\Sigma\sigma_{CF}$	$\Sigma\sigma_{ICF}$	$\sigma_{TF}$	$F_{ICF}$ (%)
$72.9 \pm 1.86$	57.05	-	57.05	-
$83.8 \pm 1.72$	510.7	54.0	564.7	9.6
$92.9 \pm 1.6$	767.1	132.3	899.4	14.7
$96.4 \pm 0.8$	864.6	206.0	1070.6	19.2
$105.0 \pm 0.78$	125.4	414.1	1369.6	30.2

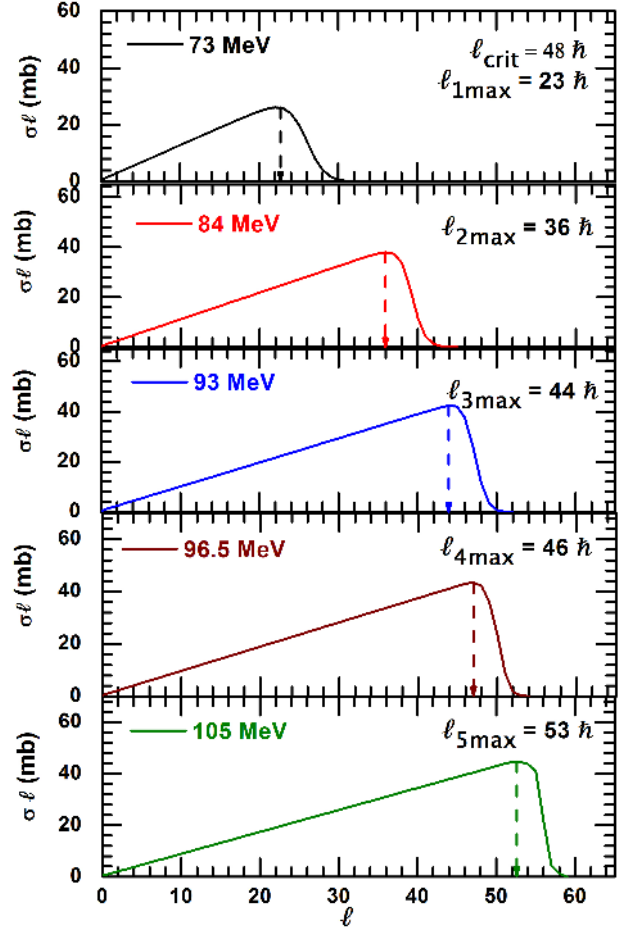
may not be possible to obtain directly the relative contribution of ICF, hence an attempt has been made to extract the ICF fraction. Therefore, for a better understanding of ICF contribution in  $\alpha$ -emitting channels,  $\Sigma\sigma_{ICF}$  has been compared with that estimated by the statistical model code PACE4,  $\Sigma\sigma_{PACE4}$  shown in Fig. 3. In this figure we can see that the sum of experimentally measured EFs of all  $\alpha$ -emitting channels is significantly higher than PACE4 predictions for the same value of level density parameter (i.e.,  $a = A/8 \text{ MeV}_{-1}$ ) has been used to compare CF residues in the present work. The observed enhancement in the measured EFs over values predicted by PACE4 may be attributed to ICF. The contribution of ICF in the production of all  $\alpha$  emitting channels has been deduced as  $\Sigma\sigma_{ICF} = \Sigma\sigma_{exp} - \Sigma\sigma_{PACE4}$ . In order to see how much ICF contributes to the total reaction cross section ( $\sigma_{TF} = \Sigma\sigma_{CF} + \Sigma\sigma_{ICF}$ ), the sum of cross sections of all CF channels ( $\Sigma\sigma_{CF}$ ) and  $\sigma_{TF}$  as a function of incident projectile energy is plotted in Fig. 3. As can be seen from this figure, the separation between  $\Sigma\sigma_{CF}$  and  $\sigma_{TF}$  increases with increasing projectile energy. Moreover, the inset of figure 3 also shows that ICF contribution increases with the incident beam energy. The percentage ICF fraction can be evaluated using the following equation

$$F_{ICF}(\%) = \frac{\Sigma\sigma_{ICF}}{\sigma_{TF}} \times 100 \quad (1)$$

The experimentally measured SsCF, SsICF, SsTF and % FICF are shown in Table II.

#### 4 Observation of incomplete fusion fraction at $\ell < \ell_{crit}$

Several recent studies [6? –8] have shown that the results of sum rule model calculation [3] are not consistent for the study of low energy incomplete fusion reaction dynamics. In this model, it is assumed that ICF channels open only for those partial waves which have  $\ell$  values equal or greater than  $\ell_{crit}$ . On the other hand partial waves which have  $\ell$  values less than  $\ell_{crit}$ , contributes to CF processes. The model contains three free parameters: the effective temperature  $T$ , the effective Coulomb interaction radius  $R_C$ , and diffuseness in  $\ell$  distribution  $\Delta$ . Wilczynski *et al.* [3], to fit the experimental data in the  $^{14}N + ^{159}Tb$  reaction at  $E_{lab} = 140 \text{ MeV}$ , used  $T = 3.5 \text{ MeV}$ ,  $R_C/(A_p^{1/3} + A_T^{1/3}) = 1.5$ , and  $\Delta = 1.7 \hbar$ . In the present work, it has been observed that the experimental cross section for fusion-evaporation channels agree reasonably well with the predictions of the sum rule model. However,


**Figure 4.** Partial  $\ell$ -distributions calculated using the code CC-FULL for  $^{16}O + ^{165}Ho$  system at studied energies. Where  $\ell$  is in units of  $\hbar$ . The value of  $\ell_{crit}$  for present system is  $48 \hbar$ .

there is a large discrepancy between measured and calculated cross-section values for ICF channels. As a typical example for ICF channels producing Ta isotopes in  $^{16}O + ^{165}Ho$  system, the sum rule calculations are lower by a factor of more than 50 in general. Similar discrepancy has also been observed by Yadav *et al.* [6] in their study on the  $^{12}C + ^{159}Tb$  system at projectile energy  $\approx 7 \text{ MeV/nucleon}$ . The underestimation of the ICF cross section by the sum rule model may be due to the assumption in the model that a major contribution to the ICF reactions comes from the collision trajectories with the angular momentum ( $\ell$ ) greater than the critical angular momentum for complete fusion ( $\ell_{crit}$ ). For a better perception about the diffuseness in  $\ell$  distribution at low incident energies, the critical angular momentum  $\ell_{crit}$  for  $^{16}O + ^{165}Ho$  system, at which the pocket in the entrance-channel potential disappears, has been calculated using the prescription of Wilczynski *et al.* [3].

$$\ell_{crit}^2 = \frac{\mu_m(C_1 + C_2)^3}{\hbar^2} \left[ 4\pi\gamma \frac{C_1 C_2}{C_1 + C_2} - \frac{Z_1 Z_2 e^2}{(C_1 + C_2)^2} \right] \quad (2)$$

Where  $C_1, C_2$  are the half density radii and  $Z_1, Z_2$  are the atomic numbers of the projectile and target nuclei, respectively and  $\mu_m$  and  $\gamma$  are the reduced mass and surface tension coefficient. The calculation gives  $\ell_{crit}$  as  $48 \hbar$  for the  $^{16}O + ^{165}Ho$  system. The fusion  $\ell$  distributions for the compound nucleus in the above mentioned interaction at studied energies have been calculated using the code CC-FULL [16], and are plotted in Fig. 4. The values of  $\ell_{max}$  at five respective energies in the present work are  $\approx 23 \hbar, 36 \hbar, 44 \hbar, 46 \hbar$  and  $53 \hbar$ , respectively. The first four values of  $\ell_{max}$  are less than the estimated value of  $\ell_{crit}$  i.e.,  $48 \hbar$ . However, from Table II, we can also see that even for these energies the ICF contribution is significant. This is an evidence of ICF contribution at  $\ell < \ell_{crit}$  for the present studied system.

## 5 Conclusions

To study the incomplete fusion reaction dynamics at low energies, an attempt has been employed to measure the EFs of twelve evaporation residues viz.,  $^{178}Re(3n), ^{175}Re(4n), ^{176}Re(5n), ^{175}Re(6n), ^{177}W(p3n), ^{176}W(p4n), ^{175}W(p5n), ^{176}Ta(\alpha n), ^{175}Ta(\alpha 2n), ^{174}Ta(\alpha 3n), ^{173}Ta(\alpha 4n)$ , and  $^{166}Tm(3\alpha 3n)$  at energies  $\approx 4-7$  MeV/nucleon. The analysis has been carried out in the frame work of the statistical model code PACE4, and EFs of all CF channels are consistent with the theoretical predictions, while for ICF channels, there is an enhancement over the theoretical predictions. It has been observed that sum rule model calculations are not consistent for ICF channels at low incident energies. Moreover, the present study also indicates the ICF contribution even at input angular momentum values  $\ell < \ell_{crit}$ .

## Acknowledgements

The authors are thankful to the Chairman, Department of Physics, Aligarh Muslim University, Aligarh (India) for providing the necessary facilities to carry out this work. Thanks are also due to the operating crew of the Pelletron facility of IUAC for providing stable and a good quality beam.

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