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Low Energy Parameters and Particle Masses in a Supersymmetric Grand Unified Model

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In the framework of the supersymmetric $SU(3)_c\times SU(2)\times U(1)$ gauge theory and the standard grand unification, the fermion mass ratio m_b/m_τ and the upper bound of the top quark mass are calculated by means of the renormalization group equations for the Yukawa couplings. It is found that the ratio m_b/m_τ remains in the phenomenologically acceptable domain for all possible values of the top quark mass whose perturbative upper bound is about 200 GeV. We also examine the structure of the Higgs potential with softly supersymmetry breaking mass terms to find some relations of the scalar masses. There should be a scalar whose mass is less than M_Z in a class of models with the soft-breaking mass terms.

§ 1. Introduction

In the grand unified gauge theory,[1] there should be at least two mass scales M_X and M_W with an enormous ratio, where M_X and M_W are masses of the grand unification (X-boson mass) and the $SU(2)\times U(1)$ breaking (W-boson mass), respectively. This has been a puzzling question known as the gauge hierarchy.[2] It is unnatural for the usual gauge theory to incorporate this enormous ratio in the scalar mass terms, i.e., it needs unnatural fine tuning.[2]

The gauge hierarchy can be "naturally" realized in the supersymmetric theory,[3] in which no quadratic divergence is present. Recently, a realistic model of the supersymmetric grand unified gauge theory has been proposed.[4] In this model, the supersymmetry is broken at energy scale of order M_W, and there is a supersymmetric "desert" in the energy region between M_W and M_X. Thus, the enormous ratio of M_X and M_W is not disturbed by radiative corrections.

In models of this type, predictions for the low energy parameters are different from those in the usual (nonsupersymmetric) grand unified models. Renormalization group analyses for the gauge couplings have been made by several authors to find that $\alpha_c(M_W)$, $\sin^2\theta_W(M_W)$ and the fermion mass ratio m_b/m_τ are consistent with the experimental data.[5] In these works, the ratio m_b/m_τ is obtained from only the QCD corrections. In this paper, we first calculate the ratio m_b/m_τ retaining also contributions from the $U(1)$ gauge and the Yukawa couplings in cases of small and large top quark mass, and find an upper bound of the top quark

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mass by means of the renormalization group equations for the Yukawa couplings. Next, we discuss the $SU(2)\times U(1)$ breaking by the Higgs scalar potential with softly supersymmetry breaking mass terms. We find constraints of the mass terms and relations of the Higgs scalar masses.

In the next section, a model we discuss in this paper is briefly described, and renormalization group analyses for the gauge couplings are made, which are consistent with Ref. 5). In § 3, the behavior of the Yukawa couplings is examined, and the effects of the Yukawa couplings on the ratio m_b/m_τ are studied. The upper bound of the top quark mass and the behavior of the quark mixing angles are also obtained. In § 4, the Higgs scalar potential is studied to find the scalar masses. The final section is devoted to the conclusion.

§ 2. A supersymmetric $SU(3)_c\times SU(2)\times U(1)$ gauge model

In this paper, we consider a supersymmetric $SU(3)_c\times SU(2)\times U(1)$ gauge model. The left-handed chiral multiplets contained in the model are listed with their representations as follows:

$$l_i \quad (\mathbf{1},\mathbf{2}, -1/2), \tag{1·a}$$

$$\bar{e}_i \quad (\mathbf{1},\mathbf{1}, +1), \tag{1·b}$$

$$q_i \quad (\mathbf{3},\mathbf{2}, +1/6), \tag{1·c}$$

$$\bar{n}_i \quad (\mathbf{3}^*,\mathbf{1}, +1/3), \tag{1·d}$$

$$\bar{p}_i \quad (\mathbf{3}^*,\mathbf{1}, -2/3), \qquad i=1,\cdots,N_g\,, \tag{1·e}$$

$$s_a \quad (\mathbf{1},\mathbf{2}, -1/2), \tag{1·f}$$

$$\bar{t}_a \quad (\mathbf{1},\mathbf{2}, +1/2), \qquad a=1,\cdots,n_H\,, \tag{1·g}$$

where s and t contain the Higgs scalar doublets, and the other multiplets contain leptons and quarks.

The Yukawa couplings are defined by

$$\mathcal{L}_{\rm int}=f_{ij}^{a*}[s_a l_i \bar{e}_j]_F+h_{ij}^{a*}[s_a q_i \bar{n}_j]_F+\tilde{h}_{ij}^{a*}[\bar{t}_a q_i \bar{p}_j]_F+\text{h.c.}, \tag{2}$$

where F means the F-component, and f, h and $\tilde{h}$ are complex Yukawa coupling constants. We make an assumption here that the Yukawa interactions of types $[ss\bar{e}]_F$, $[ll\bar{e}]_F$, $[lq\bar{n}]_F$ and $[\bar{p}\bar{n}\bar{n}]_F$, which cause the lepton and baryon number nonconservation, vanish by a certain symmetry. In the grand unified $SO(10)$ model, this is automatic.

In order to make the model realistic, we must break the supersymmetry. There are two possibilities for this breaking, that is, spontaneous or soft breaking. In this paper we take the latter, because in the former case we had to introduce disgusting complexity in the model. We assume the softly supersymmetry breaking mass terms of order M_W for all unconventional superpartners, that is, fermion fields in vector multiplets, scalar fields in (1·a)~(1·e) and fermion fields in (1·f) and (1·g). The $SU(2)\times U(1)$ is spontaneously broken by the Higgs mechanism. We assume that only the scalar fields in s and t have nonvanishing vacuum expectation values v_s and v_t. With these assumptions, the fermion mass matrices are represented as follows:

$$m_l=(1/\sqrt{2})v_s^a f^a\,, \qquad m_n=(1/\sqrt{2})v_s^a h^a\,, \qquad m_p=(1/\sqrt{2})v_t^a \tilde{h}^a\,, \tag{3}$$

where $\sum_a(v_s^{a2}+v_t^{a2})\equiv v^2=4M_W^2/g^2$.

The renormalization group equations for the gauge and the Yukawa couplings at the one-loop level are given in the Appendix. From the behavior of the gauge coupling constants g_c, g and g', we obtain the following relations:

$$\ln(M_X/M_W)=\frac{\pi}{9+n_H}[\alpha^{-1}(M_W)-(8/3)\alpha_c^{-1}(M_W)], \tag{4·a}$$

$$\sin^2\theta_W(M_W)=\frac{3}{8}-\frac{15-n_H}{27+3n_H}\left[\frac{3}{8}-\frac{\alpha(M_W)}{\alpha_c(M_W)}\right]. \tag{4·b}$$

The fermion mass ratio $R\equiv m_b/m_\tau$ is obtained from the ratio of the corresponding Yukawa coupling constants h and f. In the small Yukawa coupling limit, where the behavior of the Yukawa couplings is determined by the gauge couplings, the ratio at M_W is

$$R_0(M_W)=[\alpha_c(M_X)/\alpha_c(M_W)]^{8/(-27+6N_g)}[\alpha'(M_X)/\alpha'(M_W)]^{-10/(30N_g+9n_H)}, \tag{5}$$

where a suffix 0 means the small Yukawa coupling limit. In Ref. 5) only the contribution of α_c to $R_0(M_W)$ is taken into account. We find that the contribution of α' is about 10% to reduce the value of $R_0(M_W)$.

Before calculating Eqs. (4) and (5), we should get the phenomenological values of these parameters at M_W. At energies below M_W, the QCD and QED corrections should be taken into account. We adopt the following values:

$$\sin^2\theta_W(M_W)=0.228\pm0.010\,, \tag{6·a}$$

$$\alpha(M_W)=1/128\,, \tag{6·b}$$

$$\alpha_c(M_W)=0.11\sim0.16\,, \tag{6·c}$$

$$R(M_W)=1.7\sim2.3\,. \tag{6·d}$$

Equation (6·a) is the value obtained from the experiment.[6] Equation (6·b) is obtained in Ref. 7). Equations (6·c) and (6·d) are calculated with the QCD corrections using $m_b=(4\sim5)$GeV and $\Lambda_c=(0.1\sim0.5)$GeV.[8] We use Eq. (6·c) as an input.

From Eqs. (4), the following values are obtained:

$$n_H=1;\ \sin^2\theta_W(M_W)=0.223\sim0.233\,,$$
$$M_X=1.1\times10^{16}\sim1.3\times10^{17}\ \text{GeV}\,, \tag{7·a}$$
$$n_H=2;\ \sin^2\theta_W(M_W)=0.247\sim0.255\,,$$
$$M_X=5.6\times10^{14}\sim5.0\times10^{15}\ \text{GeV}\,, \tag{7·b}$$
$$n_H=3;\ \sin^2\theta_W(M_W)=0.266\sim0.274\,,$$
$$M_X=4.6\times10^{13}\sim3.4\times10^{14}\ \text{GeV}\,. \tag{7·c}$$

Thus, the case $n_H=1$ is the most favorable. The values of $R_0(M_W)$ in many cases are listed in Table I. We find that the cases $N_g\geqq4$ are not favorable.

Table I. $R_0(M_W)$ in the small Yukawa coupling limit.

	$N_g=3$	$N_g=4$
$n_H=1$	2.2~2.9	3.0~4.8
$n_H=2$	2.1~2.7	2.7~4.3
$n_H=3$	2.0~2.6	2.6~3.9

§ 3. Renormalization group analyses for the Yukawa couplings

The values of $R_0(M_W)$ in Table I are modified, if the contribution of the Yukawa couplings becomes nonnegligible. In this section, we mainly discuss the Yukawa couplings for the third generation, neglecting the quark mixing angles and the fermion masses of the first and second generations. The numerical results with the quark mixing angles are also presented. Here we discuss only the case $n_H=1$ and $N_g=3$. From Eqs. (A·5), we obtain the following equations in this approximation:

$$\mu\frac{df_3}{d\mu}=\frac{f_3}{(4\pi)^2}[-3g^2-3g'^2+4f_3{}^2+3h_3{}^2], \tag{8·a}$$

$$\mu\frac{dh_3}{d\mu}=\frac{h_3}{(4\pi)^2}\left[-\frac{16}{3}g_c{}^2-3g^2-\frac{7}{9}g'^2+6h_3{}^2+\tilde{h}_3{}^2+f_3{}^2\right], \tag{8·b}$$

$$\mu\frac{d\tilde{h}_3}{d\mu}=\frac{\tilde{h}_3}{(4\pi)^2}\left[-\frac{16}{3}g_c{}^2-3g^2-\frac{13}{9}g'^2+6\tilde{h}_3{}^2+h_3{}^2\right], \tag{8·c}$$

where the Yukawa coupling constants f_3, h_3 and $\tilde{h}_3$ are real (positive) numbers, and $f_3(M_X)=h_3(M_X)$.

From Eqs. (8·a) and (8·b), the evolution equation for $R\equiv h_3/f_3$ is obtained as follows:

$$\mu\frac{dR}{d\mu}=\frac{R}{(4\pi)^2}\left[-\frac{16}{3}g_c{}^2+\frac{20}{9}g'^2+3h_3{}^2-3f_3{}^2+\tilde{h}_3{}^2\right]. \quad (9)$$

As $h_3(\mu)$ is always larger than $f_3(\mu)$, the effect of Yukawa coupling terms is to reduce $R(M_W)$. In this section, the numerical calculations are made with $\sin^2\theta_W(M_W)=0.230$, where $\alpha_c(M_W)=0.122$, $M_X=2.4\times10^{16}$ GeV and $R_0(M_W)=2.37$.

Let us first examine an interesting case, where the following relation holds in the grand unification limit:

$$f_3(M_X)=h_3(M_X)=\tilde{h}_3(M_X)\equiv f_X\,. \quad (10)$$

Of course, Eq. (10) cannot hold for the Yukawa couplings of the first and second generations. With several values of f_X, the couplings $f_3(M_W)$, $h_3(M_W)$ and $\tilde{h}_3(M_W)$ are calculated from Eqs. (8), and $m_b(M_W)$ and $m_t(M_W)$ are obtained from Eqs. (3) by adjusting the ratio v_s/v_t so that we get $m_\tau(M_W)=1.8$ GeV. The numerical results are shown in Table II. The Yukawa couplings $f_3(M_W)$, $h_3(M_W)$ and $\tilde{h}_3(M_W)$ are bounded by the perturbation constraint as in the usual grand unified models.[8),9)] We find that the bound is almost saturated for $f_X\gtrsim3$. The phenomenologically desirable value of $m_b(M_W)$ is $(3.1\sim4.1)$GeV for $m_b=(4\sim5)$GeV.[8)] In order to get the desirable values of m_b and m_t in this case, the ratio v_s/v_t should be very small ($\lesssim0.05$).

Table II. The Yukawa coupling constants and the quark masses at M_W derived from Eq. (10). The ratio of the vacuum expectation values v_s/v_t is adjusted so as to give $m_\tau(M_W)=1.8$ GeV.

f_X	$f_3(M_W)$	$h_3(M_W)$	$\tilde{h}_3(M_W)$	v_s/v_t	m_b[GeV]	m_t[GeV]
0.03	0.05	0.11	0.11	0.234	4.3	19
0.1	0.15	0.34	0.35	0.071	4.2	61
0.3	0.35	0.77	0.80	0.029	3.9	139
1	0.55	1.03	1.10	0.019	3.4	191
3	0.60	1.06	1.15	0.017	3.2	199
10	0.61	1.06	1.15	0.017	3.2	200

Now, we consider the case $v_s/v_t=O(1)$. In this case, $f_3(\mu)$ and $h_3(\mu)$ are negligible, and Eq. (10) cannot hold, but $f_3(M_X)=h_3(M_X)$. Thus, we consider only gauge couplings and $\tilde{h}_3$ contribution to $R(M_W)$. As in the previous case, there is an upper bound of $\tilde{h}_3(M_W)$ corresponding to $\tilde{h}_3(M_X)\to\infty$. By a numerical calculation of Eq. (18·c), we obtain that $\tilde{h}_3(M_W)\leq1.24$. If we use a smaller value of $\sin^2\theta_W(M_W)$ than 0.23 as an input, this upper bound becomes a little larger, for example, $\tilde{h}_3(M_W)\leq1.4$ for $\sin^2\theta_W(M_W)=0.22(\alpha_c(M_W)=0.18)$. These values of the upper bound are almost the same as in the usual grand unified models.[8)]

Numerical results of $R(M_W)$ are shown in Fig. 1 with varying $\tilde{h}_3(M_W)$. More realistically, the effects of the quark mixing angles should be taken into account. In Fig. 1, we also show the results of these effects, which are calculated directly

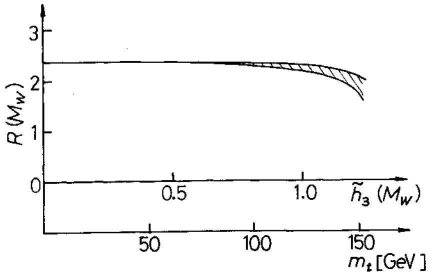

Fig. 1. $R(M_W)$ with the contribution of the Yukawa coupling $\tilde{h}_3$. m_t is calculated on the assumption that $v_s = v_t$. The shaded region corresponds to $0 < \theta_1, \theta_2, \theta_3, \delta(\text{at } M_X) < \pi/4$.

from Eqs. (A·5) with suitably small values of the other Yukawa coupling constants. The true prediction of $R(M_W)$ at the one-loop level is in the shaded region in Fig. 1. We conclude from the figure that the prediction of $R(M_W)$ is in good agreement with the phenomenological value, $1.7 \sim 2.3$, in the large Yukawa coupling cases. The values of m_t shown in the figure are calculated on the assumption that $v_s = v_t$, which is a rough estimation of m_t.

The quark mixing matrix, which is a multiple of unitary matrices diagonalizing $hh^\dagger$ and $\tilde{h}\tilde{h}^\dagger$, is also dependent on the energy scale. This is due to the cross terms of h and $\tilde{h}$ in Eqs. (A·5). In Table III, the values of the Kobayashi-Maskawa mixing angles[10] at M_X are listed with several values of m_t assuming $v_s = v_t$. In calculating these values we have used the following angles[11] at M_W as an input in order to compare with the results in Ref. 8):

$$s_1(M_W) = 0.23\,, \quad s_2(M_W) = 0.4\,, \quad s_3(M_W) = 0.3\,, \quad s_\delta(M_W) = 0.05\,. \qquad (11)$$

Furthermore, we have neglected the condition, $f(M_X) = h(M_X)$. As elements of $f(\mu)$ and $h(\mu)$ are sufficiently small, this effect on the mixing angles is not significant. The difference between the mixing angles at M_W and at M_X is somewhat larger than that in Ref. 8), but it is still too small to be observed.

In any case, the upper bound of the top quark mass is about 200 GeV. The order of magnitude of this value is the same as the prediction in Refs. 8) and 9). $R(M_W)$ and the quark mixing angles in the large Yukawa coupling limit are smaller than those in the small Yukawa coupling limit. This tendency is the same as in the usual grand unified model with two Higgs scalar doublets having the same type of the Yukawa coupling as Eq. (2).[8]

Table III. The quark mixing angles at M_X calculated from the values in Eqs. (11).

$m_t(v_s = v_t)$	$s_1(M_X)$	$s_2(M_X)$	$s_3(M_X)$	$s_\delta(M_X)$
30 GeV	0.230	0.399	0.299	0.050
60 GeV	0.229	0.394	0.296	0.050
90 GeV	0.229	0.386	0.290	0.050
120 GeV	0.227	0.369	0.277	0.049
150 GeV	0.222	0.302	0.227	0.047

§ 4. Higgs potential

We have assumed that all scalar fields contained in quark and lepton supermultiplets have masses of order M_W and vanishing vacuum expectation values. Thus, the spontaneous breakdown of $SU(2)\times U(1)$ to $U(1)_{em}$ is caused by the scalar fields of the Higgs supermultiplets, ϕ_s and ϕ_t. The quartic coupling terms of them are derived in the supersymmetric limit from the D-component as follows:[4]

$$V^{(4)}=\frac{g^2}{2}\sum_{a=1}^{3}\left(\phi_s{}^\dagger\frac{\tau_a}{2}\phi_s-\phi_t{}^\dagger\frac{\tau_a}{2}\phi_t\right)^2+\frac{g'^2}{8}(\phi_s{}^\dagger\phi_s-\phi_t{}^\dagger\phi_t)^2 . \tag{12}$$

There is no contribution from the F-component in the model described in § 2. We add the softly supersymmetry breaking mass terms to the potential as

$$V^{(2)}=m_1{}^2\phi_s{}^\dagger\phi_s+m_2{}^2\phi_t{}^\dagger\phi_t-m_3{}^2(\phi_s{}^\dagger\phi_t+\phi_t{}^\dagger\phi_s). \tag{13}$$

This is the most general form of the supersymmetry breaking mass terms invariant under $SU(2)\times U(1)$. As a phase convention, we chose $m_3{}^2\geq 0$. In this section, we discuss the Higgs potential $V=V^{(4)}+V^{(2)}$ at the tree level.

$V^{(4)}$ vanishes along the direction of $\phi_s=\phi_t$ (mod. phase). Thus, $V^{(2)}$ should be positive along this direction, i.e., the following condition should be satisfied for the vacuum stability:

$$m_1{}^2+m_2{}^2>2m_3{}^2 . \tag{14}$$

In order that ϕ_s and ϕ_t have nonvanishing vacuum expectation values, $V^{(2)}$ should take a negative value in some direction. This condition requires

$$m_1{}^2m_2{}^2<m_3{}^4 . \tag{15}$$

When these conditions, Eqs. (14) and (15), are satisfied, $SU(2)\times U(1)$ is spontaneously broken to $U(1)_{em}$.

We can rotate ϕ_s and ϕ_t to ϕ and χ so that $\langle\chi\rangle=0$. This can be done by the following transformation:

$$\phi=\phi_s\cos\theta+\phi_t\sin\theta , \qquad \chi=-\phi_s\sin\theta+\phi_t\cos\theta , \tag{16}$$

where an angle θ is related to the ratio v_s/v_t as $v_s/v_t=\cot\theta$, and defined by

$$\sin 2\theta\equiv 2m_3{}^2/(m_1{}^2+m_2{}^2),$$

$$\cos 2\theta\begin{cases}<0 & \text{for } m_1{}^2>m_2{}^2 ,\\ >0 & \text{for } m_1{}^2<m_2{}^2 .\end{cases} \tag{17}$$

Vacuum expectation values of χ and ϕ are as follows:

$$\langle\chi\rangle=0 ,$$

$$\langle\phi^\dagger\phi\rangle \equiv \frac{1}{2}v^2 = \frac{-4(m_1{}^2\cos^2\theta - m_2{}^2\sin^2\theta)}{(g^2+g'^2)\cos 2\theta}. \tag{18}$$

Now, let us parametrize the scalar fields as

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}(v+\phi_1+i\phi_2) \\ \phi^- \end{pmatrix}, \qquad \chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(\chi_1+i\chi_2) \\ \chi^- \end{pmatrix}. \tag{19}$$

The scalars ϕ_2 and $\phi^\pm$ are the Nambu-Goldstone bosons eaten by the gauge fields. χ_2 and $\chi^\pm$ are mass eigenstates, and their masses are

$$m_{\chi_2}^2 = m_1{}^2 + m_2{}^2, \qquad m_{\chi^\pm}^2 = m_{\chi_2}^2 + M_W{}^2. \tag{20}$$

ϕ_1 and χ_1 are not mass eigenstates. Their mass eigenvalues m_a and m_b are represented as follows:

$$m_{\substack{a\\b}}^2 = \frac{1}{2}\left[m_{\chi_2}^2 + M_Z{}^2 \pm \sqrt{(m_{\chi_2}^2+M_Z{}^2)^2 - 4m_{\chi_2}^2 M_Z{}^2\cos^2 2\theta}\right]. \tag{21}$$

Thus, m_b is less than m_{χ_2} and M_Z, and m_a is greater than m_{χ_2} and M_Z. Furthermore, the following relation holds:

$$m_a{}^2 + m_b{}^2 = m_{\chi_2}^2 + M_Z{}^2. \tag{22}$$

If we impose an invariance under independent phase transformation of ϕ_s and ϕ_t on the potential, that is, the global $U(1)_{PQ}$ symmetry,[12] a parameter m_3 vanishes. In this case, the solution of Eq. (17) is $\theta=0$ or $\pi/2$, namely $v_t=0$ or $v_s=0$. The $U(1)_{PQ}$ is not broken spontaneously, and the model contains massless quarks. Thus we cannot impose the $U(1)_{PQ}$ symmetry at the $SU(2)\times U(1)$ level.

§ 5. Conclusion

We have argued some features of a supersymmetric grand unified gauge model. As a summary of the renormalization group analyses at the one-loop level, $\sin^2\theta_W$ and the fermion mass ratio m_b/m_τ are in good agreement with the present phenomenology within $\alpha_c(M_W)=0.11\sim0.16$. Furthermore, we can conclude that the case $n_H=1$ and $N_g=3$ is the most favorable. The upper bound of the top quark mass is about 200 GeV. The value of M_X is $O(10^{16}\sim10^{17})$GeV, which is sufficiently large for the proton stability.

We have found that the large Yukawa coupling contributes to reduce the value of $R(M_W)$. This situation is analogous to the usual grand unified model with two Higgs scalar doublets, and in contrast to that with one Higgs scalar doublet.[8)]

We have also studied the Higgs scalar potential at the tree level to find constraints on the supersymmetry breaking mass terms, (14) and (15), and some relations of the scalar masses, Eqs. (20)~(22). There is at least one neutral scalar whose mass is less than M_Z. Whatever may be the origin of the supersymmetry breaking mass terms, they should satisfy the constraints.

From (14) and (15), a relation $m_1^2 = m_2^2$ cannot hold at M_W. What is the origin of the difference between m_1^2 and m_2^2? We can consider an interesting possibility that a relation $m_1^2 = m_2^2$ holds at M_X, and the difference at M_W is caused by the renormalization effects because of the different contribution of the Yukawa coupling to m_1^2 and m_2^2. We will discuss the renormalization group analyses of the soft-breaking mass terms in order to examine the possibility elsewhere.

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Appendix

We first describe the renormalization group equations in a supersymmetric gauge theory with a group G. The theory contains a vector multiplet in the adjoint representation and a left-handed chiral multiplet Φ in a representation R, which is reducible in general and should be anomaly free. The gauge couplings are defined in the usual way by the D-component,[3] and the Yukawa couplings are defined by the F-component as follows:

$$\mathcal{L}_{\text{int}} = \frac{1}{3!} H_{ijk} [(\Phi)_i (\Phi)_j (\Phi)_k]_F + \text{h.c.} \tag{A·1}$$

General formulas of the renormalization group equations for the gauge coupling constant g and the Yukawa coupling constant H_{ijk} at the one-loop level are

$$\mu \frac{dg}{d\mu} = \frac{g^3}{(4\pi)^2} [-3C_2(G) + T(R)], \qquad T(R)\delta^{ab} \equiv \text{Tr}(T^a T^b), \tag{A·2}$$

$$\mu \frac{dH_{ijk}}{d\mu} = \frac{1}{(4\pi)^2} [\Theta_{ii'} H_{i'jk} + \Theta_{jj'} H_{ij'k} + \Theta_{kk'} H_{ijk'}],$$

$$\Theta_{ii'} \equiv -2g^2 (T^a T^a)_{ii'} + \frac{1}{2} H_{imn} H^*_{i'mn}\,, \tag{A·3}$$

where $C_2(G)$ is the second order Casimir invariant of the group G, and T^a is a generator in a representation R.

Now, we consider the $SU(3)_c\times SU(2)\times U(1)$ model discussed in this paper. The chiral multiplets are listed in (1). We express the Yukawa coupling constants defined by Eq. (2) as $N_g\times N_g$ matrices f^a, h^a and $\tilde{h}^a$ ($a=1,\cdots,n_H$). From Eqs. (A·2) and (A·3), we obtain the following equations:

$$\mu\frac{dg_c}{d\mu}=\frac{g_c{}^3}{(4\pi)^2}(-9+2N_g), \qquad \text{(A·4·a)}$$

$$\mu\frac{dg}{d\mu}=\frac{g^3}{(4\pi)^2}(-6+2N_g+n_H), \qquad \text{(A·4·b)}$$

$$\mu\frac{dg'}{d\mu}=\frac{g'^3}{(4\pi)^2}\left(\frac{10}{3}N_g+n_H\right), \qquad \text{(A·4·c)}$$

$$\mu\frac{df^a}{d\mu}=\frac{1}{(4\pi)^2}[-(3g^2+3g'^2)f^a+f^bf^{b\dagger}f^a+2f^af^{b\dagger}f^b$$
$$+f^b\,\mathrm{Tr}(f^af^{b\dagger}+3h^ah^{b\dagger})], \qquad \text{(A·5·a)}$$

$$\mu\frac{dh^a}{d\mu}=\frac{1}{(4\pi)^2}\left[-\left(\frac{16}{3}g_c{}^2+3g^2+\frac{7}{9}g'^2\right)h^a+(h^bh^{b\dagger}+\tilde{h}^b\tilde{h}^{b\dagger})h^a\right.$$
$$\left.+2h^ah^{b\dagger}h^b+h^b\,\mathrm{Tr}(f^af^{b\dagger}+3h^ah^{b\dagger})\right], \qquad \text{(A·5·b)}$$

$$\mu\frac{d\tilde{h}^a}{d\mu}=\frac{1}{(4\pi)^2}\left[-\left(\frac{16}{3}g_c{}^2+3g^2+\frac{13}{9}g'^2\right)\tilde{h}^a+(h^bh^{b\dagger}+\tilde{h}^b\tilde{h}^{b\dagger})\tilde{h}^a\right.$$
$$\left.+2\tilde{h}^a\tilde{h}^{b\dagger}\tilde{h}^b+\tilde{h}^b\,\mathrm{Tr}(3\tilde{h}^a\tilde{h}^{b\dagger})\right]. \qquad \text{(A·5·c)}$$

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