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LOW-ENERGY THEOREMS FOR HIGGS BOSON COUPLINGS TO PHOTONS

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Abstract

The decay of a neutral Higgs boson σ into two photons is considered. The amplitude is determined by loop graphs with charged particles - leptons, quarks, W bosons, scalar particles. Their contributions are comparable to each other so that the decay is sensitive to the whole structure of the theory. Low-energy theorems are obtained, which express directly the contribution of heavy intermediate states in terms of the corresponding coefficients in the photonic Gell-Mann--Low function. These coefficients are well-known, so in the limit $m_\sigma/M \to 0$ (M stands for the mass of the intermediate state) the result can be written out immediately. By explicit computation of the relevant Feynman graphs we found also the decay rate $\sigma \to 2\gamma$ for an arbitrary value of the ratio m_σ/M. Low-energy theorems are generalized to allow for the arbitrary number of Higgs-meson and photon external legs.

I. Introduction

Scalar Higgs mesons are an indispensible part of the modern theories of weak interactions [1]. Now when the basic aspects of the unified gauge theories are confirmed experimentally it is easy to anticipate further growth of interest in the Higgs-meson phenomenology*).

One of the specific features of the Higgs interactions is that heavy intermediate states do not decouple, in general [5]. The reason is that the Higgs-meson couplings are roughly speaking proportional to the mass of the particle it couples to. So, studying the Higgs-meson decays would allow to count all heavier elementary fields.

In this paper we will consider from this point of view the $\sigma \to 2\gamma$ decay (first studied in Ref. 3). We will prove simple theorems which relate the contributions of heavy intermediate states in the $\sigma \to 2\gamma$ amplitude to the corresponding pieces in the photonic Gell-Mann-Low function.

We will also derive an effective Lagrangian for low-energy interaction of an arbitrary number of σ mesons with an arbitrary (even) number of photons. It is generated by heavy vector, fermion and scalar loops and can be extracted in a simple way from the known QED Heisenberg-Euler type Lagrangians.

It is worth emphasizing that calculations of the Feynman graphs for the $\sigma \to 2\gamma$ decay in the standard Feynman gauge has been already performed. Our results agree with

*) For earlier papers and reviews on the subject see e.g. Refs. 2-4.

those of Ellis et al [3] in the limit of vanishing σ momentum. The low-energy theorems allow to understand in a simple way the basic features of the results obtained in Ref. [3], in particular, the difference in signs of the W-boson and fermion contributions into the amplitude.

The low-energy theorems are discussed in sects. 2 and 3.

For the sake of completeness we will calculate the $\sigma \to 2\gamma$ amplitude for an arbitrary ratio of the σ-meson and intermediate masses. Our progress here as compared to previous works is minor, if any: we get a closed analitic expression for the amplitude.

The results of the calculation are presented in sect.4. It occurs that the most simple way of calculation is to use a background type gauge.

2. Low-Energy Theorems for the $\sigma \to 2\gamma$ Decay

In this section we obtain the low-energy theorems which relate the contributions of heavy intermediate states into the $\sigma \to 2\gamma$ amplitude to the coefficients in the Gell-Mann-Low function for the electric charge renormalization.

Let us start, however, with some general remarks on kinematics and notations. The amplitude of the $\sigma \to 2\gamma$ decay has the following form

$$A(\sigma \to 2\gamma) = F(G\sqrt{2})^{1/2} \frac{\alpha}{4\pi}(k_{1\mu}e_{1\nu} - k_{1\nu}e_{1\mu})(k_{2\mu}e_{2\nu} - k_{2\nu}e_{2\mu}) \quad (1)$$

where $k_{1,2}$ and $e_{1,2}$ are the 4-momenta and polarization vectors of the photons and we made explicit factors due to electromagnetic and weak couplings (α and G, respectively). Below we will compute the dimensionless amplitude F.

In terms of the amplitude F the width of the $\sigma \to 2\gamma$ decay is given by

$$\Gamma(\sigma \to 2\gamma) = |F|^2 \left(\frac{\alpha}{4\pi}\right)^2 \frac{G m_\sigma^3}{8\pi\sqrt{2}} \tag{2}$$

The amplitude (1) is contributed by the graphs of Fig.1 with vector, fermion and scalar loops and we assume now that the intermediate state is much heavier than the Higgs meson.

Our central point is that one does not need to calculate these graphs to extract the answer in this limit. The answer is in fact already known and coincides with the corresponding coefficients in the photonic Gell-Mann Low function.

To substantiate the point let us recall the reader that interaction of the Higgs meson with, say, a fermion takes the form:

$$\mathcal{L}^{\sigma f}_{int} = -(G\sqrt{2})^{1/2} M_{f\sigma}\, \sigma \bar{f} f \tag{3}$$

where f is some fermion field (lepton or quark) and $M_{f\sigma}$ is a mass parameter which characterizes the contribution of the Higgs meson considered into the fermion mass. The normalization is such that in the standard Weinberg-Salam model with a single Higgs doublet $M_{f\sigma}$ is exactly the fermion mass m_f.

For the sake of generality we reserve for the possibility of having several Higgs doublets[*] and $M_{f\sigma}$ does not necessarily coincides with m_f. Moreover, the contributions of various Higgs particles are not generally speaking of the same sign so that there can be some cancellation. The only bound on $M_{f\sigma}$ which we would like to stick to is

$$|M_{f\sigma}| \lesssim (G/\pi\sqrt{2})^{-1/2} \sim 600\ GeV \tag{4}$$

which ensures the legimacy of a perturbative treatment of inter action (3).

Let us discuss first the fermionic contribution (graph 1a). As far as the momentum of the σ boson can be neglected (compared to the mass of the fermion) one can treat the σ field in Eq. (3) simply as a constant. The net effect of the term (3) then reduces to a shift in the mass of the fermion:

$$m_f \to m_f + (G\sqrt{2})^{1/2} M_{f\sigma}\, \sigma. \tag{5}$$

Moreover, at low energies (heavy) charged particles manifest themselves only via the following effective Lagrangian:

$$\mathcal{L}^{eff} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} \sum_i \frac{b_i e^2}{16\pi^2} \ln \frac{\Lambda^2}{M_i^2} + \dots \tag{6}$$

Here $e^2 = 4\pi\alpha$, $F_{\mu\nu}$ is the photon strength tensor, M_i is the mass of the i-th particle, Λ is the ultravio-

[*]) The fact that experimental data are described well by the Weinberg-Salam model implies that the Higgs mesons belong to doublets, not higher representations.

let cut off[*]) and finally dots stand for the terms of higher orders in the electromagnetic field which are irrelevant for our present discussion. The coefficient b_i characterizes the contribution of the i-th particle into the electric charge renormalization:

$$b = -\frac{4}{3}Q^2_{lepton} \quad \text{for a lepton,}$$

$$b = -4Q^2_{quark} \quad \text{for a quark,}$$

$$b = 7 \text{ for the W boson,}$$

$$b = -1/3 \text{ for a scalar charged boson.}$$

The Lagrangian (6) represents nothing else but the good old one-loop contribution of Fig. 2.

The value of b for a charged lepton and scalar is well known from QED. The extra factor of 3 for quarks is due to colour. The coefficient $b = 7$ for W bosons was found first in Ref.7 . Nowadays, when the result of massless nonabelian vector theories are quite familiar this number can be easily understood. The coefficient of $\log\Lambda^2$ in SU(2) massless gauge theory is 22/3. For the massive quantum one should add the contribution of the zero helicity component of the vector field, which is the same as the contribution of a scalar particle, namely - 1/3. Thus one arrives at 22/3 - 1/3 = 7.

Now substitute into Eq. (6) $M = m_f + (G\sqrt{2})^{1/2} M_{f\sigma}\,\sigma$

*) It can be thought of as, say, a regulator mass.

(see Eq. (5)), expand in σ and keep only the term linear in σ. As a result the following effective Lagrangian discribing $\sigma\gamma\gamma$ low-energy interaction emerges:

$$\mathcal{L}^{\sigma\gamma\gamma} = \begin{cases} \frac{\alpha}{6\pi} Q^2_{lepton} (G\sqrt{2})^{1/2} \frac{M_{f\sigma}}{m_f} \sigma F_{\mu\nu} F_{\mu\nu} & \text{for a lepton loop} \\ \frac{\alpha}{2\pi} Q^2_{quark} (G\sqrt{2})^{1/2} \frac{M_{f\sigma}}{m_f} \sigma F_{\mu\nu} F_{\mu\nu} & \text{for a quark loop} \end{cases} \tag{7}$$

It is a trivial matter to convert the effective Lagrangian (7) into the amplitude (1). $F_{\mu\nu} F_{\mu\nu}$ goes into

$$F_{\mu\nu} F_{\mu\nu} \to -2(k_{1\mu} e_{1\nu} - k_{1\nu} e_{1\mu})(k_{2\mu} e_{2\nu} - k_{2\nu} e_{2\mu}) \tag{8}$$

and the constant F, defined in Eq. (1) is given by

$$F = \begin{cases} \left[-\frac{4}{3} Q^2_{lepton} \right] \frac{M_{f\sigma}}{m_f} & \text{for a lepton} \\ \left[-4 Q^2_{quark} \right] \frac{M_{f\sigma}}{m_f} & \text{for a quark} \end{cases} \quad (m_\sigma^2 \ll m_f^2) \tag{9}$$

Note, that the terms in the square brackets coincide with the corresponding coefficients in the Gell-Mann-Low function $\beta(e)$, defined by the following equation

$$\mu \frac{de}{d\mu} = \beta(e) \equiv -\frac{b e^3}{16\pi^2} + \dots \quad .$$

The result is readily generalized to the case of scalar and vector particles. First of all we write out the original couplings of the Higgs particle to the vector and scalar fields:

$$\mathcal{L}^{\sigma W}_{int} = 2(G\sqrt{2})^{1/2} f_{W\sigma}\, m_W^2\, \sigma W_\mu^+ W_\mu^- + \\ + g^2 \sigma^2 W_\mu^+ W_\mu^- \qquad (10)$$

$$\mathcal{L}^{\sigma S}_{int} = -2(G\sqrt{2})^{1/2} M^2_{S\sigma}\, \sigma S^+ S^- - \\ - \lambda_{S\sigma}\, \sigma^2 S^+ S^- \qquad (11)$$

Here $W_\mu^\pm$ is the W-boson field, $S^\pm$ is some (hypothetical) charged scalar field and we introduced the following notations for the parameters characterizing the Higgs' vertices: $f_{W\sigma}$, $M_{S\sigma}$, g^2, $\lambda_{S\sigma}$. In the standard minimal (Weinberg-Salam) model

$$f_{W\sigma} = 1, \\ g^2 = e^2/4\sin^2\theta_W, \\ M^2_{S\sigma} = 0, \\ \lambda_{S\sigma} = 0. \qquad (12)$$

(θ_W is the Weinberg angle). In the standard model there are no (physical) charged scalar particles $S^\pm$.

Note, that to find the answer for $\sigma\gamma\gamma$ amplitude we need only linear in σ terms in Eqs. (10), (11). Quadratic terms will be useful later.

Just in the same way as above we can consider the σ field in (10), (11) to be constant as far as the graphs 1b,c are concerned in the limit $m_\sigma^2 \ll m_W^2, m_S^2$.

The effect of $\mathcal{L}^{\sigma W}_{int}$, $\mathcal{L}^{\sigma s}_{int}$ then reduces to the following shift in the masses:

$$
\begin{aligned}
m_W^2 &\to m_W^2 + 2(G\sqrt{2})^{1/2} f_{W\sigma} m_W^2 \sigma, \\
m_s^2 &\to m_s^2 + 2(G\sqrt{2})^{1/2} M_{s\sigma}^2 \sigma .
\end{aligned}
\tag{13}
$$

These shifted masses are to be inserted in Eq. (6). We remind also that

$b = 7$ for W bosons and $b = -\frac{1}{3}$ for s bosons.

Expanding in σ we arrive at the expression

$$
\mathcal{L}^{\sigma\gamma\gamma} = \begin{cases} -\dfrac{7\alpha}{8\pi}(G\sqrt{2})^{1/2} f_{W\sigma}\, \sigma F_{\mu\nu}F_{\mu\nu} & \text{for the W-boson loop} \\[2ex] \dfrac{\alpha}{24\pi}(G\sqrt{2})^{1/2} \dfrac{M_{s\sigma}^2}{m_s^2}\, \sigma F_{\mu\nu}F_{\mu\nu} & \text{for the s-boson loop} \end{cases}
\tag{14}
$$

In terms of the form-factor F these results read:

$$
F = \begin{cases} [7] f_{W\sigma} & \text{for the W boson} \\[2ex] [-\frac{1}{3}] \dfrac{M_{s\sigma}^2}{m_s^2} & \text{for the s boson} \end{cases} \qquad (m_\sigma^2 \ll m_W^2, m_s^2),
\tag{15}
$$

where the factors in the square brackets again represent the corresponding coefficients in the Gell-Mann-Low function. Eqs. (9), (14) (together with the definitions (3), (10), (11)) are our final results in this section.

It is worth emphasizing that in the case considered the knowledge of the ultraviolet behaviour of the theory (the Gell-Mann-Low function) allows to compute the amplitude in the infrared region, i.e. for the momenta much lower than masses. The reason is that the cut-off parameter Λ enters only via log factors because of the renormalizability of the theory[*]). Moreover, on dimensional grounds alone one concludes that for vanishing external momenta the actual dependence is on the ln Λ/m so that the mass dependence is also known.

3. Effective Lagrangian of the Higgs-Meson - Multiphoton Interaction

Low-energy theorems derived in the previous section for the $\sigma \to 2\gamma$ decay can be easily extended to cover loops with arbitrary number of external Higgs-meson and photon lines. The only ingredients are QED effective Lagrangians.

Consider again the fermion loop. Then, in the absence of the σ field the effective Lagrangian of photon interaction was found first by Heisenberg and Euler. We denote it by $L_f(F_{\mu\nu}F_{\mu\nu}\ ,\ F_{\mu\nu}\widetilde{F}_{\mu\nu}\ ,\ m_f^2)$. If now a slowly varying (in the scale m_f^{-1}) field σ is switched on its effect can be absorbed again into a redifinition of the mass. Thus, the effective Lagrangian we are interested in is given by

$$\mathcal{L}_f\left(F^2,\ F\widetilde{F},\ \left(m_f + (G\sqrt{2})^{1/2} M_{f\sigma}\, \sigma\right)^2\right) \tag{16}$$

---

*) A careful reader might worry that we appeal in fact to the unitary gauge for vector fields. There is nothing wrong, however, to use this gauge for the purposes needed here. Moreover, we will confirm the result by a direct computation in an explicitly renormalizable gauge in sect. 4.

where $M_{f\sigma}$ is introduced in Eq. (3). The expansion of this Lagrangian generates the amplitudes of processes involving arbitrary number of σ 's and 2n photons (2n > 2). Similarly, the effective Lagrangian for the Higgs-meson interaction induced by the scalar and vector fields are given by

$$\mathcal{L}_S\left(F^2,\ F\tilde{F},\ \left(m_S^2+2(G\sqrt{2})^{1/2}M_{S\sigma}^2\sigma+\lambda_{S\sigma}\sigma^2\right)\right)$$

and

$$\mathcal{L}_W\left(F^2,\ F\tilde{F},\ \left(m_W^2+2f_{W\sigma}(G\sqrt{2})^{1/2}m_W^2\sigma+g^2\sigma^2\right)\right) \qquad (17a)$$

respectively (for definition of parameters see Eqs. (10),(11).

The explicit form of L_f and L_S can be found, e.g., in the book by Schwinger[7] while for a charged vector field L_W was calculated first by Vanyashin and Terentiev[6]:

$$\mathcal{L}_S(F^2, F\tilde{F}, m_S^2)=\frac{1}{16\pi^2}\int_0^\infty\frac{dt}{t}e^{-m_S^2t}\left(\frac{e^2\mathcal{E}\mathcal{H}}{\sin e\mathcal{E}t\ \sinh e\mathcal{H}t}-\frac{1}{t^2}-e^2\frac{\mathcal{E}^2-\mathcal{H}^2}{6}\right);$$

$$\mathcal{L}_f(F^2, F\tilde{F}, m_f^2)=\frac{1}{8\pi^2}\int_0^\infty\frac{dt}{t}e^{-m_f^2t}\ \times$$

$$\times\left(-Q_f^2e^2\mathcal{E}\mathcal{H}\cot Q_fe\mathcal{E}t\ \coth Q_fe\mathcal{H}t+1/t^2-(e^2Q_f^2/3)(\mathcal{E}^2-\mathcal{H}^2)\right); \qquad (17b)$$

$$\mathcal{L}_W(F^2, F\tilde{F}, m_W^2)=\frac{e^2}{4\pi^2}\int_0^\infty\frac{dt}{t}\left[e^{-im_W^2t}\mathcal{H}\left(\mathcal{E}\frac{\sin e\mathcal{H}t}{\sinh e\mathcal{E}t}-\mathcal{H}\right)-\right.$$

$$\left.-e^{-m_W^2t}\cdot\mathcal{E}\left(\mathcal{H}\frac{\sin e\mathcal{E}t}{\sinh e\mathcal{H}t}-\mathcal{E}\right)\right]+3\mathcal{L}_S(F^2, F\tilde{F}, m_W^2),$$

where

$$\mathcal{E}=\left[\left(\frac{(F^2)^2}{16}+\frac{(F\tilde{F})^2}{16}\right)^{1/2}-\frac{F^2}{4}\right]^{1/2},\quad \mathcal{H}=\left[\left(\frac{(F^2)^2}{16}+\frac{(F\tilde{F})^2}{16}\right)^{1/2}+\frac{F^2}{4}\right]^{1/2},$$

$$F^2\equiv F_{\mu\nu}F_{\mu\nu},\quad F\tilde{F}\equiv\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}.$$

Expanding Eqs. (16), (17), in the "external" σ and photon fields one finds the low energy limit of the amplitude with arbitrary number of Higgs mesons and photons. As an example of the technique developed let us present the effective Lagrangian of the $\sigma\to 4\gamma$ decay:

$$
\mathcal{L}^{\sigma 4\gamma} = \begin{cases} -\dfrac{2\alpha^2 Q_f^4 (G\sqrt{2})^{1/2} M_{f\sigma}}{45 m_f^5}\, \sigma \left[(F_{\mu\nu} F_{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu} \tilde{F}_{\mu\nu})^2 \right], \\ \qquad\qquad (\text{lepton loop}) \\ -\dfrac{\alpha^2 (G\sqrt{2})^{1/2} M_{s\sigma}^2}{90 m_s^6}\, \sigma \left[\frac{7}{4} (F_{\mu\nu} F_{\mu\nu})^2 + \frac{1}{4} (F_{\mu\nu} \tilde{F}_{\mu\nu})^2 \right], \\ \qquad\qquad (\text{s-boson loop}) \\ -\dfrac{\alpha^2 f_{w\sigma} (G\sqrt{2})^{1/2}}{10 m_W^4}\, \sigma \left[\frac{29}{4} (F_{\mu\nu} F_{\mu\nu})^2 + \frac{27}{4} (F_{\mu\nu} \tilde{F}_{\mu\nu})^2 \right]. \\ \qquad\qquad (\text{W-boson loop}) \end{cases} \tag{18}
$$

Note that, unlike the case of the two-photon decay, heavy particles do decouple so that the amplitude is dominated by relatively light intermediate states.

To complete the presentation we give also explicitly the effective Lagrangian for the transition $2\gamma - n\sigma$. The matter is that the corresponding piece of the purely electromagnetic Lagrangian usually is not given explicitly since it is included into the charge renormalization. The Lagrangian looks as follows:

$$
\mathcal{L}^{(n\sigma)2\gamma} = \begin{cases} F_{\mu\nu} F_{\mu\nu} \dfrac{\alpha Q_f^2}{6\pi} \ln \left(1 + (G\sqrt{2})^{1/2} \dfrac{M_{f\sigma}}{m_f} \sigma \right) \\ \qquad\qquad (\text{lepton loop}) \\ F_{\mu\nu} F_{\mu\nu} \dfrac{\alpha}{48\pi} \ln \left(1 + 2 (G\sqrt{2})^{1/2} \dfrac{M_{s\sigma}^2}{M_s^2} \sigma + \lambda_{s\sigma} \sigma^2 / m_s^2 \right) \\ \qquad\qquad (\text{s-boson loop}) \\ - F_{\mu\nu} F_{\mu\nu} \dfrac{7\alpha}{16\pi} \ln \left(1 + 2 (G\sqrt{2})^{1/2} f_{w\sigma} \sigma + (G\sqrt{2}) \sigma^2 \right) \\ \qquad\qquad (\text{W-boson loop}) \end{cases} \tag{19}
$$

Expanding Eqs. (19) in the σ field one can get the 2γ transition into any number of the Higgs bosons. Examples are in fact given in the previous section. Lagrangian (19) includes all the terms which survive in the limit of the intermediate mass tending to infinity.

4. Loops with Particles of Arbitrary Mass

Low-energy theorems derived above are based on very general grounds and it is nice to have them. In practical applications, however there are intermediate states of mass comparable to or lower that that of the Higgs meson. To find the amplitudes in this case there is no other way but to calculate the Feynman graphs explicitly. As was mentioned in the Introduction the $\sigma \to 2\gamma$ amplitude was evaluated in Ref. [3]. We just succeeded in getting an analytic expression for the final answer.

As far as intermediate fermions and scalar states are concerned the calculation is quite routine and we just give the final answer.

An intermediate lepton of mass m_l contributes the following amount:

$$F = Q_e^2 \frac{M_{e\sigma}}{m_e} F_e(\beta_e) \tag{20}$$

where

$$\beta_e = 4m_e^2 / m_\sigma^2$$

and

$$F_e(\beta) = \begin{cases} -2\beta\left[(1-\beta)\left(\arctan\frac{1}{\sqrt{\beta-1}}\right)^2 + 1\right], & \beta > 1 \\ -2\beta\left\{\frac{1-\beta}{4}\left[\pi^2 - \left(\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}\right)^2\right] + 1\right\} - \\ - i\pi\beta(1-\beta)\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}, & \beta < 1 \end{cases} \tag{21}$$

The amplitude associated with a scalar, charge-one particle of mass m_s looks as follows:

$$F = \left(\frac{M_{s\sigma}}{M_s}\right)^2 F_s(\beta_s) \qquad (22)$$

where $\beta_s = 4m_s^2/m_\sigma^2$,

$$F_s(\beta) = \begin{cases} \beta\left[1-\beta\left(\arctan\frac{1}{\sqrt{\beta-1}}\right)^2\right], & \beta>1 \\ \beta\left\{1-\frac{\beta}{4}\left[\pi^2-\left(\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}\right)^2\right]\right\} - \\ -i\frac{\pi}{2}\beta^2 \ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}, & \beta<1 \end{cases} \qquad (23)$$

The plots of the functions $F_e(\beta)$ and $F_s(\beta)$ are presented in Figs. 3,4.

Note that in the limit of $\beta \to \infty$ $(M_{e,s} \to \infty)$ we come again to the low-energy results summarized in the previous section, $F_e(\beta=\infty) = -\frac{4}{3}$, $F_s(\beta\ \infty) = -\frac{1}{3}$. In the opposite limit, $\beta \to 0$, the contributions of the fermion and scalar fields tend to zero. Thus, very light states are of no importance. The vanishing of the functions $F_{e,s}(\beta)$ in the limit of $\beta \to 0$ is a direct consequence of the unitarity condition for the three-point Green's function involved.

A few remarks are now in order concerning quarks. The first and trivial one is that for a quark there are three loops because of color so that a given flavor contributes three times as large as the value (20) (with $Q_e \to Q_q$). Secondly, dressing of quarks with gluons results in corrections to the bare amplitude (20). However, these corrections are proportional to a small coupling constant $\alpha_s(|4m_q^2 - m_\sigma^2|)$ provided that the mass m_σ is far enough (above or below) the flavor threshold. The line of reasoning is the same as in Ref. [8] (see also [9]).

A special case of $m_\sigma \approx 2m_q$ can be also consistently treated. Here one has to take into account mixing of the Higgs meson with resonances constructed from the quark-antiquark pair. If the quark is heavy enough (say, charmed or bottom quark) then the mixing can be evaluated by means of the QCD sum relus[9]. We would not go into details here, since the problem is specific for a particular choice of mass and here we present general equations. The conclusion, however, is that mixing does not change the results too much (the effect does not exceed, say, a factor of two).

Let us discuss now the most complicated case of vector intermediate state. In explicitely renormalizable gauges there emerge many auxilary fields (the Faddeev-Popov ghosts and unphysical charged scalars). As a result in standard gauges such as the Feynman one instead of two diagrams of Fig. 1b one has about a dozen different diagramms (see Ref. [3]). We performed the calculation of the $\sigma \to 2\gamma$ amplitude both in the Feynman gauge and in a more convenient gauge which generalizes the so called background gauge (see e.g. Ref. [10]) to theories with spontaneous symmetry breaking.

Since we want to evaluate only the $W^\pm$-boson loops it is convenient to keep electromagnetic and σ fields as external classical fields. The gauge fixing term is chosen in the form

$$\Delta \mathcal{L} = -\left(\mathcal{D}_\mu W_\mu^+ + ig\tilde{\sigma}\varphi^+\right)\left(\mathcal{D}_\mu W_\mu^- - ig\tilde{\sigma}\varphi^-\right),$$

where $\mathcal{D}_\mu$ is the covariant derivative:

$$\mathcal{D}_\mu W_\nu^\pm = (\partial_\mu \pm ie A_\mu) W_\nu^\pm ,$$

. $\psi^\pm$ are unphysical scalars and $\tilde{\sigma}$ includes vacuum expectation value

$$g^2 \tilde{\sigma}^2 = m_W^2 \left(1 + 2 (G\sqrt{2})^{1/2} f_{W\sigma}\, \sigma + (G\sqrt{2}) \sigma^2\right)$$

In this gauge the ghost term looks as

$$\mathcal{L}^{ghost} = (\mathcal{D}_\mu \eta_-^*)(\mathcal{D}_\mu \eta_+) - g^2 \tilde{\sigma}^2 \eta_-^* \eta_+ +$$

$$+ (\mathcal{D}_\mu \eta_+^*)(\mathcal{D}_\mu \eta_-) - g^2 \tilde{\sigma}^2 \eta_+^* \eta_-$$

+ other terms irrelevant to one-loop calculation .

The entire Lagrangian relevant to the one-loop calculation (i.e. quadratic in $W^\pm$ fields) is

$$\mathcal{L} = \mathcal{L}^{ghost} - (\mathcal{D}_\mu W_\nu^+)(\mathcal{D}_\mu W_\nu^-) - 2ig F_{\mu\nu} W_\mu^+ W_\nu^- +$$

$$+ g^2 \tilde{\sigma}^2 W_\mu^+ W_\mu^- + (\mathcal{D}_\mu \psi^+)(\mathcal{D}_\mu \psi^-) - g^2 \tilde{\sigma}^2 \psi^+ \psi^- -$$

$$- 2ig (\partial_\mu \tilde{\sigma})(\psi^+ W_\mu^- - \psi^- W_\mu^+) , \quad \text{where}$$

$$(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu).$$

Note that the Lagrangian is explicitly gauge invariant with respect to the electromagnetic field. As a result the number of graphs is considerably reduced as compared to the Feynman gauge. Apart from the graphs 1b there arise similar graphs with η and ψ loops (in the Feynman gauge there are

$W\psi\gamma$ vertices which increase the number of graphs).
The contribution of W into the amplitude of the $\sigma \to 2\gamma$ decay is given by

$$F = f_{W\sigma} F_W(\beta_W) , \tag{23}$$

$$\beta_W = 4m_W^2 / m_\sigma^2 ,$$

$$F_W(\beta) = \begin{cases} 3\beta(2-\beta)\left(\arctan \frac{1}{\sqrt{\beta-1}}\right)^2 + 3\beta + 2 , & \beta > 1 \\ \frac{3}{4}\beta(2-\beta)\left[\pi^2 - \left(\ln \frac{1+\sqrt{1-\beta}}{1\ \sqrt{1-\beta}}\right)^2\right] + & \\ + 3\beta + 2 + i\,\frac{3\pi}{2}\beta(2-\beta)\ln\frac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}} , & \beta < 1 . \end{cases} \tag{24}$$

The plot of function $F_W(\beta)$ is presented in Fig. 5. Note that, unlike the case of spin-1/2 or spin-0 the W bosons do not decouple in the limit $\beta_W \to 0$ ($m_W/m_\sigma \to 0$). This is due to the contribution of the longitudinally polarized vector mesons which are well-known to violate unitarity as far as the Higgs mesons themselves do not come into the game (the limit of $\beta_W \to 0$ just corresponds to a heavy Higgs boson).

5. Conclusions

In this paper we have considered the $\sigma \to 2\gamma$ decay. The decay is interesting since the amplitude i. computable within existing models of weak interactions and is sensitive to the contribution of all the charged particles, including arbitrary heavy ones. Therefore, study of this decay could distinguish between various models.

The results for the amplitude are most simple in the case of a heavy intermediate state when they are reduced to the coefficients in the Gell-Mann-Low function for photon. This case can be of most practical importance too. Indeed, there are good chances that in the near future search for the Higgs mesons will be confined to the mass interval 10-20 GeV, which lies far below the (conjectured) W-boson mass. As for the fermion contribution it dies away as far as Higgs meson is heavier than the fermion. Thus, the contribution of the known, relatively light fermions can be unimportant if the Higgs meson has the mass of, say, 10-20 GeV.

If there emerges a particular candidate for a Higgs particle, then explicit dependence on the mass of intermediate particles can be readily accounted for. In an accompaning paper we try the X(2.83) state as such a candidate.

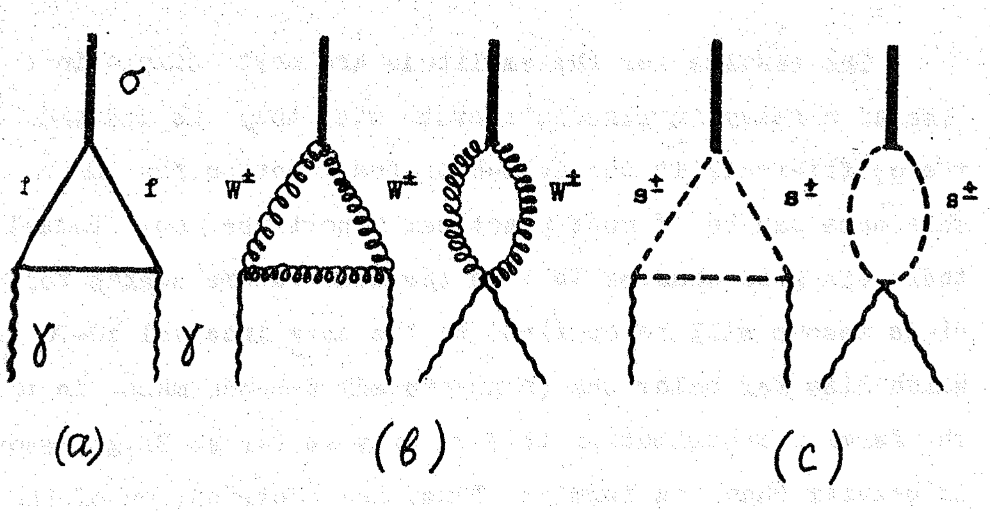

Fig.1. The $\sigma \to 2\gamma$ amplitude in the one-loop approximation:

a) Intermediate fermion state; b) W-boson contribution (unitary gauge is assumed); c) The contribution of physical scalar fields which are present in the model with at least two Higgs doublets.

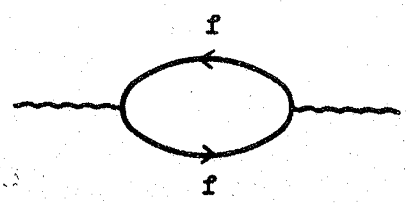

Fig. 2. An example of one-loop contribution to the renormalization of the electric charge.

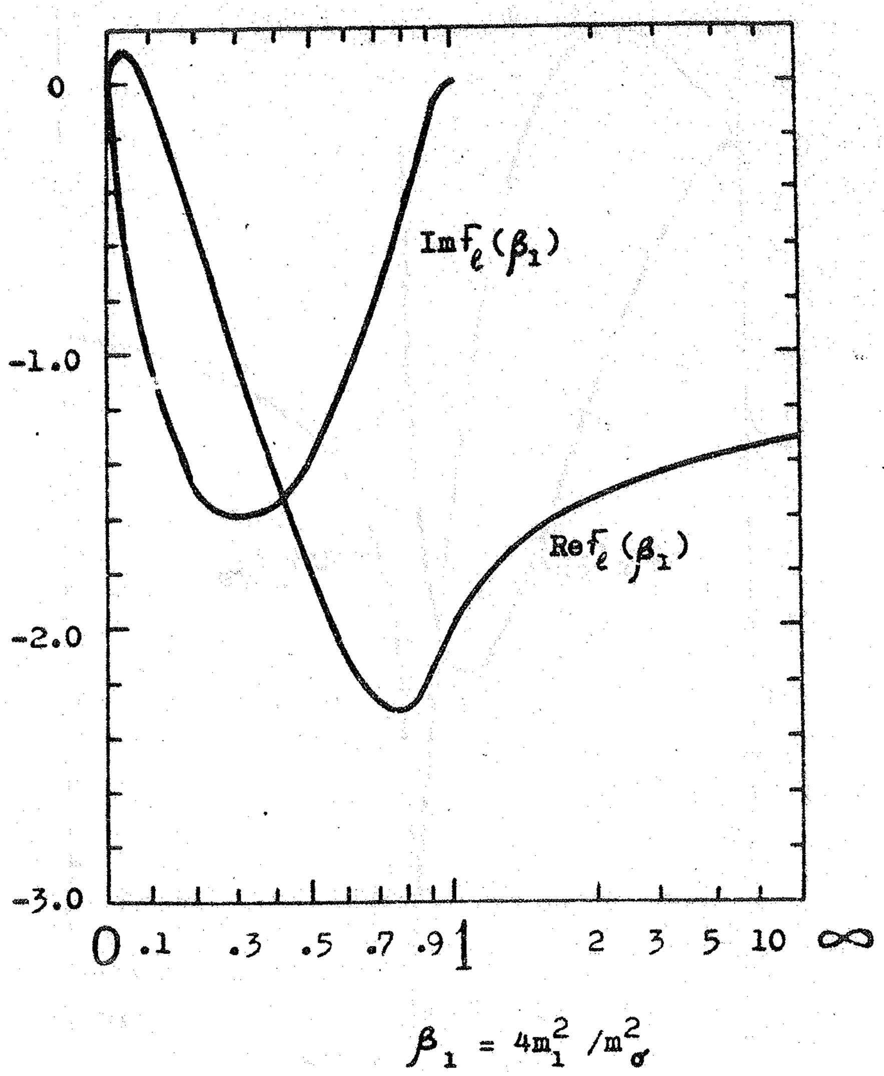

Fig. 3. Plot of the function $F_1(\beta)$ determining the contribution of intermediate lepton state into the amplitude $\sigma \to 2\gamma$. The β scale is $(2/\pi)$ arctan β.

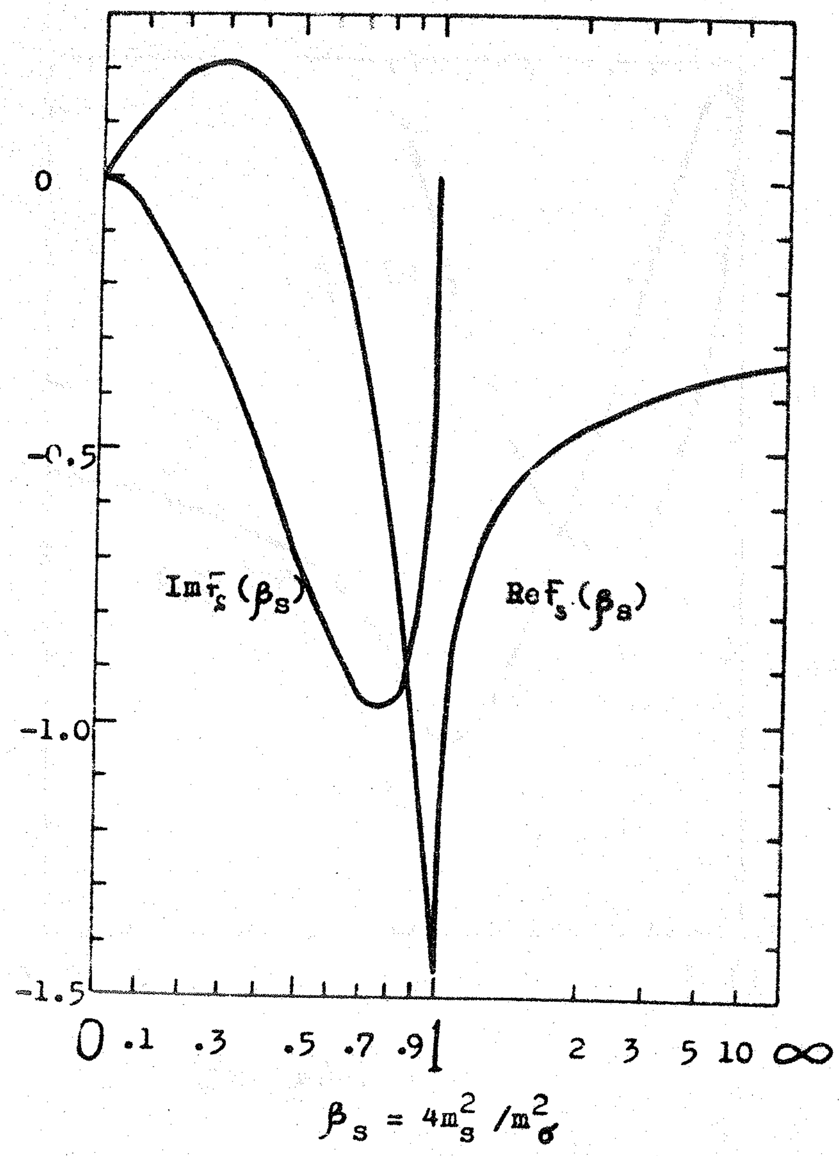

Fig. 4. Plot of the function $F_s(\beta)$ determining the contribution of intermediate scalar state into the amplitude $\sigma \to 2\gamma$. The β scale is $(2/\pi)$ arc tan β .

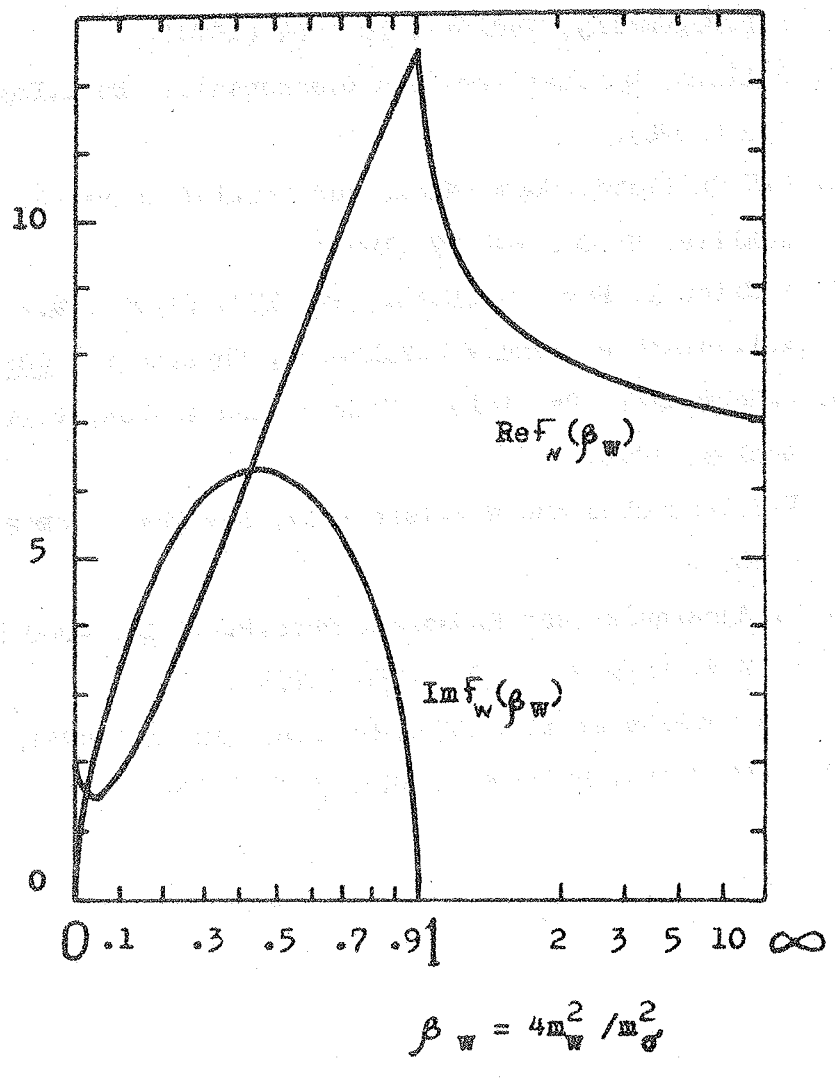

Fig. 5. Plot of the function $F_W(\beta)$ determining the contribution of intermediate W-boson state into the amplitude $\sigma \to 2\gamma$. The β scale is $(2/\pi)$ arctan β .

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