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Low frequency *1/f*-like fluctuations of the AE-index as a possible manifestation of self-organized criticality in the magnetosphere

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Abstract Low frequency stochastic variations of the geomagnetic AE-index characterized by $1/f^b$-like power spectrum (where f is a frequency) are studied. Based on the analysis of experimental data we show that the B_z-component of IMF, velocity of solar wind plasma, and the coupling function of Akasofu are insufficient factors to explain these behaviors of the AE-index together with the $1/f^b$ fluctuations of geomagnetic intensity. The effect of self-organized criticality (SOC) is proposed as an internal mechanism to generate $1/f^b$ fluctuations in the magnetosphere. It is suggested that localized spatially current instabilities, developing in the magnetospheric tail at the initial substorm phase can be considered as SOC avalanches or dynamic clusters, superposition of which leads to the $1/f^b$ fluctuations of macroscopic characteristics in the system. Using the sandpile model of SOC, we undertake numerical modeling of space-localized and global disturbances of magnetospheric current layer. Qualitative conformity between the disturbed dynamics of self-organized critical state of the model and the main phases of real magnetospheric substorm development is demonstrated. It is also shown that power spectrum of sandpile model fluctuations controlled by real solar wind parameters reproduces all distinctive spectral features of the AE fluctuations.

Key words. Magnetospheric physics (MHD waves and instabilities; solar wind – magnetosphere interactions; storms and substroms).

1 Introduction

It is known that long-period fluctuations of the geomagnetic AE-index have the power spectrum $1/f^b$ (f is a Fourier frequency) with exponent $b = 1.0$ at $f < 0.05$ mHz and $b = 2.2$–2.4 at higher frequencies, see Tsurutani *et al.*, 1990; Takalo *et al.*, 1993. This spectrum structure of AE-index fluctuations can be seen at the maximum of solar activity as well as at the periods of quiet Sun. Comparison of the AE-index spectrum with the spectrum of IMF B_s (southward component of the IMF which is one of the most important agents for AE) shows their significant dissimilarity: the dependence of spectral density of the B_s-component on the frequency is characterized by another value of the parameter b and has no break in the region $f = 0.05$ mHz (Tsurutani *et al.*, 1990).

The nature of nonlinearity of the magnetosphere reaction on the IMF-variation has not been clear till now. Among the possible causes for that were discussed: (a) the nonlinear filtering of IMF-fluctuations at $f > 0.06$ mHz by the magnetosphere, (b) the effect of saturation of the AE-variations at high B_s-values, (c) the auroral electrojets instability and its influence on the anomalous conductivity of plasma layers, and (d) the deterministic low-dimensional chaos in the magnetosphere.

From a physical point of view on the origin of $1/f^b$-fluctuations in nonequilibrium systems with many degrees of freedom, the more plausible hypotheses include the possibility of the superposition of numerous instabilities with the large relaxation time range (Hooge, 1997). Computer modeling has revealed that such an effect leads naturally to the $1/f^b$-like power spectrum of large-scale fluctuations, the mechanism proposed in frame of the theory of a self-organized criticality (SOC). This concept has been developed by P. Bak and co-workers to describe the interaction between fractal processes (fluctuations with the spectrum like $1/f^b$) and spatial fractal structures in complex dynamical systems (Bak *et al.*, 1987, 1988). Then the SOC-theory was successfully used for describing flicker-noise effects and turbulence in fluid and solid chaotic media. It was shown that the empirical Gutenberg-Richter law (the power dependence between earthquake frequency and

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their energy) is a sequence of the fact that the earth's crust as a metastable system of interacting blocks is in the SOC-state (Bak and Tang, 1989). However, the question of the existence of SOC in Earth's magnetosphere has not been considered before.

The aim of this work is to give the experimental and numerical evidence for the existence of SOC in the magnetosphere (Uritsky, 1996). In this connection special attention is paid to the investigation of the origin of scaling characteristics of the AE-index dynamics. In addition to the differences quoted in the literature of AE-index and IMF B_s spectra, the comparison of spectra of solar wind velocity and the coupling function of Akasofu with the spectral structure of the AE-index was completed. To prove the conformity of AE-index to scale-invariant criterion of the SOC avalanche size distribution, a special exploration is made, and the results are presented later. Then we demonstrate on the base of a sandpile cellular automaton how the SOC principles can be used to model some important statistical features of AE response to the solar wind disturbances.

2 Method and results of data processing

The hourly averaged values of AE-index, IMF B_z (where B_z is Z-component of IMF in GSM coordinate system) and solar wind velocity v for two periods (1968–1969 and 1973–1974) were analyzed.

The Fourier power spectra have been calculated using standard FFT algorithm. The data time series to be analyzed were divided for a few time intervals each of which had its own power spectrum. Then all these power spectra were averaged. This procedure allows us to increase significantly the accuracy of spectrum calculation without using the frequency smoothing. In accordance with the length of time series and the number of chosen intervals in it, the lower analyzed frequency equals $1.0 \cdot 10^{-6}$ Hz and the highest frequency is $1.4 \cdot 10^{-4}$ Hz.

The time-dependent solar wind-magnetosphere coupling function of Akasofu $\varepsilon(t)$ was calculated according to formulas:

$$\varepsilon(t) = v \cdot B^2 \cdot \sin^4(\theta/2) \ ,$$

$$B^2 = B_x^2 + B_y^2 + B_z^2 \ ,$$

$$\theta = \mathrm{arctg}(B_y/B_z) \ ,$$

where B_x and B_y are X- and Y-components of IMF correspondingly.

To analyze the distribution density of AE-index, the total range of its variations was divided into 100 equal subdiapasons. The number of AE-values (N) corresponding to each subdiapason was calculated.

For convenience in visualizating the dependency, the Fourier power spectra $S(f)$ and the distribution of AE-index values were presented using double-logarithmic scale. The power functions approximation used for the definition of scaling exponents was made by the least square method.

The power spectra of hourly-averaged AE-index values, B_z-component of IMF, the solar wind velocity v, and the function of Akasofu $\varepsilon(t)$ for period 1973–1974 are presented in Fig. 1. From the figure it can be seen that AE-spectrum differs from B_z and v spectra. It may be shown that the power spectrum of the AE-index can be fitted by function $1/f^b$ with $b = 0.95$ at $f = 1.0 \cdot 10^{-6} - 5.6 \cdot 10^{-5}$ Hz and $b = 2.1$ at $f > 5.6 \cdot 10^{-5}$ Hz. Spectra of B_z and v are characterized by other values of b and by other intervals of frequencies where b is stationary: for B_z-component $b = 0.4$ at $f = 1.0 \cdot 10^{-6} - 1.1 \cdot 10^{-5}$ Hz and $b = 1.0$ at higher frequencies; for v-spectrum $b = 2.2$ at $f = 1.0 \cdot 10^{-6} - 6.9 \cdot 10^{-5}$ Hz and $b \approx 0.5$ at $f > 7.0 \cdot 10^{-5}$ Hz. The Fourier power spectrum of $\varepsilon(t)$ differs considerably from AE spectrum as well: although

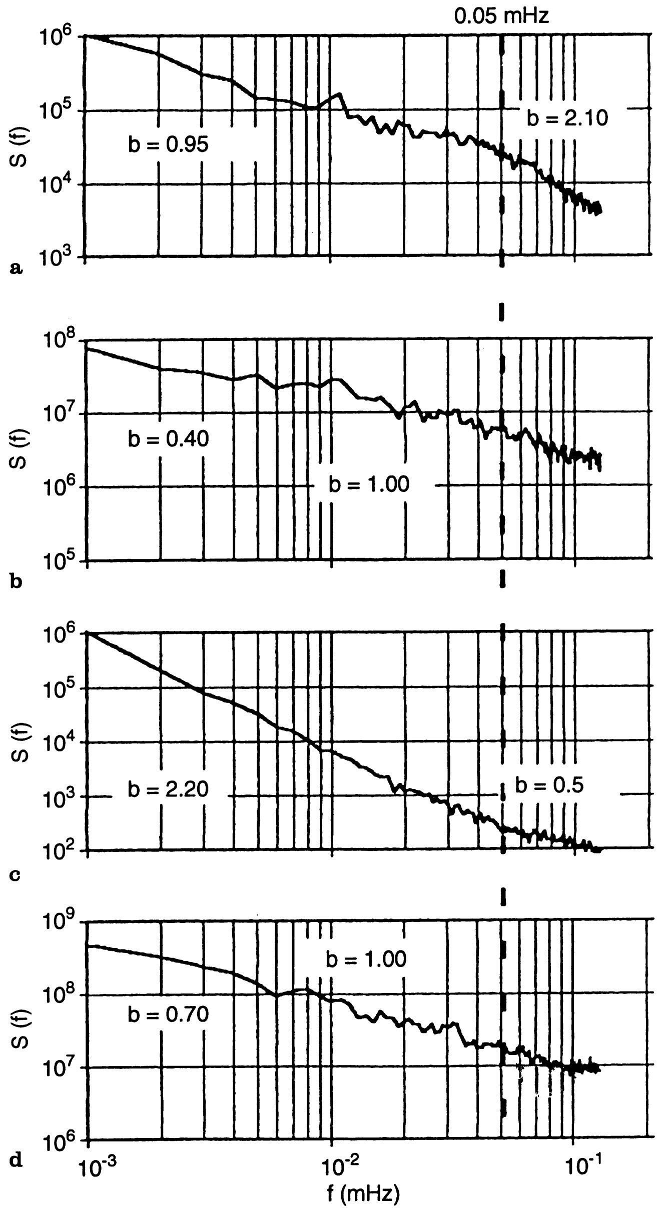

Fig. 1a–d. The power spectra of hourly-averaged AE-index values, B_z-component of IMF, solar wind velocity v, and function of Akasofu $\varepsilon(t)$ for period 1973–1974

its low frequency slope is close to that of the AE fluctuations, at higher frequency the spectrum of the Akasofu function does not show a transition to $b \approx 2$, and has no peculiarities in the region $f = 0.05$ mHz where the break of the AE spectrum is observed (Fig. 1).

An analogous investigation was done for period 1968–1969 (maximum of solar activity). The results were similar to those for the period of low solar activity (1973–1974) considered already.

The observed difference between low-frequency spectra of the AE-indices and spectra of solar wind parameters might be partly explained by the inadequacy of the AE-indices as the characteristic of long-period variations of geomagnetic activity (Tsurutani *et al.*, 1990), but this is not the case. In Fig. 2 there are power-spectra of geomagnetic field fluctuations observed over 20–25 days at the high and middle latitudes. Because of the small extent of the data, the accuracy of spectral density estimation at low frequencies in this case is less than for AE-index. Nevertheless, it is clear that the variations of geomagnetic activity (like AE-index variations) are characterized by spectrum $1/f^b$ with b changing from $b = 1$ to $b > 2$ near $f = 0.05$ mHz. It means that AE-spectrum describes the real reaction of the magnetosphere on the IMF-variations for the considered frequency interval.

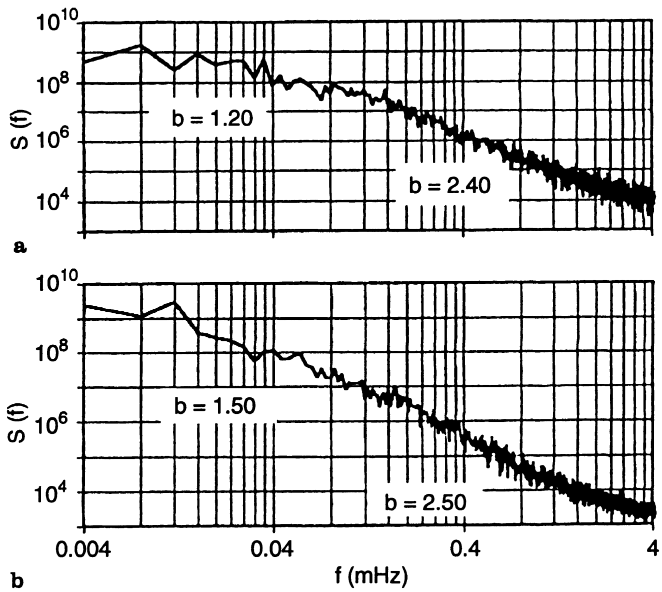

Fig. 2a, b. The power-spectra of geomagnetic field fluctuations observed for 20–25 days at the high and middle latitudes

The mentioned regularities of spectra presented in Figs. 1, 2 show that the connection of AE-index with B_z and υ is strongly nonlinear. Really, contrary to these solar wind parameters, AE-index fluctuations have the form of classical flicker-noise with $b = 1$, besides, at the high frequencies $b \geq 2.0$. The low frequency behavior of AE could probably be explained by variations of the solar wind-magnetosphere coupling function having similar spectral dependence, but the nature of AE dynamics at frequencies higher than 0.05 mHz seems not to be clear. The remarkable stability of AE-spectrum for both frequency intervals at the significant variations of solar activity level supposes that the physical processes responsible for the $1/f^b$-like geomagnetic fluctuations have entirely magnetospheric origin.

It will be shown below that there are reasons to speak on the magnetosphere as on an open non-equilibrium system which exists in the state of the self-organized criticality (SOC) forced by internal- and external influences, and the observed scale-invariant form of AE-index fluctuations can be considered as a result of superposition of great number of turbulent instabilities of all sizes developing in plasma sheet.

In the frame of this hypothesis, one has to expect the statistical distribution of AE-values to be described by the expression $N \sim 1/AE^{\kappa}$, where $\kappa = 1$ (this value is predicted by SOC-theory for 2 D-systems (Bak *et al.*, 1987)). In Fig. 3, the distribution of AE-values for the period of solar maximum activity is presented (the analogous distribution takes place for 1973–1974). For the both periods (maximum and minimum of solar activity) an interval (for AE = 60–400 nT) in which the AE distribution function may be approximated by power dependence. This interval includes 60–70% of all AE-values, so the fractal statistics of AE-index is dominant. Violation of the power form distribution at high AE-values means that the magnetosphere is in the SOC state at the moderate magnetosphere disturbances. Parameter κ, calculated as a tangent of the angle produced by the distribution curve in the log-log coordinates, varies in the range 0.9–1.1 for different time intervals with its mean value being equal to 1.0. These peculiarities of AE-values distribution as well as its $1/f^b$-like behavior in the frequency domain are

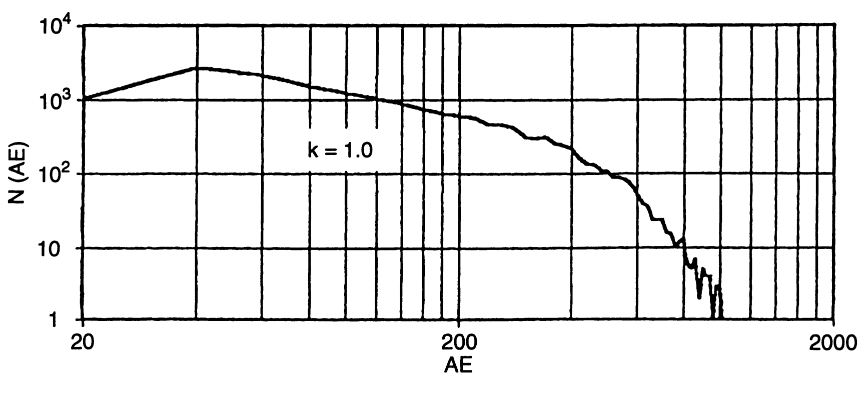

Fig. 3. The distribution of AE-values for the period of solar maximum activity (the analogous distribution takes place for 1973–1974)

in a good agreement with predictions of the SOC-theory.

3 Application of self-organizing sandpile model for studying magnetospheric substorms

3.1 The model

The basic principles and the results of SOC theory may be used for different classes of open systems consisting of the numerous nonlinear elements (large interactive systems). It was shown that in these systems under certain conditions there appear cascades of hierarchically organized instabilities each of which develops as a chain reaction: a lack of stability of any system element can result in disturbing the equilibrium of a few neighboring elements which, in their turn, can provoke the further increase of an instability cluster known as an, avalanche. The growth of clustering stops when the probability of chain reaction expansion becomes less than probability of its damping. If the geometrical dimension of a system is not less than two, its dynamic has a nontrivial attractor: statistics for avalanches are globally stable in spite of the system being close to the critical point (Bak *et al.*, 1987, 1988). A critical entity of the dynamics manifests itself in the scale-invariant behavior of system macroscopical characteristics observed in space, time and functional domain (Bak *et al.*, 1988; Robinson, 1994) similar to critical phenomena appearing in thermodynamics of phase transition. Thus, the time fluctuations of system under the SOC conditions have the character of flicker-noise with $1/f^b$ spectral density, size and energy distributions of the avalanches following the power law scaling relations.

Let us consider a two-dimensional self-organized sandpile model (Bak *et al.*, 1987, 1988) as a tool for numerical study of the sequences of spontaneous and triggered reconnections in the magnetosphere current sheet. The current sheet is considered as rectangular grid of elements each of which has some reserve of free magnetic energy Z. The state of an element with coordinates (x, y) is characterized by energy $Z_t(x, y)$ at the moment t. When $Z(x, y) \leq Z_c$, where Z_c is some critical threshold value, the element is stable (see Eq.1); when the energy of the element is greater than Z_c, 4 energy units transfer to the 4 surrounding elements, so that at the next moment $t = t + 1$

$$Z_{t+1}(x, y) = Z_t(x, y); Z \leq Z_c \quad (1)$$

$$\begin{cases} Z_{t+1}(x, y) = Z_t(x, y) - 4 \ ; \\ Z_{t+1}(x \pm 1, y \pm 1) = Z_t(x \pm 1, y \pm 1) + 1; Z > Z_c. \end{cases} \quad (2)$$

In this model the law of local energy conservation is fulfilled, and transfer of energy from element to element is nonlinear due to the threshold condition "switching" the Eqs (1) or (2). In hydrodynamical limits these rules lead to a singular diffusion-like behavior, with a coefficient of diffusion depending both on Z_c and local Z-value (Carlson *et al.*, 1990, 1993).

Closed boundary conditions are used: excessive energy reaching the edges of the grid is removed from the system. As an initial condition $Z \gg Z_c$ is taken for each element of the grid.

After the initial relaxation, the sandpile model reaches to the SOC-state. As mentioned earlier, in a system which is in this state, the various clusters of the macroscopic region (of instability developing according to chain reaction) can appear in response to weak external perturbations.

3.2 Avalanche development and plasma sheet instabilities

At the beginning, we considered the reaction of the system to weak local disturbance. As an example, in Fig. 4 a cluster is shown provoked by adding a unit energy into the center of the system (32 × 32 array). The area of the avalanche (shaded) is proportional to the response energy spreading on the grid as the chain reaction. As usual, the latter is much more than the energy of influence, and the system works as an amplifier of weak disturbances (Uritsky and Muzalevskaya, 1997). The local disturbance considered can be caused not only by the energy input from the external source but also by the energy redistribution between the neighboring system elements as well as by the decrease of the threshold of the instability Z_c.

It can be shown that a non-stable avalanche evolution has much in common with dynamics of current instabilities of magnetosphere plasma sheet resulting in

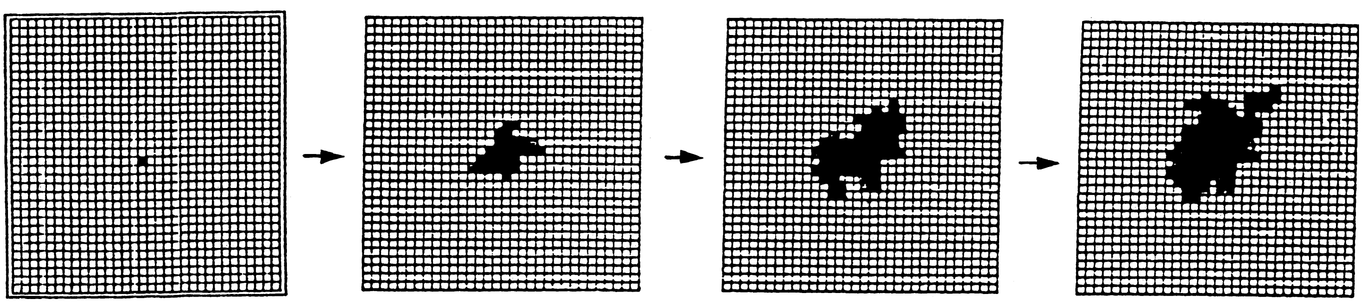

Fig. 4. An example of a cluster provoked by adding a unit energy into the center of the system in SOC regime (32 × 32 array). The *shaded area* is proportional to the reaction energy spreading on the grid as the chain reaction. As usual, the latter is much more than the added energy, and so the system is working as an amplifier of weak perturbations

the appearance of anomalous resistivity regions (Liperovsky and Pudovkin, 1983; Pudovkin and Semenov., 1985; Pudovkin, 1991). Really, the electric currents in the equatorial plane of magnetospheric tail exist in form of a rather long-spread, relatively thin and weakly inhomogeneous current sheet. At the initial substorm phase, the intensity of these currents and the amount of magnetic energy stored in the magnetic lobes increase significantly and reach (at the beginning of break-up) some critical level close to the threshold of plasma instability development. As a consequence of this, the bursts of anomalous resistivity together with impulses of magnetic reconnection appear in localized regions of the current sheet.

Development of the anomalous resistivity results in the re-distribution of electric currents and of the magnetic field in those regions, and thereby in generation of induced electric fields. The latters are carried by the magnetohydrodynamic waves (magnetosonic and Alfvenic ones) to neighboring elements of the current sheet initiating there subsequent bursts of magnetic field reconnection. Thus, each impulse of reconnection may be identified with the avalanches in SOC models. Localization of the bursts in space (in limits of auroral magnetosphere) and in time at the initial substorm phase seems to be random. The frequency of impulses which can be identified with the avalanches in SOC models, increases with the development of disturbance and they merge into the unified and rather complicated system of magnetospheric substorm.

3.3 Modeling the substorms

To show the similarity between the magnetospheric substorms and the processes in the model system it was necessary to study an evolution of system's state in time, in a regime of continuos disturbances. The basic time-dependent quantity of the sandpile is the number $F(t)$ of active sites (elements) which represents dissipation of energy at time t. During an individual relaxation event, $F(t)$ increases to some level connected with an avalanche size and returns to zero as the avalanche ceases. A macroscopical evolution of the sandpile is thought to be a result of the superposition of separate relaxation events acting concurrently and independently (Bak *et al.*, 1988). This representation is somewhat contradictory because the avalanches are allowed to interact in time, but not in space. The simplest way to overcome this difficulty is to record the individual dissipation functions of the independent clusters and to superimpose them at the end of the experiment, such that the consecutive relaxation curves start one after another with a time step of unity. Although the $F(t)$ function thus obtained does not represent the dynamics of the model in a real time regime, its believed to mimic the essential statistical properties of the macrosopic evolution in non-interactive avalanche approximation; in particular, a power spectrum of F is similar to $1/f^b$.

The physical meaning of $F(t)$ is that this function describes fluctuations of integral level of sandpile activity. So, in a frame of qualitative and simplified interpretation F-value can be regarded as the analogue of AE-index characterizing global geomagnetic activity. This implies that we consider here only the contribution of the plasma sheet instabilities to the AE fluctuations. The role of ionospheric and ionosphere-magnetosphere coupling processes is also important, but its discussion is beyond our scope.

Typical samples of $F(t)$ fluctuations modeling a reaction of the magnetosphere to a small disturbance are presented in Fig. 5. The avalanches constituting these samples were initiated by adding a unit portion of energy to Z-value of randomly chosen element of a 50×50 array according to rule:

$$Z_{t+1}(x, y) = Z_t(x, y) + 1 \quad . \tag{3}$$

After the end of each relaxation, the energy injection Eq. (3) was repeated. To reconstruct the $F(t)$-function, the separate relaxation curves were combined using the

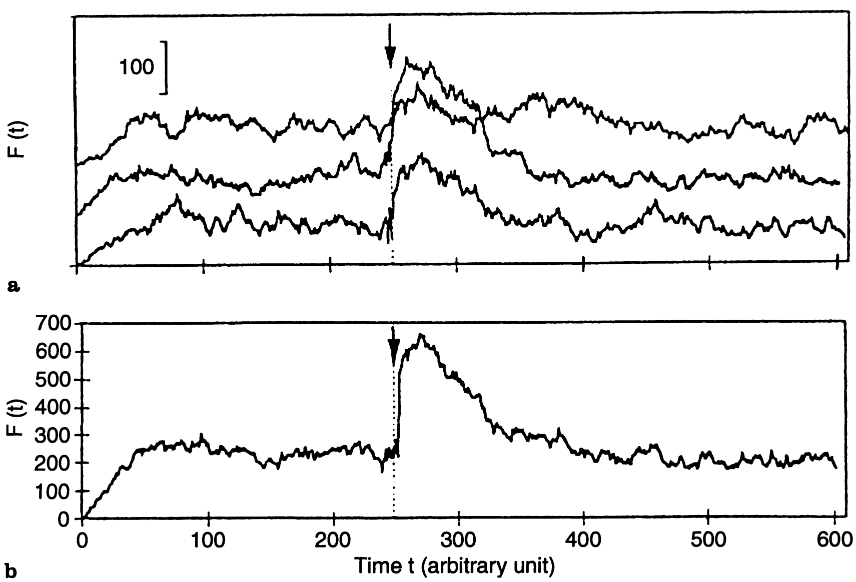

Fig. 5a,b. A reaction of $F(t)$-fluctuations to the increase of the input energy for a 50×50 array (a model of moderate geomagnetic disturbance, see the text). **a** Three typical samples of $F(t)$; **b** sum of samples **a**. *Arrow* shows time of external perturbation method already mentioned. For avalanche beginning at the moment $t = 240$, the quantity of input energy was increased up to 20 for only one time step, and then it again decreased to the previous level. It is seen from Fig. 5, that the reaction of the F function to the influence from outside is similar to the AE-index behavior during a moderate substorm. In this numerical experiment the energy of external disturbance amounted to the value of 30–50% the energy of inner noise of system [a mean-square deviation of $F(t)$ before the discussed event], and the energy of reaction was higher than this level (see Fig. 5b). Thus, as it was supposed that a large interactive system being in the SOC-state, increases weak local disturbances, preserving its global stability at the same time. Notice that such behavior is typical of the magnetosphere during periods of moderate solar activity.

Another kind of non-stationary processes which can develop in the sandpile model is the induced instability that appear in many elements of a macroscopical part of the array simultaneously. These events take place during the active phase of induced magnetospheric substorm, when the northward turnings of the IMF decrease the current sheet instability level sharply, and, as a result, large regions of anomalous resistivity appear in the magnetospheric tail. Under these conditions, the destabilization of current sheet can be modeled by a decreasing of the threshold level of interaction of all SOC-model elements.

Let us discuss qualitatively possible consequences of the Z_c decrease. If the initial state of the model is the state of self-organized criticality, the decrease of interaction threshold transforms the most part of elements into an excited state, so that the entire system enters into the supercritical regime (Bak and chen, 1991). When the system disperses the excess of energy away through its boundaries, and the mean $Z(x, y)$ value is in accordance with the new Z_c-value, the dynamic equilibrium of the system can be restored. When Z_c returns to its previous value (this situation takes place at a rapid second change of IMF sign), the subcritical type of instability connected with the energy deficiency of the system arises. Then the system, having accumulated a sufficient amount of energy, can go to its initial SOC-state.

The results of investigations of the model reaction to the single reversible decrease of Z_c (Fig. 6) agree with those presented. In this model experiment (array 50×50, the initial Z_c-value was 4, and at the moment $t = 80$ it decreased by one unit so that the avalanche initiated at this time started to develop under the supercriticality conditions. At the next moment Z_c returned to the previous value, and a subsequent relaxation of this cluster passed into the usual regime. In addition, an external energy was put into random elements of the system during the entire period of simulation (see Eq. 3). The evaluation of fractal dimensions of the $F(t)$ curve using the Hurst method (Hurst *et al*, 1965) shows that the system really is going through two nonstable intermediate regimes – supercritical and subcritical. The ability of the system to react to the second change of the threshold depends on the whole restoration of the SOC-state.

3.4 Evolution of fractal exponents of AE time series during the substorm

We think that the considered stages of the evolution of a large interactive system correspond rather well to the main phases of an induced substorm. Namely, the initial and the active substorm phases are like the SOC-regime formation and the transition of the model into supercritical state, whereas the recovery phase should be completed by the subcriticality recovering gradually to the SOC-state before the next substorm onset. It has to be noted that in our model as in the real magnetosphere all three phases can be seen under the conditions when the external disturbances follow each other in time intervals sufficient to allow the system to recover.

Table 1 shows the results of statistical study of distribution of the Hurst exponent H over the main phases of the magnetospheric substorms development (about 25 distinctive samples of substorms that occurred in 1978 were selected for this analysis). The data suggest that the H value considerably increases with substorm onset (H index of the initial phase is found to be much higher than that of background AE-variations) and returns to its initial level during the relaxation phase. A temporary decrease in H during the active phase should be considered as a specific transient effect resulting from the intensive reconnection pulses at the "peak" of substorm comprising all the system. Under this condition magnetosphere operates as a whole, low-dimen-

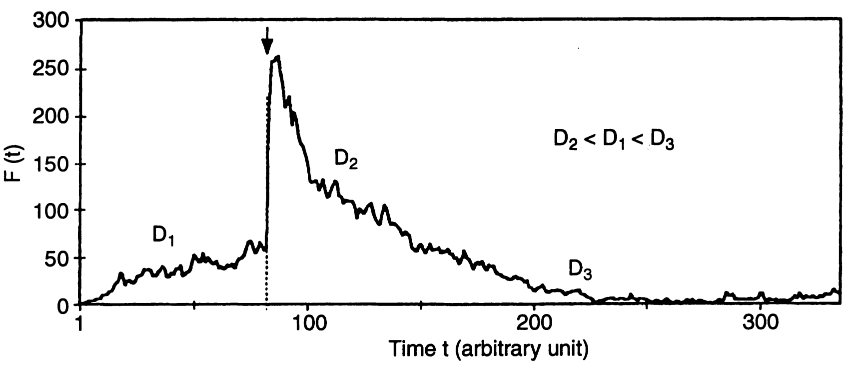

Fig. 6. The result of investigation of the model reaction to the single reversible decrease of Z_c using a 50×50 array. Time of Z_c change is marked by *arrow*

Table 1. Distribution of Hurst's exponent H of AE-variations over the phases of magnetospheric substorm

Substorm phase	Hurst's exponent (H ± confid. interval, $p = .05$)	Classification of the phase in terms of the SOC theory
Background variations with low AE	0.40 ± 0.08	Subcriticality
Initial phase of substorm	0.59 ± 0.07	Supercriticality
Active phase	0.44 ± 0.06	Subcriticality (transient regime)
Relaxation phase	0.49 ± 0.06	Moderate supercriticality
After – effect	0.38 ± 0.05	Subcriticality

sional system and ceases to be an SOC system with many independent degrees of freedom.

Fractal dimension D related with H as $D = 2\text{-}H$ changes during the substorm onset as well (note that this is not a correlation dimension in phase space coordinates but the geometrical dimension of time traces of studied time series). As an illustration, in Fig. 7, the AE-index variations during substorms on 01.07.1978 15:00–02.07.1978 15:00 are shown. One can see that the structure of AE-index fluctuations follows in details the behavior of the model presented in Fig. 6. In particular, the fractal dimension of AE-index fluctuations decreases sharply at the active period of disturbance (supercritical regime) and significantly increases to the end of recovery phase (subcritical regime).

3.5 Comparison of F(t) and AE power spectra

To compare spectral properties of fluctuations of sandpile integral activity $F(t)$ with the spectrum of the AE-index variations it was of interest to see how the model reacts to the parameters of real solar wind. For this purpose, we have taken a time series of hourly averaged values of the *IMF* B_z and the Akasofu function in 1978 and use them as controlling influences. Both types of sandpile response to external perturbations studied were considered. The input energy was assumed to be proportional to time-dependent value $\varepsilon(t)$ of the Akasofu function so that a series of avalanche reactions similar to those shown in Fig. 5 occurred. A coefficient of proportionality between $\varepsilon(t)$ and injected energy was chosen to prevent the saturation of the system dynamics due to the too large clusters. Moreover, a threshold Z_c depended on the sign of the B_z-component: for periods of negative B_z the value of Z_c was 5.0, for positive B_z it decreased slightly to $Z_c = 4.95$.

In contrast to other of our simulations, evolution of $F(t)$ in the sandpile controlled by the solar wind parameters was modeled and observed in real time. This implied that the avalanches were allowed to interact and to compete with each other. As was shown such a regime can be used successfully for simulating SOC effects in continuously disturbed sandpiles (Ginzburg *et al.*, 1997).

In Fig. 8 there are Fourier power spectra corresponding to modeled and natural 1-min AE fluctuations registered during 1978. It can easily be seen that spectral curves of $F(t)$ function and the AE fluctuations look similar. Model spectrum has a power-law form with a characteristic break at some frequency which seems to depend on the array size. The spectral exponent b of $F(t)$ variations changes from the value 1.0 at frequencies below the break to the value 2.2–2.5 for higher frequencies. Therefore, the sandpile cellular automaton reproduces essential features of the AE fluctuations spectrum which were difficult to predict on the base of the solar wind fluctuations analysis. This result gives an important evidence for the hypothesis of SOC in Earth's magnetosphere.

4 Conclusions

The data presented allow us to arrive at the following conclusions:

1. The AE-index fluctuations are characterized by a fractal structure stable for the long time intervals of the moderate solar activity. The power spectra of long-period variations of geomagnetic field intensity show the same peculiarities as AE-index spectrum independently of the latitude of observation point.
2. The spectrum structure of the IMF B_z -component, the solar wind velocity, and the solar wind –

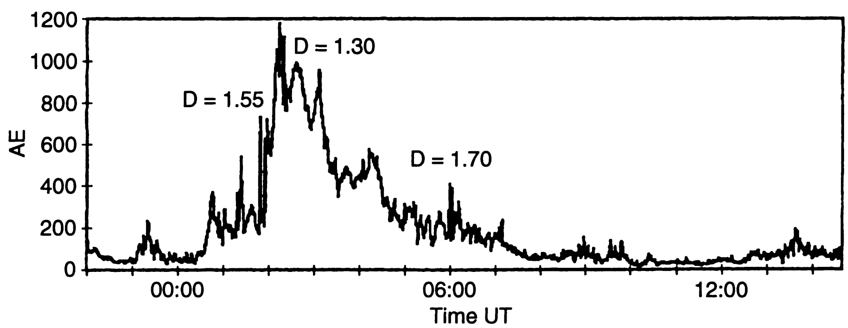

Fig. 7. Example of temporary evolution AE index during a real magnetospheric substorm. *Values* of fractal dimension D of AE trace corresponding to three stages of the substorm development are shown

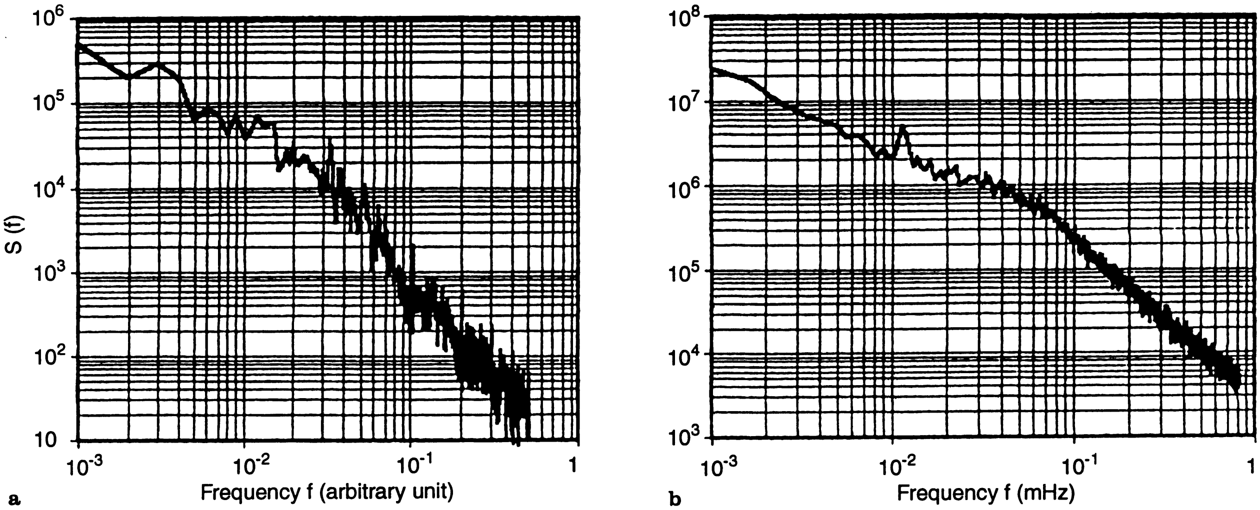

Fig. 8a, b. The Fourier power spectra for model process $F(t)$ obtained in system controlled by **a** the solar wind parameters and **b** for the real AE fluctuations in 1978

magnetosphere coupling function of Akasofu differ greatly from that of AE-index. The solar wind parameters do not demonstrate spectral break at 0.05 mHz, and for higher frequencies they show spectral slopes other than the slope of the AE spectrum. Thus, there is no reason to consider these parameters as a direct cause of the observed fractal dynamics of AE-index at frequencies above 0.05 mHz.

3. Using a two-dimensional sandpile computer model it was shown that the theory of self-organized criticality (SOC) can explain fractal dynamics of AE-index. The effect of SOC may be responsible for the variety of substorm intensity caused by the same variations of solar wind parameters, and high sensitivity of processes in magnetospheric tail to external perturbations.
4. The qualitative conformity between disturbed dynamics of self-organized critical state of the model and the main phases of real magnetospheric substorm development was demonstrated. It was shown that the evolution of fractal dimension of modeled and experimental time series of the AE-index during the substorm has the same form.
5. The power spectrum of sandpile model fluctuations controlled by real solar wind parameters reproduces distinctive features of the AE fluctuations spectrum.

Thus, the application of SOC-theory to the investigation of solar wind-magnetosphere coupling seems to allow one to model the essential statistical features of the geomagnetic activity. In addition the result suggests that the criterion of decreased nonstationary fractal dimension of AE time series can be used for better identifying the initial substorm phases. For further continuing this research, a more detailed quantitative analysis of relations between the large-scale magnetospheric processes and the parameters of the sandpile model is needed.

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