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# LOW-FREQUENCY SCATTERING FROM TWO-DIMENSIONAL PERFECT CONDUCTORS

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We have obtained exact expressions for the leading terms in the low-frequency expansions of the far fields scattered from three different types of two-dimensional perfect conductors: the cylinder with finite cross section, the cylindrical bump on an infinite ground plane, and the cylindrical dent in an infinite ground plane [1]. By inserting the low-frequency expansions of the incident fields and Green's functions into exact integral equations for the surface current, we obtain integral equations for the leading terms in the low-frequency expansions of the surface currents. The leading terms in the low-frequency expansions of the scattered fields are obtained from simple integrations of these low-frequency expansions for the currents.

For the cylinder with finite cross section the low-frequency scattered field for TM polarization is independent of the shape of the cross section of the cylinder and of order $1/\ln kd$ where d is a characteristic dimension of the cylinder. This low-frequency result can not be obtained from a corresponding statics field problem.

For TE polarization the scattered field is of order $(kd)^2$ and it consists of a contribution from a magnetic dipole along the axis of the cylinder and an electric dipole in a direction normal to the axis of the cylinder. The magnetic dipole moment is found directly from the area of the cross section of the cylinder. The electric dipole moment is found by solving two electrostatic problems and performing an integration of these two electrostatic solutions around the cylinder. These electrostatic solutions are determined from simple integral equations and depend only on the shape of the cylinder.

For the two-dimensional bump on a ground plane the low-frequency diffracted field for TM polarization is of order $(kd)^2$, where d is a characteristic dimension of the bump. The low-frequency field is that of a magnetic dipole in the direction normal to the axis of the cylinder and parallel to the ground plane.

For the TE polarization the low-frequency diffracted field is also of order $(kd)^2$. It consists of a contribution from a magnetic dipole in the cylinder direction and an electric dipole in the direction normal to both the axis of the cylinder and the ground plane. The TE magnetic dipole moment is found directly from the area of the cross section of the bump.

Both the TM magnetic dipole field and the TE electric dipole field are written as a constant times a known simple function. It is proven that, remarkably, this constant, which depends only on the shape of the bump, is the same for the two polarizations. This constant can be found by solving either a magnetostatic or electrostatic field problem and performing an integration of these solutions over the bump. This means that the TM-TE low-frequency diffracted fields for an arbitrarily shaped bump are completely determined by calculating one constant.

The low-frequency results for the two-dimensional dent in a ground plane, even more remarkably, are qualitatively the same as the results for the bump. That is, the TM-TE low-frequency diffracted fields for an arbitrarily shaped dent are completely determined by calculating one constant that depends only on the shape of the dent.

These results are all derived for the case where the incident plane wave propagates in a direction normal to the axis of the cylinder. However, they can easily be extended to oblique incidence using the standard procedure [2] based on the facts that the scatterer is perfectly conducting and the obliquely reflected field can be obtained from the normally reflected field by the same procedure.

We include a summary of the equations for the low-frequency far fields produced by TM and TE polarized plane waves incident normally on a cylinder with finite cross section, a two-dimensional bump on an infinite ground plane, and a two-dimensional dent in an infinite ground plane. With $e^{j\omega t}$ time dependence:

I. Cylinder of finite cross section

A. TM polarization ($\bar{E}^i = \hat{z}\, e^{jk(x \cos\phi^i + y \sin \phi^i)}$)

$$E_z^s(\bar{r}) = \frac{1}{\ln kd}\, e^{-j\frac{\pi}{4}} \sqrt{\frac{\pi}{2}}\, \frac{e^{-jkr}}{\sqrt{kr}} \tag{1}$$

B. TE polarization ($\bar{H}^i = \hat{z}\, e^{jk(x \cos\phi^i + y \sin \phi^i)}$)

$$H_z^s(\bar{r}) = -\,(kd)^2\, \frac{e^{-j\frac{\pi}{4}}}{2\sqrt{2\pi}}\, \frac{e^{-jkr}}{\sqrt{kr}}\, \frac{1}{d^2}\Big[A_s + C_1 \sin\phi \sin\phi^i \tag{2}$$

$$+ C_2 \cos\phi \cos\phi^i + C_3 \sin(\phi + \phi^i)\Big],$$

$$C_1 = \int_s \frac{\sigma^{ox}(\bar{r}')}{\varepsilon}\, x' ds', \quad C_2 = \int_s \frac{\sigma^{oy}(\bar{r}')}{\varepsilon}\, y' ds', \quad C_3 = -\int_s \frac{\sigma^{oy}(\bar{r}')}{\varepsilon}\, x' ds' \tag{3a,b,c}$$

$A_s$ is the area of the cross section of the cylinder and $\frac{\sigma^{ox}}{\varepsilon}$ and $\frac{\sigma^{oy}}{\varepsilon}$ are the solutions to the static integral equations

$$\frac{\partial x}{\partial s} = -\int_s \frac{\partial}{\partial s'}\, G^o(\bar{r},\bar{r}')\, \frac{\sigma^{ox}(\bar{r}')}{\varepsilon}\, ds', \quad \bar{r} \in s \tag{4a}$$

$$\frac{\partial y}{\partial s} = - \int_{S} \frac{\partial}{\partial s'} G^{o}(\bar{r},\bar{r}') \frac{\sigma^{oy}(\bar{r}')}{\epsilon} ds' , \quad \bar{r} \in s \tag{4b}$$

These expressions for the narrow cylinder of finite cross section rigorously confirm the previous results of Van Bladel [3].

II. Two-dimensional bump on an infinite ground plane

A. TM polarization ($\bar{E}^{i} = \hat{z}\, e^{jk(x \cos\phi^{i} + y \sin \phi^{i})}$)

$$E_{z}^{d}(\bar{r}) = -(kd)^{2} e^{-j\frac{\pi}{4}} \sqrt{\frac{2}{\pi}} \frac{e^{-jkr}}{\sqrt{kr}} \sin\phi \sin \phi^{i} \frac{B_{o}}{d^{2}} \tag{5}$$

B. TE polarization ($\bar{H}^{i} = \hat{z}\, e^{jk(x \cos\phi^{i} + y \sin \phi^{i})}$)

$$H_{z}^{d}(\bar{r}) = -(kd)^{2} e^{-j\frac{\pi}{4}} \sqrt{\frac{2}{\pi}} \frac{e^{-jkr}}{\sqrt{kr}} \frac{1}{d^{2}} \left[A_{B} - B_{o} \cos\phi \cos\phi^{i}\right] \tag{6}$$

$$B_{o} = \int_{B} K_{z}^{oB}(\bar{r}') y' ds' \tag{7}$$

$A_{B}$ is the area of the cross section of the bump and $K_{z}^{oB}$ is the solution to the static integral equation

$$-y = \int_{BUB_{i}} K_{z}^{oB}(\bar{r}') G^{o}(\bar{r},\bar{r}') ds', \quad \bar{r} \in BUB_{i} \tag{8}$$

III. Two-dimensional dent in an infinite ground plane

A. TM polarization ($\bar{E}^{i} = \hat{z}\, e^{jk(x \cos\phi^{i} + y \sin \phi^{i})}$)

$$E_{z}^{d}(\bar{r}) = -(kd)^{2} e^{-j\frac{\pi}{4}} \sqrt{\frac{2}{\pi}} \frac{e^{-jkr}}{\sqrt{kr}} \sin\phi \sin \phi^{i} \frac{D_{o}}{d^{2}} \tag{9}$$

B. TE polarization ($\bar{H}^{i} = \hat{z}\, e^{jk(x \cos\phi^{i} + y \sin \phi^{i})}$)

$$H_{z}^{d}(\bar{r}) = (kd)^{2} e^{-j\frac{\pi}{4}} \sqrt{\frac{2}{\pi}} \frac{e^{-jkr}}{\sqrt{kr}} \frac{1}{d^{2}} \left[A_{D} + D_{o} \cos\phi \cos\phi^{i}\right] \tag{10}$$

$$D_o = \int_D K_z^{oD}(\bar{r}')y'ds' \qquad (11)$$

$A_D$ is the area of the cross section of the dent and $K_z^{oD}$ is the solution to the coupled static integral equations

$$\int_D \frac{\partial}{\partial y'} G^o(\bar{r},\bar{r}')\, K_z^{oD}(\bar{r}')\, ds' = \frac{1}{2} - H_x^{oD}(\bar{r}) \;,\; \bar{r} \in A \qquad (12a)$$

and

$$\int_D G_N^o\, (\bar{r},\bar{r}')\, K_z^{oD}(\bar{r}')\, ds' + 2\int_A G^o(\bar{r},\bar{r}')\, H_x^{oD}(\bar{r}')\, ds' = 0,\; \bar{r} \in D \qquad (12b)$$

The expressions (1), (2), (5), (6), (9), and (10) are used in the following companion paper to find the incremental length diffraction coefficients for narrow perfectly conducting cylinders, ridges on ground planes, and channels in ground planes.

## References


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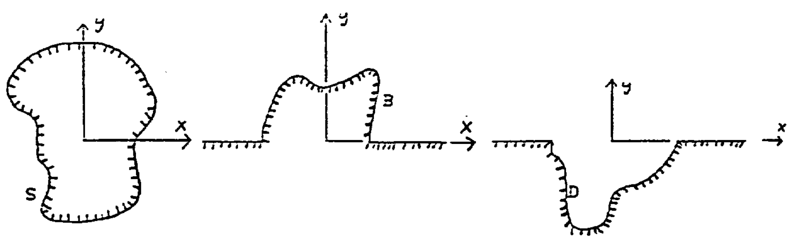


cylinder bump dent