Low-latency Transmission of Low-Rate Analog Sources

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Abstract— **Asymptotically optimal schemes for both single and dual-source cases with low-latency are addressed in this research. Single-source two-way protocol with non-coherent reception is introduced and its asymptotic behaviour is studied. The protocol consists of a data phase and a control phase which can go on up to N rounds. An upper bound on the distortion is derived for a two-round protocol. It is also extended to the case where there are two highly correlated analog sources one of which is uniformly distributed and the other one with a contaminated uniform distribution in the presence of two-sided feedback. Total energy used by protocol is fixed and the energy used by each source in both phases are derived individually. We have shown that it is possible to achieve the distortion bound of the single-source with two highly correlated sources in two rounds.**

I. Introduction

The optimization of wireless digital transmission of analog sources is an area of research that has received attention since the origins of communication theory. A modern example of a system using joint source-channel approaches would be the current WHDI standard on top of OFDM transmission used for short-range wireless transmission of high-definition video with sub-1ms latency. This makes use of variable signal levels in the transformed source signal (audio/video) with unequal error protection at the physical layer for the different levels of importance of the source signals. Here the *analog* information is not encoded using a source code at all aside from scalar quantization. The most important remark to stress is that this approach is used to *minimize latency.*

Another reason motivating the use of novel joint-source and channel coding paradigms would be the time scales corresponding to the source and channel bandwidths. In sensor networks, for instance, the sources may be characterized by a few independent samples of an analog phenomenon that needs to be transmitted very sporadically across a wideband channel. This would be the case arising, for instance, when we integrate low-cost/power analog sensors to cellular radio infrastructures.

Finally, transmission of multiple spatially distributed samples of a slowly time-varying random field is another instance where joint-source and channel coding can be beneficial. This is clearly an important remote sensing problem where the coding aspect needs to be combined with multiple-access of correlated observations. In [1], a coding strategy based on separate source and channel coding is introduced for a network information flow with discrete correlated sources. In the described model, the authors set the conditions for which perfect reconstruction of the messages from the encoder nodes can be achieved.

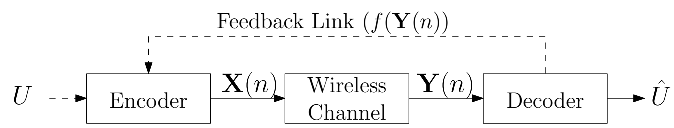

Fig. 1. System Model

The single-source model considered here is depicted in Fig. 1. The encoder maps one realization of the source U into $\mathbf{X} \triangleq (X_1, \ldots, X_N)$ where N denotes the dimension of the channel input. $\mathbf{X}$ is then sent across the channel corrupted by a white Gaussian noise sequence $\mathbf{Z}$, and is received as $\mathbf{Y}$. The receiver is a mapping function which tries to construct an estimate $\widehat{U}$ of U given $\mathbf{Y}$. The fidelity criterion that we wish to minimize is the MSE distortion defined as $D \triangleq \mathbb{E}[(U - \widehat{U})^2]$, under the mean energy constraint $\mathbb{E}[||\mathbf{X}||^2] \leq E$. It is well-known that the linear encoder (i.e. $X = \sqrt{E}U$) achieves the best performance under the mean energy constraint for the special case $N = 1$ [2], [3], [4]. An important generalization the case of multiple sensing nodes with spatially-correlated information as shown in Fig. 2. In fact, a lower bound on the distortion over all possible encoders and decoders is easily derived in [2] using classical information theory, and given by

$$D \geq e^{-2E/N_0} \tag{1}$$

where $N_0/2$ is the variance of the channel noise per dimension.

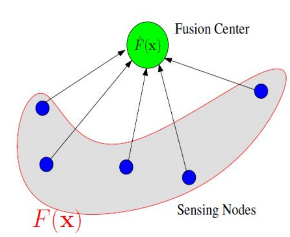

Fig. 2. Multisensor Sampling and Transmission of a Random-Field

For an upper-bound on the optimal performance, many achievable schemes based on separated or joint source-channel encoders can be found. A simple scheme recently described in [5] combines a scalar quantizer with an orthogonal modulation and MAP receiver or an MMSE estimator. Here, the Gaussian (or uniform) source is quantized in b bits which are mapped onto an appropriate orthogonal modulation before being transmitted over the channel. The distortion is caused by the quantization process and the noisy channel. An increase in the number of quantization bits per source sample has the effect of reducing the quantization error and simultaneously increasing the error induced by the channel; decreasing it will have the opposite effect. Thus, the number of quantization bits has to be optimized as a function of the energy. It is reasonably straightforward to show that a scheme such as this achieves a distortion (for large SNR or reconstruction fidelity) behaving as $\mathrm{e}^{-E/3N_0}$, both for coherent or non-coherent detection, which is significantly worse than the lower bound in (1). Such optimization for a different power constraint can be found in the literature for example in [6] and [7], where the authors try to bound the optimal number of quantization bits that minimizes distortion. Another classical example of a joint-source channel mapping is the coherent PPM scheme with ML detection [8, pp. 623], which gives a similar $\mathrm{e}^{-E/3N_0}$ behavior.

A comparison in [9] with best-known joint medium-resolution source-channel codes [10] for high channel to source bandwidth ratios shows that simple hybrid (yet separated) joint-source channel techniques can outperform non-linear mappings.

In two-way systems, for example cellular networks, we could clearly imagine the use of reliable feedback from the downlink, with vanishing probability of error (i.e. perfect feedback). The main drawback is the requirement of energy for receiver which would impact the overall energy budget of the sensing node. Although it is difficult to model, protocol latency becomes an issue for overall energy consumption. Through two-way communication, stochastic control approaches [11], [12] can achieve, at least asymptotically, the lower bound on distortion in (1). This comes at the expense of delay, since, as in many adaptive systems, the feedback system must converge to minimize distortion. It is reasonable to assume that both can be extended to non-coherent detection and even broadband frequency-selective channels for diversity. However, the underlying estimation strategies will quickly become quite involved. With respect to multiple-source systems, in [13], the authors derive a threshold SNR ratio through the correlation between the sources so that below this threshold, minimum distortion is attained by uncoded transmission in a Gaussian multiple access channel in the presence of feedback. It was also proved that the effect of feedback in this case is useless.

The outline of the paper is as follows. In Section II, we present new results which aim to show the benefit of feedback regarding optimality, yet with minimal latency through a two-phase protocol. In Section III, the results are extended to the case of two correlated analog sources again through a two-round two-way protocol. This is the first step to a more general system with many sensors with correlated measurements. In both cases we analyze the asymptotic performance of the protocols.

II. Asymptotic Optimality of Simple Two-Way Protocols with Non-coherent Detection

Let us consider now an adaptation of Yamamoto's protocol [14] for transmitting isolated analog samples. This will serve as a motivating example for the use of feedback with low-latency achieving asymptotically optimal distortion performance. Here we focus on a simple AWGN channel with one degree of freedom. Yamamoto's protocol consists of two phases, a data phase and a control phase. A source sample quantized to B bits is encoded into one of 2^B N-dimensional messages $\mathbf{S}_m$, with $m = 1, 2, \cdots, 2^B$ and each message is transmitted with equal energy $\sqrt{\mathcal{E}_{\mathrm{D},1}}$, where $\mathcal{E}_{\mathrm{D},1}$denotes the energy of the data phase on the first round. Upon reception, the receiver computes the maximum-likelihood (or MAP if source is non-uniform) message, $\hat{m}(\mathbf{Y}_d)$, based on the N-dimensional observation $\mathbf{Y}_d = \sqrt{\mathcal{E}_{\mathrm{D},1}}\mathrm{e}^{j\Phi_1}\mathbf{S}_m + \mathbf{Z}$. The random phase sequence ϕ_i is assumed to be i.i.d with uniform distribution on $[0, 2\pi)$. The N-dimensional vector noise sequence z_i is complex, circularly symmetric, with zero-mean and autocorrelation $N_0\mathbf{I}_{N\times N}$. After the first data phase, the receiver feeds $\hat{m}$ back to the encoder via the noiseless feedback link. Let the corresponding error event be E_1 where the subscript indicates to the first round of the protocol. After the data phase, the encoder enters the control phase and informs the receiver whether or not its decision was correct via a signal $\sqrt{\mathcal{E}_{\mathrm{C},1}}\mathbf{S}_c$ of energy $\sqrt{\mathcal{E}_{\mathrm{C},1}}$ if the decision is incorrect and $\mathbf{0}$ if the decision was correct. $\mathcal{E}_{\mathrm{C},1}$ here is the notation for the energy of the control phase in the first round. During the control phase the receiver observes $\mathbf{Y}_c$. Let $y_c = \mathbf{Y_c}^H\mathbf{S}_c$ and assume a detector of the form

$$\mathrm{e} = I\left(|y_c|^2 > \lambda\mathcal{E}_{\mathrm{C},1}\right) \tag{2}$$

where $I(\cdot)$ is the indicator function and λ is a threshold to be optimized and included within the interval $[0, 1)$. Let the error events be denoted by $E_{\mathrm{c}\to\mathrm{e},1}$ and $E_{\mathrm{e}\to\mathrm{c},1}$ as described and analyzed in [14]. $E_{\mathrm{e}\to\mathrm{c},1}$ corresponds to an uncorrectable error since it acknowledges an error as correct decoding. On the other hand, $E_{\mathrm{c}\to\mathrm{e},1}$ represents a misdetected acknowledged error declaring correct decoding as incorrect. This on-off signaling guarantees that with probability $\Pr(E_1^c)(1 - \Pr(E_{\mathrm{c}\to\mathrm{e},1}))$ the transmitter will not expend more than $\mathcal{E}_{\mathrm{D},1}$ joules, which should be close to one. If the receiver correctly decodes the control signal and it signals that the data phase was correct, with probability $\Pr(E_1^c)(1 - \Pr(E_{\mathrm{c}\to\mathrm{e},1}))$, the protocol halts, otherwise another identical round is initiated by the receiver. The retransmission probability is $\Pr(E_1)(1 - \Pr(E_{\mathrm{e}\to\mathrm{c},1}))$. After each data phase, the receiver computes the ML or MAP message $\hat{m}_i(\mathbf{Y}_1, \cdots, \mathbf{Y}_i)$ based on all observations up to round i with error event E_i.

The same control phase is repeated and the protocol is terminated after N rounds. The resulting average distortion of the protocol is investigated in the following subsection for two rounds together with the probability of error and the average energy used by protocol.

A. Two-rounds

The mean squared-error distortion for a uniform source U on $(-1, 1)$ is given as

$$D\left(\mathcal{E}, N_0, N, \lambda\right) = \frac{2^{-2B}}{3}(1 - P_e) + \frac{2}{3}P_e. \tag{3}$$

The above given expression of distortion is obtained through

$$D = D_q(1 - P_e) + D_e P_e \tag{4}$$

and can be bounded further as

$$D \le D_q + D_e P_e \tag{5}$$

where P_e is the total probability of error, D_q represents the distortion caused by the quantization process and D_e corresponds to the MSE distortion for the case where an error was made.

In the following, we will consider the case where the protocol is repeated up to two rounds, i.e. $N = 2$. The probability of error and the average energy used by the protocol are given by

$$\begin{aligned} P_{\mathrm{e}} &= \Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))\Pr(E_2|E_1) \\ &\quad + (1-\Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1})\Pr(E_2|E_1^c) \\ &\le \Pr(E_1)\Pr(E_{\mathrm{e}\to\mathrm{c},1}) + \Pr(E_2) \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{E} = \mathcal{E}_{\mathrm{D},1} + \Pr(E_1)\mathcal{E}_{\mathrm{C},1} + (\Pr(E_1)(1-\Pr(E_{\mathrm{e}\to\mathrm{c},1}))+ \\ (1-\Pr(E_1))\Pr(E_{\mathrm{c}\to\mathrm{e},1}))\mathcal{E}_{\mathrm{D},2}, \end{aligned} \tag{7}$$

respectively. $\mathcal{E}_{\mathrm{D},2}$ here denotes the required energy for retransmission, which is the energy to be used in the data phase of the second round. Clearly if $\Pr(E_{\mathrm{e}\to\mathrm{c},1})$ and $\Pr(E_{\mathrm{c}\to\mathrm{e},1})$ are small then the protocol achieves marginally more than $\mathcal{E}_{\mathrm{D},1}$ joules per source symbol. The probability of an uncorrectable error is obtained as

$$\begin{aligned} \Pr(E_{\mathrm{e}\to\mathrm{c},1}) &= \Pr\left(|\sqrt{\mathcal{E}_{\mathrm{C},1}} + z_c|^2 \le \lambda\mathcal{E}_{\mathrm{C},1}\right) \\ &= 1 - \mathrm{Q}_1\left(\sqrt{\frac{2\mathcal{E}_{\mathrm{C},1}}{N_0}}, \sqrt{\frac{2\lambda\mathcal{E}_{\mathrm{C},1}}{N_0}}\right), \end{aligned} \tag{8}$$

where $\mathrm{Q}_1(\alpha, \beta)$ is the first-order Marcum-Q function and $z_c = \mathbf{S}_c^H\mathbf{Z}$ is a circularly-symmetric Gaussian zero-mean random variable with variance N_0. Furthermore, we have the recent bound on the $\mathrm{Q}_1(\alpha, \beta)$ for $\alpha > \beta$ from [15, eq. 12] which is very useful for bounding (8) as follows

$$\begin{aligned} \Pr(E_{\mathrm{e}\to\mathrm{c},1}) &\le \frac{\arcsin\left(\sqrt{\lambda}\right)}{\pi}\left[\exp\left(-\left(1+\lambda+2\sqrt{\lambda}\right)\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}\right)\right. \\ &\quad \left. - \exp\left(-\left(1+\lambda+2\sqrt{\lambda}\right)\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}\right)\right] \\ &\le \frac{\arcsin\left(\sqrt{\lambda}\right)}{\pi}\exp\left(-\left(1-\sqrt{\lambda}\right)^2\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}\right). \end{aligned} \tag{9}$$

The probability of a misdetected acknowledged error is obtained as

$$\Pr(E_{\mathrm{c}\to\mathrm{e},1}) = \Pr\left(|z_c|^2 > \lambda\mathcal{E}_{\mathrm{C},1}\right) = \mathrm{e}^{-\frac{\lambda\mathcal{E}_{\mathrm{C},1}}{N_0}}. \tag{10}$$

Lastly, bounds on the error probabilities of both rounds are derived using [16, p. 686] and given by

$$\Pr(E_1) \le 2^{B-1}\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}}, \tag{11}$$

$$\Pr(E_2) \le 2^{B-3}\left(4 + \frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{N_0}\right)\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}}. \tag{12}$$

B. Asymptotic Performance

In this part, we will make use of the expressions of the error probabilities and the average energy derived in the previous subsection, to bound the distortion (3) and to observe its asymptotic performance. Applying (5) to (3) and combining it with (6), (9), (11) and (12), the distortion is bounded as

$$\begin{aligned} D(\mathcal{E}, N_0, 2, \lambda) \le K_1\mathrm{e}^{-2B\ln 2} + K_2\mathrm{e}^{B\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}}{2N_0} - \left(1-\sqrt{\lambda}\right)^2\frac{\mathcal{E}_{\mathrm{C},1}}{N_0}} \\ + K_3\mathrm{e}^{B\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1}+\mathcal{E}_{\mathrm{D},2}}{2N_0}}, \end{aligned} \tag{13}$$

where K_1 and K_2 are $O(1)$, while K_3 is $O(\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2})$. By equating coefficients in the three exponentials of (13) we have that $\mathcal{E}_{\mathrm{C},1} = \frac{\mathcal{E}_{\mathrm{D},2}}{2(1+\lambda-2\sqrt{\lambda})}$. In order for $\Pr(E_1)$ to be very close to zero so that $\mathcal{E}$ can be made arbitrarily close to $\mathcal{E}_{\mathrm{D},1}$, we define $\mathcal{E}_{D,2} = (2-\mu)\mathcal{E}_{D,1}$ where μ is an arbitrary constant satisfying $\mu \in (0, 2)$. Finally, we obtain the bound on the distortion at the end of the second round as given by

$$D(\mathcal{E}, N_0, 2) \le K_D\mathrm{e}^{-\frac{\mathcal{E}_{\mathrm{D},1}(1+\mu/3)}{N_0}} \tag{14}$$

with $K_D \sim O(\mathcal{E}_{\mathrm{D},1})$. It is worth mentioning, the asymptotic of the bound (15) in [17] is achieved here only in two rounds.

III. The Dual-Source Case

As in [14] and its non-coherent version studied earlier in Section II, the protocol comprises a data phase and a control phase, which can be repeated up to two rounds. The total energy to be used by protocol is fixed and we will denote the energy used in data phase in the i^{th} round by the j^{th} source by $\mathcal{E}_{\mathrm{D},i,j}$, where $i, j = 1, 2$. In the same way, $\mathcal{E}_{\mathrm{C},i,j}$ denotes the energy used in the control phase in the i^{th} round by the j^{th} source. The quantized source sample of the j^{th} source is encoded into 2^{B_j} messages with dimension N. For simplicity, we will assume that B_j's are equal to the same value B. In the data phase, the first source sends its message $m_1(U_1)$ to the receiver with energy $\mathcal{E}_{\mathrm{D},1,1}$. The receiver detects $\hat{m}_1$ and feeds it back. And the second source sends $m_2(\hat{m}_1, U_2)$ with energy $\mathcal{E}_{\mathrm{D},1,2}$. This encoding rule allows the second source to exploit the correlation of its sample with that of its peer and the energy used is chosen according to the likelihood of the estimate fed back from the receiver. After the estimation and feedback of $\hat{m}_2$, data phase of the first round ends and the encoders enter the control phase to inform the receiver about the correctness of its decision, as in the single source case. To do that, each source sends ACK/NACK regarding its own message to the decoder. According to the control signals, either the protocol halts or goes on one more round. For the second data phase, the destination instructs the sources to retransmit and re-detect its message. The first source U_1 is defined to be uniformly distributed over $(-\sqrt{3}, \sqrt{3})$ and the second source U_2 is defined as $U_2 = \rho U_1 + \sqrt{1-\rho^2}U_2'$ based on U_1 and an auxiliary random variable U_2' which is also uniform on $(-\sqrt{3}, \sqrt{3})$. Depending on the value of ρ, the distribution of the second source U_2 can be either a triangular distribution or a contaminated uniform distribution. In the case of a very high correlation, i.e. ρ is very close to 1, the effect of the auxiliary random variable U_2' will be very small. On the contrary, for a low correlation between U_1 and U_2, U_2' will have a significant effect so the second source will have a triangular distribution as a sum of two uniform random variables. We will focus on the case of a very high correlation between the two sources. So, here we have one uniform and one almost uniform source having covariance equal to the correlation coefficient ρ between them. Extending the output signal based on the N dimensional observation to this case, output signal of the j^{th} source is $\mathbf{Y}_d = \sqrt{\mathcal{E}_{\mathrm{D},1,j}} \mathrm{e}^{j\Phi_j} \mathbf{\Psi}_{m_j} + \mathbf{Z}_j$. We assume the random phases Φ_j to be distributed uniformly on $[0, 2\pi)$, the channel noise $\mathbf{Z}_j$ to have zero mean and equal autocorrelation $N_0 \mathbf{I}_{N\times N}$ for $j = 1, 2$ and $\mathbf{\Psi}_{m_j}$ are the N-dimensional messages, with $m = 1, 2, \cdots, 2^B$ and $j = 1, 2$.

We denote the error events for the first round and the j^{th}source with $E_{1,j}$. Let $e_{1,j}$ and $c_{1,j}$ denote erroneous and correct decoding in the first round on U_j, respectively. $E_{c\to e,1}$ and $E_{e\to c,1}$ are used to denote a misdetected acknowledged error and an uncorrectable error, respectively. The probability of the error is bounded by

$$\begin{aligned} P_e = &\Pr(E_{1,1}) \Pr(E_{e\to c,1,1}) + \Pr(E_{1,2}) \Pr(E_{e\to c,1,2}) \\ &+ (\Pr(E_{1,1}, E_{1,2})(1 - \Pr(E_{e\to c,1,1}))(1 - \Pr(E_{e\to c,1,2})) \\ &+ \Pr(E_{1,1}, E^c_{1,2})(1 - \Pr(E_{e\to c,1,1})) \\ &+ \Pr(E^c_{1,1}, E_{1,2})(1 - \Pr(E_{e\to c,1,2}))) \Pr(E_2|E_1) \\ &+ (\Pr(E^c_{1,1}, E^c_{1,2}) \Pr(E_{c\to e,1,1}) \Pr(E_{c\to e,1,2}) \\ &+ \Pr(E_{1,1}, E^c_{1,2}) \Pr(E_{c\to e,1,2}) \\ &+ \Pr(E^c_{1,1}, E_{1,2}) \Pr(E_{c\to e,1,1})) \Pr(E_2|E^c_1) \\ &\le \Pr(E_{1,1}, E_{1,2}) \Pr(E_{e\to c,1,}) + \Pr(E_2) \end{aligned} \tag{15}$$

where the probability of an uncorrectable error in the first round is taken as the sum of the probability of errors of each source and given by

$$\Pr(E_{e\to c,1}) = \Pr(E_{e\to c,1,1}) + \Pr(E_{e\to c,1,2}). \tag{16}$$

The total probability of misdetected acknowledged error in the first round is obtained in the same way as

$$\Pr(E_{c\to e,1}) = \Pr(E_{c\to e,1,1}) + \Pr(E_{c\to e,1,2}). \tag{17}$$

The average energy used by the protocol is given by the following equality

$$\begin{aligned} \mathcal{E} = &\mathcal{E}_{\mathrm{D},1,1} + E(\mathcal{E}_{\mathrm{D},1,2}(\hat{m}_1, U_2)) + \mathcal{E}_{\mathrm{C},1,1} \Pr(E_{1,1}) \\ &+ \mathcal{E}_{\mathrm{C},1,2} \Pr(E_{1,2}) + \mathcal{E}_{\mathrm{D},2}[\Pr(E_{1,1})(1 - \Pr(E_{e\to c,1,1})) \\ &+ \Pr(E_{1,2})(1 - \Pr(E_{e\to c,1,2})) + (1 - \Pr(E_{1,1})) \\ &\Pr(E_{c\to e,1,1}) + (1 - \Pr(E_{1,2})) \Pr(E_{c\to e,1,2})] \end{aligned} \tag{18}$$

which can be bounded as

$$\begin{aligned} \mathcal{E} \le &\mathcal{E}_{\mathrm{D},1} + \Pr(E_{1,1}, E_{1,2})\mathcal{E}_{\mathrm{C},1} + \mathcal{E}_{\mathrm{D},2}[\Pr(E_{1,1}, E_{1,2}) \\ &(1 - \Pr(E_{e\to c,1})) + (1 - \Pr(E_{1,1}, E_{1,2})) \Pr(E_{c\to e,1})] \end{aligned} \tag{19}$$

where $E(\mathcal{E}_{\mathrm{D},1,2}(\hat{m}_1, U_2))$ is the expected energy to be used in the data phase of the first round by the second source. The total energy for a certain phase and round is obtained by taking the sum over the both sources. The energy in the control phase of the i^{th} round is defined as $\mathcal{E}_{\mathrm{C},i} = \sum_{j=1}^{2} \mathcal{E}_{\mathrm{C},i,j}$ and the total energy in the data phase of the first round is $\mathcal{E}_{\mathrm{D},1} = \sum_{i=1}^{2} \mathcal{E}_{\mathrm{D},1,j}$.

We have the same form of detector described in (2) for the j^{th} source as $e_j = I\left(|y_{c,j}|^2 > \lambda_j \mathcal{E}_{\mathrm{C},1,j}\right)$ with $y_{c,j} = \mathbf{Y_{c,j}}^H \mathbf{\Psi}_{c,j}$ λ_1 and λ_2 are threshold values to be optimized and included within the interval $[0, 1)$. For simplification, we will assume λ_1 and λ_2 to be equal to the same value λ. The probability of error of an uncorrectable error $E_{e\to c}$ for U_j is given by

$$\begin{aligned} \Pr(E_{e\to c,1,j}) &= \Pr(|\sqrt{\mathcal{E}_{\mathrm{C},1,j}} + z_{c,j}|^2 \le \lambda \mathcal{E}_{\mathrm{C},1,j}) \\ &= 1 - Q_1\left(\sqrt{\frac{\mathcal{E}_{\mathrm{C},1,j}}{N_0/2}}, \sqrt{\frac{\lambda\mathcal{E}_{\mathrm{C},1,j}}{N_0/2}}\right). \end{aligned} \tag{20}$$

Using the bound on the $Q_1(\alpha, \beta)$ given in [18, eq:4], the probability of error of an uncorrectable error can be bounded as

$$\Pr(E_{e\to c,1,j}) \le 1/2 \exp\left(-\frac{(\sqrt{\lambda} - 1)^2 \mathcal{E}_{\mathrm{C},1,j}}{N_0}\right). \tag{21}$$

And the probability of error of a misdetected acknowledged error $E_{c\to e}$for U_j is $\Pr(E_{c\to e,1,j}) \le \exp\{-\frac{\lambda\mathcal{E}_{\mathrm{C},1,j}}{N_0}\}$. The joint probability of $E_{1,1}, E_{1,2}$ and the probability of error of the second round E_2 is bounded as in (11) and (12) using [16, p. 686] and given by

$$\begin{aligned} &\Pr(E_{1,1}, E_{1,2}) \\ &\le \left\lceil 2^B \sqrt{\frac{1-\rho^2}{3}} \right\rceil 2^{B-3} \exp\left\{-\frac{\mathcal{E}_{\mathrm{D},1}}{2N_0}\right\} \left(4 + \frac{\mathcal{E}_{D,1}}{N_0}\right) \\ &+ \left\lceil 2^B \sqrt{\frac{1-\rho^2}{3}} \right\rceil 2^{-1} \left(\exp\left\{-\frac{\mathcal{E}_{D,1,1}}{2N_0}\right\} + \exp\left\{-\frac{\mathcal{E}_{D,1,2}}{2N_0}\right\}\right) \end{aligned} \tag{22}$$

At the end of the second round, the protocol is terminated with distortion bounded as

$$D(\mathcal{E}, N_0, 2, \lambda) \le 2^{-2B}(1 + \rho^2)(1 - P_e) + 4P_e. \tag{24}$$

To obtain (23), we used the same approach as in the single source case and generalized (4) for two sources. By combining (15), (21),(22), (23) and (24), we have the following bound on distortion

$$\begin{aligned} &D(\mathcal{E}, N_0, 2, \lambda) \le K_1 e^{-2B\ln 2 + \ln(1+\rho^2)} \\ &+ \left(K_2 \sqrt{\frac{1-\rho^2}{3}} e^{B\ln 2} + K_3 \epsilon(\rho)\right) e^{(B-1)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{C},1}(\sqrt{\lambda}-1)^2}{2N_0}} \\ &+ \left(K_4 \sqrt{\frac{1-\rho^2}{3}} e^{B\ln 2} + K_5 \epsilon(\rho)\right) e^{2\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{C},1}(\sqrt{\lambda}-1)^2}{4N_0}} \\ &+ \left(K_6 \sqrt{\frac{1-\rho^2}{3}} e^{B\ln 2} + K_7 \epsilon(\rho)\right) e^{(B-3)\ln 2 - \frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2}}{2N_0}} \\ &+ \left(K_8 \sqrt{\frac{1-\rho^2}{3}} e^{B\ln 2} + K_9 \epsilon(\rho)\right) e^{-\frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{D},2}}{4N_0}} \end{aligned} \tag{25}$$

where K_1, K_4, K_5 are $O(1)$, K_2, K_3 are $O(\mathcal{E}_{\mathrm{D},1})$, K_6, K_7, K_8, K_9 are $O(\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2})$ with $\epsilon(\rho) \in [0, 1)$ which arose from the ceiling functions in (22) and (23). To simplify the calculations the energy used by a source on a particular phase is assumed to be half of the energy on the corresponding round, for example $\mathcal{E}_{D,1} = 2\mathcal{E}_{D,1,1} = 2\mathcal{E}_{D,1,2}$.

$$\Pr(E_2) \leq \left\lceil 2^B \sqrt{\frac{1-\rho^2}{3}} \right\rceil 2^{-3} \left(\exp\left\{ -\frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{D},2,2}}{4N_0} \right\} \left(4 + \frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{D},2,2}}{2N_0} \right) + \exp\left\{ -\frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{D},2,1}}{4N_0} \right\} \left(4 + \frac{\mathcal{E}_{\mathrm{D},1} + 2\mathcal{E}_{\mathrm{D},2,1}}{2N_0} \right) \right) + \left\lceil 2^B \sqrt{\frac{1-\rho^2}{3}} \right\rceil 2^{B-5} \exp\left\{ -\frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2}}{2N_0} \right\} \left(7.5 + \frac{\mathcal{E}_{\mathrm{D},1} + \mathcal{E}_{\mathrm{D},2}}{N_0} \right) \quad (23)$$

For a very weak correlation between U_1 and U_2 the terms with $\epsilon(\rho)$ in (25) become insignificant, so the bound on distortion for this case is obtained as

$$D_{low}(\mathcal{E}, N_0, 2) \leq K_{D_{low,1}} \sqrt{\frac{1-\rho^4}{6}} e^{-\frac{\mathcal{E}_{\mathrm{D},1} - \mu/2}{2N_0}} + K_{D_{low,2}} (1+\rho^2)^{1/3} (\frac{1-\rho^2}{3})^{1/2} e^{-\frac{\mathcal{E}_{\mathrm{D},1} - 2\mu/3}{2N_0}}. \quad (26)$$

with $\mathcal{E}_{\mathrm{C},1} = \frac{\mathcal{E}_{\mathrm{D},2}}{(1-\sqrt{\lambda})^2}$ and $\mathcal{E}_{\mathrm{D},2} = (1-\mu)\mathcal{E}_{D,1}$ where μ is a non-negative and a non-zero arbitrary constant. This relationship is obtained through equating coefficients of the exponentials in (25). On the other hand, with a high correlation between the sources through (22), i.e. when $\sqrt{\frac{1-\rho^2}{3}} < \theta 2^{-B}$ the distortion (25) becomes

$$D_{high}(\mathcal{E}, N_0, 2) \leq K_{D_{high,1}} (1+\rho^2)^{1/3} e^{-\frac{\mathcal{E}_{\mathrm{D},1} - (2\mu)/3}{N_0}} + K_{D_{high,2}} (1+\rho^2) e^{-\frac{5\mathcal{E}_{\mathrm{D},1} - 2\mu}{4N_0}} \quad (27)$$

with $\mathcal{E}_{C,1} = \frac{\mathcal{E}_{D,2}}{(1-\sqrt{\lambda})^2}$ and $\mathcal{E}_{D,2} = (2-\mu)\mathcal{E}_{D,1}$ where μ is an arbitrary constant satisfying $\mu \in (0, 2)$. The average energy $\mathcal{E}$ of the protocol can be made arbitrarily close to $\mathcal{E}_{\mathrm{D},1}$ with a vanishing $\Pr(E_{1,1}, E_{1,2})$, guaranteed by the interval in which $\epsilon(\rho)$ is defined.

The two extremes considered here show the effect of correlation on the reconstruction fidelity at the receiver. The high correlation case yields the exponential behaviour of the single-source case and benefits from energy accumulation, or the collaboration of the two sources. Low-correlation results insignificantly reduced energy-efficiency. In a large network scenario, nodes with highly-correlated samples (in the above sense) would collaborate through joint detection at the receiver in order to optimize the energy efficiency of the network.

IV. Conclusions

We introduced a low-latency two-way protocol achieving asymptotically optimal distortion for a single-source using digital transmission over a wideband channel with non-coherent detection. Its extension to the two-source case with correlation is studied for both low and high correlation levels again with analog sources in the presence of feedback. We have shown that with highly correlated sources, the same asymptotic performance of a single-source can be achieved in terms of distortion. Although not shown here, lower bounds on the achievable distortion can be derived in the multiple-sensor case which exhibit the same asymptotic behaviour. Our ongoing work focuses on practical larger-scale algorithms for exploiting correlation for energy-efficiency in sensor networks.

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