Low-temperature behavior of the vortex lattice in unconventional superconductors

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(Received 4 December 1997)

We study the effect of the superconducting gap nodes on the vortex lattice properties of high-temperature superconductors at very low temperatures. The nonlinear, nonlocal, and nonanalytic nature of this effect is shown to have measurable consequences for the vortex lattice geometry and the effective penetration depth in the mixed state as measured by muon-spin-rotation experiments. [S0163-1829(98)03833-8]

I. INTRODUCTION

The presence of nodes in the superconducting gap is probably one of the most significant features of high-T_c superconductors that has attracted considerable theoretical and experimental attention in recent years. Many experiments[1,2] have confirmed the existence of gap nodes. These experiments most commonly indicate an order parameter with $d_{x^2-y^2}$ symmetry.[2] The fourfold symmetric nature of the d-wave order parameter, together with the presence of gap nodes on the Fermi surface, opens possibilities for novel effects to be observable in cuprates. An early theoretical investigation of the weak-field response of a $d_{x^2-y^2}$ superconductor by Yip and Sauls[3] predicted a direction-dependent nonlinear Meissner effect, associated with the quasiclassical shift of the excitation spectrum due to the superflow created by the screening currents. Maeda *et al.*[4] reported experimental evidence for such an effect in $Bi_2Sr_2CaCu_2O_y$, but subsequent experiments failed to confirm their findings and the situation remains controversial. A similar effect was also studied independently by Volovik.[5] In the mixed state, he predicted a contribution to the residual density of states (DOS) proportional to the intervortex distance $\sim\sqrt{H}$. Such contribution was identified in the specific-heat measurements on $YBa_2Cu_3O_{7-\delta}$ (YBCO) by Moler *et al.*,[6] but later this interpretation was disputed by Ramirez,[7] who found evidence for a similar effect in a conventional superconductor V_3Si, and by others.[8] Kosztin and Leggett[9] predicted that the nonlocal response at very low temperatures will lead to a T^2 dependence of the penetration depth in clean samples, in contrast to the linear T dependence obtained from the local theory for d-wave materials.

In the mixed state also, it is conceivable that these effects still present themselves in some measurable properties such as vortex lattice geometry, magnetic-field distribution, etc. Neutron-scattering[10] and scanning tunnel microscope[11] experiments on the high-T_c compound YBCO revealed a vortex lattice structure different from the hexagonal—which is expected for an isotropic superconductor. However, this may be explainable by penetration depth anisotropy and twin-boundary pinning without involving any effects associated with gap nodes.[12] Muon-spin-rotation (μSR) experiments,[13–17] on the other hand, have demonstrated an unusual magnetic-field dependence in their line shapes for the magnetic-field distribution. This has been attributed to a field-dependent penetration depth—which is expected in the Meissner state because of quasiparticle generation at gap nodes.[3] It was also modeled using an approach based on the Bogoliubov–deGennes (BdG) equations in a square lattice tight-binding model.[18]

At intermediate fields $H_{c1}<H\ll H_{c2}$, properties of the flux lattice are determined primarily by the superfluid response of the condensate, i.e., by the relation between the supercurrent $\vec{j}$ and the superfluid velocity $\vec{v}_s$. In conventional isotropic strongly type-II superconductors, this relation is to a good approximation that of simple proportionality,

$$\vec{j}=-e\rho_s\vec{v}_s, \tag{1}$$

where ρ_s is a superfluid density. More generally, however, this relation can be both *nonlocal* and *nonlinear*. The concept of nonlocal response dates back to the ideas of Pippard[19] and is related to the fact that the current response must be averaged over the finite size of the Cooper pair given by the coherence length ξ_0. In strongly type-II materials the magnetic field varies on a length scale given by the London penetration depth λ_0, which is much larger than ξ_0 and therefore nonlocality is typically unimportant unless there exist strong anisotropies in the electronic band structure.[20] Nonlinear corrections arise from the change of quasiparticle population due to superflow which, to leading order, modifies the excitation spectrum by a quasiclassical shift[19]

$$\mathcal{E}_k=E_k+\vec{v}_f\cdot\vec{v}_s, \tag{2}$$

where $E_k=\sqrt{\epsilon_k^2+\Delta_k^2}$ is the BCS energy. Again, in clean, fully gapped conventional superconductors, this effect is typically negligible except when the current approaches the pair breaking value. In the mixed state, this happens only in the close vicinity of the vortex cores that occupy a small fraction of the total sample volume at fields well below H_{c2}. The situation changes dramatically when the order parameter has nodes, such as in $d_{x^2-y^2}$ superconductors. Nonlocal corrections to Eq. (1) become important for the response of electrons with momenta on the Fermi surface close to the gap nodes, even for strongly type-II materials. This can be understood by realizing that the coherence length, being inversely proportional to the gap,[19] becomes very large close to the node and formally diverges at the nodal point. Thus, quite generally, there exists a locus of points on the Fermi surface where $\xi \gg \lambda_0$ and the response becomes highly nonlocal. This effect had been first discussed by Kosztin and Leggett[9] for the Meissner state and by us[22] in the mixed state. Similarly, the nonlinear corrections become important in a d-wave superconductor. Equation (2) indicates that finite areas of gapless excitations appear near the node for arbitrarily small v_s. This effect has been studied in the Meissner state,[3] but its consequences have remained largely unexplored for the mixed state.

In our previous works,[21,22] we discussed the effect of fourfold anisotropies associated with a *nonlocal* response on the field and temperature dependence of a vortex lattice structure. In Ref. 21, starting from a phenomenological Ginzburg-Landau (GL) theory, we derived the leading fourfold anisotropic corrections to the London equation, making the usual assumption that the free energy was an analytic functional of the order parameter and field. We showed that such corrections result in a field-driven continuous transition from triangular to square vortex lattice. Being derived from a GL theory, this London equation is expected to lose its validity at low temperatures and a microscopic theory is needed to address this regime. In Ref. 22, we investigated the effect of the nonlocality due to the presence of the nodes in the superconducting gap using a simple weak-coupling microscopic model. At high temperatures, a nonlocal correction similar to the one suggested in Ref. 21 was obtained. At low temperatures, however, a novel singular behavior was found, directly related to the nodal structure of the gap that completely changes the form of the London equation. This singular behavior has profound implications for the structure of the vortex lattice which, as a function of decreasing temperature, undergoes a series of sharp structural transitions and attains a universal limit at $T=0$.

In Ref. 22, we neglected the effect of the *nonlinear* terms resulting from the excitation of quasiparticles at the gap nodes, assuming that they are small compared to the nonlocal corrections. In this paper, we consider both the nonlinear and the nonlocal terms, assuming that the effects are additive and do not affect each other. We show that, as we claimed in Ref. 22, the dominant effect that determines the vortex lattice geometry and the effective penetration depth as defined in μSR experiments is the nonlocal corrections, while the nonlinear corrections play a secondary role at low T.

II. GENERALIZED LONDON EQUATION

A. Nonlinear corrections

Let us for the moment neglect any nonlocality effect and focus on the quasiparticles generated at the gap nodes. Excitation of quasiparticles at the gap nodes produces a current

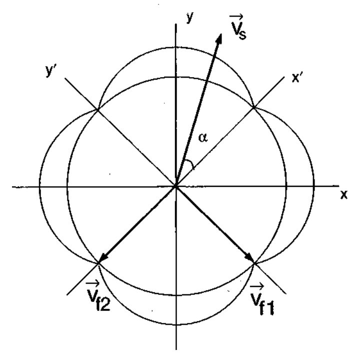

FIG. 1. Circular Fermi surface with a $d_{x^2-y^2}$ gap. Quasiparticles will be excited at nodes marked by $\vec{v}_{f1}$ and $\vec{v}_{f2}$ opposite to $\vec{v}_s$.

density flowing in the direction opposite to the superfluid velocity—sometimes called backflow. The total current can then be written as

$$\vec{j} = -e\rho_s\vec{v}_s + \vec{j}_{qp}, \tag{3}$$

where ρ_s is the superfluid density and $\vec{v}_s$ is the superfluid velocity defined by[23]

$$\vec{v}_s = \frac{1}{2}\left(\frac{2e}{c}\vec{A} - \nabla\phi\right). \tag{4}$$

$\vec{A}$ is the vector potential and ϕ is the phase of the order parameter. The contribution of the quasiparticles generated at the nodes to the total current is given by[3]

$$\vec{j}_{qp} = -4eN_f\int_{FS} ds\,\vec{v}_f(s)\int_0^\infty d\xi f[\sqrt{\xi^2+\Delta(s)^2} + \vec{v}_f(s)\cdot\vec{v}_s], \tag{5}$$

where N_f is the density of states at the Fermi surface and s is a parameter that represents a point on the Fermi surface. $\Delta(s)$ is the superconducting gap which, in general, can have s-wave, d-wave, or other symmetries. At zero temperature, Eq. (5) leads to

$$\vec{j}_{qp} = -4eN_f\int ds\,\vec{v}_f(s)\,\theta(-\vec{v}_f\cdot\vec{v}_s - |\Delta|)\sqrt{(\vec{v}_f\cdot\vec{v}_s)^2 - \Delta^2}. \tag{6}$$

At higher temperatures however, this gives the first term in the Sommerfeld expansion, which is a good approximation as long as $T < T^\star = T_c(H/H_0)$, where H_0 is of order of the thermodynamic critical field H_c.[3] The presence of the θ function in Eq. (6) results in excitations only at the nodes in the opposite direction to $\vec{v}_s$. Figure 1 illustrates a circular Fermi surface with a d-wave gap. Quasiparticles are excited at the nodes marked by $\vec{v}_{f1}$ and $\vec{v}_{f2}$ in the opposite direction to $\vec{v}_s$. For a small enough v_s, the excitations stay very close to the gap nodes. Therefore, one can linearize the gap function near the nodes, writing $\Delta(\theta) \simeq \gamma\Delta_0\theta$ with Δ_0 the maximum gap and γ defined by

$$\gamma = \frac{1}{\Delta_0}\left[\frac{d}{d\theta}\Delta(\theta)\right]_{\text{node}}. \tag{7}$$

The component of $\vec{j}_{qp}$ along the x' direction, which is diagonal to the x and y (a and b) directions (as illustrated in Fig. 1), is then

$$j_{qpx'}=4eN_f v_f\int_{-\theta_c}^{\theta_c}\frac{d\theta}{2\pi}\sqrt{(v_f v_s\cos\alpha)^2-|\gamma\Delta_0\theta|^2}$$
$$=e\rho_s\frac{v_s^2}{v_0}\cos\alpha|\cos\alpha|=e\rho_s\frac{v_{sx'}|v_{sx'}|}{v_0}. \quad (8)$$

α is the angle between v_s and x' axis, $\rho_s=N_f v_f^2$ is the superfluid density, $v_0=\gamma\Delta_0/v_f$ is some characteristic velocity, and θ_c is a cutoff imposed by the θ function in Eq. (6). Similarly, the y' component is

$$j_{qpy'}=e\rho_s\frac{v_{sy'}|v_{sy'}|}{v_0} \quad (9)$$

and the total current thereby becomes

$$\vec{j}=-e\rho_s[\vec{v}_s-(v_{sx'}|v_{sx'}|\hat{x}'+v_{sy'}|v_{sy'}|\hat{y}')/v_0]. \quad (10)$$

The nonanalytic nature of the effect is evident from this equation.

It is now possible to write a free energy in such a way that Eq. (10) can be obtained by minimization with respect to $\vec{A}$. Keeping in mind that $\vec{j}=(c/4\pi)\nabla\times\vec{B}=(c/4\pi)\nabla\times\nabla\times\vec{A}$ and $\partial/\partial\vec{v}_s=(c/e)\partial/\partial\vec{A}$, the corresponding free-energy density can be written as

$$f=\rho_s\left[\frac{1}{2}v_s^2-\frac{1}{3v_0}(|v_{sx'}|^3+|v_{sy'}|^3)\right]+\frac{B^2}{8\pi}. \quad (11)$$

In general, it is possible to solve Eq. (10) for $\vec{v}_s$ in terms of $\vec{j}=(c/4\pi)\nabla\times\vec{B}$, substitute it into Eq. (11), and write a London free energy only in terms of $\vec{B}$ and its derivatives. However, instead of solving Eq. (10) exactly, we find $\vec{v}_s$ perturbatively, assuming that the nonlinear part is much smaller than the linear part. This results in a polynomial correction to the London equation that is more convenient for the numerical purposes. To first order in perturbation theory:

$$\vec{v}_s=\frac{c}{4\pi e\rho_s}\Bigg(\nabla\times\vec{B}+\frac{c}{4\pi e\rho_s v_0}[(\nabla\times\vec{B})_{x'}|\nabla\times\vec{B}|_{x'}\hat{x}'$$
$$+(\nabla\times\vec{B})_{y'}|\nabla\times\vec{B}|_{y'}\hat{y}']\Bigg). \quad (12)$$

Substituting this into Eq. (11) and keeping the lowest-order terms, the London free-energy density becomes

$$f_L=\frac{1}{8\pi}\Bigg[B^2+\lambda_0^2(\nabla\times\vec{B})^2+\left(\frac{2\pi}{3\gamma}\right)\frac{\xi_0\lambda_0^2}{B_0}[|(\nabla\times\vec{B})_{x'}|^3$$
$$+|(\nabla\times\vec{B})_{y'}|^3]\Bigg], \quad (13)$$

where $\lambda_0=\sqrt{c^2/4\pi e^2\rho_s}$ is the zeroth order penetration depth, $\xi_0=v_f/\pi\Delta_0$ is the coherence length, $B_0\equiv\phi_0/2\pi\lambda_0^2$ is a characteristic field of the order of H_{c1}, and $\phi_0=\pi c/e$ is the flux quantum. The first two terms in Eq. (13) are the ordinary London free-energy terms, while the remaining two terms constitute the leading nonlinear correction. For magnetic fields in the z direction, the London free-energy density becomes

$$f_L=\frac{1}{8\pi}\left[B^2+\lambda_0^2(\nabla B)^2+\left(\frac{2\pi}{3\gamma}\right)\frac{\xi_0\lambda_0^2}{B_0}(|\partial_{x'}B|^3+|\partial_{y'}B|^3)\right] \quad (14)$$

and the corresponding London equation will be

$$-\lambda_0^2\nabla^2 B+B-\left(\frac{2\pi}{\gamma}\right)\frac{\xi_0\lambda_0^2}{B_0}(\partial_{x'}^2B|\partial_{x'}B|+\partial_{y'}^2B|\partial_{y'}B|)=0. \quad (15)$$

A similar London equation is also derived by Zutic and Valls who investigated the effect in the Meissner state.[24]

The most commonly used form for a d-wave gap is $\Delta(\theta)=\Delta_0\cos(2\theta)$. In this case, Eq. (7) leads to $\gamma=2$. As discussed in more detail below, in order to find the magnetic-field distribution in a vortex lattice, one has to insert a source term $\Sigma_j\rho(\vec{r}-\vec{r}_j)$ on the right-hand side of Eq. (15) with $\vec{r}_j$ being the position of the vortices in the lattice. The function $\rho(\vec{r})$ takes into account the vanishing of the order parameter at the center of the vortex cores. Numerically, it is more convenient to work in Fourier space rather than in real space. However, because of the nonanalyticity of the nonlinear term, the Fourier transformation of this term cannot be done by simple convolution integrals and numerical techniques such as fast Fourier transformation (FFT) are required. Fourier transforming Eq. (15) with a proper source term on the right-hand side yields

$$B_{\vec{k}}+\lambda_0^2k^2B_{\vec{k}}-G_{\rm nl}(\vec{k},B_{\vec{k}})=\bar{B}F(\vec{k}), \quad (16)$$

where $\vec{k}$ is a reciprocal lattice wave vector, $G_{\rm nl}$ is the Fourier transform of the nonlinear term, and $\bar{B}$ is the average magnetic field. The cutoff function $F(\vec{k})$ comes from the Fourier transformation of the source term and removes the divergences by cutting off the momentum sums.

B. Nonlocal corrections

One can add the nonlocal effect to Eq. (16) by replacing the second term with a nonlocal one as discussed in Ref. 22. The resulting London equation will be

$$B_{\vec{k}}+\mathcal{L}_{ij}(\vec{k})k_ik_jB_{\vec{k}}-G_{\rm nl}(\vec{k},B_{\vec{k}})=\bar{B}F(\vec{k}). \quad (17)$$

Sums over i and j are implicit here. $\mathcal{L}_{ij}(\vec{k})$ is related to the electromagnetic response tensor $\hat{Q}(\vec{k})$ defined in Ref. 22 by

$$\mathcal{L}_{ij}(\vec{k})=Q_{ij}(\vec{k})/\det\hat{Q}(\vec{k}). \quad (18)$$

The electromagnetic response tensor describes the linear response of a superconductor to an applied magnetic field. It has a particularly simple form in the gauge where $\nabla\phi=0$ and $\vec{v}_s$ is proportional to the vector potential $\vec{A}$. At $T=0$ one then obtains[22]

$$Q_{ij}(\vec{k})=\frac{1}{\lambda_0^2}\int ds\,\hat{v}_{fi}\hat{v}_{fj}\frac{2\,\mathrm{arcsinh}\,y}{y\sqrt{1+y^2}}, \quad (19)$$

where $y=\vec{v}_s\cdot\vec{k}/2\Delta(s)$. The expression for $\hat{Q}(\vec{k})$ becomes more complicated in an arbitrary gauge and was discussed by Millis.[25] In the present context, however, it is easy to verify that, because of the particular way $\hat{Q}(\vec{k})$ enters the London equation (17), this is gauge invariant as required for an equation containing an observable quantity.

In the local case we have $\mathcal{L}_{ij}(\vec{k})=\lambda_0^2\delta_{ij}$, which leads back to Eq. (16). Taking $G_{\rm nl}$ to the right-hand side of Eq. (17), $B_{\vec{k}}$ can be obtained by

$$B_{\vec{k}}=\frac{\overline{B}F(\vec{k})+G_{\rm nl}(\vec{k},B_{\vec{k}})}{1+\mathcal{L}_{ij}(\vec{k})k_ik_j}. \tag{20}$$

We use this equation to find $B_{\vec{k}}$ iteratively for a specific lattice geometry. Having $B_{\vec{k}}$, the free energy can be easily calculated using

$$\mathcal{F}=\mathcal{F}_{\rm nl}+\sum_{\vec{k}}\,[1+\mathcal{L}_{ij}(\vec{k})k_ik_j]B_{\vec{k}}^2, \tag{21}$$

where $\mathcal{F}_{\rm nl}$ is the free energy due to the nonlinear parts as in Eq. (14).

C. Vortex source

The functional form of $\rho(\vec{r})$, and thereby $F(\vec{k})$, depends on the detailed form of the order parameter at the center of the vortex and therefore requires a more fundamental theory to be evaluated. In the GL limit, there have been some conjectures about the profile of the order parameter at the vortex center and the form of the source term.[26–29] Brandt[26] suggests a Gaussian form for the source term

$$\rho(\vec{r})=(\phi_0/2\pi\xi_0^2)e^{-r^2/2\xi_0^2}, \tag{22}$$

which leads to

$$F(k)=e^{-\xi^2k^2/2}. \tag{23}$$

Clem,[27] on the other hand, assumes the order parameter to vanish as $r/\sqrt{r^2+\xi_v^2}$ at the center of the vortex core near T_c, where ξ_v is a variational parameter proportional to ξ_0. It is not difficult to show that this form of the order parameter leads to a squared Lorentzian source term[32]

$$\rho(r)=\frac{\Phi_0}{\pi}\frac{\xi_v^2}{(\xi_v^2+r^2)^2},\qquad F(k)=uK_1(u), \tag{24}$$

where K_1 is the modified Bessel function. In the extreme type-II case, $\xi_v=\sqrt{2}\xi_0$ and $u=\sqrt{2}\xi_0k$. This form of the source term shows quite a good agreement with the exact solution of the GL equation in the vortex state.[30] At low temperatures, however, the GL equation is not applicable. The most commonly assumed form for the order parameter near the vortex core that gives a good fit to the numerical solutions of the BdG equation[31] is $\psi(r)\propto\tanh(r/\xi)$. The cutoff function resulting from this form of the order parameter fits quite well with the Gaussian form of Eq. (23) up to high Fourier modes.[32] It is worth pointing out that none of the models for the source terms discussed above consider the d-wave nature of the order parameter or other anisotropies that might be important for the vortex lattice properties. Recent numerical computations within the self-consistent BdG theory[33] for a d-wave vortex indicate an order parameter with a relatively weak fourfold anisotropy and an amplitude relaxing to its bulk value as $\sim 1/r^2$ far from the core even at $T=0$. Therefore, a more careful consideration is required to formulate a reasonable model for the vortex core in the London theory. We leave a detailed discussion of the vortex core to a separate paper[32] and from now on assume the Gaussian form of Eq. (22) for our vortex source.

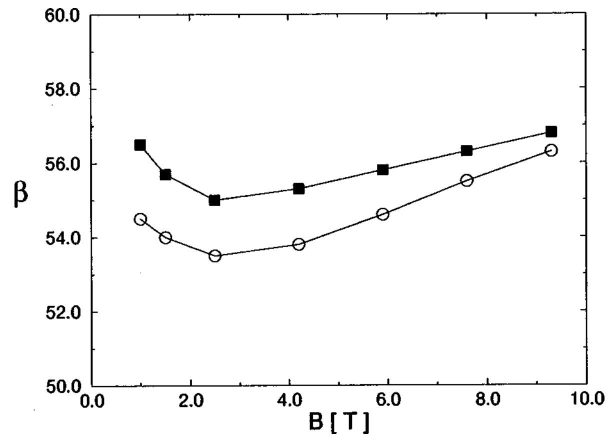

FIG. 2. Apex angle β as a function of magnetic field B at $T=0$. Circles represent the result of the calculation using only the nonlocal corrections (Ref. 22). Squares correspond to the calculations considering both nonlinear and nonlocal corrections.

III. NUMERICAL CALCULATIONS

The numerical calculations are performed by employing Eq. (20) to calculate $B_{\vec{k}}$ iteratively. At each step, $G_{\rm nl}$ is calculated numerically using FFT. In our numerical calculation, we use $\lambda_0=1400$ Å, $\kappa=\lambda_0/\xi_0=68$, and $\gamma=2$. Unlike the Ginzburg-Landau free energy, the London free energy cannot completely determine the vortex lattice by a simple minimization. Instead, one has to impose a set of source terms located at the position of the vortices. The functional form of the source terms does not come from the London theory and requires more fundamental treatment. The free energy must then be minimized with respect to the positions of the vortices (determined by the source terms). In general, a two-dimensional lattice can be determined by four parameters. However, a centered rectangular lattice is energetically more favorable than an oblique lattice. On the other hand, the vortex lattice spacing is fixed by the average magnetic field $\overline{B}$ ($\overline{B}\approx H$ away from H_{c1}). Thus we are left with two variational parameters, i.e., the lattice orientation with respect to a and b directions and the apex angle β—the angle between the two basic vectors of the lattice. We therefore find the vortex lattice geometry by minimizing $\mathcal{F}$ in Eq. (21) with respect to the apex angle β for different orientations of the lattice. At $T=0$, the stable orientation of the lattice is aligned with the a and b axes. Figure 2 shows the results of our numerical calculations for β as a function of magnetic field. The upper curve (squares) is the result of the combined calculations considering both nonlocal and nonlinear corrections, i.e., using Eqs. (20) and (21). The lower curve (circles), on the other hand, corresponds to taking into ac-

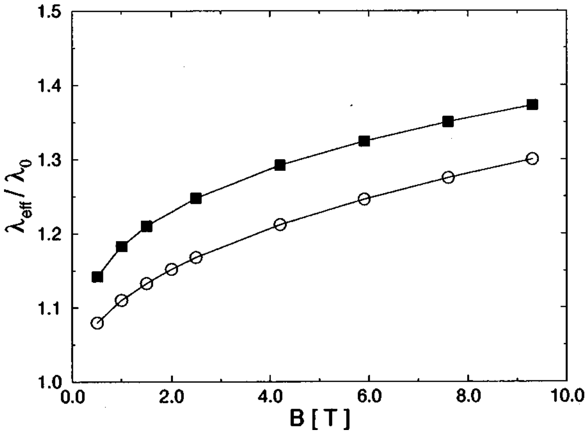

FIG. 3. The effective penetration depth as a function of the magnetic field. Circles represent the result of the calculations considering only the nonlocal effects whereas squares are the result of combined calculations considering both nonlocal and nonlinear effects.

count only the nonlocal corrections, i.e., neglecting $\mathcal{F}_{nl}$ in Eq. (21) and G_{nl} in Eq. (20). As is clear from Fig. 2, the difference between the two cases is small and about 1° or 2°. Therefore, the phase diagram given in Ref. 22 retains its validity qualitatively even after adding the nonlinear correction to the London free energy.

We also calculate the effective penetration depth λ_{eff} for different magnetic fields in almost the same way as it is calculated from μSR data.[13] In these experiments, the muon precession signal obtained from the experiment is fit to a signal obtained by Fourier transforming the magnetic-field distribution calculated on a hexagonal vortex lattice with the same average magnetic field using the ordinary London model with some cutoff function. The λ that provides the best fit to the data is considered as λ_{eff}. Here, we calculate λ_{eff} in a different way (but similar in spirit) using the fact that in the ordinary London model, for a hexagonal lattice and for a large enough field,

$$\overline{(B-\overline{B})^2} \equiv \overline{\Delta B^2} = \overline{B}^2 \sum_{\vec{k}\neq 0} \frac{e^{-\xi^2 k^2}}{(1+\lambda^2 k^2)^2}$$

$$\simeq \lambda^{-4} \overline{B}^2 \sum_{\vec{k}\neq 0} \frac{e^{-\xi^2 k^2}}{k^4} \propto \lambda^{-4}. \qquad (25)$$

Therefore, associating all the field dependence of $\overline{\Delta B^2}$ in our calculation with the field dependence of an effective penetration depth λ_{eff}, we can define λ_{eff} by

$$\frac{\lambda_{eff}}{\lambda_0} = \left(\frac{\overline{\Delta B_0^2}}{\overline{\Delta B^2}} \right)^{1/4}, \qquad (26)$$

where $\overline{\Delta B_0^2}$ is the mean squared value of the magnetic field $B_0(\vec{r}) - \overline{B}$ obtained by applying the ordinary London model on a hexagonal lattice with the same average field $\overline{B}$ and with the penetration depth λ_0.

Figure 3 shows the result of our numerical calculation for λ_{eff} using Eq. (26). The lower curve corresponds to the calculations including only the nonlocal correction. The upper curve corresponds to the result of the calculations using both nonlinear and nonlocal terms. The effect of the nonlinear term to the field dependence of λ_{eff} is almost nothing but an overall shift. Figure 4 exhibits magnetic-field distribution $n(B)$ defined by

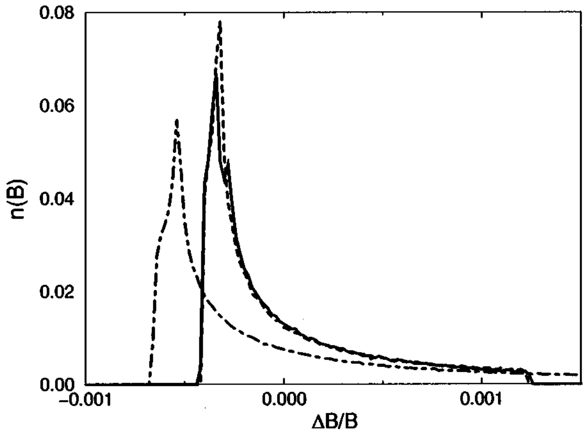

FIG. 4. Solid line: Magnetic-field distribution obtained from the nonlinear-nonlocal London equation with $\overline{B} = 5.9$ T and $\lambda_0 = 1400$ Å. Dot-dashed line: Field distribution obtained from an ordinary London equation on a hexagonal lattice with the same $\overline{B}$ and λ_0. Dashed line: Result of the same ordinary London calculation but with $\lambda_0 = 1850$ Å.

$$n(B') = \frac{1}{A} \int d^2 r\, \delta[B' - B(\vec{r})] \qquad (27)$$

at the average magnetic field $\overline{B} = 5.9$ T, with A being the area of a unit cell. The solid line in Fig. 4 represents the magnetic-field distribution calculated from the nonlinear-nonlocal London equation. This line shape is then compared with another line shape (dashed line) obtained from an ordinary London calculation but with a larger value of λ_0. μSR experiments also produce the same kind of line shape but with some additional broadening due to lattice disorder, interaction of muons with nuclear dipolar fields, and the finite lifetime of muons. The resolution of the magnetic field as a result of this broadening is $\delta B \sim 10^{-3}$ T. The difference between the solid line and dashed line in Fig. 4, as well as the double peak feature of the solid line, is therefore not observable by μSR experiments because of these broadening effects. Thus, as far as these line shapes are concerned, it is difficult to distinguish a nonlinear-nonlocal effect from a simple shift in the magnetic penetration depth in the ordinary London model. Figure 5 compares the effect of including both nonlinear and nonlocal corrections to the London equation with the effect of including only the nonlocal term. Comparing the two line shapes, it is apparent that the effect of the nonlinear term is small compared to the nonlocal term, as was emphasized before.

IV. DISCUSSION

The ordinary London equation is not adequate to describe all the different properties of a vortex lattice in high-T_c superconductors, especially the properties resulting from the

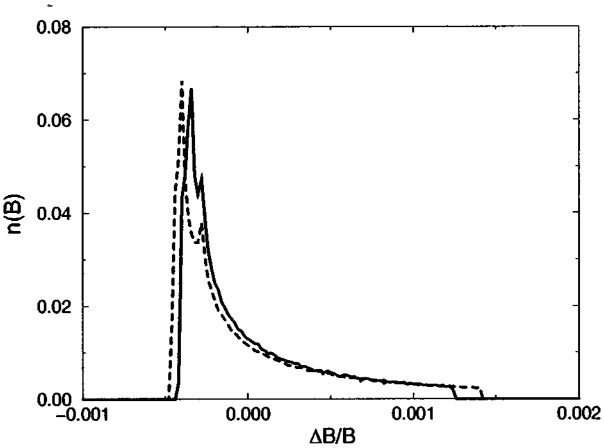

FIG. 5. Solid line: Magnetic-field distribution obtained from the nonlinear-nonlocal London equation. Dashed line: Magnetic-field distribution obtained from a London equation including only the additional nonlocal term using the same parameters.

presence of the superconducting gap nodes or other anisotropies on the Fermi surface. However, as we discussed in this paper and also in our previous publications,[21,22] a generalized London model with appropriate higher-order corrections that take into account these anisotropic effects can still provide a fairly simple way to calculate different properties of a vortex lattice.

In Ref. 21, we established a generalized London equation that could describe the structure of the vortex lattice below T_c down to intermediate temperatures. Our results in Ref. 21 were in agreement with the Ginzburg-Landau calculations.[34] At low temperatures, however, the suggested generalized London model ceases to be valid because of the nonanalyticities resulting from the presence of the nodes in the superconducting gap. In the present paper and also in Ref. 22, we generalized our London approach to describe the nonanalytic and also nonlinear and nonlocal nature of a d-wave vortex lattice at low temperatures. Our numerical calculations indicate that at both high-temperature and low-temperature regimes, the nonlocal parts of the free energy play the dominant role in determining the vortex lattice properties.

The equilibrium vortex lattice geometry exhibits novel field and temperature dependence owing to the fourfold anisotropic effects expressed by the corrections to the London equation (Fig. 2; see also Refs. 21 and 22). The numerical calculation of the lattice geometry is rather insensitive to the details of the vortex cores.[35] The reason is that the details of the magnetic field inside the vortex cores mainly affect the self-energies of the vortices. In the magnetic fields far below H_{c2}, the vortex lattice geometry is mostly determined by the magnetic interaction energy between vortices which is insensitive to the precise shape of the core. Therefore, our replacement of the vortex core by a Gaussian source term should be adequate for the vortex lattice structure calculations.

The effective penetration depth λ_{eff} also exhibits field dependence at low temperatures as is illustrated in Fig. 3. Some important points need to be emphasized here.

(i) The field dependence of λ_{eff} is not linear. The variation of λ_{eff} with the magnetic field is faster at lower fields. μSR data are only available for a limited range of the magnetic field and the uncertainty of the experimental data is noticeable. Thus, it is hard to judge from the μSR data about the linearity of the field dependence although a negative curvature comparable to our result in Fig. 3 is evident from the μSR data for a detwinned YBCO single crystal in Ref. 16. At low magnetic fields where the μSR data are available, the relative variation of the effective penetration depth in our calculation is about 7% for an increase of 1 T in the magnetic field, which is close to a 7.3% variation obtained from μSR data for an optimally doped, and a 9.5% variation for a detwinned underdoped, YBCO single crystal using the same cutoff function as Eq. (23) for fitting calculations.[13,16]

(ii) More importantly, this field dependence of λ_{eff} has a predominantly nonlocal origin rather than a nonlinear one, contrary to what is generally believed. The contribution of the nonlinear term to the total (minimized) free energy is almost one order of magnitude smaller than the nonlocal term. What is more important, however, is the field dependence and β dependence of these terms, not their orders of magnitude at fixed β and B. As we mentioned earlier, we consider β as a variational parameter that has to be fixed by minimizing the London free energy. As can be inferred from Figs. 2 and 3, the field dependence and β dependence of the nonlinear term in the free energy are also smaller than the nonlocal term. It is worth noting that in the Meissner state, a linear nonlocal term can never produce field dependence in the penetration depth (as it is usually defined in that state) and therefore a nonlinear term is necessary for such an effect.[3] In the vortex state, however, a nonlocal term can result in a field-dependent effective penetration depth. To understand this, let us neglect the nonlinear term and assume a linear but nonlocal London equation. In that case, the total magnetic field will be the superposition of the fields around individual vortices. The magnetic field around an isolated vortex is given by

$$B(\vec{r})=\Phi_0\int\frac{d^2k}{(2\pi)^2}\frac{F(k)e^{i\vec{k}\cdot\vec{r}}}{1+\mathcal{L}_{ij}(\vec{k})k_ik_j},\tag{28}$$

where Φ_0 is the flux quantum and $F(k)$ is the cutoff function resulting from the source term. $\mathcal{L}_{ij}(\vec{k})$ is defined in Eq. (18). For small values of k, $\mathcal{L}_{ij}(\vec{k})\approx\lambda_o^2\delta_{ij}$. Since small k corresponds to large r, one expects an isotropic field, similar to the local London case, far away from the vortex core. For large values of k, however, $\mathcal{L}_{ij}(\vec{k})$ has strong k dependence with fourfold anisotropy. Thus, the closer to the vortex core, the more deviation from an isotropic ordinary London single vortex is expected, as is clearly shown in Fig. 6. At low magnetic fields, the vortices are far apart and their magnetic fields overlap in regions far away from their cores. The properties of the vortex lattice should then be similar to the ordinary London hexagonal lattice. As the magnetic field is increased, the vortices come closer to each other. Although the profile of the magnetic field around each vortex remains unchanged (the nonlinear term is neglected), the overlap regions will be closer to the vortex cores and will be more affected by the nonlocal term. Therefore, it is conceivable that at large magnetic fields, the vortex lattice properties, such as the magnetic-field distribution, will be affected by the nonlocal term in the London equation. The magnetic field

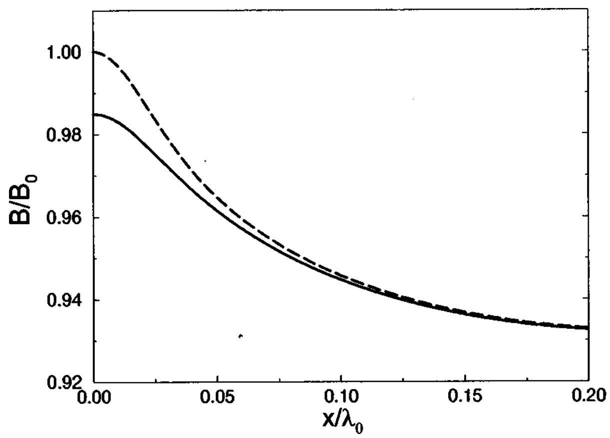

FIG. 6. Magnetic field as a function of the distance from the center of the vortex (a direction) for an isolated vortex. The solid line corresponds to the nonlocal London equation whereas the dashed line represents an ordinary London calculation with the same value of λ_0. $B_0 = B(r=0)$ for the ordinary London vortex.

near the vortex core is reduced by the nonlocal term, as is clear in Fig. 6. This is because $\mathcal{L}_{ij}(\vec{k}) - \lambda_0^2 \delta_{ij}$ is a positive definite tensor for all k and therefore the denominator of the integrand in Eq. (28) is always larger than the corresponding ordinary London one. As a result, the magnitude of $\overline{\Delta B^2}$ is smaller for the nonlocal case and therefore $\lambda_{\rm eff}$ defined by Eq. (26) tends to be larger. This explains why $\lambda_{\rm eff}/\lambda_0$ is always greater than 1 in Fig. 3. Figure 4 exhibits the resemblance between a change in the magnetic-field distribution due to the nonlocal term and due to a shift in the ordinary London penetration depth. The slight difference between the solid and dashed lines in Fig. 4 would be unobservable in μSR experiments as a result of the broadening effects. Since no field dependence due to the nonlocal term is expected in the Meissner state, the $\lambda_{\rm eff}$ as defined here and also in μSR experiments is expected to be conceptually different from what is usually defined as penetration depth in the Meissner state, although they are closely related.

(iii) The magnitude and field dependence of $\lambda_{\rm eff}$ are not so sensitive to the apex angle β. In other words, a few degrees' change in the variational parameter β does not modify the magnetic-field distribution as much as a variation in the average magnetic field does.

(iv) Calculation of $\lambda_{\rm eff}$ is rather sensitive to the form of the vortex source term. The importance of the source term in the calculation of $\overline{\Delta B^2}$ has already been emphasized by Yaouanc, Dalmas de Réotier, and Brandt.[29] In Ref. 13, $\lambda_{\rm eff}$ is obtained by fitting to the μSR data using both a Gaussian cutoff [Eq. (23)] and also the cutoff function proposed by Hao *et al.*[28,29] The difference between the two cases is significant and about 30% for the magnitude of $\lambda_{\rm eff}$ and even more (for the detwinned sample) for the relative variation with respect to the magnetic field. This can explain the importance of the source term in calculations of the effective penetration depth.

μSR experiments[13,17] on $NbSe_2$, which is believed to be a conventional superconductor, also show a field dependence in the effective penetration depth, although it is much weaker than what is observed in high-T_c compounds. Since there is no node in the superconducting gap of these materials, the theory presented in this paper cannot explain this field dependence. However, since the size of the vortex core in these materials is large and comparable to the vortex lattice spacing for the magnetic fields of experimental interest, it is conceivable that a significant effect can come from the cores, as is pointed out in Ref. 29. Thus, a more careful consideration of the vortex core might be necessary in order to have a better quantitative explanation of the experimental results.[32]

V. CONCLUSION

We investigated the effect of the superconducting gap nodes on the vortex lattice properties at very low temperatures by a generalized London approach with higher-order corrections to the free energy. We found that nonlocal effects, arising from the diverging coherence length near the gap node, are predominantly responsible for the unusual behavior of the vortex lattice geometry and the effective penetration depth. The nonlinear effects associated with the shift of the quasiparticle excitation spectrum play only a secondary role, resulting in small (but not negligible) corrections at low T. Contrary to common belief, the effective penetration depth, as defined in a μSR experiment, is not a linear function of the magnetic field and is mainly affected by the nonlocal effects. This is in marked contrast to the nonlinear Meissner effect[3] in a d-wave superconductor where the correction to $\lambda_{\rm eff}$ arises strictly from the nonlinear term in the London free energy and is linear in the field.

ACKNOWLEDGMENTS

We would like to thank P. Stamp, W. Hardy, R. Kiefl, J. Sonier, I. Herbut, M. Sigrist, R. Heeb, and Z. Tešanović for useful discussions, M. Kohmoto and P. Muzikar for correspondence, and M. Nekovee for numerical advice. This work was supported by NSERC, the CIAR, and NSF Grant No. DMR-9415549 (M.F.).

[1] W. N. Hardy, D. A. Bonn, D. C. Morgan, R. Liang, and K. Zhang, Phys. Rev. Lett. **70**, 3999 (1993).

[2] For a review, see D. J. Van Harlingen, Rev. Mod. Phys. **67**, 515 (1995).

[3] S. K. Yip and J. A. Sauls, Phys. Rev. Lett. **69**, 2264 (1992); D. Xu, S. K. Yip, and J. A. Sauls, Phys. Rev. B **51**, 16 233 (1995).

[4] A. Maeda, Y. Iino, T. Hanaguri, N. Motohira, K. Kishio, and T. Fukase, Phys. Rev. Lett. **74**, 1202 (1995); A. Maeda, T. Hanaguri, Y. Iino, S. Masuoka, Y. Kokata, Junichi Shimoyama, K. Kioshio, H. Asaoka, Y. Matsushita, M. Hasegawa, and H. Takei, J. Phys. Soc. Jpn. **65**, 3638 (1996).

[5] G. E. Volovik, J. Phys. C **21**, L221 (1988); Zh. Eksp. Teor. Fiz. **58**, 457 (1993) [Sov. Phys. JETP **58**, 469 (1993)].

[6] K. A. Moler, D. J. Baar, J. S. Urbach, R. Liang, W. N. Hardy, and A. Kapitulnik, Phys. Rev. Lett. **73**, 2744 (1994).

[7] A. P. Ramirez, Phys. Lett. A **211**, 59 (1996).
[8] R. A. Fisher, J. E. Gordon, S. F. Reklis, D. A. Wright, J. P. Emerson, B. F. Woodfield, E. M. McCarro III, and N. E. Phillips, Physica C **252**, 237 (1995).
[9] I. Kosztin and A. J. Leggett, Phys. Rev. Lett. **79**, 135 (1997).
[10] B. Keimer, J. W. Lynn, R. W. Erwin, F. Dogan, W. Y. Shih, and I. A. Aksay, J. Appl. Phys. **76**, 6788 (1994); B. Keimer, W. Y. Shih, R. W. Erwin, J. W. Lynn, F. Dogan, and I. A. Aksay, Phys. Rev. Lett. **73**, 3459 (1994).
[11] I. Maggio-Aprile, Ch. Renner, A. Erb, E. Walker, and O. Fischer, Phys. Rev. Lett. **75**, 2754 (1995).
[12] M. B. Walker and T. Timusk, Phys. Rev. B **52**, 97 (1995).
[13] J. E. Sonier, Ph.D. thesis, University of British Columbia, 1998.
[14] J. E. Sonier, R. F. Kiefl, J. H. Brewer, D. A. Bonn, J. F. Carolan, K. H. Chow, P. Dosanjh, W. N. Hardy, R. Liang, W. A. MacFarlane, P. Mendels, G. D. Morris, T. M. Riseman, and J. W. Schneider, Phys. Rev. Lett. **72**, 744 (1994).
[15] J. E. Sonier, R. F. Kiefl, J. H. Brewer, D. A. Bonn, S. R. Dunsiger, W. N. Hardy, R. Liang, W. A. MacFarlane, T. M. Riseman, D. R. Noakes, and C. E. Stronach, Phys. Rev. B **55**, 11 789 (1996).
[16] J. E. Sonier, R. F. Kiefl, J. H. Brewer, D. A. Bonn, W. N. Hardy, R. Liang, W. A. MacFarlane, R. I. Miller, T. M. Riseman, D. R. Noakes, C. E. Stronach, and M. F. White, Phys. Rev. Lett. **79**, 2875 (1997).
[17] J. E. Sonier, R. F. Kiefl, J. H. Brewer, J. Chakhalian, S. R. Dunsiger, W. A. MacFarlane, R. I. Miller, A. Wong, G. M. Luke, and J. W. Brill, Phys. Rev. Lett. **79**, 1742 (1997).
[18] Y. Wang, Ph.D. thesis, Indiana University, 1996; A. H. MacDonald and Y. Wang (unpublished).
[19] See, e.g., M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, FL, 1975).
[20] V. G. Kogan, M. Bullock, B. Harmon, P. Miranović, Lj. Dobrosavljević-Grujić, P. L. Gammel, and D. J. Bishop, Phys. Rev. B **55**, R8693 (1997).
[21] I. Affleck, M. Franz, and M. H. S. Amin, Phys. Rev. B **55**, R704 (1997).
[22] M. Franz, I. Affleck, and M. H. S. Amin, Phys. Rev. Lett. **79**, 1555 (1997).
[23] There is a factor of $-\frac{1}{2}$ difference from our previous notation (Ref. 21). Here, we follow Ref. 3 by setting the electron mass equal to 1.
[24] I. Zutic and O. T. Valls, Phys. Rev. B **54**, 15 500 (1996); **56**, 11 279 (1997); J. Comput. Phys. **136**, 337 (1997).
[25] A. J. Millis, Phys. Rev. B **35**, 151 (1987).
[26] E. H. Brandt, J. Low Temp. Phys. **24**, 709 (1977); **73**, 355 (1988); Phys. Rev. B **37**, 2349 (1988).
[27] J. R. Clem, J. Low Temp. Phys. **18**, 427 (1975).
[28] Z. Hao, J. R. Clem, M. W. McElfresh, L. Civale, A. P. Malozemoff, and F. Holtzberg, Phys. Rev. B **43**, 2844 (1991).
[29] A. Yaouanc, P. Dalmas de Réotier, and E. H. Brandt, Phys. Rev. B **55**, 11 107 (1997).
[30] E. H. Brandt, Phys. Rev. Lett. **78**, 2208 (1997).
[31] F. Gygi and M. Schluter, Phys. Rev. B **43**, 7609 (1991).
[32] I. Affleck, M. Franz, and M. H. S. Amin (unpublished).
[33] M. Franz and Z. Tešanović, Phys. Rev. Lett. **80**, 4763 (1998).
[34] A. J. Berlinsky, A. L. Fetter, M. Franz, C. Kallin, and P. I. Soininen, Phys. Rev. Lett. **75**, 2200 (1995); M. Franz, C. Kallin, P. I. Soininen, A. J. Berlinsky, and A. L. Fetter, Phys. Rev. B **53**, 5795 (1996); J. H. Xu, Y. Ren, and C.-S. Ting, *ibid.* **53**, R2991 (1996); R. Heeb, A. Van Otterlo, M. Sigrist, and G. Blatter, *ibid.* **54**, 9385 (1996).
[35] The calculations presented here and in Refs. 21 and 22 have used isotropic core cutoff functions. For *d*-wave superconductors the actual vortex cores will not necessarily be isotropic [J. Shiraishi, M. Kohmoto, and K. Maki, cond-mat/9709310, (unpublished)]. We are presently investigating the effect of core anisotropy on vortex lattice geometry using the modified London free-energy approach.