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LOWER-LIMIT BARRIER IN THE PROBLEM OF THE IDENTIFICATION OF A BARRIER IN THE FUNCTIONING OF A CERTAIN INVENTORY STORAGE AND ISSUE SYSTEM

The paper investigates a certain inventory system whose input is a non-aggregated dynamic-parameter process. The authors derive equations that define the distribution of conditional probabilities for the case of a lower-limit barrier in subsystem L . They depend on the parameters of the functioning of transport subsystem and the parameters of the process of product supply to finite-volume storage.

Keywords: *system, barrier, inventory*

1. Introduction

Paper [1] defined the notion of a barrier in the functioning of an inventory storage and issue system with non-aggregated dynamic-parameter input and presented the general operating principles of such a system and the conditional distributions for intermediate states of the subsystem L . These distributions were used in paper [2] to derive relations satisfied by the density functions for intermediate states.

Continuing the investigations reported in [1], [2], this paper contains derivation of the analytical forms of the conditional probabilities in the case of a lower-limit barrier in a process controlled by a dynamic-parameter input.

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2. Conditional probabilities in the case of a lower-limit barrier in the subsystem L

The lower-limit barrier of the subsystem L at time $t \in T_1$ is defined by the random event $z(t) = 0$. The state can be characterized by probabilities of the form:

$$Q_k^{ul}(\{0\}, t) = P(z(t) = 0, x(t) = x_k, v(t) = u), t \in T_l, u = 1, 0. \quad (1)$$

These probabilities satisfy the following relations:
for $u = 0$,

$$\begin{aligned} Q_k^{0l}(\{0\}, t + \tau) &= P(z(t + \tau) = 0, x(t + \tau) = x_k, v(t + \tau) = 0) = \\ &= \sum_i \int_0^{V_1} q_{ik}^{00l}(z, \{0\}; \tau, t) Q_i^{0l}(dz, t) + \int_0^{V_1} q_{kk}^{10l}(z, \{0\}; \tau, t) Q_k^{1l}(dz, t), \end{aligned} \quad (2)$$

where

$$\begin{aligned} q_{ik}^{00l}(z, \{0\}; \tau, t) &= P(x(t) \text{ has one jump for } t + \theta \in (t, t + \tau), v(s) = 0 \text{ for } s \in [t, t + \tau], \\ & z - d\tau \leq 0 \mid x(t) = x_i, v(t) = 0) + o_1^{(l)}(\tau); \\ & i \neq k, z = z(t) \in [0, V_1], t \in T_l, t + \tau \in T_l, \end{aligned} \quad (3)$$

$$\begin{aligned} q_{kk}^{00l}(z, \{0\}; \tau, t) &= P(x(s) = x_k, s \in [t, t + \tau], v(s) = 0 \text{ for } s \in [t, t + \tau], \\ & z - d\tau \leq 0 \mid x(t) = x_k, v(t) = 0) + o_1^{(l)}(\tau); \\ & z = z(t) \in [0, V_1], t \in T_l, t + \tau \in T_l, \end{aligned} \quad (4)$$

$$\begin{aligned} q_{kk}^{10l}(z, \{0\}; \tau, t) &= P(x(s) = x_k, \text{ for } s \in [t, t + \tau], v(t) \text{ has one jump for } t + \theta \in (t, t + \tau), \\ & h(z + x_k \theta) - d(\tau - \theta) \leq 0 \mid x(t) = x_k, v(t) = 1) + o_1^{(l)}(\tau); \\ & z = z(t) \in [0, V_1], t \in T_l, t + \tau \in T_l, \end{aligned} \quad (5)$$

for $u = 1$,

$$\begin{aligned} Q_k^{1l}(\{0\}, t + \tau) &= P(z(t + \tau) = 0, x(t + \tau) = x_k, v(t + \tau) = 1) = \\ &= \sum_i \int_0^{V_1} q_{ik}^{11l}(z, \{0\}; \tau, t) Q_i^{1l}(dz, t) + \int_0^{V_1} q_{kk}^{01l}(z, \{0\}; \tau, t) Q_k^{0l}(dz, t), \end{aligned} \quad (6)$$

where

$$\begin{aligned} q_{ik}^{11l}(z, \{0\}; \tau, t) &= P(x(t) \text{ has one jump for } t + \theta \in (t, t + \tau), v(s) = 1 \text{ for } s \in [t, t + \tau], \\ & h(z + \theta x_i) - (\tau - \theta)x_k \leq 0 \mid x(t) = x_i, v(t) = 1) + o_1^{(l)}(\tau); \\ & i \neq k, z = z(t) \in [0, V_1], t \in T_l, t + \tau \in T_l, \end{aligned} \quad (7)$$

$$\begin{aligned}
q_{kk}^{11l}(z, \{0\}; \tau, t) &= P(x(s) = x_k, v(s) = 1 \text{ for } s \in [t, t + \tau], \\
z + \alpha x_k &\leq 0 \mid x(t) = x_k, v(t) = 1) + o_1^{(l)}(\tau); \\
z = z(t) &\in [0, V_1], t \in T_l, t + \tau \in T_l,
\end{aligned} \tag{8}$$

$$\begin{aligned}
q_{kk}^{01l}(z, \{0\}; \tau, t) &= P(x(s) = x_k \text{ for } s \in [t, t + \tau], v(t) \text{ has one jump for } t + \theta \in (t, t + \tau), \\
h(z - d\theta) + x_k(\tau - \theta) &\leq 0 \mid x(t) = x_k, v(t) = 0) + o_1^{(l)}(\tau); \\
z = z(t) &\in [0, V_1], t \in T_l, t + \tau \in T_l.
\end{aligned} \tag{9}$$

In order to find the probabilities $Q_k^{1l}(\{0\}; t + \tau)$ and $Q_k^{0l}(\{0\}; t + \tau)$, we have to find the conditional probabilities $q_{ik}^{11l}(z, \{0\}; \tau, t)$, $q_{kk}^{11l}(z, \{0\}; \tau, t)$, $q_{kk}^{01l}(z, \{0\}; \tau, t)$, $q_{ik}^{00l}(z, \{0\}; \tau, t)$, $q_{kk}^{10l}(z, \{0\}; \tau, t)$, $q_{kk}^{00l}(z, \{0\}; \tau, t)$. Taking into account relations (3) and (6) from [1], we obtain

$$q_{ik}^{11l}(z, \{0\}; \tau, t) = \exp(-\pi_1^{*l} \tau) \int_B \frac{\pi_{ik}^{(l)}}{\pi_i^{(l)}} d(1 - \exp(-\pi_i^{(l)} \theta)) + o_1^{(l)}(\tau), \tag{10}$$

where

$$\begin{aligned}
B &= \{\theta: 0 < \theta < \tau, h(z + \alpha x_i) + (\tau - \theta)x_k \leq 0\}, \\
i &\neq k, z = z(t) \in [0, V_1], t \in T_l, t + \tau \in T_l.
\end{aligned} \tag{11}$$

If $x_i > 0$, $x_k < 0$, τ small, $0 < \theta < \tau$, $t \in T_l$, $t + \tau \in T_l$, then

$$\begin{aligned}
\{\theta: h(z + \alpha x_i) + (\tau - \theta)x_k \leq 0\} &= \{\theta: z + \theta x_i + (\tau - \theta)x_k \leq 0\} \cap \{\theta: z + \alpha x_i < V_1\} = \\
&= \begin{cases} \left\{ \theta: 0 \leq \theta \leq \frac{-z - \alpha x_k}{x_i - x_k} \right\} \cap \left\{ \theta: \theta < \frac{V_1 - z}{x_i} \right\}, & z < -\alpha x_k, \\ \emptyset, & \text{other } z. \end{cases}
\end{aligned}$$

Thus

$$B = \begin{cases} \left\{ \theta: 0 < \theta \leq \frac{z + \alpha x_k}{x_k - x_i} \right\}, & 0 \leq z < -\alpha x_k, \\ \emptyset, & \text{other } z. \end{cases}$$

Hence, from (10), by integration, we get

$$q_{ik}^{11l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \frac{z + \alpha x_k}{x_k - x_i} + o^{(l)}(\tau, z), & 0 \leq z < -\alpha x_k, \\ 0, & \text{other } z. \end{cases} \tag{12}$$

Similarly it can be shown that the conditional probabilities are given by the following formulas:

for $x_k > 0$, x_i – any state,

$$q_{ik}^{11l}(z, \{0\}; \tau, t) = o_1^{(l)}(\tau), \quad 0 \leq z \leq V_1, \quad (13)$$

for any x_k, x_i ,

$$q_{ik}^{00l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_0^{*l} \tau) \pi_{ik}^{(l)} \tau + o^{(l)}(\tau), & 0 \leq z < d\tau, \\ 0, & \text{other } z, \end{cases} \quad (14)$$

$$q_{kk}^{00l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_0^{*l} \tau)(1 - \pi_k^{(l)} \tau) + o^{(l)}(\tau), & 0 \leq z < d\tau, \\ 0, & \text{other } z, \end{cases} \quad (15)$$

for $x_i < 0$, $x_k = 0$,

$$q_{ik}^{11l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \left(z + \frac{z}{x_i} \right) + o^{(l)}(\tau; z), & 0 \leq z < -\alpha x_i, \\ 0, & \text{other } z, \end{cases} \quad (16)$$

for $x_i < 0$, $x_k < 0$,

$$q_{ik}^{11l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \tau + o^{(l)}(\tau), & x_i > x_k, 0 \leq z < -\alpha x_i, \\ (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \frac{z + \alpha x_k}{x_k - x_i} + o^{(l)}(\tau; z), & x_i > x_k, -\alpha x_i < z < -\alpha x_k, \\ 0, & x_i > x_k, \text{ other } z, \\ (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \tau + o^{(l)}(\tau), & x_i < x_k, 0 \leq z < -\alpha x_k, \\ (1 - \pi_1^{*l} \tau) \pi_{ik}^{(l)} \left(\tau - \frac{z + \alpha x_k}{x_k - x_i} \right) + o^{(l)}(\tau; z), & x_i < x_k, -\alpha x_k < z < -\alpha x_i, \\ 0, & x_i < x_k, \text{ other } z, \end{cases} \quad (17)$$

for $x_k \leq 0$,

$$q_{kk}^{11l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau)(1 - \pi_1^{*l} \tau) + o^{(l)}(\tau), & 0 \leq z < \alpha x_k, \\ 0, & \text{other } z, \end{cases} \quad (18)$$

for $x_k > 0$,

$$q_{kk}^{01l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = 0, \quad 0 \leq z \leq V_1, \quad (19)$$

for $x_k = 0$,

$$q_{kk}^{01l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau) \pi_0^{*l} \left(\tau - \frac{z}{d} \right) + o^{(l)}(\tau; z), & 0 \leq z < \tau d, \\ 0, & \text{other } z, \end{cases} \quad (20)$$

for $x_k < 0$,

$$q_{kk}^{01l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau) \pi_0^{*l} \tau + o^{(l)}(\tau), & d + x_k \leq 0, 0 \leq z < d\tau, \\ 0, & d + x_k \leq 0, \text{ other } z, \\ (1 - \pi_k^{(l)} \tau) \pi_0^{*l} \tau + o^{(l)}(\tau), & d + x_k > 0, 0 \leq z < -x_k \tau, \\ (1 - \pi_k^{(l)} \tau) \pi_0^{*l} \left(\tau - \frac{z + \tau x_k}{d + x_k} \right) + o^{(l)}(\tau; z), & d + x_k > 0, -x_k \tau < z < d\tau, \\ 0, & d + x_k > 0, \text{ other } z, \end{cases} \quad (21)$$

for $x_k < 0$,

$$q_{kk}^{10l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau) \pi_1^{*l} \tau + o^{(l)}(\tau), & d + x_k \leq 0, 0 \leq z < d\tau, \\ 0, & d + x_k \leq 0, \text{ other } z, \\ (1 - \pi_k^{(l)} \tau) \pi_1^{*l} \tau + o^{(l)}(\tau), & d + x_k > 0, 0 \leq z < -x_k \tau, \\ (1 - \pi_k^{(l)} \tau) \pi_1^{*l} \frac{d\tau - z}{d + x_k} + o^{(l)}(\tau; z), & d + x_k > 0, -x_k \tau < z < d\tau, \\ 0, & d + x_k > 0, \text{ other } z, \end{cases} \quad (22)$$

for $x_k > 0$,

$$q_{kk}^{10l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau) \pi_1^{*l} \frac{d\tau - z}{d + x_k} + o^{(l)}(\tau; z), & 0 \leq z < d\tau, \\ 0, & \text{other } z, \end{cases} \quad (23)$$

for $x_k = 0$,

$$q_{kk}^{10l}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = \begin{cases} (1 - \pi_k^{(l)} \tau) \pi_1^{*l} \frac{d\tau - z}{d} + o^{(l)}(\tau; z), & 0 \leq z < d\tau, \\ 0, & \text{other } z, \end{cases} \quad (24)$$

for $x_k > 0$,

$$q_{kk}^{111}(z, \{0\}; \tau, t) - o_1^{(l)}(\tau) = 0, \quad 0 \leq z \leq V_1. \quad (25)$$

The components $o^{(l)}(\tau; \dots)$, $o_1^{(l)}(\tau)$, which appear in equations (12)–(25) are in the vicinity of $\tau = 0$ infinitely small quantities of an order higher than τ . The relations (12)–(25) will be used in the authors' next paper to derive relations satisfied by the probabilities $Q_k^{ul}(\{0\}; t + \tau)$. These will be used for quantitative identification of a barrier in the functioning of the system under consideration.

References

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Bariera dolna procesu w zagadnieniu identyfikacji i bariery funkcjonowania pewnego systemu gromadzenia i wydawania zapasów

Badana jest bariera działania pewnego systemu gospodarki zapasami, którego wejście jest procesem niezagregowanym o dynamicznych parametrach. Wyprowadzono wzory wyrażające warunkowe rozkłady prawdopodobieństwa w przypadku bariery dolnej podsystemu L . Zależą one od parametrów funkcjonowania podsystemu transportowego oraz parametrów procesu podaży produktu do magazynu o skończonej objętości.

Słowa kluczowe: *system, bariera, zapas*