Progress of Theoretical Physics, Vol. 112, No. 1, July 2004

Lowest Order Constrained Variational Calculation for Nuclear and Neutron Matter with a New Charge-Dependent Reid Potential

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(Received January 31, 2004)

The lowest order constrained variational (LOCV) method is reformulated to calculate the equation of state of nuclear and neutron matter with a new charge-dependent Reid potential (*Reid*93). The state-dependent correlation functions are calculated by performing a full functional minimization for each $JLSTM_T$-channel and the resulting Euler Lagrange equations are solved up to $J = 9$. The correlation functions are compared with those coming from LOCV calculation with the old *Reid*68 potential. It is shown that unlike Δ-*Reid*68 and AV_{18} interactions, the new *Reid*93 over binds nuclear matter at much larger saturation density than *Reid*68. Finally, the results are being compared with similar calculations with other potentials and many-body techniques.

§1. Introduction

In order to put microscopic nuclear many-body calculation to a reliable test, one needs a perfect nucleon-nucleon potential such that the discrepancies in the prediction cannot be blamed on a bad fit of NN data.[1)] In 1990's, Nijmegen,[2)] Argonne[3)] and Bonn[4)] groups made a quantitative analysis of the NN potentials. Before, that the best NN models fit NN data typically with a χ^2 datum about 2. But all of the new potentials such as *Reid*93[2)] or AV_{18}[3)] have in common that they use the about 45 parameters and fit the pruned 1992 Nijmegen data base[2)] with a χ^2 datum about 1. The *Reid*93 and Argonne AV_{18} potentials do not use meson-exchange for intermediate and short range. The AV_{18} uses local functions of Woods-Saxon while similar to the *Reid*68[5)] potential, *Reid*93 applies local Yukawa functions. Both of the above potentials are local and regularized either by exponential (AV_{18}) or by dipole (*Reid*93) form factors. Because of the well-known breaking of charge-independence in the *np* and *pp* partial waves, these potentials have different format in each isospin projection. So in order to perform nuclear many-body calculations with these potentials one should treat explicitly this charge-dependence in the nuclear many-body wave-functions.

In a series of papers the lowest order constrained variational (LOCV) method was developed[6),7)] for calculating the properties of homogeneous nuclear fluids with realistic nucleon-nucleon interactions.[5),8)] In 1997, this approach was further generalized[9)] to include more sophisticated interactions such as the UV_{14},[10)] the AV_{14}[11)] and the new argonne AV_{18}[3)] as well as the *Reid*68[5)] and Δ-*Reid*68[8)] potentials. For a wide range of models our LOCV calculations agree well with the results of variational fermion hypernetted chain (VHC) calculations where these have performed

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and for a number of central potentials, there is agreement with the essentially exact numerical solutions obtained by Monte Carlo technique.[6),7)] Despite this agreement for model problems, there has been some dispute about the convergence of LOCV results in calculations, employing realistic nucleon-nucleon interactions which are strongly spin-dependent and which, in particular, contain a sizeable tensor force. This argument was tested, by calculating the energy of the three-body cluster contribution in nuclear matter and the normalization integral $\langle\psi \mid \psi\rangle$ both at zero and finite temperatures.[12),13)] It was shown that $\langle\psi \mid \psi\rangle$ is normalized $(\chi_2 = 0)$[12)] within one percent (χ_3)[12)] and the three-body cluster energy is less than 1 MeV for $k_F \leq 1.6$ fm^{-1}. These results are given in Table I.

Table I. The nuclear matter saturation properties.[12)]

	*Reid*68		*Δ-Reid*68		
	E_1+E_2	$E_1+E_2+E_3$	E_1+E_2	$E_1+E_2+E_3$	Empirical
Density (fm^{-3})	0.28	0.21	0.25	0.2	0.17
Binding energy (MeV)	22.54	21.85	16.28	15.52	15.86
Compressibility (MeV)	340	298	300	277	200–300
χ_2	0.0	0.0	0.0	0.0	—
χ_3	0.127	0.085	0.093	0.062	—

Our LOCV calculation is a fully self-consistent technique with state-dependent correlation functions (without any free parameter, except the NN potential) and it is capable of dealing with the well-defined phenomenological potentials such as the *Δ-Reid*68 (the modified Reid potential with an allowance of $\Delta(1234)$ degree of freedom, see Refs. 7) and 8)) potential. The Δ state, being the most important configuration that modifies the nuclear force, might be the key to the understanding of three-body forces.[14)] The results suggest that the LOCV method reasonably describes the nucleonic-matter properties at zero and finite temperatures.[6),7),9),12),15)] Furthermore, our recent calculations at zero temperature with the UV_{14} and AV_{18} potentials[9)] show a good agreement with more sophisticated calculation such as the variational fermion hypernetted chain method (VHC)[9),16)] (compare table 7 of Ref. 9) (BM,1997) with table 6 of Ref. 16) (APR,1998)). On the other hand the LOCV method considers constraint in the form of a normalization condition[17)] to keep the higher-order terms as small as possible and it also assumes a particular form for the long-range behavior of the correlation functions in order to perform an exact functional minimization of the two-body energy with respect to the short-range parts of correlation functions. The functional minimization procedure represents an enormous computational simplification over the unconstrained methods, i.e. to parameterized the short-range behavior of the correlation functions, that attempt to go beyond the lowest-order.[16)] This difference has been pointed out by Moroni et al.,[18)] that the optimized two-body correlation functions obtained by solving numerically the hypernetted chain Euler equations, have provided, improved energy upper bounds with respect to parameterized two-body correlations functions. Finally as already discussed, in the LOCV framework the higher-order terms at and beyond nuclear matter saturation densities are reasonably small.[12),13)]

The LOCV method has been also developed for calculating the various properties of homogeneous nuclear fluids such as hot and frozen neutron, nuclear and β-stable matter with realistic nucleon-nucleon interactions.[9),13),15)] In these works the liquid-gas phase transition and corresponding critical temperature were found.

With respect to the above arguments, in this work we shall attempt to develop LOCV method for the new *Reid*93 potential which has state and charge-dependence with the operator parameterizations, S_{12}^I and $\boldsymbol{L}\cdot\boldsymbol{S}$.

In recent years several many-body methods such as variational fermion hypernetted chain (VHC), the correlated basis function (CBF), the Brueckner-Bethe (BB) and Brueckner-Hartree-Fock (BHF) approaches have been also applied to the new and other potentials. We intend to discuss their results in this work as well.

There is also a lowest-order constrained variational method, which was originally developed by Pandharipande.[19)] This method was used with some changes in the constraint by Mittet and Ostaard.[19)] Both methods have been only applied to the *Reid*68 potential (see Ref. 9) for discussion about the validity and criticism about these approaches).

So the paper is planned as follows: A short description of the lowest order constrained variational method for the state and charge-dependence potential and evaluation of the energy for nuclear and neutron matter is given in §2. Section 3 is devoted to results and discussion for binding energies of nuclear and neutron matter and the properties of the two-body correlation functions. Finally, our summary and conclusions are presented in §4.

§2. The LOCV formalism

In the LOCV method, we use an ideal Fermi gas type wave functions, ϕ_i, for the single particle states and variational techniques, to find the wave function of interacting system[6),7),9)]

$$\psi = \mathcal{F}\Phi, \tag{1}$$

where ($\mathcal{S}(\mathcal{A})$ is a symmetrizing (anti-symmetrizing) operator)

$$\mathcal{F} = \mathcal{S}\prod_{i>j} f(ij) \tag{2}$$

and

$$\Phi = \mathcal{A}\prod_{i} \phi_i. \tag{3}$$

The Jastrow correlation functions $f(ij)$ are operators and they are written as:

$$f(ij) = \sum_{\alpha,k} f_\alpha^{(k)}(ij) O_\alpha^{(k)}(ij). \tag{4}$$

In the above equation $\alpha = \{S, L, J, T, M_T\}$, $k = 1-3$ and

$$O_\alpha^{k=1-3} = 1, \left(\frac{2}{3} + \frac{1}{6}S_{12}^I\right), \left(\frac{1}{3} - \frac{1}{6}S_{12}^I\right). \tag{5}$$

For spin-singlet channels with orbital angular momentum $L = J$, the spin-triplet channels with $L \neq J \pm 1$ and ${}^3P_0$ channel, etc. (non-tensor channels), k is superfluous and set only to unity, while for $L = J \pm 1$ channels (tensor channels) it takes values of 2 and 3.

The choice of O_α^k (ignoring $\boldsymbol{L} \cdot \boldsymbol{S}$ operator in the correlation functions) has been investigated in Ref. 7) and has been found that it is good approximation to ignore the spin-orbit correlation.

In general, each of the correlation functions $f_\alpha^{(1)}$, $f_\alpha^{(2)}$ and $f_\alpha^{(3)}$ is required to heal to the modified Pauli function $f_P^{M_T}(r)$,

$$\begin{aligned} f_P^{M_T}(r) &= \left[1 - \frac{1}{2}(l(k_F r))^2\right]^{-\frac{1}{2}} \qquad n\text{-}n \ \text{ and } \ p\text{-}p \ \text{ channels } \quad (M_T = 1, -1), \\ &= 1 \qquad\qquad\qquad\qquad\qquad\quad n\text{-}p \ \text{ channels } \quad (M_T = 0), \end{aligned} \tag{6}$$

with

$$l(x) = \frac{3}{x^3}(\sin x - x \cos x), \tag{7}$$

where $k_F = ((\frac{6\pi^2}{\nu})\rho)^{\frac{1}{3}}$ and $\nu = 4$ and 2 for nuclear and neutron matter respectively.

The many-body energy term $\mathcal{E}[f]$ is calculated by constructing a cluster expansion for the expectation value of our Hamiltonian:

$$\mathcal{H} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j}^{A} V(ij), \tag{8}$$

where

$$V(12) = \sum_{i,\alpha} V_\alpha^i(12) \mathcal{O}_\alpha^i |\alpha > \langle \alpha|, \tag{9}$$

$$\mathcal{O}_\alpha^{i=1-3} = 1, S_{12}^I, \boldsymbol{L} \cdot \boldsymbol{S}, \tag{10}$$

$$V_\alpha(12)^{i=1-3} = V_\alpha^c(12), V_\alpha^T(12), V_\alpha^{LS}(12), \tag{11}$$

and $V_\alpha^c(12), V_\alpha^T(12)$ and $V_\alpha^{LS}(12)$ are the central tensor and spin-orbit channel-parts of $Reid93$ interaction.[2)] Then, we keep only the first two terms in a cluster expansion of the energy functional:

$$\mathcal{E}[f] = \frac{1}{A} \frac{\langle \Psi | \mathcal{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \cdots . \tag{12}$$

The one body term $\mathcal{E}_1$ is independent of the f and is simply the Fermi-gas kinetic energy, $\mathcal{E}_1 = \frac{3}{5} \frac{\hbar^2 k_F^2}{2m}$, while the two-body energy term is defined as,

$$\mathcal{E}_2 = (2A)^{-1} \sum_{ij} \langle ij | \mathcal{V} | ij \rangle_a, \tag{13}$$

where

$$\mathcal{V}(12) = -\frac{\hbar^2}{2m}[f(12), [\nabla_{12}^2, f(12)]] + f(12) V(12) f(12) \tag{14}$$

and the two-body antisymmetrized matrix element $\langle ij|\mathcal{V}|ij\rangle_a$ are taken with respect to the single-particle functions composing Φ, i.e. plane-waves. By inserting a complete set of two-particle state twice in Eq. (13) and performing some algebra we can rewrite the two-body term as follows:

$$\mathcal{E}_2 = \mathcal{E}_{\rm central} + \mathcal{E}_{\rm tensor}, \tag{15}$$

where

$$\begin{aligned}\mathcal{E}_{\rm central} = {} & \frac{1}{2\pi^5\rho}\sum_{\alpha,m_{\tau_1}m_{\tau_2}} |\langle m_{\tau_1}m_{\tau_2}|T,m_{\tau_1}+m_{\tau_2}\rangle|^2(2J+1)\frac{1}{2}\{1-(-1)^{L+S+T}\} \\ & \times\int_0^\infty dr\frac{\hbar^2}{2m}\left\{f_\alpha^{(1)'^2}+\frac{2m}{\hbar^2}V_\alpha^c f_\alpha^{(1)^2}\right\}a^{(1)^2}_{\alpha,m_{\tau_1},m_{\tau_2}}(r), \qquad (16)\\ \mathcal{E}_{\rm tensor} = {} & \frac{1}{2\pi^5\rho}\sum_{\alpha,m_{\tau_1}m_{\tau_2}} |\langle m_{\tau_1}m_{\tau_2}|T,m_{\tau_1}+m_{\tau_2}\rangle|^2(2J+1)\frac{1}{2}\{1-(-1)^{J+T}\} \\ & \times\int_0^\infty dr\frac{\hbar^2}{2m}\left[\left\{f_\alpha^{(2)'^2}+\frac{2m}{\hbar^2}(V_\alpha^c+2V_\alpha^T-V_\alpha^{LS})f_\alpha^{(2)^2}\right\}a^{(2)^2}_{\alpha,m_{\tau_1},m_{\tau_2}}(r)\right. \\ & +\left\{f_\alpha^{(3)'^2}+\frac{2m}{\hbar^2}(V_\alpha^c-4V_\alpha^T-2V_\alpha^{LS})f_\alpha^{(3)^2}\right\}a^{(3)^2}_{\alpha,m_{\tau_1},m_{\tau_2}}(r) \\ & \left.+\left\{r^{-2}(f_\alpha^{(2)}-f_\alpha^{(3)})^2+\frac{2m}{\hbar^2}V_\alpha^{LS}f_\alpha^{(2)}f_\alpha^{(3)}\right\}b^2_{\alpha,m_{\tau_1},m_{\tau_2}}(r)\right] \qquad (17)\end{aligned}$$

and (i and j stand for protons or neutrons)

$$a^{(1)^2}_{\alpha,i,j}(r,\rho) = r^2 I^\alpha_{i,j;J}(r,\rho), \tag{18}$$

$$a^{(2)^2}_{\alpha,i,j}(r,\rho) = r^2(2J+1)^{-1}[(J+1)I^\alpha_{ij;J-1}(r,\rho)+JI^\alpha_{ij;J+1}(r,\rho)], \tag{19}$$

$$a^{(3)^2}_{\alpha,i,j}(r,\rho) = r^2(2J+1)^{-1}[JI^\alpha_{ij;J-1}(r,\rho)+(J+1)I^\alpha_{ij;J+1}(r,\rho)], \tag{20}$$

$$b^2_{\alpha,i,j}(r,\rho) = r^2(2J+1)^{-1}[2J(J+1)(I^\alpha_{ij;J-1}(r,\rho)-I^\alpha_{ij;J+1}(r,\rho))], \tag{21}$$

$$I^\alpha_{ij;J}(x,\rho) = \int d\vec{q}\,\mathcal{J}_J^2(xq)\mathcal{P}_{ij}(q), \tag{22}$$

with

$$\mathcal{P}_{ij}(q) = \frac{2}{3}\pi\left[{k_F^i}^3+{k_F^j}^3-\frac{3}{2}({k_F^i}^2+{k_F^j}^2)q-\frac{3}{15}({k_F^i}^2-{k_F^j}^2)^2q^{-1}+q^3\right] \tag{23}$$

for $\frac{1}{2}|k_F^i-k_F^j|\langle q\langle|k_F^i+k_F^j|$ and

$$\mathcal{P}_{ij}(q) = \frac{4}{3}\pi\min({k_F^i}^3,{k_F^j}^3) \tag{24}$$

for $q\langle\frac{1}{2}|k_F^i-k_F^j|$ and

$$\mathcal{P}_{ij}(q) = 0 \tag{25}$$

for $q>\frac{1}{2}|k_F^i+k_F^j|$. $\mathcal{J}_J(x)$ are the familiar Bessel functions.

As we pointed out before, we impose the normalization condition:[17)]

$$\rho \int (\mathcal{G}(r) - 1) d\vec{r} = -1, \tag{26}$$

where $\mathcal{G}(r)$ is the two-body radial distribution function. This condition also plays the role of smallness parameter in the cluster expansion.[1)]

The channel break down of the above normalization constraint[9)] has the following form:

$$\frac{1}{\pi^5 \rho} \sum_{\alpha,k} (2J+1) \frac{1}{2} [1 - (-1)^{(L+S+T)}] |\langle m_{\tau_1} m_{\tau_2} | T, m_{\tau_1} + m_{\tau_2} \rangle|^2 \times \int_0^\infty dr [f_\alpha^{(k)^2}(r) - f_p^{M_T\,2}(r)] a^{(k)^2}_{\alpha, m_{\tau_1}, m_{\tau_2}}(r) = -1. \tag{27}$$

Minimizing the two-body energy $\mathcal{E}_2$ subject to the constraint of Eq. (27), we find the following sets of uncoupled,

$$g^{(1)''}_{\alpha,i,j} - [a^{(1)''}_{\alpha,i,j}/a^{(1)}_{\alpha,i,j} + m\hbar^{-2}(V^c_\alpha + \lambda)] g^{(1)}_{\alpha,i,j} = 0 \tag{28}$$

and coupled,

$$g^{(2)''}_{\alpha,i,j} - [a^{(2)''}_{\alpha,i,j}/a^{(2)}_{\alpha,i,j} + m\hbar^{-2}(V^c_\alpha + 2V^T_\alpha - V^{LS}_\alpha + \lambda) + r^{-2} b^2_{\alpha,i,j}/a^{(2)^2}_{\alpha,i,j}] g^{(2)}_{\alpha,i,j} + \left(r^{-2} - \frac{1}{2} m\hbar^{-2} V^{LS}_\alpha \right) b^2_{\alpha,i,j} \{a^{(2)}_{\alpha,i,j} a^{(3)}_{\alpha,i,j}\}^{-1} g^{(3)}_{\alpha,i,j} = 0, \tag{29}$$

$$g^{(3)''}_{\alpha,i,j} - [a^{(3)''}_{\alpha,i,j}/a^{(3)}_{\alpha,i,j} + m\hbar^{-2}(V^c_\alpha - 4V^T_\alpha - 2V^{LS}_\alpha + \lambda) + r^{-2} b^2_{\alpha,i,j}/a^{(3)^2}_{\alpha,i,j}] g^{(3)}_{\alpha,i,j} + \left(r^{-2} - \frac{1}{2} m\hbar^{-2} V^{LS}_\alpha \right) b^2_{\alpha,i,j} \{a^{(2)}_{\alpha,i,j} a^{(3)}_{\alpha,i,j}\}^{-1} g^{(2)}_{\alpha,i,j} = 0. \tag{30}$$

Euler-Lagrange differential equations, where

$$g^{(k)}_{\alpha,i,j} = a^{(k)}_{\alpha,i,j} f^{(k)}_\alpha. \tag{31}$$

The Lagrange multiplier λ have been introduced to satisfy the normalization condition. The constraint is incorporated by solving the above E-L equations only out to certain distances, until the logarithmic derivative of correlation functions matches those of $f_p^{M_T}(r)$ and then we set the correlation functions equal to $f_p^{M_T}(r)$. So as we pointed out before there is no free parameter in our LOCV formalism, i.e. the healing distance is determined directly by the constraint and the initial conditions.

§3. Nuclear and neutron matter results

The results of our LOCV calculations for nuclear matter are presented in Fig. 1. *Reid*93 ($J < 3$) is a calculation without the $J \geq 3$ channels of *Reid*93 interaction. *Reid*68 ($+J > 2$) is the LOCV results with assumption of one pion exchange part of

*Reid*68 in $J \geq 3$ channels (see Ref. 6) for its detail) of the *Reid*68 potential. Our previous calculations with AV_{18}, UV_{14} and Δ-*Reid*68 potentials are also given for comparison.[9] It is seen that like *Reid*68, *Reid*93 over binds nuclear matter. Its saturation energy is about -32.7 MeV at $\rho = 0.48$ fm^{-3}. It is interesting that $E(\rho)$ for *Reid*93 ($J < 3$) and old *Reid*68 (obviously for $J < 3$) are very similar. They only show some differences at very high densities. This is expected, since *Reid*68 is more "singular" near the origin, than that of *Reid*93.[2] On the other hand, there is much differences between the $E(\rho)$ of *Reid*68 ($+J > 2$) and *Reid*93. This shows that the simple naive approximation of OPE interaction in $J \geq 3$ channels is not enough and the higher partial waves should be treated in the same footing as the low partial waves (i.e. fitting the phase-shift in $J \geq 3$ partial waves explicitly). It also shows that different channels should be treated separately both in the interactions and correlation functions. We will come back to this point later on.

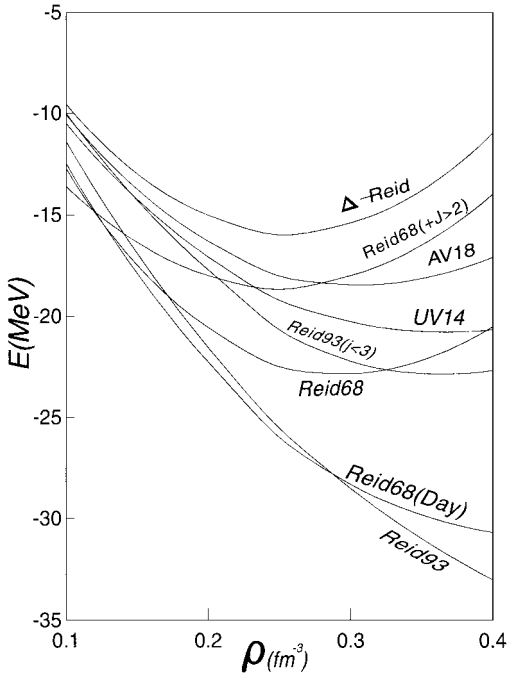

Fig. 1. The LOCV prediction of nuclear matter saturation curves for *Reid*93 interaction and other potentials discussed in the text. *Reid*93 ($J < 3$) is the same calculation but by switching off the $J > 2$ channels in *Reid*93.

We have also performed a LOCV calculation by using the *Reid*68 (Day) potential.[5] This potential has been defined up to $J = 5$ partial waves. It is seen that we get a similar result to those of *Reid*93.

In Table II we present the difference between the LOCV calculation for nuclear matter with *Reid*93, *Reid*68, *Reid*68 (Day), UV_{14} and AV_{18} up to $J = 9$ ($J = 5$ for *Reid*68 (Day)) and $J = 2$. It is seen that the higher partial waves are much more negative in *Reid*93 and *Reid*68 (Day) than the other interactions. This again shows the importance of channel-dependent calculations (note that UV_{14} and AV_{18} depend only explicitly on (S,T)-channels).

Table II. The difference between the LOCV nuclear matter energies with $J \leq 9$ and $J \leq 2$ for various potentials.

ρ (fm^{-3})	*Reid*93	*Reid*68	*Reid*68 (Day)	UV_{14}	AV_{18}
0.1	-1.41	-0.81	-1.22	1.9	0.74
0.2	-3.42	2.55	-2.99	2.23	1.72
0.3	-6.25	4.61	-5.42	2.05	2.77
0.4	-9.27	6.35	-7.52	1.2	3.73

Table III. Comparison of channel breakdown of energy for *Reid*93, *Reid*68 (Day) and *Reid*68 interactions at $\rho = 0.3$ fm^{-3}.

		*Reid*93			*Reid*68	*Reid*68 (Day)
channel	$M_T = -1$	$M_T = 0$	$M_T = 1$	$\sum M_T$		
$1S_0$	-7.67	-7.95	-7.67	-23.29	-23.89	-23.12
$3P_0$	-1.82	-1.71	-1.82	-5.35	-5.35	-5.38
$1P_1$	–	8.26	–	8.26	4.84	4.54
$3P_1$	6.25	6.08	6.25	18.58	19.64	18.84
$3S_1$	–	-25.67	–	-25.67	-23.62	-23.77
$3D_1$	–	2.40	–	2.40	2.63	2.53
$1D_2$	-1.97	-1.94	-1.97	-5.88	-5.65	-5.67
$3D_2$	–	-8.19	–	-8.19	-8.94	-8.94
$3P_2$	-4.92	-4.86	-4.92	-14.7	-14.51	-14.18
$3F_2$	-0.47	-0.42	-0.47	-1.36	-1.33	-1.34
$2 < J < 5$	0.59	-7.43	0.59	-6.25	–	-5.42
$5 < J < 9$	-0.11	-0.43	-0.11	-0.65	–	–
Total	-10.12	-4.86	-10.12	-62.12	-56.18	-61.92

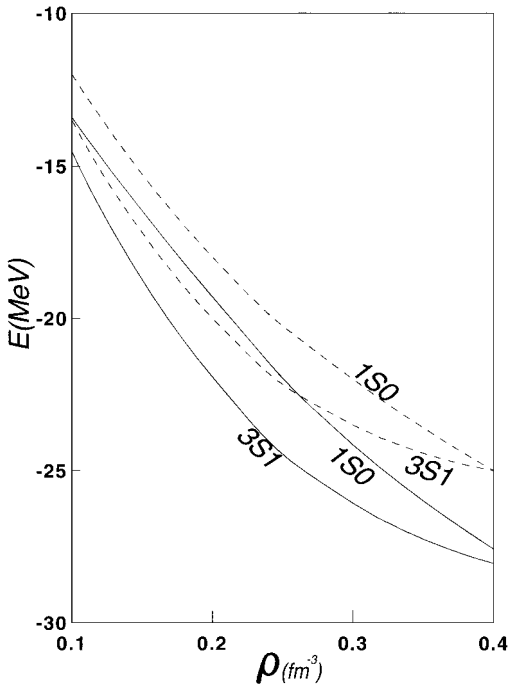

Fig. 2. Comparison of $^1S_0$ and $^3S_1$ channels contributions to the nuclear matter binding energy for *Reid*93. LOCV (full curves) and BHF[20] (dashed curves).

The channel contributions to energy for *Reid*93, *Reid*68 and *Reid*68 (Day) interactions at $\rho = 0.3$ fm^{-3} are presented in Table III. As we stated before (also see Fig. 1), for $J \leq 2$ the contribution of each channel is approximately the same for both interactions. These results show that the old *Reid*68 potential has been quite well fitted for $J \leq 2$ channels and the other channels ($J \geq 3$) plus the three-body force or Δ excitations of nucleons may improve the N-N interaction. On the other hand, there is not much difference between the energies of various iso-spin projection in different channels regarding *Reid*93 interaction.

Figure 2 shows the density dependence of $^1S_0$ and $^3S_1$ channels for *Reid*93 interaction. The results of BHF calculations[20] are also presented for comparison. In general BHF gives less binding than LOCV. Both calculations have the same density dependence up to $\rho = 0.25$ fm^{-3}. Especially BHF $^3S_1$ channel starts to saturate at high densities, i.e. 0.4 fm^{-3}, which is well known in BHF calculation. As they have pointed out,[20] their results are reliable below nuclear matter density.

The saturation properties of nuclear matter predicted by various techniques and N-N interactions are given in Table IV. As we pointed out before, we get very close results to that of VHC. The BHF calculation shows less binding for most of the interactions except that of *Reid*93. It seems that only $UV_{14} + TNI$ interaction gets proof from different many-body techniques as a true N-N interaction, since it gives result close to the empirical prediction. However TNI has been adjusted

Table IV. Nuclear matter saturation properties calculated with different many-body techniques and interactions.

Potential	Method	Author	ρ_0 (fm^{-3})	$E_0(\rho_0)$ (MeV)
AV_{18}	LOCV	BM[9]	0.31	−18.46
AV_{18}	BHF	EHMMP[20]	0.24	−16.09
AV_{18}	VHC	APR[16]	0.29	−18.6
AV_{14}	LOCV	BM[9]	0.29	−15.99
AV_{14}	VHC	WFF[16]	0.32	−15.6
AV_{14}	BB	DW[21]	0.28	−17.8
AV_{14}	BHF	BBB[22]	0.26	−18.26
UV_{14}	LOCV	BM[9]	0.36	−20.81
UV_{14}	VHC	CP[16]	0.35	−20.0
UV_{14}	VHC	WFF[16]	0.33	−17.1
$UV_{14} + TNI$	LOCV	BM[9]	0.17	−17.33
$UV_{14} + TNI$	VHC	WFF[16]	0.16	−16.6
$UV_{14} + TNI$	CBF	FFP[23]	0.16	−18.3
Δ-*Reid*68	LOCV	MI[7]	0.25	−16.28
Δ-*Reid*68	LOCV+$\mathcal{E}_3$	MI[12]	0.2	−15.52
*Reid*68	LOCV	OBI[6]	0.29	−22.54
*Reid*68	LOCV	MO[19]	0.23	−14.58
Reid-V_{12}	LOCV	BM[9]	0.29	−19.89
Reid-V_{12}	VHC	LP[16]	0.28	−17.0
Reid-V_8	LOCV	BM[9]	0.29	−18.19
Reid-V_8	VHC	LP[16]	0.37	−21.37
BJ-II-V_8	LOCV	BM[9]	0.22	−12.87
BJ-II-V_8	VHC	LP[16]	0.18	−8.98
*Reid*93	BHF	EHMMP[20]	0.24	−15.36
*Reid*93	LOCV	This work	0.48	−32.07
Empirical	–	–	0.17	−15.68

Table V. LOCV calculation up to various J values for *Reid*93 and *Reid*68 (Day) interactions at $\rho = 0.3$ fm^{-3}.

	*Reid*93	*Reid*68 (Day)
$E(J \leq 2)$	−22.65	−22.62
$E(J \leq 3)$	−26.29	−26.82
$E(J \leq 4)$	−30.21	−29.85
$E(J \leq 5)$	−28.48	−27.72
$E(J \leq 6)$	−29.25	–
$E(J \leq 7)$	−28.77	–
$E(J \leq 8)$	−28.85	–
$E(J \leq 9)$	−28.72	–

to reproduce the saturation curve. This shows that one should take into account the three-body forces or Δ degrees of freedom (also see the LOCV result with Δ-*Reid*68 interaction) in the N-N potential in order to get results close to the empirical saturation properties.

In order to see the effect of potentials up to various partial waves, we have perform LOCV calculations for *Reid*93 and *Reid*68 (Day) potentials with maximum J values between 2 and 9. The results are presented in Table V. It is seen that the LOCV calculations in larger J values give more binding energy respect to smaller J values.

In Fig. 3 our two-body kinetic and potential ($-V_{\rm Potential}$) energies are plotted against density for different interactions, i.e. *Reid*68, *Reid*68 (Day), *Reid*93, UV_{14} and AV_{18}. For most of the interactions the result of LOCV calculation varies between 15 to 20 MeV. Only the *Reid*93 and *Reid*68 (Day) results have large differences from the other interactions. The results of APR[16] calculations with AV_{18} are also presented for comparison. Circles (triangles (squares)) are their two-body kinetic (minus two-body potential (many-body)) energies. Their results are very similar to ours. Their potential energies are the same as ours and our kinetic energies are equal

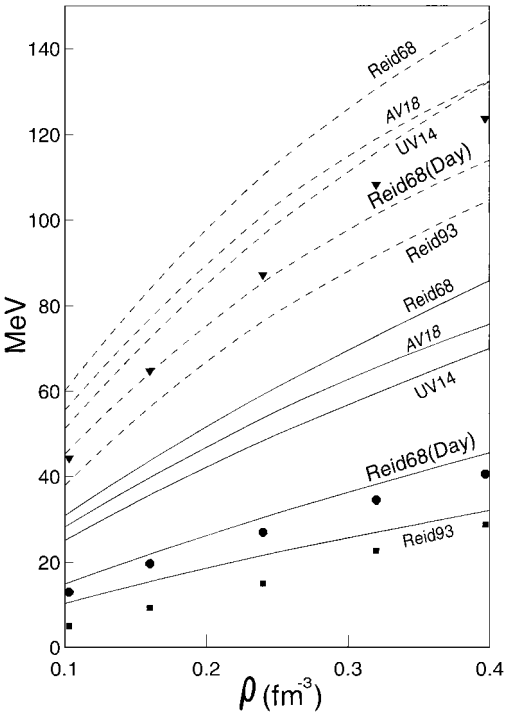

Fig. 3. The LOCV calculations with different potentials for the two-body kinetic (full curves) and potential (dotted curves) energies. Triangles (circles (squares)) are the APR[16] two-body potential (two-body kinetic (many-body)) energies with AV_{18} interaction.

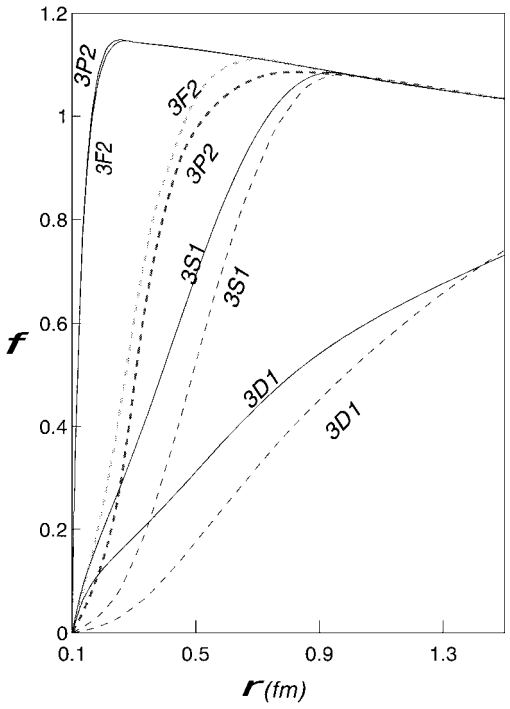

Fig. 4. The np correlation functions for the coupled $J < 3$ channels. Full curve (Reid93) and dotted curve (Reid68).

to the sum of their two-body kinetic and many-body energies.

The *np* coupled correlation functions (CCF) for $J \leq 2$ channels at $\rho = 0.3$ fm^{-3} are plotted in Fig. 4. Dotted curves are the *Reid*68 results (obviously it is averaged over iso-spin projections). It is seen that only $^3P_2$-$^3F_2$ channel has different correlation range with respect to *Reid*68 one. Other *np* CF's for coupled channels ($J \geq 3$) are presented in Fig. 5. Their behaviors are very different from $J \leq 2$ CCF's. But on average, as the J increases their correlation ranges become shorter. They reflect exactly the structure of N-N interactions.[2)] For comparison, in Fig. 6 we have plotted the CCF's of *nn* or *pp* for $J \geq 3$ channels. Their structures are not too different from the corresponding CCF's of *np* channels.

Figure 7 shows the *np* CF's of uncoupled channels for $J \leq 2$. The *Reid*68 results are also given for comparison. There are large differences, especially between the $^1P_1$ CF's of *Reid*93 and *Reid*68. This is expected since in Reid93, the r^{-1} singularities have been completely removed in these channels. The $J \geq 3$ *np* uncoupled CF's are presented in Fig. 8. Similar to $J \geq 3$ CCF's, they have different structure from the $J \leq 3$ CF's. Only the $^1F_3$ CF's has correlation range as long as $J \leq 2$ CF's. As we will mention later on the $J = 3$ channel in *Reid*93 potential has different behavior from other channels (i.e. it is more attractive). Figure 9 shows *nn* or *pp* uncoupled CF's for $J \leq 2$ channels which can be compared with the corresponding CF's for *np* channels presented in Fig. 7. There is not much difference between them, which indicates that the charge dependence of N-N interaction is not as important as the

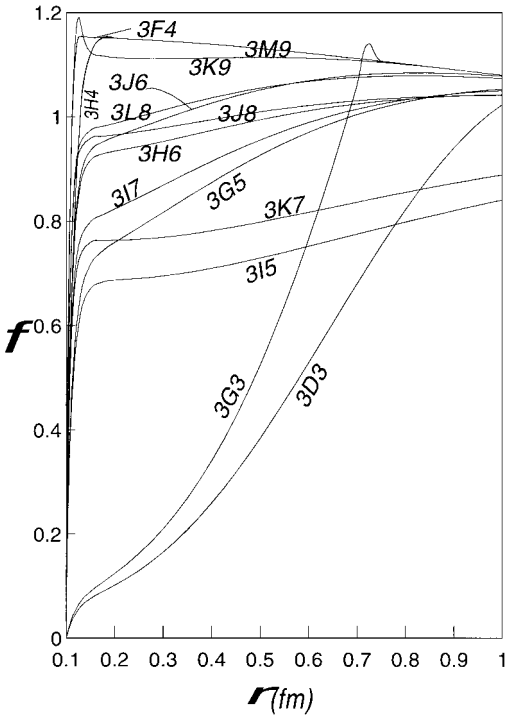

Fig. 5. As Fig. 4 but for $J > 2$.

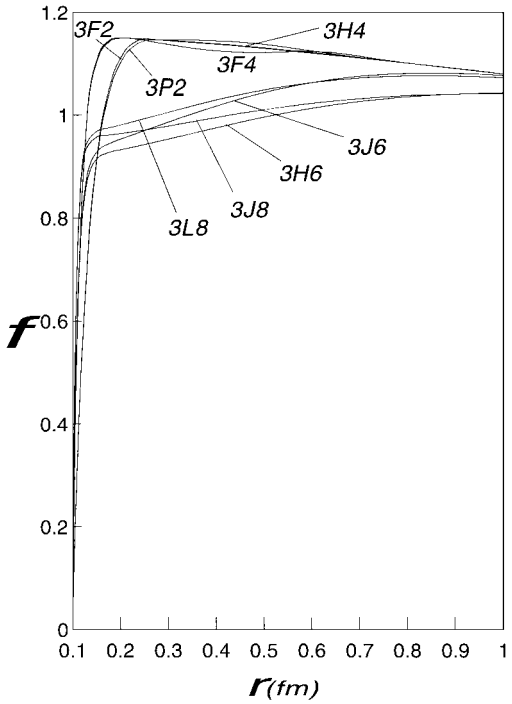

Fig. 6. As Fig. 4 but for any J and *nn* or *pp* channels .

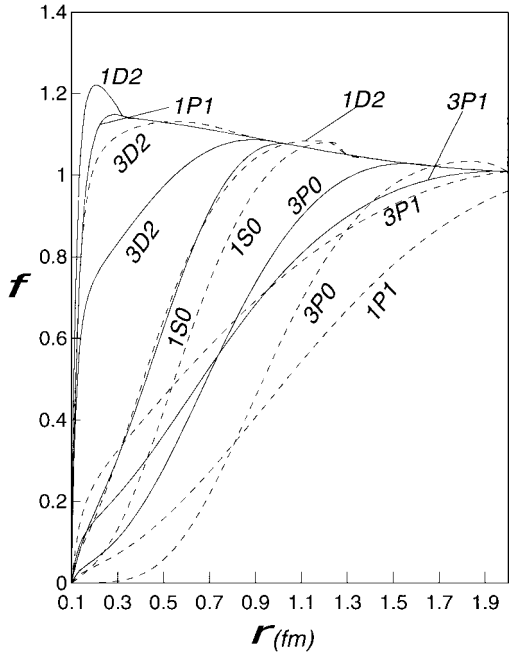

Fig. 7. As Fig. 4 but for uncoupled channels.

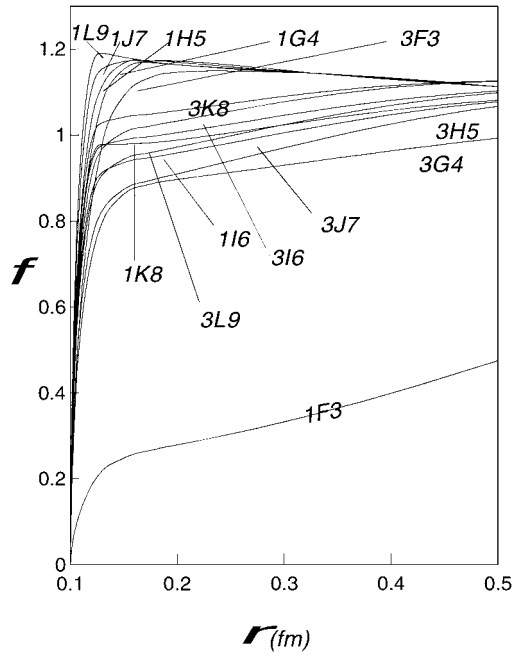

Fig. 8. As Fig. 5 but for uncoupled channels.

higher partial waves ($J \geq 3$). The *pp* or *nn* uncoupled CF's with $J \geq 3$ are also plotted in Fig. 10. The same conclusions, as above, can be made for them as well.

In Fig. 11, we present the results of our LOCV calculations for pure neutron matter (PNM) with *Reid*93 interaction. The similar calculations but with other potentials are also given for comparison. The mark styles are the results of different many-body techniques with the interactions we have considered here. Similar to our nuclear matter calculations, we find much difference between the results of PNM with *Reid*93 and *Reid*68 interactions. It seems that in general, the LOCV calculation with *Reid*93 shows completely different density dependence from other interactions. Again, like nuclear matter, LOCV and BHF results with *Reid*93 are not close. They also show different behavior respect to other *N*-*N* potentials as well. At least in Ref. 20), a very simple model has been used for the BHF calculation and as it has been pointed out in this reference, the calculation is not very accurate for $\rho \geq 0.17$ fm^{-2}. There is not any VHC calculation available with *Reid*93. But PNM results of LOCV and VHC are very close for other potentials (especially for AV_{18}).

Finally in order to see why in general *Reid*93 gives more binding than other interactions, such as AV_{18}, in Fig. 12 we have compared the central and tensor parts of *Reid*93, *Reid*68 and AV_{18} interactions in $J = 2$ and $J = 3$ channels, because we found from Table V that the main difference between the LOCV results with these interactions comes from $J = 3$ channel. It is seen from this figure that the potentials are roughly the same in $J = 2$ channels especially for r larger than 0.3 fm, but in $J = 3$ channel *Reid*93 and *Reid*68 (Day) are attractive where as AV_{18} is repulsive. So we expect that we get more binding with the Reid potentials respect to AV_{18}

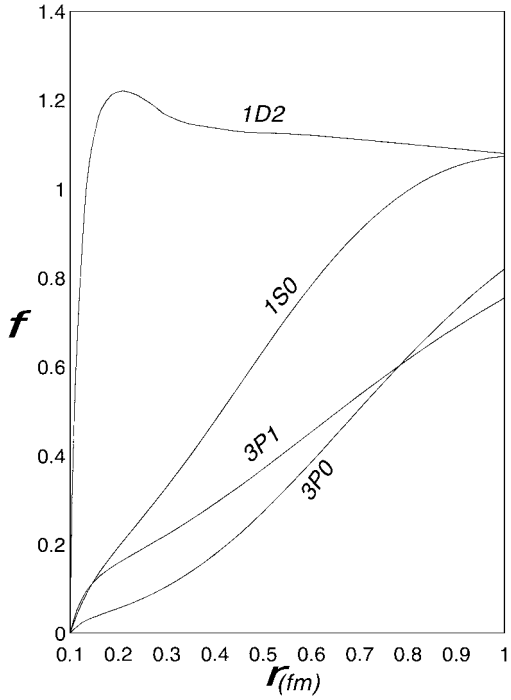

Fig. 9. As Fig. 8 but for *nn* or *pp* channels with $J \leq 2$.

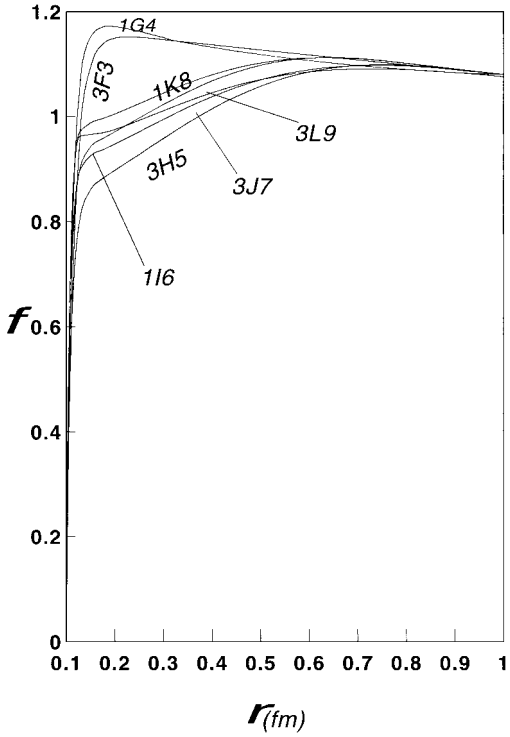

Fig. 10. As Fig. 8 but for *nn* or *pp* channels with $J \geq 3$.

interaction.

§4. Summary and conclusions

We have developed the LOCV formalism for the new *Reid*93 potential which is charge-dependent and has been fitted very accurately to the partial wave phase shift up to $J = 9$. It is important that we could solve the corresponding E-L (coupled and uncoupled) differential equations with appropriate constraint up to $J = 9$ states. It was found that the nuclear and neutron matter results are very different from those have been previously obtained using the same techniques but with different potentials. It was shown that while the $J \geq 3$ partial waves have a large effect on our results, the charge-dependent properties of *Reid*93 do not change the nuclear and neutron matter energies significantly in different channels.

Our results for *Reid*93 are very different from those of BHF calculations. This difference is in the same direction respect to the same calculations but with other N-N interactions. On the other hand, it is encouraging that the LOCV and VHC predict very close results for nuclear and neutron matter. Although it can be argued that it is not known how large is the higher order terms that LOCV ignores in the cluster expansion. As we explained before, our state-averaged calculation shows that the many-body terms are very small beyond the nuclear matter density. However,

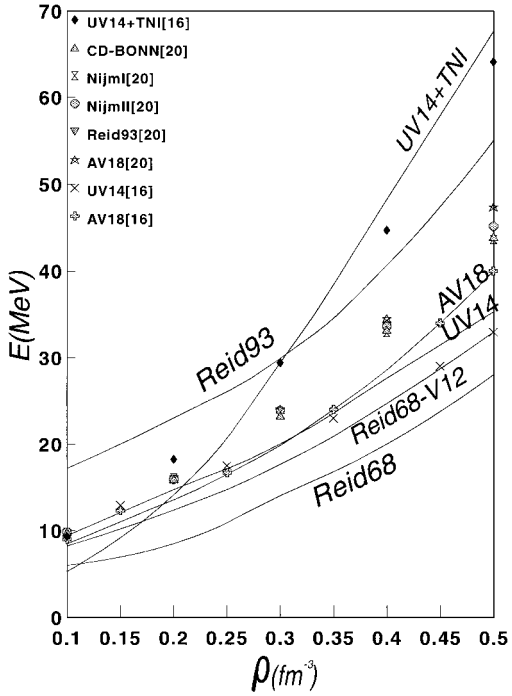

Fig. 11. The results of pure neutron matter calculations for various many-body techniques and N-N interactions. Full curves are the LOCV results with different potentials.

this dependence is under investigation,[24] since we believe that the explicit treatment of state-dependent correlation functions is very important. VHC also implies a state-averaged approximation (for example with this approximation the contribution of $\boldsymbol{L}\cdot\boldsymbol{S}$ terms in the diagonal matrix elements becomes zero). Beside this, as we stated before, rather than solving Euler-Lagrange differential equation, VHC uses the parameterized correlation functions. In VHC the neglecting higher-order terms estimated to be as much as 1 to 4 MeV (the normalization constraint is uncertain within 10%) and the accuracy of spin-orbit correlations is difficult to estimate.[16] So we would like to repeat this remark that the LOCV method is in good agreement with other variational calculations which claim that they include many-body cluster contributions. Therefore we believe LOCV method, especially because of its great simplicity, will be useful tool in the study of the other properties of nuclear matter, finite nuclei, Helium liquid, etc.

We have tested several N-N interactions during last two decades. It seems that none of the nucleon-nucleon potentials can produce satisfactory results. Of course, new potentials such as the $Reid93$, AV_{18}, etc. are more reliable than old interactions since they fit the N-N data more accurately. But still there are several points that should be considered to get a reliable N-N interactions.[1] We can summarized these effects as follows: (1) relativistic effect, (2) medium effect, (3) many-body forces, (4) uncertainty in the precise values of πNN coupling, (5) contradiction in the np scattering length, (6) incomplete understanding of charge-dependence especially in

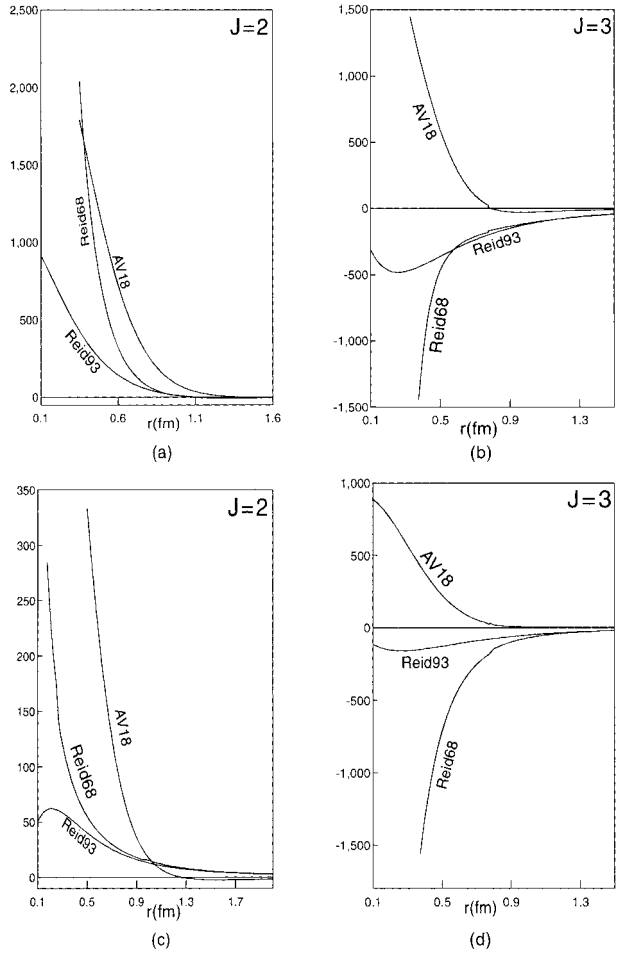

Fig. 12. The comparison of central (a,b) and tensor (c,d) parts of *Reid*93, *Reid*68 (Day) and AV_{18} interactions in MeV.

$^1S_0$ channel, (7) nonlocality, (8) position of multi-meson exchange diagram, and (9) the theoretical understanding of N-N interaction, starting from the quark-gluon picture.

Finally, we hope we could develop the LOCV formalism to perform nuclear matter calculations with the non-local N-N potentials.

Acknowledgements

We would like to thank Tehran University for supporting us under the grant provided by the Research Council.

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