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L^p BOUNDS FOR PARAMETRIC MARCINKIEWICZ INTEGRALS WITH MIXED HOMOGENEITY

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Abstract. In this paper we consider the parametric Marcinkiewicz integrals with mixed homogeneity along certain compound surfaces. Under the rather weakened size conditions on the integral kernels both on the unit sphere and in the radial direction, the L^p boundedness for such operators are given. As applications, the corresponding results for parametric Marcinkiewicz integral operators related to area integrals and Littlewood-Paley g_λ^* functions are also obtained.

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Paper MIA-18-34

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