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LRS Bianchi type-V Universe with Variable Modified Chaplygin Gas in a Scalar Tensor Theory of Gravitation

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Abstract

The spatially homogeneous and anisotropic LRS Bianchi type-V Universe filled with variable modified Chaplygin gas in the framework of Brans-Dicke [1] scalar-tensor theory of gravitation has been studied. To obtain a determinate solution of the field equations we have used the condition that scalar field (ϕ) is a function of average scale factor. Some physical and kinematical properties of the model are discussed. We have also analyzed the stability of the solution.

Key words: Bianchi type-V metric, Brans-Dicke theory, Chaplygin gas cosmology, dark energy.

1 Introduction

Recent observations of type Ia supernovae (Bahchall et al. [2]; Perlmutter et al. [3], [4]; Riess et al. [5]) and cosmic microwave background (CMB) observations of a peak in the angular power spectrum on degree scales (Berrnardis et al. [6]; Lange et al. [7]; Balbi et al. [8]) indicate that the expansion of the Universe is accelerating rather than slowing down and strongly suggest that the Universe is spatially flat with approximately 1/3 of the critical energy density being in non-relativistic matter and 2/3 in a smooth component with large negative pressure. This acceleration is caused by some unknown matter is known as *dark energy* (Griddle et al. [9]; Spergel et al. [10]; Peebles and Ratra [11]; Caldwell et al. [12]). Recent WMAP (Bennet et al. [13]) and Chandra X-ray observations (Allen et al. [14]) strongly indicate that our universe is undergoing an accelerating phase. The most appealing and simplest candidate for dark energy is the cosmological constant Λ which is characterized by the equation of state $p = \omega\rho$ with $\omega = -1$. There are various candidates to play the role of dark energy, which is the dominant part of the Universe. Some of them are quintessence (Peebles and Ratra [11]), K-essence Armendariz-Picon et al. [15]), Chaplygin gas (Kamenshchik et al. [16]), its modification known as modified Chaplygin gas (Debnath [17]), tachyonic field (Sen [18]), holographic dark energy (Li [19]), DBI-essence Martin and (Yamaguchi [20]) etc.

The Chaplygin gas having equation of state (EoS) $p = \frac{-\beta}{\rho}$, $\beta > 0$ (Kamenshchik et al. [16]) acts as a pressureless fluid for small values of the scale factor and tends to accelerated expansion for large values of the scale factor. The generalization of the Chaplygin gas model is known as the *generalized Chaplygin gas*, which satisfies $p = \frac{-\beta}{\rho^{\alpha}}$, $0 \leq \alpha \leq 1$ (Gorini et al. [21];

Alam et al. [22]; Bento et al. [23]). This model also is modified to the *modified Chaplygin gas*, having the EoS $p = \mu\rho - \frac{\beta}{\rho^{\alpha}}$, $0 \leq \alpha \leq 1$, $\mu > 0, \beta > 0$ (Debnath et al. [17]; Sahni et al. [24]). That illustrates a radiation era ($\mu = \frac{1}{3}$), while the scale factor is vanishingly small and the ΛCDM model is for an infinitely large scale factor. Further Guo and Zhang [25] established the *variable Chaplygin gas* with EoS $p = \frac{-\beta}{\rho}$, where $\beta = \beta(a)$, 'a' is the average scale factor and β is a positive function of the scale factor. Subsequently Debnath [26] provided the *variable modified Chaplygin gas*, with EoS $p = \mu\rho - \frac{\beta(a)}{\rho^{\alpha}}$ for the accelerating phase of the Universe. Bhadra and Debnath [27] have studied accretion of new variable modified Chaplygin gas and generalized cosmic Chaplygin gas onto Schwarzschild and KerrNewman black holes. Ranjit et al. [28] have discussed variable modified Chaplygin gas in anisotropic Universe with Kaluza-Klein metric. Samanta [29] has investigated Universe described by variable modified Chaplygin gas with statefinder diagnostic in general relativity.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the Universe is studied through the passage of time. The anomalies found in the cosmic microwave background (CMB) and large scale structure observations stimulated a growing interest in anisotropic cosmological model of Universe. Here we confine ourselves to models of Bianchi type-V. For studying the possible effects of anisotropy in the early Universe on present day observations many researchers, Lima and Maia [30], Chimento et al. [31], Pradhan and Singh [32], Saha [33], Rao et al. [34]-[36], Akarsu and Kilinc [37, 38], Yadav et al. [39], Amirhashchi et al. [40], Reddy et al. [41], Rao et al. [42] and Yadav & Srivastava [43] have investigated anisotropic Bianchi models from different point of view.

Several theories have been proposed as alternatives to Einsteins theory. Brans and Dicke [1] formulated a scalar-tensor theory of gravitation which is supposed to be the best alternative to Einstein's theory. Rao and Vijaya Santhi [44]-[46], Naidu et al. [47], Vidya Sagar et al. [48], Das and Abdulla [49] and Rao & Jayasudha [50] are some of the authors who have investigated several aspects of this theory.

Motivated by the above investigations, in this paper, we have studied spatially homogeneous and anisotropic LRS Bianchi type-V Universe filled with variable modified Chaplygin gas in Brans-Dicke [1] scalar-tensor theory of gravitation. This paper is organized as follows: In section 2, we have obtained Brans-Dicke field equations with the help of LRS Bianchi type-V metric. Section 3, deals with the solution of the field equations and the variable modified Chaplygin gas model. The last section contains discussion and conclusions of the obtained model.

2 Metric and the field equations

We consider the LRS Bianchi type-V space-time in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{2x}(dy^2 + dz^2), \tag{1}$$

where scale factors A and B are functions of cosmic time t only. Brans-Dicke field equations for combined scalar and tensor fields are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi^{,k}_{;k}\right), \tag{2}$$

and scalar field ϕ satisfies the following condition

$$\phi^{,k}_{;k} = 8\pi T(3+2\omega)^{-1} \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is the Einstein tensor, ω is a dimensionless coupling constant, T_{ij} is energy momentum tensor of matter and is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}. \tag{4}$$

Here ρ, p are energy density and pressure of the matter. Also, we have energy conservation equation as

$$T^{ij}_{;j} = 0, \tag{5}$$

which is a consequence of field equations (2) and (3).

In a comoving coordinate system, Brans-Dicke field equations (2) and (3) for the metric (1) with the help of (4) can be, explicitly, written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + 2\frac{\dot{\phi}\dot{B}}{\phi B} + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi p}{\phi} \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\ddot{\phi}}{\phi} = -\frac{8\pi p}{\phi} \tag{7}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{8\pi\rho}{\phi} \tag{8}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{8\pi}{3+2\omega}(\rho - 3p). \tag{9}$$

The energy conservation equation, $T^{ij}_{;j} = 0$ leads to

$$\dot{\rho} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(\rho + p) = 0 \tag{10}$$

where overhead dot stands for ordinary differentiation with respect to t.

3 Solution of the field equations

To get deterministic solution of the model we assume the shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials A and B, i.e.

$$A = B^k, \tag{11}$$

where $k \neq 0$ is an arbitrary constant.
Now from equations (6), (7) and (11), we have

$$\frac{\ddot{B}}{B} + (k+1)\frac{\dot{B}^2}{B^2} + \frac{\dot{B}\dot{\phi}}{B\phi} = 0. \tag{12}$$

We assume that scalar field ϕ is a function of average scale factor(a) Johri and Kalyani [51],

$$i.e.\ \ \phi = \phi_0 a^n \tag{13}$$

where $\phi_0 > 0$ and n are arbitrary constants.
From equations (11), (12) and (13), we get

$$\begin{aligned} A &= (c_3 t + c_4)^{\frac{kc_1}{c_3}} \\ B &= (c_3 t + c_4)^{\frac{c_1}{c_3}} \end{aligned} \tag{14}$$

where c_1, c_2 are integration constants and $c_3 = c_1(k+2)(n+1)$, $c_4 = c_2(k+2)(n+1)$.
From equation (13), we have the scalar field ϕ as

$$\phi = \phi_0 (c_3 t + c_4)^{\frac{n(k+2)c_1}{c_3}}. \tag{15}$$

Now the metric (1) can be written as

$$ds^2 = dt^2 - (c_3 t + c_4)^{\frac{2c_1}{c_3}} dx^2 - (c_3 t + c_4)^{\frac{2kc_1}{c_3}} e^{2x}(dy^2 + dz^2). \tag{16}$$

Here we assume that the Universe is filled with variable modified Chaplygin gas having equation of state proposed by Debnath [26] as

$$p = \mu\rho - \frac{\beta(a)}{\rho^\alpha} \tag{17}$$

with $0 \leq \alpha \leq 1$, and μ is a positive constant. Where β is a positive function of the average scale factor a(i.e. $\beta = \beta(a)$). Now, for simplicity, assume $\beta(a)$ is of the form

$$\beta(a) = \beta_0 a^{-\gamma} \tag{18}$$

where $\beta_0 > 0$ and γ are constants. The average scale factor 'a' for our metric is defined as $a = (AB^2)^{1/3}$. From equation (10) using equations (17) and (18), we have

$$\rho = \left[\frac{3\beta_0(1+\alpha)}{(3(1+\mu)(1+\alpha)-\gamma)\, a^\gamma} + \frac{c_5}{a^{3(1+\mu)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{19}$$

where $c_5 > 0$ is an integration constant and $3(1+\mu)(1+\alpha) > \gamma$, for positivity of first term. Here γ must be positive, because otherwise, $a \to \infty$ implies $\rho \to \infty$, which is not the case for expanding Universe.

Thus the metric (16) together with equations (15), (17) and (19) constitutes Bianchi type-V variable modified Chaplygin gas cosmological model in Brans-Dicke scalar-tensor theory of gravitation.

(I). Radiating era: For small values of scale factors A and B from equation (19), we get

$$\rho \approx \frac{c_5^{\frac{1}{1+\alpha}}}{a^{3(1+\mu)}} \tag{20}$$

which is very large and corresponds to the Universe dominated by an equation of state $p = \mu\rho$, i.e. for $\mu = \frac{1}{3}$ radiation dominated Universe.

From equations (14) and (20), we get the energy density as

$$\rho \approx \frac{c_5^{\frac{1}{1+\alpha}}}{(c_3 t + c_4)^{\frac{c_1(k+2)(1+\mu)}{c_3}}}. \tag{21}$$

For $\mu = \frac{1}{3}$, we get the pressure as

$$p \approx \frac{c_5^{\frac{1}{1+\alpha}}}{3(c_3 t + c_4)^{\frac{c_1(k+2)(1+\mu)}{c_3}}}. \tag{22}$$

The volume and average scale factor of the model are given by

$$V = (c_3 t + c_4)^{\frac{c_1(k+2)}{c_3}} \tag{23}$$

$$a = (c_3 t + c_4)^{\frac{c_1(k+2)}{3c_3}}. \tag{24}$$

The directional Hubble's parameters and average Hubble's parameter are respectively given by

$$H_x = \frac{kc_1}{c_3 t + c_4} \tag{25}$$

$$H_y = H_z = \frac{c_1}{c_3 t + c_4} \tag{26}$$

$$H = \frac{(k+2)c_1}{3(c_3 t + c_4)}. \tag{27}$$

The expansion scalar (θ), shear scalar (σ^2) and deceleration parameter (q) are given by

$$\theta = \frac{(k+2)c_1}{(c_3 t + c_4)} \tag{28}$$

$$\sigma^2 = \frac{7(k+2)^2 c_1^2}{18(c_3 t + c_4)^2} \tag{29}$$

$$q = 2 + 3n \tag{30}$$

from equation (30) we observed that the deceleration parameter q is negative for $n < \frac{-2}{3}$, which shows that the Universe is accelerating.

The mean anisotropy parameter of expansion (A_m) is defined by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 2\left(\frac{k-1}{k+2}\right)^2. \tag{31}$$

Jerk parameter (j) is defined as a dimensionless third derivative of the scale factor a with respect to cosmic time, and is given by

$$j = \frac{\dddot{a}}{aH^3} = 18n^2 + 27n - 10 \tag{32}$$

it is observed that the above jerk parameter value overlap with the approximate value of 2.16 obtained from the three kinematical data sets: the gold sample of type Ia supernovae [52], the SNIa data from the SNLS project [53] and the X-ray galaxy cluster distance measurements [54] for $n = 0.249$ and $n = -1.749$.

We investigate stability of the models. There are numerous methods available for this purpose. Among them, we use sound speed given as

$$C_s^2 = \frac{dp}{d\rho}. \tag{33}$$

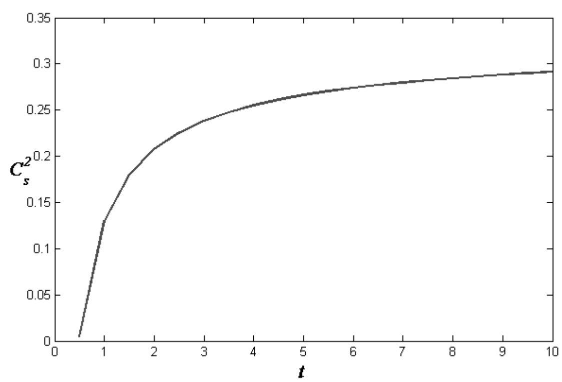

Figure 1: Plot of sound speed(C_s^2) versus t for $\alpha = 0.5$, $k = 2$, $\mu = 1/3$, $c_5 = 10$, $\gamma = 2$, $\beta_0 = 1$ and $n = -0.6$.

For a model to be physically acceptable, $C_s^2 \le 1$. From equations (17), (19) and (23) we obtain C_s^2 and plotted in terms of time t in figure 1. From figure 1 it is observed that $0 \le C_s^2 \le 1$ so, we can say that variable modified Chaplygin gas model of Universe within the framework of Brans-Dicke theory of gravitation is completely stable.

The look-back time, $\triangle t = t_0 - t(z)$ is the difference between the age of the Universe at present time (z=0) and the age of the Universe when a particular light ray at redshift z, the expansion scalar of the Universe $a(t_z)$ is related to a_0 by $1+z=\frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (24), we get

$$\triangle t = \frac{c_1(k+2)H_0^{-1}}{3c_3}\left(1-(1+z)^{\frac{-3c_3}{c_1(k+2)}}\right) \tag{34}$$

as $z \to \infty$ in equation (34), we get age of the Universe as $t_0 = \frac{c_1(k+2)}{3c_3}H_0^{-1}$.

The luminosity distance (d_L) of a light source is derived as the ratio of the detected energy flux, L and the apparent luminosity l_* i.e., $d_L^2 = \frac{L}{4\pi l_*}$. It can be written in terms of redshift(z) as (Rudhra [55])

$$d_L = a_0(1+z)r_1(z) \tag{35}$$

where $r_1(z)$ is the radial coordinate distance of the object at light emission and is given by

$$r_1(z) = \int_t^{t_0}\frac{dt}{a} = \frac{a_0^{-1}H_0^{-1}c_1(k+2)}{3c_3-c_1(k+2)}\left(1-(1+z)^{-1+\frac{3c_3}{c_1(k+2)}}\right) \tag{36}$$

Using equations (35) and (36), we get

$$d_L = \frac{H_0^{-1}c_1(k+2)(1+z)}{3c_3-c_1(k+2)}\left(1-(1+z)^{-1+\frac{3c_3}{c_1(k+2)}}\right) \tag{37}$$

(II). Quintessence era: For large values of scale factors A and B from equation (19), we get

$$\rho \approxeq \left(\frac{3\beta_0(1+\alpha)}{3(1+\mu)(1+\alpha)-\gamma}\right)^{\frac{1}{\alpha+1}} a^{\frac{-\gamma}{1+\alpha}} \tag{38}$$

$$p \approxeq \left(-1+\frac{\gamma}{3(1+\alpha)}\right)\rho \tag{39}$$

which correspond to quintessence model (i.e., dark energy with constant equation of state) for $\left(-1+\frac{\gamma}{3(1+\alpha)}\right) < \frac{-1}{3}$. So in the variable modified Chaplygin gas scenario, the equation (19) interpolates between a radiation dominated phase ($\mu = 1/3$) and quintessence dominated phase $\left(\left(-1+\frac{\gamma}{3(1+\alpha)}\right) < \frac{-1}{3}\right)$.

From equations (14), (38) and (39), we get the energy density and pressure as

$$\rho \approxeq \left(\frac{3\beta_0(1+\alpha)}{3(1+\mu)(1+\alpha)-\gamma}\right)^{\frac{1}{\alpha+1}} (c_3t+c_4)^{\frac{-\gamma c_1(k+2)}{3(1+\alpha)c_3}} \tag{40}$$

$$p \approxeq \left(-1+\frac{\gamma}{3(1+\alpha)}\right)\left(\frac{3\beta_0(1+\alpha)}{3(1+\mu)(1+\alpha)-\gamma}\right)^{\frac{1}{\alpha+1}} (c_3t+c_4)^{\frac{-\gamma c_1(k+2)}{3(1+\alpha)c_3}}. \tag{41}$$

It is observed that the remaining properties in this case are same as in radiating era.

If $\gamma = 0$, equation (39) gives $p = -\rho$, which corresponds to the original modified Chaplygin gas scenario (Debnath et al. [17]), in which the modified Chaplygin gas behaves initially radiation and later as a cosmological constant. From equations (40) and (41) for $\gamma = 0$, we get

$$\rho = \left(\frac{\beta}{1+\mu}\right)^{\frac{1}{1+\alpha}} \tag{42}$$

$$p = -\left(\frac{\beta}{1+\mu}\right)^{\frac{1}{1+\alpha}} \tag{43}$$

which means that the pressure and energy density are remains constant.

4 Discussion and Conclusions

In this paper, we have investigated spatially homogeneous and anisotropic LRS Bianchi type-V Universe with variable modified Chaplygin gas in the scalar-tensor theory of gravitation formulated by Brans and Dicke [1]. The equation of state of this cosmological model is valid from the radiation era to the quintessence model. In this paper a detailed description of the Universe has been given from radiation era ($\mu = \frac{1}{3}$ & ρ is very large) to quintessence era (ρ is very small). As compared to Chaplygin gas models, this model describes the Universe to a large extent.

In case of small values of scale factors we obtained radiating model of the Universe. The spatial volume is vanishes at $t = \frac{-c_4}{c_3}$, and the model has initial singularity of point type at $t = \frac{-c_4}{c_3}$ for $n > -1$. At this initial singularity the physical parameters $p, \rho, \theta, \sigma^2$ and H tend to infinity whereas these parameters tend to zero as $t \to \infty$. Thus the rate of expansion slows down with increase in time, also the model would essentially give an empty Universe for large time t. The model is anisotropic throughout the evolution of Universe for $k \neq 1$. Thus, it is concluded from these observations that the model starts its expansion with zero volume at $t = \frac{-c_4}{c_3}$ and it continues to expand up to infinitely large volume for $n > -1$. The derived model represents an acceleration Universe which is in good agreement with recent observations. The cosmic jerk parameter in our model is also found to be in good agreement with the recent data of astrophysical observations. We have also obtained and presented the expressions for look back time and luminosity distance versus red shift (z). For large values of scale factors the physical quantities p, ρ are satisfies the quintessence EoS parameter (i.e., $p = \omega\rho$ where $\omega < \frac{-1}{3}$) and other parameters are same as in case of small values of the scale factors. If $\gamma = 0$, one can get original modified Chaplygin gas with EoS parameter varying from radiating era to cosmological constant era. For our model speed of the sound (C_s^2) is in the range $[0, 1]$, therefore, we conclude that variable modified Chaplygin gas model of the Universe within the framework of Brans-Dicke theory are completely stable.

Finally, the solutions presented in this paper are more general and may be one of the potential candidates to describe the observed Universe.

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