# METHODS \& DESIGNS Luminance profiles demonstrate nonlinearities of brightness perception 

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#### Abstract

A projector and cylindrical lens are used for displaying gratings and for simple demonstrations of Weber's law, Fechner's law, simultaneous contrast, and Fourier synthesis effects in vision.


It is well known that perceived brightness is not a linear function of physical intensity. For instance, Fechner's law states that, other things being equal, brightness is a logarithmic function of luminance. Furthermore, the geometry of the stimulus alters brightness perception in luminance gratings and Mach band phenomena, as well as in simultaneous contrast, which is affected by the luminance of neighboring areas. In addition, the spatial luminance profile of a field can greatly influence certain aftereffects produced by timevarying its luminance (Anstis, 1967).

All these phenomena can be demonstrated with suitable luminance profiles and an overhead projector, using a technique described by Anstis and Comerford (1975). The desired luminance profile is cut as an aperture out of opaque paper and laid on the horizontal bed of an overhead projector, so that it casts a sharply focused image on the projection screen. (Alternatively, the desired pattern can be made into a $35-\mathrm{mm}$ slide and projected.) The projected image is "smeared" vertically by covering the projection lens with one or two thicknesses of horizontally ribbed or beaded Plexiglas (Edmund lenticular screen No. 80130, price about \$3; also obtainable from Lamac International, 12 West 18th Street, New York, New York 10011). A sheei of this material is really a Fresnel cylindrical lens, being embossed with tiny hemicylindrical ribs, each .5 mm wide (like the material used on 3-D postcards). It acts like an optician's "Maddox rod"; a point source of light, such as a small flashlamp filament, viewed through the lens is turned into a long vertical streak. If this material is unobtainable, a powerful astigmatizing cylindrical lens can be used instead (e.g., Edmund cylinder lens No. 30240, price about $\$ 4.50$ ). This astigmatizing lens or Plexiglas is placed in the light path just in front of the projection lens. It converts the vertical height of any aperture on the projector bed into a zone of appropriate luminance. For example, the vertical slots of Figure 2, when laid on the bed and astigmatized, are converted

[^0]into a set of vertical bars, all of which stretch the full height of the projection screen and beyond, and have a luminance exactly proportional to the slot height: the long slots producing bright bars, the short slots dim ones. Thus, the geometrical pattern of the opening is transformed into a pattern of light density. The slots can conveniently be cut in heavy graph paper, and a subject can vary the brightness of each bar independently by uncovering or covering various amounts of each slot with pieces of card. To insure accurate luminance values, the projection screen should be shielded from any ambient light scattered directly forward from the projector bed; the stimulus slots should be confined to the central areas of the bed, since some makes of projector suffer from a fall-off in picture brightness (vignetting) toward the edges of the field.

The reader can simply look at the printed figures through a piece of astigmatizing sheet, but results are far better with a projector. A technique similar to the one described here has been used for some years at UCLA by Jim Thomas (Note 1). Other methods of generating luminance profiles have been reviewed by Ratliff (1965).

Weber's law is an example of nonlinearity in brightness perception. It states that a just noticeable difference in brightness is a fixed percentage of the background luminance, or $\mathrm{dI} / \mathrm{I}=$ constant. A transparency of Figure 1 is laid on the projector bed. First, the triangular aperture (Figure 1) is kept covered with a piece of card on the bed, and the row of rectangular spots projects dim bars, each of luminance dI, which are all equally visible when viewed on their own. But when the triangular aperture is uncovered, the astigmatizing lens acts as a near-perfect optical adder and masks the bars with a background luminance wedge, whose intensity I is graded from dark to light. Near the lefthand end of the wedge, the background $I$ is low, and the incremental bars dI are clearly visible; near the righthand end, the background $I$ is high and the value of $\mathrm{d} / / \mathrm{I}$ is low, so the incremental bars fall to near threshold or below.

Fechner's law, obtained by integrating Weber's law, states that just noticeable differences represent equal steps of brightness sensation and, hence, that


Figure 1. Weber's law: dI/I = constant. The small rectangular spots at the top produce dim vertical bars when astigmatized, all of the same low luminance dI. The triangle adds a luminance wedge which is progressively brighter (I increasing) toward the right, where it tends to mask the dim bars ( $\mathrm{dI} / \mathrm{I}$ is small), driving them below threshold.
sensation $=k \log \mathrm{I}$. This logarithmic relationship between physical intensity and perceived brightness can be shown by a bisection method (Woodworth and Schlosberg, 1954, Chapter 9). Five vertical slots, each .5 cm wide, are laid on the projector bed (see Figure 2). Slot 1 is .5 cm high, giving a dim vertical bar on the screen, and Slot 5 is 8 cm high, giving a bright bar. First, Slots 2 and 4 are kept covered, and the subject adjusts the height of Slot 3 (by covering part of it with a card) until it projects a bar apparently halfway in brightness between Bars 1 and 5. He then makes further bisection judgments by setting Bars 2 and 4 to lie apparently halfway in brightness between 1 and 3 and 3 and 5, respectively. The mean settings for a group of subjects should converge on the heights shown in Figure 2, namely, $.5,1,2,4,8 \mathrm{~cm}$. These form a logarithmic series, in accordance with Fechner's law, in which each bar is twice the height of its neighbor. In comparison, a row of slots that are actually graded linearly in height (right segment of Figure 2) does not look graded linearly in brightness.when projected through the astigmatizing lens. The intermediate bars $(2,3,4)$ all look much too bright.

Simultaneous brightness contrast can be demonstra-
ted with Figure 3. This consists of six vertical slots of equal height which are astigmatized into long vertical bars, all of equal luminance. However, they are surrounded by a triangular wedge that is astigmatized into a spatial luminance wedge surround, graded from dark on the left to bright on the right. Simultaneous contrast makes the bars appear to be graded in subjective brightness, from bright on the left to dark on the right. Figure 3 resembles the Ponzo illusion, in that the sloping lines in the triangular background make the bars on the right look shorter than the bars on the left. This "height contrast" is transformed by astigmatizing into brightness contrast. Figure 4 shows how contrast can be measured by a matching method. A test bar (upper right slot) of fixed luminance is surrounded by a background (lower right) whose luminance is increased in steps on successive trials. It does not matter that the background slots are lower than the test bar slot, because the astigmatizing process acts as an optical adder, spreading the surround upward and the test bar downward and bringing them into effective alignment. For each background luminance, the subject estimates the apparent brightness of the test bar by setting a comparison bar (upper left slot) to the same apparent brightness. This procedure could be useful for student laboratory classes.

Simultaneous brightness contrast can also be shown with a luminance profile shaped like a staircase (Figure 5). Each bar or stair is of uniform luminance, but it appears darker near its left-hand edge, where it abuts a brighter neighbor, than near its right-hand edge, where it abuts a dimmer neighbor.

Further ways in which stimulus geometry can affect


Figure 2. Fechner's law: sensation $=\mathbf{k} \log \mathrm{I}$. The five slots on the left represent a logarithmic series, each being twice as tall as its left-hand neighbor. When astigmatized, the bars form a series of logarithmically increasing luminances, but they have linearly increasing subjective brightnesses. The five slots on the right increase linearly in luminance but nonlinearly in subjective brightness. Bars 2, 3, 4 look too bright.


Figure 3. The six slots are of equal height and, when astigmatized, give long vertical bars, all of the same luminance. The triangle, when astigmatized, adds a background of graded luminance, from dark on the left to bright on the right. Simultaneous contrast against the background makes the bars appear to be of different brightnesses, with the bar on the left looking brightest and the bar on the right looking darkest.
the perception of brightness can be demonstrated with luminance gratings. Each of the computer-generated stimuli in Figure 6 produces, when astigmatized, a different kind of grating with vertical bars. Each pattern took $20-30$ min to draw on a CalComp plotter and $8-15 \mathrm{~h}$ to blacken in by hand. Figure 6 a produces a simple grating of vertical bars, of contrast .7. This pattern consists of a set of horizontal slots: Each slot is sinusoidally modulated in width and produces a grating with a sinusoidal luminance profile. One slot would produce a grating, but the presence of many replicated slots makes the grating much brighter. Figure 6 b produces a grating whose spatial frequency is logarithmically swept over a $1: 20$ frequency range. Figure 6 c produces a grating with a logarithmic sawtooth luminance profile.

Figure 6d shows the Fourier components of a square wave, up to the 59th harmonic. These consist of the odd harmonics of a fundamental sine wave with frequencies $\mathrm{f}, 3 \mathrm{f}, 5 \mathrm{f}, 7 \mathrm{f}, \ldots 57 \mathrm{f}, 59 \mathrm{f}$, and relative amplitudes 1 , $1 / 3,1 / 5,1 / 7, \ldots 1 / 57,1 / 59$. Each sine wave alone would
produce a separate grating, each of a different frequency, and the astigmatizing lens adds them all together optically to produce a single square-wave grating.

Figure 6e shows the Fourier components of a sawtooth wave, up to the 60th harmonic. These consist of the first 60 harmonics of a fundamental sine wave with frequencies $\mathrm{f}, 2 \mathrm{f}, 3 \mathrm{f}, 4 \mathrm{f}, \ldots 59 \mathrm{f}, 60 \mathrm{f}$ and relative amplitudes $1,1 / 2,1 / 3,1 / 4, \ldots 1 / 59,1 / 60$. When astigmatized, these add together to form a single grating with a sawtooth luminance profile.

It is often convenient to project these patterns as $35-\mathrm{mm}$ slides. However, it is more versatile to make them into transparencies for an overhead projector, since the individual components can then be manipulated independently. For instance, a variable contrast grating can be generated by printing the alternate black wavy-edged horizontal bars in Figure 6a on two separate transparencies, superimposing them on the bed of an overhead projector, and then moving the transparencies slowly in opposite directions, so that the even-numbered black horizontal bars move to the left and the oddnumbered bars move to the right, as indicated by the arrows in Figure 6a. In effect, the two transparencies produce two superimposed identical gratings of vertical bars which are optically summed or added. As they are moved gradually from an in-phase to an out-of-phase spatial position, they add together to produce a single sinusoidal grating whose contrast falls gradually from a maximum to near zero (Anstis \& Comerford, 1975).


Figure 4. Measurement of simultaneous contrast. The standard slot 4 cm high (top right) produces a bright bar, which looks apparently dimmer because of surround (bottom right), that is preset to different values, in this case, 4.5 cm high. The subject adjusts the height of the comparison slot (top right) until its apparent bright tness matches standard. In this case, he sets it to about 1.8 cm high.


Figure 5. Simultaneous contrast. When astigmatized, each step of the luminance "staircase" looks apparently dimmer at its left edge, where it abuts a brighter neighbor, than at its right edge, where it abuts a dimmer neighbor.

If a transparency of Figure $6 e$ is made, the individual components of the Fourier sawtooth series can be manipulated. A mask of horizontal black strips which block out the even harmonics will pass only the odd harmonics and generate a square-wave grating identical to Figure 6 c . The photographic negative of such a mask will block out the odd harmonics and pass only the even harmonics of frequencies $2 \mathrm{f}, 4 \mathrm{f}, 6 \mathrm{f}, 8 \mathrm{f}, \ldots$. Note that this is mathematically identical to $2 \mathrm{x}(\mathrm{f}, 2 \mathrm{f}, 3 \mathrm{f}, 4 \mathrm{f}, \ldots$ ). In other words, the even harmonics of a sawtooth grating, on their own, will produce a second sawtooth grating of twice the spatial frequency (and half the amplitude). Figure $6 f$ shows graphically that a sawtooth waveform is equal to the sum of a square wave (odd harmonics) and a doubled-frequency sawtooth wave (even harmonics).

The human visual system is relatively insensitive to gratings with sinusoidal luminance profiles if they have a spatial frequency below about 1 cycle/deg of visual angle. Campbell, Howell, and Robson (1971) have demonstrated this by showing that low-amplitude lowfrequency square-wave gratings are seen only because
they contain harmonic spatial components. Their fundamental frequency sine wave can be optically subtracted without altering their appearance, since the sine-wave grating is below threshold. This can be demonstrated with Figure 7.

Richards (1968) has described an "illusory reversal of brightness contrast." When spun on a disk, or projected and astigmatized, his luminance profile (Figure 8) gives a ring or band whose borders are formed by two spikes. The ring or band appears darker than the rest on close viewing (at large visual angles) but brighter when viewed from further away (at small visual angles). Richards points out that this profile is really a low-frequency pattern having one contrast relationship superimposed on a high-frequency pattern with the reverse contrast. It is really two Cornsweet illusions placed back to back (Cornsweet, 1970). Viewed from a distance, only the lower spatial frequencies are observed, and the band appears as light on a dark gray. Viewed from close up, the high-frequency spikes are seen and the lower frequency gradient is obscured, so the band then looks dark. The two frequency components are displayed separately on the right in Figure 8. The astigmatizing lens adds them together so that the left and right segments of Figure 8 give identical patterns when projected.

The overhead projector method has various advantages over existing methods of generating luminance profiles. If only simple sinusoidal gratings are required, the projection method has no particular advantages, apart from increased luminance, over the conventional technique of modulating the brightness of a television raster on an oscilloscope (Campbell \& Green, 1965; Schade, 1956). But it comes into its own for patterns of complex waveforms made up of many sine waves, such as Figure 6d, which would require many generators and create considerable problems in triggering an oscilloscope. It is also good for complicated luminance profiles, which are hard to generate electronically but can easily be made with paper and scissors. The overhead projector method is also more convenient than a rotating color wheel because it requires no moving parts, and patterns can be adjusted in real time on the projector bed while being viewed on the screen. It is useful for simultaneous contrast experiments where the luminance of several patches of light may need to be adjusted independently. There is no need for a separate projector, dimmer, and screen for each patch of light; instead, the luminance of each area can be adjusted with a moveable piece of card and the luminance read off a calibrated graph-paper scale on the projector bed. Finally, the overhead projector method is helpful in teaching undergraduates because they can see directly what the luminance profile is by looking at the projector bed or by removing the astigmatizing lens and looking at the screen.















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Figure 6a. This computer-plotted stimulus produces a grating with sinusoidal luminance profile and contrast of .7. If alternate horizontal black zones are moved to left and right, as indicated by arrows at right of figure, the contrast can be varied (see text)





Figure 6d. This computer-plotted stimulus produces Fourier components of a square wave (odd harmonics from the 1 st to the 59 th).


Figure 6 e . This computer-plotted stimulus produces Fourier components of a sawtooth wave (all harmonics, from the 1 st to the 60th). When astigmatized, this gives a grating with a sawtooth luminance profile. If covered with Mask 1 , only the odd harmonics are passed, which gives the same square wave as Figure 6d. If covered with Mask 2, only the even harmonics are passed, which gives a sawtooth grating of twice the spatial frequency.


Figure 6f. A sawtooth is composed of a square wave plus a double-frequency saw tooth.


Figure 7. Upper edge generates a visible low-contrast squarewave grating. Lower edge generates its fundamental Fourier component, a sine wave of the same frequency, which is below threshold (invisible) owing to poor visual sensitivity to lowfrequency sine waves. Thus, square-wave grating is seen in left segment, nothing is visible in right segment. In center segment, sine wave is in antiphase and therefore subtracts from the square wave: But this makes no difference to the appearance of the square-wave grating, which is still visible (Campbell, Howell, \& Robson, 1971).


Figure 8. Illusory contrast reversal (Richards, 1968). On left segment, luminance profile looks like a dark band on close viewing, but looks like a bright band when viewed from further away. Right segment, the profile consists of a high-frequency pattern somewhat like a square wave (bottom edge), with a lowfrequency pattern somewhat like a sine wave (upper edge) subtracted from it.

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