Lumpy Price Adjustments: A Microeconometric Analysis

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Abstract

This paper presents a simple model of state-dependent pricing that allows identification of the relative importance of the degree of price rigidity that is inherent to the price setting mechanism (intrinsic) and that which is due to the price's driving variables (extrinsic). Using two data sets consisting of a large fraction of the price quotes used to compute the Belgian and French CPI, we are able to assess the role of intrinsic and extrinsic price stickiness in explaining the occurrence and magnitude of price changes at the outlet level. We find that infrequent price changes are not necessarily associated with large adjustment costs. Indeed, extrinsic rigidity appears to be significant in many cases. We also find that asymmetry in the price adjustment could be due to trends in marginal costs and/or desired mark-ups rather than asymmetric cost of adjustment bands.

JEL Classifications: *C51, C81, D21.*

Keywords: Sticky prices, nominal intrinsic and extrinsic rigidities, micro non-linear panels

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1 Introduction

Following the seminal contributions of Cecchetti (1986) on newspaper prices, Kashyap (1995) on catalog prices (both using US data), and Lach and Tsiddon (1992) on meat and wine prices in Israel, a recent wave of empirical research has provided new evidence on consumer and producer price stickiness at the micro level. Bils and Klenow (2004) and Klenow and Kryvstov (2005) provide studies on consumer prices in the US and Dhyne *et al.* (2006) give a synthesis of the recent studies carried out for the euro area countries. Studies of producer prices include Alvarez *et al.* (2006), Cornille and Dossche (2006), Loupias, Heckel and Sevestre (2007) and Sabbatini *et al.* (2005) among many others.

One of the main conclusions of these studies is the existence of a significant degree of heterogeneity in the degree of price flexibility across different product categories. Some products are characterized by a high frequency of price changes, with outlets reseting their prices almost on a continuous basis (for instance, oil products and perishable goods), whilst other product categories are characterized by a very low frequency of price changes (for instance, in some services). Aucremanne and Dhyne (2004) also document a high degree of heterogeneity in the duration of price spells (and hence in the frequency of price changes) even within relatively homogeneous product categories. Indeed, several studies have shown that the frequency of consumer price changes not only differs across product categories, but also varies across categories of retailers.[1] Hyper and super-markets also tend to change their prices more frequently than local corner shops.

These studies are, however, silent as to the reasons for such infrequent price changes. A low frequency of price change has sometimes been taken as evidence of nominal or intrinsic price rigidity, namely price rigidity that is inherent to the price-setting mechanism. This ignores the role of extrinsic rigidity in price stickiness, namely the type of price rigidity that is induced by a low degree of volatility of either common or idiosyncratic shocks to the marginal cost and/or the desired mark-up.[2] Indeed, infrequent price changes are not necessarily due to high cots of price adjustments (i.e. nominal or intrinsic rigidities). When marginal costs and other market conditions do not vary, firms have little or no incentive to change their prices. In this paper, we aim at identifying the respective contributions of intrinsic and extrinsic rigidities to the observed price stickiness. For that purpose, we develop a state dependent price-setting model, close in spirit to Cec-

[1] See Baudry *et al.* (2007), Fougère, Le Bihan and Sevestre (2007), Jonker, Blijenberg and Folkertsma (2004), and Veronese *et al.* (2005).

[2] Here we are adopting a terminology used in Altissimo, Erhmann and Smets (2006) to characterize the different sources of inflation persistence.

chetti (1986), that relates price changes to the variations in an unobserved optimal price reflecting common and idiosyncratic movements in marginal costs and/or in the *desired* mark-up, but where price changes are subject to price adjustment costs.[3] Compared to the existing literature, we argue and show that the frequency of price changes may be a poor indicator of intrinsic price rigidities. Our estimates reveal that the scarcity of price changes for some services in particular originates essentially from extrinsic rigidities rather than from high intrinsic rigidities.

The outline of the rest of the paper is as follows. We first present the theoretical model in Section 2. We then discuss the estimation procedure in Section 3. Section 4 describes the micro price data sets used and presents the estimation results. Section 5 concludes.

2 A Canonical Model of Sticky Prices

It is now a well-established stylized fact that most consumer prices remain unchanged for periods that can last several months (e.g. see Bils and Klenow, 2004, Dhyne *et al.,* 2006, among many others). Indeed, for a number of reasons (physical menu costs, fear of consumer anger, etc.), retailers may be reluctant to immediately adjust their prices to changes in their environment (costs increases/decreases, demand variations, changes in local competition, etc.). Such a behavior can be modelled assuming fixed price adjustment costs that do not depend on the size of the price change,[4] leading to an optimal price strategy of the (s, S) variety. See, for example, Sheshinski and Weiss (1977, 1983), Cecchetti (1986), and Gertler and Leahy (2006).

A simple representation of this behavior can be written as:

$$p_{it}^{(j)} = \begin{cases} p_{i,t-1}^{(j)} & \text{if } \left| p_{it}^{(j)*} - p_{i,t-1}^{(j)} \right| \leq c_{it}^{(j)}, \\ p_{it}^{(j)*} & \text{if } \left| p_{it}^{(j)*} - p_{i,t-1}^{(j)} \right| > c_{it}^{(j)}, \end{cases} \tag{1}$$

where $p_{it}^{(j)}$ is the (log) observed price of a product j in outlet i at time t, $p_{it}^{(j)*}$ is the (log) optimal price that would be set in the absence of any adjustment costs, and $c_{it}^{(j)}$ denotes

[3] The use of state dependent price-setting rules by firms seem to be supported by surveys. Indeed, Fabiani *et al.* (2005) report that in the euro area 66% of firms consider pure or mixed state dependent pricing rules in order to decide when to change their prices.

[4] Several papers have found evidence of fixed physical menu costs of price adjustment (Levy *et al.*, 1997, Zbaracki *et al.*, 2004). However, Zbaracki *et al.* (2004) argue that, in addition to these fixed physical menu costs, managerial and customers costs are convex in the price change, while survey responses discussed in Blinder *et al.* (1998) suggest that price adjustment costs might be fixed.

the thresholds beyond which outlets find it profitable to adjust their prices in response to a shock, i.e. the extent to which price changes are costly; $c_{it}^{(j)}$ essentially represents the costs incurred by the outlet when changing its price.[5] In what follows to simplify the notation we drop the superscript j and continue to refer to c_{it} as the adjustment cost, although it is clear that c_{it} goes beyond physical menu cost (see below), and represents all types of costs associated with the price change by outlet i in period t. We shall also refer to the condition

$$|p_{it}^{*} - p_{i,t-1}| \geq c_{it}, \tag{2}$$

as the 'price change trigger' condition. The magnitude of c_{it} characterizes the extent of intrinsic price rigidity. The larger it is, the lower the likelihood of a price change in response to a given shock.

This model is very close in spirit to the econometric model proposed by Rosett (1959) for the analysis of frictions in yield changes. However, we depart from Rosett's model in that, in our model, the adjustment threshold, c_{it}, only affects the decision to change prices but not the level of the newly set prices, p_{it}^{*}. Indeed, we consider that when firms decide to adjust their prices, they fully adjust to the optimal price while in Rosett's model, agents are assumed to reduce the magnitude of their effective adjustment by the amount of the adjustment cost they incur. Denoting by $I(A)$ an indicator function that takes the value of unity if $A > 0$ and zero otherwise, model (1), can be written as:

$$\begin{aligned} p_{it} &= p_{i,t-1} + (p_{it}^{*} - p_{i,t-1})I(p_{it}^{*} - p_{i,t-1} - c_{it}) \\ &\quad + (p_{it}^{*} - p_{i,t-1})I(p_{i,t-1} - p_{it}^{*} - c_{it}). \end{aligned} \tag{3}$$

This formulation is reasonably general and allows the adjustment cost to vary both over time and across outlets. Assuming constant and identical adjustment costs might be considered as a too strong assumption since, as documented in Aucremanne and Dhyne (2004) and Fougère, Le Bihan and Sevestre (2007) among others, price setting can be strongly heterogeneous across outlets, even within relatively homogeneous product categories. At the outlet level, some price trajectories may be characterized by very frequent price changes, while others may be characterized by infrequent price changes. Moreover, as described in Campbell and Eden (2005), some price trajectories at the micro level exhibit long periods of price stability followed by periods of frenetic price changes. As noted by Caballero and Engel (2006), this pattern of price changes suggests that c_{it} is

[5] For the sake of simplifying notations, we will not, in the sequel, use anymore the index j for products since we estimate this model for each product separately.

best modelled as a stochastic process. Another argument for adopting such an approach lies in the synchronization of price changes within stores. Midrigan (2006) documents that a lot of price changes are particularly small compared to the average magnitude of price changes.[6] Following Lach and Tsiddon (2005), he rationalizes these small price changes by the existence of economies of scales in price changes for multi-product sellers. This may be accounted for by allowing for some variability in adjustment costs.

As mentioned above, c_{it} is only partly determined by the narrow traditional notion of menu costs (the cost of changing posted prices, including managerial and decision costs), but it is also intended to reflect a broader notion of costs of price adjustments. For instance, the magnitude of c_{it} may reflect the specific marketing policy of outlets, regarding sales or promotions. They may also capture the degree of customers anger against price changes, as in Rotemberg (2003). If a firm fears to lose a significant fraction of its customers when it changes its prices, it will keep its prices constant as long as the expected loss induced by a non optimal price is smaller than the expected loss associated with customers anger. Interpreting the adjustment costs as a proxy of the importance of customer relationship instead of traditional price adjustment costs is supported by surveys on price setting behavior. Indeed, Fabiani *et al.* (2005) for the euro area, Aucremanne and Druant (2005) for Belgium or Loupias and Ricart (2006) for France, on the basis of surveys about firms' price setting behavior, indicate that a major source of price stickiness lies in customer relationships (existence of implicit or explicit contracts), while physical menu costs are not considered as a major source of nominal rigidity.[7]

Now, the question arises as whether we can also identify extrinsic rigidities, i.e. those corresponding to the low variability of the fundamentals underlying prices such as changes in marginal costs caused by input price variations or demand variations, changes in the mark-up caused by varying market competition, etc. Consider that, for a given product line, retailer i that operates on a market characterized by imperfect competition sets optimally its price at its marginal cost, MC_{it}, augmented by its desired mark-up rate (MU_{it}):

$$P^*_{it} = MC_{it} \times (1 + MU_{it}).$$

[6] Using US data, Midrigan (2006) indicates that 30% of the observed price changes are smaller than half of the average absolute size of price changes. For Belgium, 34% of the observed price changes fullfill a similar condition. This proportion is close to 50% in France.

[7] Although these studies relate to producer prices, one can expect these particular observations to be also relevant for consumer prices.

Using logarithms, the (log) optimal price may be written as:

$$p_{it}^{*} = mc_{it} + \mu_{it}.$$

Unfortunately, despite their size and coverage, the data sets available on consumer prices do not provide any information on costs and demand conditions faced by outlets. In spite of this, it is possible, as we shall show below, to extract information on the probability distribution of p_{it}^{*}, using a non-linear unobserved common factor model. To this end, we have decomposed the (unobserved) optimal price p_{it}^{*} as follows:

$$p_{it}^{*} = f_t + \mathbf{x}_{it}'\boldsymbol{\beta} + v_i + \varepsilon_{it}, \tag{4}$$

where f_t represents the unobserved common component of p_{it}^{*}, $\mathbf{x}_{it}$ is a vector of observable retail-specific variables, v_i are retail-specific time-invariant unobservable effects, while ε_{it} accounts for firm-specific idiosyncratic shocks. The common component, f_t, can be viewed as the (log) producer price paid by all outlets, apart from a scaling constant. The remaining terms in (4) are intended to capture the differences in marginal costs and mark-ups across the outlets. The above decomposition also allows us to distinguish between extrinsic and intrinsic sources of price rigidities. Changes in the marginal costs as well as other changes in the market conditions (competition, demand variations) that are common to *all outlets,* as reflected in f_t, can be viewed, as a first source of extrinsic rigidity.

The variables $\mathbf{x}_{it}$ are introduced to control for the possible effects of store type (such as hyper or supermarket versus corner shop) or geographical location (city centre or suburbs), and other observable characteristics on price setting behavior of the outlets. The retail-specific unobservable effects, v_i, account for the heterogeneity in the level of observed prices at the product category level that cannot be traced to observables (product differentiation and/or the ability of retailer i to consistently price above or below the common component f_t, e.g. because of local competitive conditions). ε_{it} accounts for idiosyncratic shocks to marginal costs and/or to the *desired* mark-up that depend on some particular factors such as specific changes in (local) competition conditions, rebates on the wholesale price obtained by large retailer chains, management quality, quality of customer relations. This component also includes outlet specific seasonal patterns arising from specific sales and other forms of market promotions. The magnitude of idiosyncratic shocks, as measured by the standard deviation of ε_{it}, say σ_{ε}, is then also informative about the extent of extrinsic rigidity. For example, everything else being equal, we would expect products with low estimates of σ_ε to have also relatively low frequency of price changes. This factor may also be an important source of infrequent price changes if we consider the results reported in Fabiani *et al.* (2005), Aucremanne and Druant (2005) or Loupias and Ricart (2006). Indeed, these papers show that, in addition to customer relationship, what is considered as a major source of price rigidity by firms is the fact that their marginal costs are relatively stable. Finally, following Golosov and Lucas (2003), this idiosyncratic component might be a crucial factor in capturing the very diverse price dynamics that are observed even for relatively homogenous product categories. This point is illustrated in the price trajectories for oranges in Belgium and men's socks in France displayed in figures Figures 1.A and 1.B, respectively.

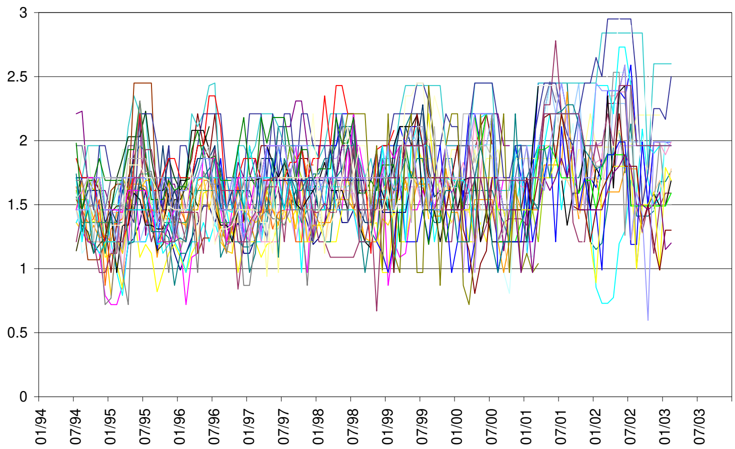

FIGURE 1.A. - 50 PRICE TRAJECTORIES - ORANGES (IN EUR/KG) - BELGIAN CPI

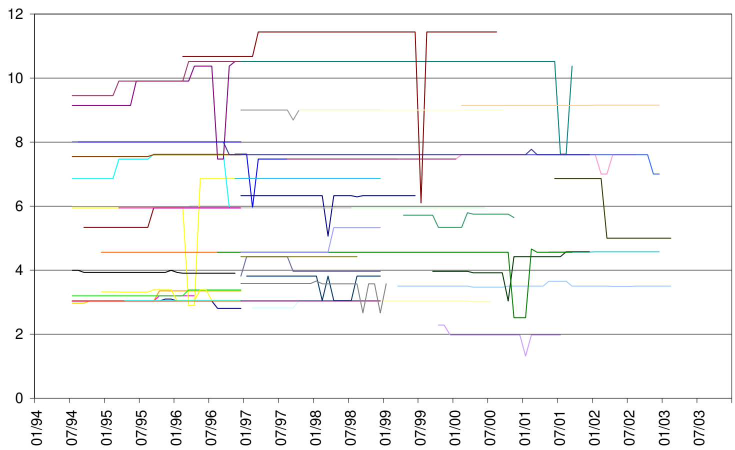

FIGURE 1.B - 50 PRICE TRAJECTORIES - MEN'S SOCKS (IN EUR) - FRENCH CPI

Although our model is relatively close to the one presented for instance in Tsiddon (1993) or Ratfai (2006), we depart from the existing empirical literature in several ways.

First, rather than using a proxy for the common component f_t (a sectoral producer price index is often used in this respect; see Ratfai, 2006), we estimate it out of the micro price data. One important advantage of proceeding in this way is to ensure the coherency of this common component with the dynamics of micro price decisions as stated by our model.

Second, we also depart from the existing empirical literature in the information used in our estimation procedure. Most of the literature estimates state-dependent pricing model using binary response or duration models (Cecchetti, 1986, Aucremanne and Dhyne, 2005, Campbell and Eden, 2006, Fougère, Le Bihan and Sevestre, 2007, Ratfai, 2006) and therefore neglects the information contained in the magnitude of price changes. However, as we show below, this information is crucial in order to identify the volatility of the idiosyncratic shocks and for disentangling the idiosyncratic shocks (to marginal costs and/or desired mark-ups) from the stochastic price adjustment costs.

2.1 Extensions to the basic model

The above model can be generalized in a number of ways. Here, we discuss two important extensions.

2.1.1 Gradual adjustment

One important extension is to allow for only a partial adjustment of prices to their optimal values. While the basic model assumes that, once the retailers decide to adjust their prices, they fully adjust to the optimal price p_{it}^{*}, retailers may possibly decide to proceed only with a partial adjustment of their prices, setting their new price p_{it} as $(1-\lambda)\, p_{it}^{*} + \lambda p_{i,t-1}$, where λ is the partial adjustment coefficient ($0 \leq \lambda < 1$). Such a partial adjustment process may be motivated on several grounds. First, uncertainty surrounding the evaluation of the size and source (common or idiosyncratic) of the shocks to the marginal cost and/or desired mark-up may lead firms to adopt a conservative attitude towards price changes. Indeed, competition on the product market may induce firms to proceed only to partial price adjustments in response to shocks, in order to keep their market shares when they do not know about their competitors' reaction. Secondly, under consumers' inattention (Levy *et al.*, 2005), it may be more profitable for outlets to perform gradual adjustments to the optimal price level rather than a single large price change. Thirdly, if the information gathering process is costly as in Mankiw and Reis (2002), some firms may consider as more profitable to base their current price decision partly on past information.

In that case, the price change trigger condition becomes:

$$\left|(1-\lambda)\, p_{it}^{*}+\lambda p_{i,t-1}-p_{i,t-1}\right|>c_{it},$$

or, equivalently,

$$(1-\lambda)\left|p_{it}^{*}-p_{i,t-1}\right|>c_{it}.$$

A non zero λ parameter introduces an additional source of rigidity due to price level persistence, and accordingly adds a backward-looking component in the model.

2.1.2 Asymmetric adjustment costs

Another natural extension of the basic model is to allow for asymmetric price adjustments, by allowing the size of the adjustment costs for downward and upward price movements to be different. This is justified in theory where firms discount future profits, or if the profit function and the distribution of shocks themselves are asymmetric. Indeed, Aucremanne and Dhyne (2004) and Baudry *et al.* (2007), among others, have highlighted that price decreases are less frequently observed than price increases, especially in the service sector. This could result from asymmetric price adjustment costs and, more specifically, from stronger downward intrinsic rigidities (as discussed in Hall and Yates, 1998, and Yates, 1998). In order to test this assumption, one can extend our basic specification and write:

$$\begin{aligned} p_{it} &= p_{i,t-1}+(p_{it}^{*}-p_{i,t-1})I(p_{it}^{*}-p_{i,t-1}-c_{Uit}) \\ &\quad +(p_{it}^{*}-p_{i,t-1})I(p_{i,t-1}-p_{it}^{*}-c_{Lit}). \end{aligned} \qquad (5)$$

If $c_{Lit}>c_{Uit}$, this model will produce more price increases than price decreases, for given values of f_t. However, it is important to stress that asymmetric thresholds do not necessarily reflect the asymmetry in strictly defined adjustment costs. Other sources of asymmetry such as the asymmetry of the profit function, of the probability distribution of shocks or the fact that firms discount future profits, all could contribute to asymmetric price adjustments. The range of inaction will then be asymmetric even if price adjustment costs are similar upwards and downwards.

It is also worth mentioning that asymmetry in the thresholds of inaction is sufficient but not necessary for generating more price increases than price decreases. Our baseline model, with $c_{Lit}=c_{Uit}=c_{it}$, will generate more price rises than price falls, so long as f_t

exhibits a positive drift, as in Ball and Mankiw (1994).

These are important extensions, but to keep the computations manageable, in the empirical section we shall focus on the symmetric case.

3 Estimation of the Model

One can combine equations (3) and (4) representing our baseline price-setting model into the following econometric representation:

$$\begin{aligned} p_{it} - p_{i,t-1} &= (f_t + \mathbf{x}_{it}'\boldsymbol{\beta} + v_i + \varepsilon_{it} - p_{i,t-1}) I(f_t + \mathbf{x}_{it}'\boldsymbol{\beta} + v_i + \varepsilon_{it} - p_{i,t-1} - c_{it}) \\ &\quad + (f_t + \mathbf{x}_{it}'\boldsymbol{\beta} + v_i + \varepsilon_{it} - p_{i,t-1}) I(p_{i,t-1} - f_t - \mathbf{x}_{it}'\boldsymbol{\beta} - v_i - \varepsilon_{it} - c_{it}). \end{aligned} \tag{6}$$

There are essentially two groups of parameters to estimate in this model. First, the unobserved common components, f_t, which can also be viewed as unobserved time effects. Second, the other structural parameters: c and σ_c which respectively denote the mean and standard deviation of c_{it}, σ_ε, the standard deviation of the idiosyncratic shocks ε_{it}, σ_v, the standard deviation of the firm specific random effect, v_i, and $\boldsymbol{\beta}$, the parameters associated with the observed explanatory variables, $\mathbf{x}_{it}$.

The estimation of the baseline model can be carried out in two ways. One can use an iterative procedure that combines the estimation of the f_t's using the cross-sectional dimension of the data with the maximum likelihood estimation of the remaining structural parameters, conditional on the first-stage estimate of f_t. Alternatively, one can use a standard maximum likelihood procedure, where the f_t's are estimated simultaneously with the other parameters. The two procedures lead to consistent estimates, provided N and T are sufficiently large. It is worthwhile noting that if N is small, one would face the well-known incidental parameters problem: the bias in estimating f_t, due to the limited size of the cross-sectional dimension, would contaminate the other parameter estimates. In the alternative situation where T happens to be small, the problem of the initial observation would then become an important issue. Therefore, our estimation procedure is essentially valid for relatively large N and T. Fortunately, in our context, prices of most of the products we consider have been observed monthly over the period 1994:7 - 2003:2 (i.e. more than 100 months), and the number of outlets selling the various products we consider are also relatively large, being only slightly less tgab 300, both in Belgium and in France.

3.1 Estimation of f_t using cross-cectional averages

As mentioned above, f_t is in practice an unobserved time effect that needs to be estimated along with the other unknown parameters. It reflects the common component in the marginal cost and *desired* mark-up for each particular product for which we estimate the model. Thanks to the very large size and high degree of disaggregation of our data, we can split our data sets according to a very detailed definition of the products while keeping, at the same time, a large number of price trajectories in the sub-samples to be analyzed.

Moreover, because we are able to consider precisely defined types of products sold in a particular outlet, it is reasonable to assume that any remaining cross-sectional heterogeneity in the price level can be modelled through the observable outlet-specific characteristics, $\mathbf{x}_{it}$, and through random specific effects (accounting for outlets unobserved characteristics). Accordingly, we assume that, conditional on $\mathbf{h}_{it} = (f_t, \mathbf{x}'_{it}, p_{i,t-1})'$, $(c_{it},\, v_i, \varepsilon_{it})'$ are distributed independently across i, and that c_{it} and ε_{it} are serially uncorrelated. Due to the non-linear nature of the pricing process and to make the analysis tractable, we shall also assume that

$$\begin{pmatrix} c_{it} \\ v_i \\ \varepsilon_{it} \end{pmatrix} |\mathbf{h}_{it} \backsim i.i.d.N\left(\begin{pmatrix} c \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 \end{pmatrix}\right).$$

The assumption of zero covariances across the errors is made for convenience and can be relaxed.

Before discussing the derivation of f_t we state the following lemma, established in the Appendix, which provides a few results needed below.

Lemma 1 Suppose that $y \backsim N(\mu, \sigma^2)$ then

$$E\left[yI(y+a)\right] = \sigma\phi\left(\frac{a+\mu}{\sigma}\right) + \mu\Phi\left(\frac{a+\mu}{\sigma}\right),$$

$$E\left[\phi\left(\frac{y+a}{b}\right)\right] = \frac{b}{\sqrt{b^2+\sigma^2}}\phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right),$$

$$E_y\left[\Phi\left(\frac{y+a}{b}\right)\right] = \Phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right),$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the density and the cumulative distribution function of the standard normal variate, and $I(A)$ is the indicator function defined above.

Let

$$d_{it} = f_t + \mathbf{x}_{it}'\boldsymbol{\beta} - p_{i,t-1}, \quad \xi_{it} = v_i + \varepsilon_{it} \backsim N(0, \sigma_\xi^2),$$

and note that $\sigma_\xi^2 = \sigma_v^2 + \sigma_\varepsilon^2$. Consider now the baseline model, (6), and using the above, write it as

$$\Delta p_{it} = (d_{it} + \xi_{it}) I(d_{it} + \xi_{it} - c_{it}) + (d_{it} + \xi_{it}) I(-d_{it} - \xi_{it} - c_{it}),$$

or

$$\Delta p_{it} = (d_{it} + \xi_{it}) + (d_{it} + \xi_{it}) \left[I(d_{it} + \xi_{it} - c_{it}) - I(d_{it} + \xi_{it} + c_{it}) \right].$$

Denote the unknown parameters of the model by $\boldsymbol{\theta} = (c, \boldsymbol{\beta}', \sigma_c^2, \sigma_v^2, \sigma_\varepsilon^2)'$, and note that

$$E\left(\Delta p_{it} \left| \mathbf{h}_{it}, \boldsymbol{\theta} \right.\right) = d_{it} + g_{it},$$

where $g_{it} = g_{1,it} + g_{2,it}$, with

$$g_{1,it} = d_{it} E\left[I(d_{it} + \xi_{it} - c_{it}) - I(d_{it} + \xi_{it} + c_{it}) \left| \mathbf{h}_{it}, \boldsymbol{\theta} \right.\right],$$

and

$$g_{2,it} = E\left[\xi_{it} I(d_{it} + \xi_{it} - c_{it}) - \xi_{it} I(d_{it} + \xi_{it} + c_{it}) \left| \mathbf{h}_{it}, \boldsymbol{\theta} \right.\right].$$

Also, under our assumptions

$$\begin{pmatrix} c_{it} \\ \xi_{it} \end{pmatrix} | \mathbf{h}_{it} \backsim i.i.d.N \left(\begin{pmatrix} c \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_v^2 + \sigma_\varepsilon^2 \end{pmatrix} \right).$$

It is easily seen that

$$\begin{aligned} & E\left[I(d_{it} + \xi_{it} - c_{it}) - I(d_{it} + \xi_{it} + c_{it}) \left| \mathbf{h}_{it}, \boldsymbol{\theta} \right.\right] \\ = \; & \Phi\left(\frac{d_{it} - c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}} \right) - \Phi\left(\frac{d_{it} + c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}} \right). \end{aligned}$$

Using the results in Lemma 3.1 and noting that $\xi_{it} \left| \mathbf{h}_{it}, \boldsymbol{\theta} \right. \backsim N(0, \sigma_\xi^2)$, then

$$E\left[\xi_{it} I(d_{it} + \xi_{it} - c_{it}) \left| \mathbf{h}_{it}, \boldsymbol{\theta}, c_{it} \right.\right] = \sigma_\xi \phi\left(\frac{d_{it} - c_{it}}{\sigma_\xi} \right).$$

Hence, taking expectations with respect to c_{it}, we have

$$E\left[\xi_{it} I(d_{it} + \xi_{it} - c_{it})\,|\mathbf{h}_{it}, \boldsymbol{\theta}\right] = \sigma_\xi E\left[\phi\left(\frac{d_{it} - c_{it}}{\sigma_\xi}\right)|\mathbf{h}_{it}, \boldsymbol{\theta}\right].$$

Again using the results in Lemma 3.1 we have

$$E\left[\phi\left(\frac{d_{it} - c_{it}}{\sigma_\xi}\right)|\mathbf{h}_{it}, \boldsymbol{\theta}\right] = \frac{\sigma_\xi}{\sqrt{\sigma_c^2 + \sigma_\xi^2}} \phi\left(\frac{d_{it} - c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right),$$

and therefore,

$$E\left[\xi_{it} I(d_{it} + \xi_{it} - c_{it})\,|\mathbf{h}_{it}, \boldsymbol{\theta}\right] = \frac{\sigma_\xi^2}{\sqrt{\sigma_c^2 + \sigma_\xi^2}} \phi\left(\frac{d_{it} - c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right).$$

Similarly,

$$E\left[\xi_{it} I(d_{it} + \xi_{it} + c_{it})\,|\mathbf{h}_{it}, \boldsymbol{\theta}\right] = \frac{\sigma_\xi^2}{\sqrt{\sigma_c^2 + \sigma_\xi^2}} \phi\left(\frac{d_{it} + c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right).$$

Collecting the various results we obtain

$$g_{1,it} = d_{it}\left[\Phi\left(\frac{d_{it} - c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right) - \Phi\left(\frac{d_{it} + c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right)\right],$$

and

$$g_{2,it} = \frac{\sigma_\xi^2}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\left[\phi\left(\frac{d_{it} - c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right) - \phi\left(\frac{d_{it} + c}{\sqrt{\sigma_c^2 + \sigma_\xi^2}}\right)\right].$$

Note that $g_{1,it}$ and $g_{2,it}$ are non-linear functions of f_t and depend on i only through the observable, $p_{i,t-1}$ and $\mathbf{x}_{it}$. It is therefore possible to compute f_t for each t in terms of $p_{i,t-1}$, $\mathbf{x}_{it}$ and $\boldsymbol{\theta}$. Then, following Pesaran (2006), the cross-sectional average estimator of f_t, denoted by $\tilde{f}_t$, can be obtained as the solution to the following non-linear equation

$$\bar{p}_t = \tilde{f}_t + \bar{\mathbf{x}}_t' \boldsymbol{\beta} + \bar{g}_t(\tilde{f}_t), \tag{7}$$

where

$$\bar{p}_t = \sum_{i=1}^{N} w_{it}\, p_{it},\ \bar{\mathbf{x}}_t = \sum_{i=1}^{N} w_{it}\, \mathbf{x}_{it},\ \text{and } \bar{g}_t(f_t) = \sum_{i=1}^{N} w_{it}\, g_{it},$$

and $\{w_{it}, i = 1, 2, .., N\}$ represent a predetermined set of weights such that

$$w_{it} = O(N^{-1}), \text{ and} \sum_{i=1}^{N} w_{it}^2 = O(N^{-1}).$$

For a given value of $\boldsymbol{\theta}$ and each t, (7) provides a non-linear function in $\tilde{f}_t$. This equation clearly shows that unlike the linear models considered in Pesaran (2006), here the solution to the common component f_t does not reduce to an average of (log) prices. In particular, $\tilde{f}_t$ also accounts for the dynamic feature of the price-setting behavior through the $\bar{g}_t$ component, which depends on $p_{i,t-1}$. Equation (7) has a unique solution as long as $c > 0$. A proof is provided in Appendix A. It is also easily seen that under the cross-sectional independence of v_i and ε_{it}, $\bar{g}_t(f_t) \rightarrow E(g_{it})$ and $\tilde{f}_t - f_t \xrightarrow{p} 0$, as $N \rightarrow \infty$.[8]

3.2 Conditional likelihood estimation with no individual effects

In this section, we derive the maximum likelihood estimation of the structural parameters, $\boldsymbol{\theta}$, conditional on f_t and assuming there are no firm-specific effects, so that $\sigma_v^2 = 0$, and hence in this case $\boldsymbol{\theta} = (c, \boldsymbol{\beta}', \sigma_c^2, \sigma_\varepsilon^2)'$. Given the distributional assumptions stated in Section 3.1, and defining ζ_{it} as $c_{it} - c$, our baseline model can be rewritten as

$$\Delta p_{it} = d_{it} + \varepsilon_{it} + (d_{it} + \varepsilon_{it}) \left\{ I\left[d_{it} + \varepsilon_{it} - \zeta_{it} - c\right] - I\left[d_{it} + \varepsilon_{it} + \zeta_{it} + c\right] \right\},$$

where

$$\begin{pmatrix} \zeta_{it} \\ \varepsilon_{it} \end{pmatrix} \backsim iid\ N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_c^2 & 0 \\ 0 & \sigma_\varepsilon^2 \end{pmatrix} \right), \text{ for } i = 1, 2, ..., N;\ t = 1, 2, ..., T.$$

Equivalently

$$\Delta p_{it} = d_{it} + \varepsilon_{it} + (d_{it} + \varepsilon_{it}) \left\{ I\left[d_{it} - c + \varepsilon_{1it}\right] - I\left[d_{it} + c + \varepsilon_{2it}\right] \right\},$$

where

$$\varepsilon_{1it} = \varepsilon_{it} - \zeta_{it},\ \varepsilon_{2it} = \varepsilon_{it} + \zeta_{it},$$

[8] For the sake of simplicity, we assume here that the panel data sample is balanced: all outlets are observed over the full time period. This is not the case in practice. However, the result can be easily generalized to unbalanced panels assuming that $N_t \rightarrow \infty$ for each t (see the Appendix A).

with

$$\begin{pmatrix} \varepsilon_{1it} \\ \varepsilon_{2it} \\ \varepsilon_{it} \end{pmatrix} \sim iidN \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varepsilon^2 + \sigma_c^2 & \sigma_\varepsilon^2 - \sigma_c^2 & \sigma_\varepsilon^2 \\ . & \sigma_\varepsilon^2 + \sigma_c^2 & \sigma_\varepsilon^2 \\ . & . & \sigma_\varepsilon^2 \end{pmatrix} \right), \text{for } i = 1, 2, ..., N; \ t = 1, 2, ..., T.$$

Let

$$\begin{aligned} \tau_{1it} &= \begin{cases} 1 \text{ if } \Delta p_{it} = 0 \text{ for } i = 1, 2, ..., N \text{ and } t = 1, 2, ..., T, \\ 0 \text{ otherwise} \end{cases} \\ \tau_{2it} &= \begin{cases} 1 \text{ if } \Delta p_{it} > 0 \text{ for } i = 1, 2, ..., N \text{ and } t = 1, 2, ..., T, \\ 0 \text{ otherwise} \end{cases} \\ \tau_{3it} &= \begin{cases} 1 \text{ if } \Delta p_{it} < 0 \text{ for } i = 1, 2, ..., N \text{ and } t = 1, 2, ..., T, \\ 0 \text{ otherwise} \end{cases} \end{aligned}$$

Then conditional on $f_t, t = 1, 2, ..., T$ and the initial value p_{i0}, the log-likelihood function of the model for each i can be written as

$$\begin{aligned} L_i(\boldsymbol{\theta} \,|\mathbf{f}\,) &= \Pr\left(\Delta p_{i1} \,|p_{i0}\,\right) \Pr\left(\Delta p_{i2} \,|p_{i0}, p_{i1}\,\right) \\ &\quad \times \Pr\left(\Delta p_{i,T} \,|p_{i0}, p_{i1}, ..., p_{i,T-1}\,\right) \times \Pr\left(p_{i0}\right) \end{aligned}$$

where $\mathbf{f} = (f_1, f_2, ..., f_T)'$. In view of the first-order Markovian property of the model we have

$$\begin{aligned} L_i(\boldsymbol{\theta} \,|\mathbf{f}\,) &= \Pr\left(\Delta p_{i1} \,|p_{i0}\,\right) \Pr\left(\Delta p_{i2} \,|p_{i1}\,\right) \\ &\quad \times \Pr\left(\Delta p_{i,T} \,|p_{i,T-1}\,\right) \times \Pr\left(p_{i0}\right). \end{aligned}$$

When T is small, the contribution of $\Pr\left(p_{i0}\right)$ could be important. In what follows we assume that p_{i0} is given and T reasonably large so that the contribution of the initial observations to the log-likelihood function can be ignored.

To derive $\Pr\left(\Delta p_{it} \,|p_{i,t-1}, f_t\,\right)$ we distinguish between cases where $\Delta p_{it} = 0$, $\Delta p_{it} > 0$ and $\Delta p_{it} < 0$, noting that

$$\begin{aligned} & \Pr\left(\Delta p_{it} \,|\Delta p_{it} = 0, p_{i,t-1}, f_t\,\right) = \Pr\left(\varepsilon_{1it} \leq c - d_{it} \ ; \varepsilon_{2it} \geq -c - d_{it}\right) \\ = \ & \Pr\left(\varepsilon_{1it} \leq c - d_{it}\right) - \Pr\left(\varepsilon_{1it} \leq c - d_{it} \ ; \varepsilon_{2it} \leq -c - d_{it}\right) \\ = \ & \Phi\left(\frac{c - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}\right) - \Phi_2\left(\frac{c - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}; \frac{-c - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}; \frac{\sigma_\varepsilon^2 - \sigma_c^2}{\sigma_\varepsilon^2 + \sigma_c^2}\right) = \pi_{1it}, \end{aligned}$$

where $\Phi_2(x; y; \rho)$ is the cumulative distribution function of the standard bivariate normal. Similarly

$$\begin{aligned} & \Pr(\Delta p_{it} \,|\Delta p_{it} > 0, p_{i,t-1}, f_t) = \Pr(\varepsilon_{it} = \Delta p_{it} - d_{it}) \Pr(\varepsilon_{1it} \geq c - d_{it} \; ; \varepsilon_{2it} > -c - d_{it} \,|\varepsilon_{it}) \\ = \; & \frac{1}{\sigma_\varepsilon}\phi\left(\frac{\Delta p_{it} - d_{it}}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{-c + \Delta p_{it}}{\sigma_c}\right) - \Phi\left(\frac{-c - \Delta p_{it}}{\sigma_c}\right)\right] = \pi_{2it}, \end{aligned}$$

and

$$\begin{aligned} & \Pr(\Delta p_{it} \,|\Delta p_{it} < 0, p_{i,t-1}, f_t) = \Pr(\varepsilon_{it} = \Delta p_{it} - d_{it}) \Pr(\varepsilon_{1it} < c - d_{it} \; ; \varepsilon_{2it} \leq -c - d_{it} \,|\varepsilon_{it}) \\ = \; & \frac{1}{\sigma_\varepsilon}\phi\left(\frac{\Delta p_{it} - d_{it}}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{-c - \Delta p_{it}}{\sigma_c}\right) - \Phi\left(\frac{-c + \Delta p_{it}}{\sigma_c}\right)\right] = \pi_{3it}. \end{aligned}$$

Hence

$$\ell(\boldsymbol{\theta}, \mathbf{f}) = \sum_{i=1}^{N} \ln L_i(\boldsymbol{\theta}, \mathbf{f}) = \sum_{i=1}^{N}\sum_{t=1}^{T} \left[\tau_{1it} \ln(\pi_{1it}) + \tau_{2it} \ln(\pi_{2it}) + \tau_{3it} \ln(\pi_{3it})\right]. \tag{8}$$

The ML estimator of $\boldsymbol{\theta}$ is given by

$$\hat{\boldsymbol{\theta}}_{ML}(\mathbf{f}) = \arg\max_{\boldsymbol{\theta}} \ell(\boldsymbol{\theta}, \mathbf{f})$$

and for N and T sufficiently large we have:

$$\sqrt{NT}\left(\hat{\boldsymbol{\theta}}_{ML}(\mathbf{f}) - \boldsymbol{\theta}\right) \overset{a}{\backsim} N(0, \mathbf{V}_{\boldsymbol{\theta}}),$$

where $\mathbf{V}_{\boldsymbol{\theta}}$ is the asymptotic variance of the ML estimator and can be estimated consistently using the second derivatives of the log likelihood function.

Remark 1 *In the case where f_t, $t = 1, 2, ..., T$ are estimated, the ML estimators will continue to be consistent as both N and T tend to infinity. However, the asymptotic distribution of the ML estimator is likely to be subject to the generated regressor problem. The importance of the generated regressor problem in the present application could be investigated using a bootstrap procedure.*

3.3 Conditional likelihood estimation with random effects

Consider now the random effects specification where $p^*_{it} = f_t + \mathbf{x}'_{it}\boldsymbol{\beta} + v_i + \varepsilon_{it}$, and note that

$$Cov(p^*_{it}, p^*_{it'} \,|\mathbf{x}_{it}, \mathbf{x}_{it'}) = \sigma^2_v \text{ for all } t \text{ and } t', \; t \neq t'.$$

Under this model, the probability of no price change in a given period, conditional on the previous price, $p_{i,t-1}$, will not be independent of episodes of no price changes in the past. So we need to consider the joint probability distribution of successive unchanged prices. For example, suppose that prices for outlet i have remained unchanged over the period t and $t+1$, then the relevant joint events of interest are

$$\mathcal{A}_{it} : \{-c - \zeta_{it} - d_{it} \leq \varepsilon_{it} + v_i \leq c + \zeta_{it} - d_{it}\},$$

and

$$\mathcal{A}_{i,t+1} : \left\{-c - \zeta_{i,t+1} - d_{i,t+1} \leq \varepsilon_{i,t+1} + v_i \leq c + \zeta_{it} - d_{i,t+1}\right\}.$$

An explicit derivation of the joint distribution of A_{it} and A_{it+1} would seem rather difficult. An alternative strategy is to use the conditional independence property of successive price changes, and note that for each i, and conditional on $\mathbf{v} = (v_1, v_2, ..., v_N)'$ and $\mathbf{f}$, the likelihood function will be given by

$$L(\boldsymbol{\theta}, \mathbf{v}, \mathbf{f}) = \prod_{i=1}^{N} \prod_{t=1}^{T} [\pi_{1it}(v_i)]^{\tau_{1it}} [\pi_{2it}(v_i)]^{\tau_{2it}} [\pi_{3it}(v_i)]^{\tau_{2it}},$$

where

$$\begin{aligned}
\pi_{1it}(v_i, f_t) &= \Phi\left(\frac{c - v_i - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}\right) - \Phi_2\left(\frac{c - v_i - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}; \frac{-c - v_i - d_{it}}{\sqrt{\sigma_\varepsilon^2 + \sigma_c^2}}; \frac{\sigma_\varepsilon^2 - \sigma_c^2}{\sigma_\varepsilon^2 + \sigma_c^2}\right), \\
\pi_{2it}(v_i, f_t) &= \frac{1}{\sigma_\varepsilon}\phi\left(\frac{\Delta p_{it} - v_i - d_{it}}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{-c + \Delta p_{it}}{\sigma_c}\right) - \Phi\left(\frac{-c - \Delta p_{it}}{\sigma_c}\right)\right]
\end{aligned}$$

and

$$\pi_{3it}(v_i, f_t) = \frac{1}{\sigma_\varepsilon}\phi\left(\frac{\Delta p_{it} - v_i - d_{it}}{\sigma_\varepsilon}\right)\left[\Phi\left(\frac{-c - \Delta p_{it}}{\sigma_c}\right) - \Phi\left(\frac{-c + \Delta p_{it}}{\sigma_c}\right)\right].$$

The random effects can now be integrated out with respect to the distribution of v_i [assuming $v_i \sim N(0, \sigma_v^2)$, for example] and then the integrated log-likelihood function, $E_{\mathbf{v}}[\ell(\boldsymbol{\theta}, \mathbf{v}, \mathbf{f})]$, maximized with respect to $\boldsymbol{\theta}$.[9]

[9] A further extension of the model would consist of including also a firm specific effect into the menu cost. However, the estimation of this model would then requires a double integration with respect to the distribution of the two individual effects.

3.4 Full maximum likelihood estimation

In the case where N and T are sufficiently large, the incidental parameters problem does not arise and the effects of the initial distributions, $\Pr(p_{i0})$, on the likelihood function can be ignored. Then, the maximum likelihood estimators of $\boldsymbol{\theta}$ and $\mathbf{f}$ can be obtained as the solution to the following maximization problem:

$$\left(\widehat{\mathbf{f}}_{ML}, \widehat{\boldsymbol{\theta}}_{ML}\right) = \arg\max_{\mathbf{f},\boldsymbol{\theta}} \sum_{t=1}^{T}\sum_{i=1}^{N} \left[\tau_{1it}\ln(\pi_{1it}) + \tau_{2it}\ln(\pi_{2it}) + \tau_{3it}\ln(\pi_{3it})\right]. \quad (9)$$

Note that for a given value of $\boldsymbol{\theta}$ the ML estimator of f_t can be obtained as

$$\hat{f}_t(\boldsymbol{\theta}) = \arg\max_{f_t} \sum_{i=1}^{N} \left[\tau_{1it}\ln(\pi_{1it}) + \tau_{2it}\ln(\pi_{2it}) + \tau_{3it}\ln(\pi_{3it})\right],$$

and will be consistent as $N \to \infty$, since conditional on $\boldsymbol{\theta}$ and f_t, the elements in the above sum are independently distributed. Also for a given estimate of $\mathbf{f}$, the optimization problem defined by (9) will yield a consistent estimate of $\boldsymbol{\theta}$ as N and $T \to \infty$. Iterating between the solutions of the two optimization problems will deliver consistent estimates of $\boldsymbol{\theta}$ and $f_1, f_2, ..., f_T$, even though the number of incidental parameters, $f_t, t = 1, 2, ..., T$, is rising without bounds as $T \to \infty$. This is analogous to the problem of estimating time and individual fixed effects in standard linear panel data models. Individual fixed effects can be consistently estimated from the time dimension and time effects from the cross section dimension.

3.5 Some monte carlo simulations

In order to evaluate the performance of the two alternative estimation procedures (that is, the iterative procedure based on the cross-sectional estimates of f_t and the Full Maximum Likelihood estimation of the model), we carried out a limited number of Monte Carlo simulations. We generated the log price series according to the baseline model, (6), by setting $c = 0.15$, $\sigma_\varepsilon = 0.05$, $\sigma_c = 0.01$ and simulating the common factors as the first order autoregressive process

$$f_t = \rho_0 + \rho_1\ f_{t-1} + \omega_t,\ \omega_t \backsim i.i.d.N(0, \sigma_\omega^2),$$

with $\rho_0 = 0.05$, $\rho_1 = 0.90$, and $\sigma_\omega = 0.10$. In Table 1, we report the average (across R replications[10]) of the point estimates of c, σ_ε, σ_c and σ_v and their average standard errors in different setups. Concerning the estimation of f_t, we compute the RMSE with respect to the true f_t and compare the standard deviation of the true f_t with that of the estimated f_t. In our reference case, the sample size is set at $N = 50$, $T = 50$.

		c	σ_ε	σ_c	σ_v	$RMSE(\widetilde{f}_t)$	$\frac{RMSE(\widetilde{f}_t)}{RMSE(f_t)}$	R
Average frequency of price changes ~ 0.27								
With random effects	True value	0.15	0.05	0.01	0.025			
N=50, T=50, full ML	ML(.)	0.150	0.049	0.011	0.027	0.00020	1.0011	500
	std(.)	0.0014	0.0011	0.0013	0.0030			
No random effects	True value	0.15	0.05	0.01	0			
N=50, T=50, full ML	ML(.)	0.150	0.049	0.007		0.00014	1.0018	500
	std(.)	0.0014	0.0011	0.0013				
N=25, T=50, full ML	ML(.)	0.150	0.048	0.006		0.00029	1.0051	500
	std(.)	0.0019	0.0015	0.0018				
N=50, T=25, full ML	ML(.)	0.150	0.049	0.003		0.00014	1.0022	500
	std(.)	0.0019	0.0015	0.0018				
N=50, T=25, iterative ML	ML(.)	0.148	0.051	0.006		0.00017	0.9907	500
	std(.)	0.0019	0.0016	0.0017				
Average frequency of price changes ~ 0.12								
With random effects	true value	0.300	0.050	0.100	0.025			
N=50, T=50, full ML	ML(.)	0.302	0.047	0.103	0.029	0.0005	1.0042	500
	std(.)	0.0070	0.0017	0.0055	0.0037			
With random effects	true value	0.300	0.100	0.125	0.250			
N=100, T=100, full ML	ML(.)	0.307	0.099	0.131	0.247	0.0055	1.1720	500
	std(.)	0.0105	0.0026	0.0078	0.0242			

R is the number of replications, ML(.) is the average of the point estimates, std(.) is the average of the standard deviation of the estimated coefficient, $\frac{RMSE(\widetilde{f}_t)}{RMSE(f_t)}$ stands for the ratio of the standard deviation of the estimated f_t over the standard deviation of the true f_t.

Table 1 - Monte Carlo Simulations

Under both estimation procedures, initial values for the estimation of f_t are set to $\overline{p}_t$. In the iterative procedure, a first set of estimates for the remaining parameters of the model, $\boldsymbol{\theta}$, are then obtained by maximum likelihood, which is in turn used to compute another estimate of the unobserved common components, and the procedure is iterated until convergence. The standard errors of the parameter estimates are computed from the second derivatives of the full log-likelihood function.

The estimation of the models with and without random effects by the Full Maximum Likelihood roughly leads to similar results The point estimates and precision of the es-

[10] Because the estimation procedure with random effects takes much more time, we ran most simulations without random effects, and the number of replications is limited to 500.

timators are of the same order of magnitude, although the estimation of σ_c appears to improve in a model with random effects. Considering the model without random effects, the estimates of the parameters c and σ_ε obtained by Full ML are essentially unbiased. However, σ_c appears slightly underestimated in the simulations without random effects, contrary to the case with random effects. The unobserved component, f_t, is also very precisely estimated, and its volatility is only 0.14% higher than that of the true f_t.

Unsurprisingly, the precision of the estimates increases with the total size of the sample $N \times T$, as suggested by a comparison of the standard errors of the coefficients c, σ_ε and σ_c, in the three alternative sets of simulations without random effects. However, increasing N and T do not play a symmetrical role in improving the precision of the point estimates. For small values of N there seems to be a downward bias in estimating σ_ε. Furthermore, the RMSE of $\widehat{f_t}$ is higher and its volatility relative to that of the true f_t increases[11]. Decreasing T from 50 to 25 does not seem to have any significant impact on the estimates, except for σ_c which is now more severely underestimated. It might be for only quite low values of T that the impact of ignoring the initial observations in the likelihood function could be non negligible.

We also report a comparison of the full ML and iterative estimation procedures. The results suggest that the point estimates of the coefficients are very close, and that the iterative procedure delivers a smoother f_t than the full ML.[12] The full ML may produce slightly better results in the sense that, as compared to the iterative procedure, the difference between the point estimate of c and its true value is smaller, the RMSE of the estimated f_t as compared to the true f_t is lower, and its volatility is closer to the true one. Finally, in practice, the iterative procedure is much more time consuming than the "Full Maximum Likelihood" method. Therefore, we chose to estimate our baseline pricing model using the Full Maximum Likelihood method. Indeed, given the above Monte-Carlo results and the large size (in both N and T) of our samples, we know that the two methods will not differ in any significant way and that the estimates obtained with the Full ML will be consistent and have a good precision.

In the above exercises, the parameters chosen lead to a frequency of price changes of around 27%. This is close to the frequency of price changes reported by Bils and

[11]When the number of trajectories is small, the unobserved component f_t is poorly estimated, because the cross-sectional dimension is too small for the idiosyncratic shocks, ε_{it}, to cancel out by aggregation. This results in excessive volatility in the estimated f_t. Consequently, in order for the model to be in line with the observed frequency of price changes, the volatility of the idiosyncratic shock has to diminish.

[12]Iterative estimations made on real data for a limited number of products also produce less or equally volative f_t as compared to the full ML estimate of f_t. The estimates of the other parameters are similar in the two estimation procedures.

Klenow (2004) or Klenow and Kryvstov (2006) for the US. For a better comparison with our results and data sets, we also carried out a set of experiments where the frequency of price changes was set to around 12%, which is closer to the frequencies observed in Europe (Dhyne *et al.*, 2006). For these experiments, c is set equal to 0.30 (and σ_c is also increased to 0.10). As expected, the precision of the estimates is reduced when less price changes are observed. This is particularly true for c and σ_c, which appear only in the part of the model corresponding to the price change trigger condition. Deviations from the true values, although larger than for higher frequencies of price changes, remain limited. Finally, we also report simulations for parameter values and sample size that are closer to our estimates based on Belgian and French data. Compared to the preceding case, the size of the idiosyncratic shock, σ_ε, and random effects, σ_v, are increased, while that of the common shock, σ_ω is reduced to 0.025. N and T are set to 100. Results are reported in the last panel of Table 1. They are of the same order of magnitude. Differences with the true values are slightly reduced,except for c and σ_c.In this setup where the idiosyncratic shock plays a dominant role, with a reduced volatility of the true f_t, $\widehat{f_t}$ is less precisely estimated and its volatility is larger as compared to the that of the true f_t.[13]

4 Estimation Results

The estimates of our baseline model, (6), are based on individual consumer price quotes compiled by the Belgian and French statistical institutes for the computation of their consumer price indices.[14] These data refer to monthly price series of individual products sold in a particular outlet. The period covered has been restricted to the intersection of the two databases, that is July 1994 - February 2003. See Appendix B for further details about the two data sets.

Since we want to estimate our model for narrowly defined products, price series have been grouped into 368 product categories for Belgium and 305 for France. However, as the estimation procedure is particularly time consuming,[15] the estimation has only been conducted on a subset of randomly selected product categories, with price trajectories of at least 20 months.[16] As a result we end up estimating our baseline model for 98 product

[13] σ_ε is now four times larger as σ_ω while in the preceding exercise, σ_ε was one half of σ_ω.

[14] Each of these two datasets contains more than 10 millions observations. They are described in detail in Aucremanne and Dhyne (2004) for Belgium and in Baudry *et al.* (2007) for France.

[15] The estimation of our model for a typical product category, using S.A.S. 8.02 on a 1.6 Ghz P4 computer takes between 3 to 5 days.

[16] We define a price trajectory as a continuous sequence of price reports referring to one particular product sold in store i. The prices we refer to are (logs of) prices per unit of product so that promotions

categories in Belgium and for 93 categories in France. Extended versions of the model (that allow for gradual or asymmetric adjustment costs) are also estimated with Belgian data for some selected product categories.

As stated above, we have opted, for practical reasons, for the "Full Maximum Likelihood" estimator so that we have simultaneously estimated, for each product category, the unobserved common component, f_t, as well as the other parameters, namely, the average level of the adjustment cost, c, and its variability, σ_c, the magnitude of the idiosyncratic shocks, σ_ε, and the variability of firms specific desired mark-up, σ_v. Finally, as we lack information on local competition or other factors that might affect the (log) optimal price, the outlet specific regressors, $\mathbf{x}_{it}$, included in the model only contain a dummy variable corresponding to the nature of the outlet: the dummy takes the value of 1 whenever the outlet is a supermarket and 0 otherwise.

The response of actual prices to changes in the common component of the "optimal" price clearly depends on the profile of this common component. Variations in this common component are likely to induce price changes, even though they are partly predictable. For instance, changes in conventional wages are a good example of such predictable changes that induce variations in the optimal prices which in turn, are likely to lead to changes in actual prices. Such wage increases are largely predictable[17] and have a clear impact on prices (e.g. see Loupias, Heckel and Sevestre (2007) for a study of French industrial price movements and Stahl (2005) for a study on German industrial prices).

Obviously, unpredictable common shocks (such as the impact of the "mad cow disease" on the demand for beef, the variations in the price of raw materials, or bad weather conditions affecting the harvest of vegetal products) may also have an impact on the likelihood of a price change.

In order to help interpret the impact on price changes of the variations in f_t, we propose a decomposition of these variations into several components: a trend, an autoregressive component and a random component. More specifically, for each product category, the estimates $\widehat{f}_1, \widehat{f}_2, ..., \widehat{f}_T$ are used to fit an $AR(K)$ model[18]

$$\widehat{f}_t = \beta_0 + \beta_1 t + \sum_{k=1}^{K} \rho_k \widehat{f}_{t-k} + \omega_t, \quad \omega_t \backsim i.i.d.N\left(0, \sigma_\omega^2\right).$$

in quantities are also captured in our analysis.

[17]For instance, in France, changes in the minimum wage are decided by the government and are put into effect annually in July. In Belgium, conventional wage changes for the next two years are negotiated every two years.

[18]For each product category, K is selected to eliminate any serial correlation in ω_t, using AIC applied to autoregressions with the maximum value of K set to 12.

To characterize the magnitude of common variations in the optimal price, p^*_{it}, in the following subsections, we use two different measures : the unconditional standard deviation of f_t, σ_f and the standard error of shocks to the common factors, σ_ω. The tables also provide some basic statistics such as the sum of the autoregressive coefficients, $\overline{\rho} = \sum_{k=1}^{K} \rho_k$. and the autocorrelation coefficients of orders 1, 2, 3, 4, 6 and 12 of the estimated f_t's.

Table 2 below presents a summary of the estimates by broad product categories.[19]

4.1 Assessing intrinsic rigidities

Overall, the estimates obtained for Belgium and France lead to similar conclusions. The average level of the adjustment cost is estimated to represent one third of the price level (36% in Belgium and 31% in France). These estimates are comparable to the relative magnitude of the estimated menu costs reported in Levy *et al.* (1997) for the US. Indeed, Levy *et al.* (1997), using a data set on prices, sales and costs in 5 large multi-store chains, report estimates of menu costs in the US retail grocery trade, ranging from $0.46 to $1.33 per price change; which represent 27% to 40% of the average price level.

Since numerous studies point to a remarkable ranking of the frequency of price changes according to the product category (e.g. see Bils and Klenow, 2004, for the US and Dhyne *et al.*, 2006, for the euro area), it is also worth considering the average adjustment costs by product categories. These are given in the first column of Table 2.[20] The most striking conclusion from the simple comparison of the price change frequencies with the estimated adjustment costs is that indeed, the incidences of less frequent price changes are often associated with larger estimates of the adjustments costs.

The relatively high frequency of price changes observed for energy and especially oil products can partly be explained by relatively low adjustment costs: the mean adjustment cost estimate, $\hat{c}$, for oil energy products is on average in the range 0.012 - 0.014 for Belgium and 0.004 - 0.007 for France, compared to sample averages for the product categories as a whole. of 0.365 for Belgium and 0.328 for France. Similarly, numerous

[19] Tables A and B in the appendix first present detailed results for the estimated structural parameters and the time-series representation of the estimated common component. These tables also include some basic statistics that characterize the price setting behavior of each product category (frequency of price changes, average absolute size of price changes, share of price increases) and indicators of the ability of the model to replicate these characteristics. In the case of Belgium, the correlations between $\widehat{f_t}$ and $\overline{p_t}$ and between $\widehat{f_t}$ and the log of the product category price index, $ln(IP_t)$, are also provided. Tables C and D in Appendix C provide further statistics associated with the estimated common component.

[20] The figures in this table are unweighted. They have been computed after the exclusion of 8 products for Belgium and 2 products for France for which the model appeared to fit particularly badly to the data. See Section 4.4 below.

price changes of perishable food products are associated with low mean adjustment costs. Our estimates for these products are very close to the numbers reported in Ratfai (2006) for meat products in Hungary. At the opposite, manufactured goods and services exhibit higher mean adjustment costs that explain, at least partly, the often underlined stronger stickiness of their prices.

Product type	$\widehat{c}$	$\widehat{\sigma_\varepsilon}$	$\widehat{\sigma_c}$	$\widehat{\sigma_u}$	$\widehat{\sigma_f}$	$\widehat{\sigma_\omega}$	$\widehat{\rho}$	*Freq*	$\overline{\lvert\Delta p\rvert}$	*%up*
Energy (BE - 3 product categories ; FR - 2 product category)										
Average - Belgium	0.014	0.030	0.006	0.091	0.176	0.038	0.866	0.731	0.043	0.535
Average - France	0.005	0.026	0.004	0.155	0.112	0.018	0.794	0.799	0.023	0.572
Perishable food (BE - 24 product categories ; FR - 13 product categories)										
Average - Belgium	0.274	0.097	0.143	0.154	0.073	0.030	0.674	0.230	0.128	0.648
Average - France	0.196	0.097	0.136	0.267	0.067	0.015	0.901	0.254	0.107	0.574
Non perishable food (BE - 12 product categories ; FR - 11 product categories)										
Average - Belgium	0.309	0.080	0.173	0.202	0.055	0.018	0.802	0.127	0.104	0.627
Average - France	0.190	0.067	0.125	0.239	0.064	0.014	0.806	0.198	0.059	0.589
Non durable goods (BE - 15 product categories ; FR - 31 product categories)										
Average - Belgium	0.375	0.079	0.178	0.233	0.064	0.013	0.852	0.147	0.089	0.686
Average - France	0.430	0.108	0.219	0.433	0.074	0.043	0.283	0.119	0.180	0.551
Durable goods (BE - 17 product categories ; FR - 13 product categories)										
Average - Belgium	0.547	0.079	0.264	0.228	0.057	0.014	0.739	0.055	0.076	0.613
Average - France	0.314	0.076	0.180	0.420	0.077	0.030	0.785	0.137	0.083	0.487
Services (BE - 19 product categories ; FR - 21 product categories)										
Average - Belgium	0.400	0.049	0.178	0.162	0.107	0.009	0.743	0.040	0.062	0.836
Average - France	0.370	0.074	0.177	0.274	0.066	0.023	0.612	0.083	0.054	0.744
Full basket (BE - 90 product category - FR - 91 product categories)										
Average - Belgium	0.365	0.076	0.178	0.187	0.077	0.019	0.754	0.146	0.092	0.681
Average - France	0.328	0.087	0.176	0.341	0.071	0.028	0.593	0.157	0.109	0.595

TABLE 2 - ESTIMATION RESULTS BY BROAD PRODUCT CATEGORIES

Another striking result is that, for all product types, except for oil products, the average adjustment costs are larger than the average size of price changes. Initially, this may be considered as a rather puzzling result. However, it can be rationalized noting the stochastic nature of the adjustment cost variable, c_{it}. Indeed, since the distribution of c_{it} is symmetric around its mean, c, the likelihood that a price change occurs is larger the lower the realized adjustment cost i.e., for negative values of $(c_{it} - c)$, as compared to the positive case where $c_{it} - c$ is positive. Therefore, small price changes are more likely than large ones, which lowers the average size of price changes.[21] This may explain why, despite significant average adjustment costs, a large number of small price changes are observed.

[21] We thank H. LeBihan for this insight. This is easy to check with a simulation where, setting σ_c^2 equal to 0 leads to the expected result: the average size of price changes is larger than c.

Although we observe globally the expected negative correlation between the frequency of price changes and the estimated mean adjustment cost, the observed differences in the frequency of price changes across products are not fully explained by those in the estimated c. This can be illustrated by the following two examples. First, the monthly frequency of price changes associated with beef sirloin (14.9%) in the Belgian data set represents only a fourth of the frequency of price changes of Kiwis (54,2%). However, the adjustment costs of these two products are of the same order of magnitude ($\hat{c}$ equal to 0.166 for sirloin compared to 0.141 for Kiwis). Therefore, differences in the frequency of price changes must originate in differences in the size of the common and/or idiosyncratic shocks. A second interesting example relates to men's suit and sugar in France. While the observed frequencies of price changes of these two products are quite similar (16.7% and 17.0%, respectively), their estimated adjustment costs differ markedly as their respective estimates are 0.33 for the former product and only 0.13 for the latter.

4.2 Assessing extrinsic rigidities

Our estimates show that extrinsic rigidity (the magnitude of shocks, both common and idiosyncratic, to the optimal price) does play an important role in explaining the frequency of price changes. This result can be readily illustrated using the two examples discussed above. In the case of men suits and sugar in France, we observe strong differences in the profile and magnitude of the shocks affecting the optimal prices of these two product categories. First, the overall variability of the common component f_t (as measured by σ_f) appears to be larger for men suits than for sugar. Interestingly, the profiles over time of these two common components differ markedly. Indeed, the autocorrelation profile of the estimated f_t's for men suits exhibit a high degree of autocorrelation at lag orders 6 and 12, suggesting strong seasonal effects in prices. A reasonable interpretation of this result lies in the prevalence of promotion sales that strongly affect prices of clothing. This is a situation where the profile of the common component contributes to the understanding of the observed frequency of price changes. Second, idiosyncratic shocks affecting men suit optimal prices are of a much larger magnitude than those affecting sugar prices, explaining why men suit prices vary as much as sugar prices over time, despite its higher adjustment costs. The importance of the idiosyncratic component may reflect the outlet specific "marketing policy" regarding sales. Consider now Kiwis and sirloin in Belgium. While the frequencies of their price changes are quite different, these two products exhibit very similar mean adjustment cost estimates Then, this difference must stem from differences in the magnitude of idiosyncratic shocks affecting the price of these two products (σ_ε equals 0.058 for sirloin compared to 0.203 for Kiwis) and/or from differences in the the unconditional variability of the common factors associated with these two product categories (σ_f equals 0.020 for sirloin compared to 0.172 for Kiwis).

Overall, our estimates clearly show the relative importance of idiosyncratic shocks for our understanding of the price change frequencies. With a very few exceptions (mainly energy products), the magnitude of idiosyncratic shocks is generally larger than the (unconditional) variability of the common component σ_f. Over the entire range of products, the ratio of $\widehat{\sigma}_\varepsilon$ to $\widehat{\sigma}_f$ takes values above one for 60% of the product categories in Belgium and in 70% of cases in France.[22] Considering $\widehat{\sigma}_\omega$ instead of the unconditional standard deviation of the f_t's obviously yields much larger values for this ratio. This result is in line with the conclusion of Golosov and Lucas (2003) who state that price trajectories at the micro level are largely affected by idiosyncratic shocks.

4.3 Intrinsic and extrinsic rigidities and the frequency of price changes

Our main findings so far, can be summarized as follows: the relatively high frequency of price changes observed for energy, and especially oil products, can be explained by the low values of the mean adjustment costs parameter, but also by a significant variability of $\widehat{f_t}$ for this product category. Indeed, for Belgium, the unconditional standard deviation of $\widehat{f_t}, \widehat{\sigma}_f$, lies between 0.114 and 0.263 for the three energy products considered (resp. between 0.091 and 0.133 in France) while it averages to only 0.077 for the set of products as a whole (resp. 0.071 in France). Both in Belgium and France, the consumer prices of the energy products is thus largely determined by the common movements in marginal costs (which are highly correlated with the price of oil products on the international markets as illustrated in Figure 2). The contribution of idiosyncratic shocks and the dispersion of firm specific mark-ups is of second order importance, compared to what is observed in the other product categories.[23]

[22] The average value of this ratio over the 88 product categories considered in the Belgian sample is 1.74 and it is 1.59 in the French sample.

[23] In the case of Belgium, this might be due to the fact that oil prices at the gas station are regulated (there is an agreement between the government and oil companies to set up the maximum prices of oil product).

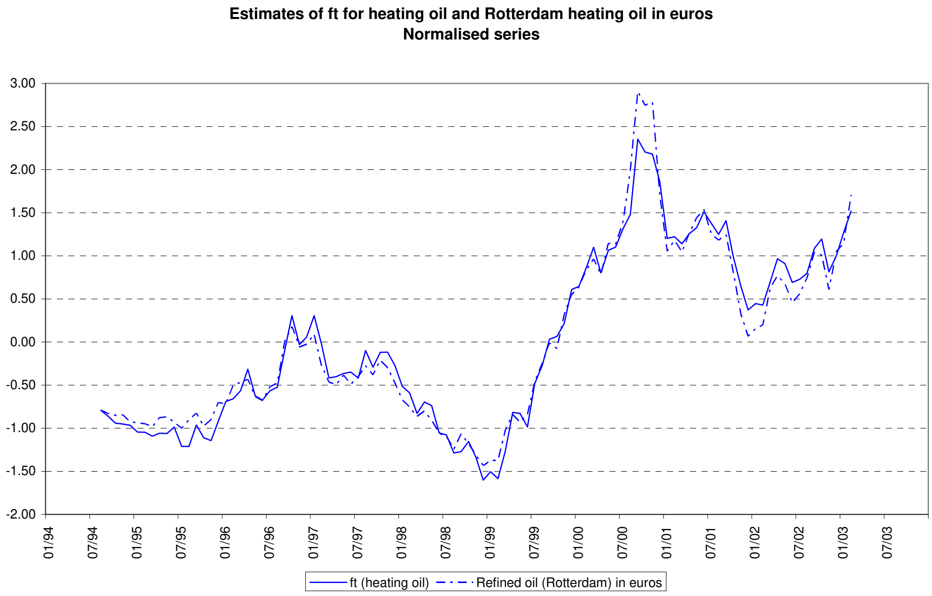

Figure 2 - Evolution of common component $\widehat{f_t}$ for heating oil and of refined oil on Rotterdam Market

The perishable food product categories, which rank second in terms of the frequency of price changes both in Belgium and in France, are characterized by medium sized adjustment costs (c is estimated to be 0.274 in Belgium, 0.196 in France) but these product categories are affected by relatively important common and idiosyncratic shocks. In other words, intrinsic rigidities appear here again to be the main reason for the observed "mild" stickiness of these product prices. It is worth noticing that for France, the slightly lower frequency of price changes observed for non perishable food products seems to be only the consequence of lower idiosyncratic shocks, all the other parameters being quite close to those obtained for non perishable food products. This is another clear illustration of the role of extrinsic rigidity. At the opposite side of the spectrum, the most sticky components of the CPI in Belgium (services and durable goods) and in France (services) are characterized by higher adjustment costs but also, in Belgium, by smaller idiosyncratic and common shocks. Some services in France are also characterized by smaller shocks but there seems to be a significant heterogeneity in this respect. Finally, the frequency of price changes for the remaining categories (non perishable food and non durable industrial goods in Belgium; durable and non durable goods in France) is driven by both slightly larger than average adjustment costs and a lower variability of the idiosyncratic and common components of the optimal price. Then, the relative stickiness of these prices are due to both intrinsic and extrinsic rigidities, where the latter seems to be more "concentrated" in the common component of the price, while idiosyncratic shocks appear to be an important factor of price variability in those sectors.

In conclusion, the frequency of price changes seems, unsurprisingly, to be closely related to the ratio of the variability of the optimal price over time, as measured by $\sqrt{\sigma_\varepsilon^2 + \sigma_f^2}$, to the mean adjustment cost parameter c. Indeed, the simple correlation between the frequency of price changes and this ratio is 0.708 for Belgium, and 0.818 for France.

For a deeper understanding of the link between the frequency of price changes and the structural parameters of the model, we have estimated a simple equation relating the frequency of price changes to the estimated adjustment costs parameter, $\hat{c}$, the volatility of the idiosyncratic and the common shocks, $\hat{\sigma}_\varepsilon$ and $\hat{\sigma}_\omega$, respectively. Two groups of regressions are run. First, three linear regressions explaining the observed frequency of price changes ($freq_i$) are estimated by OLS. A second set of regressions with the dependent variable defined as the logit transformation of the frequencies (i.e. $\ln\left[freq_i/(1-freq_i)\right]$) is also estimated by the quasi maximum likelihood (QML) estimation procedure proposed by Papke and Wooldridge (1996). These regressions are run on the sample of product categories for which the quality of the fit was good (see below), i.e., 90 product categories for Belgium and 91 product categories for France. Table 3 reports the results (with standard errors in brackets). The QML and OLS provide qualitatively similar results, although the QML procedure provides a better fit,[24] which favours a non-linear relation between the structural parameters and the frequency of price changes.

	OLS			QML		
	(1)	(2)	(3)	(4)	(5)	(6)
const	0.252 (0.019)	0.146 (0.011)	0.154 (0.012)	−1.044 (0.240)	−1.732 (0.112)	−1.673 (0.104)
France	−0.050 (0.015)	−0.012 (0.008)	−0.014 (0.008)	0.054 (0.106)	0.165 (0.057)	0.152 (0.053)
$\hat{c}$	−0.715 (0.043)	−0.409 (0.026)	−0.433 (0.029)	−6.169 (0.476)	−4.171 (0.210)	−4.607 (0.287)
$\widehat{\sigma_\varepsilon}$	1.643 (0.184)	1.121 (0.098)	1.223 (0.119)	10.000 (1.592)	9.205 (0.898)	11.579 (1.167)
$\widehat{\sigma_\omega}$	1.603 (0.344)	0.493 (0.186)	0.441 (0.202)	10.980 (2.590)	4.929 (2.223)	3.197 (2.331)
$\frac{\sqrt{\widehat{\sigma_\varepsilon^2}+\widehat{\sigma_\omega^2}}}{\hat{c}}$	-	0.101 (0.004)	-	-	0.416 (0.048)	-
$\frac{\widehat{\sigma_\varepsilon}}{\hat{c}}$	-	-	0.069 (0.014)	-	-	0.030 (0.124)
$\frac{\widehat{\sigma_\omega}}{\hat{c}}$	-	-	0.075 (0.017)	-	-	0.639 (0.176)
R^2	0.649	0.906	0.898	0.800	0.940	0.955

Table 3 - Relation between frequency of price changes and structural parameters

[24]This is particularly true of the specification that excludes the $\hat{c}/\hat{\sigma}_\varepsilon$.

These regressions confirm that the frequency of price changes is strongly influenced by the size of the shocks, as estimated by $\hat{\sigma}_\varepsilon$ and $\hat{\sigma}_\omega$, relative to the adjustment costs parameter, $\hat{c}$. If larger adjustment costs tend to significantly reduce the frequency of price changes, this effect can be partly offset by larger shocks to the marginal costs/desired mark-up. Introducing the relative importance of idiosyncratic shocks and common shocks separately also indicates that it is mostly the relative size of the common shock that determines the frequency of price changes.[25]

4.4 Evaluating the fit of the model

In order to assess how well the model fits the data, we compare the realized frequency and average size of price changes with those obtained by simulating the model. More precisely, for each product, we simulate an unbalanced panel of price trajectories starting with p_{i0}, the observed initial value of each price trajectory i, using the estimated values of c, f_t and randomly generated ε_{it}'s and c_{it}'s with respective standard-errors $\widehat{\sigma}_\varepsilon$, $\widehat{\sigma}_c$ as well as an estimate of u_i. Indeed, as the true initial value p_{i0} is used as starting value of the i^{th} price trajectory, the true u_i should be used to simulate the subsequent price observation of that trajectory. Since u_i is unknown, the simulation exercise is based on an estimated $\widehat{u}_i$ which is computed by re-estimating our baseline model with trajectory specific fixed effects, keeping the other parameters of the model ($\widehat{c}$, $\widehat{\sigma}_\varepsilon$, $\widehat{\sigma}_c$, $\widehat{f_t}$) as given. The time dimension of the simulated trajectory i is set to coincide with the length of the associated realized price trajectory. The number of price trajectories in the simulated panels is given by the number of trajectories in the observed panels. The experiment is repeated 1000 times for each trajectory.

For each product category and their simulated counterparts, the frequency of price changes, the average (absolute) size of price changes and the share of price increases are computed. Scatter plots of these statistics for the 98 product categories in the Belgian CPI are presented in Figure 3. Similar graphs would be obtained using the French estimates. Figures 3a shows that, except for a small number of products (8 out of 98), the observed frequencies of price changes match the simulated ones quite well. The same is also true for France where except for 2 product categories (out of 93), the actual and simulated frequencies match very closely.[26] The few cases where the simulations do not match the

[25] Using the standard deviation of $\hat{f}_t$ instead of $\hat{\sigma}_\omega$ does not induce any change in the conclusions.

[26] The 10 product categories for which our estimated parameters do not allow to replicate the characteristics of the observed price trajectories are, for Belgium, "Dining room oak furniture", "Cup and saucer", "Parking spot in a garage", "Fabric for dress", "Wallet","Small anorak (9 month)"; "Men T Shirt" and "Hair spray 400 ml", and for France, "classic lunch in a restaurant" and "pasta". These

realizations are confined to product categories with relatively rigid prices. In the case of these products our simulations lead to an overestimation of the frequency of price changes, and to an underestimation of the average size of the price change. Moreover, these product categories are characterized by a very high degree of heterogeneity in the price dynamics, which translates into a large degree of heterogeneity in the adjustment costs parameter, c_{it}. When σ_c is very large as compared to c, our model could, in principle generate negative menu costs. This leads to a failure of the simulated samples to reproduce the data characteristics.[27]

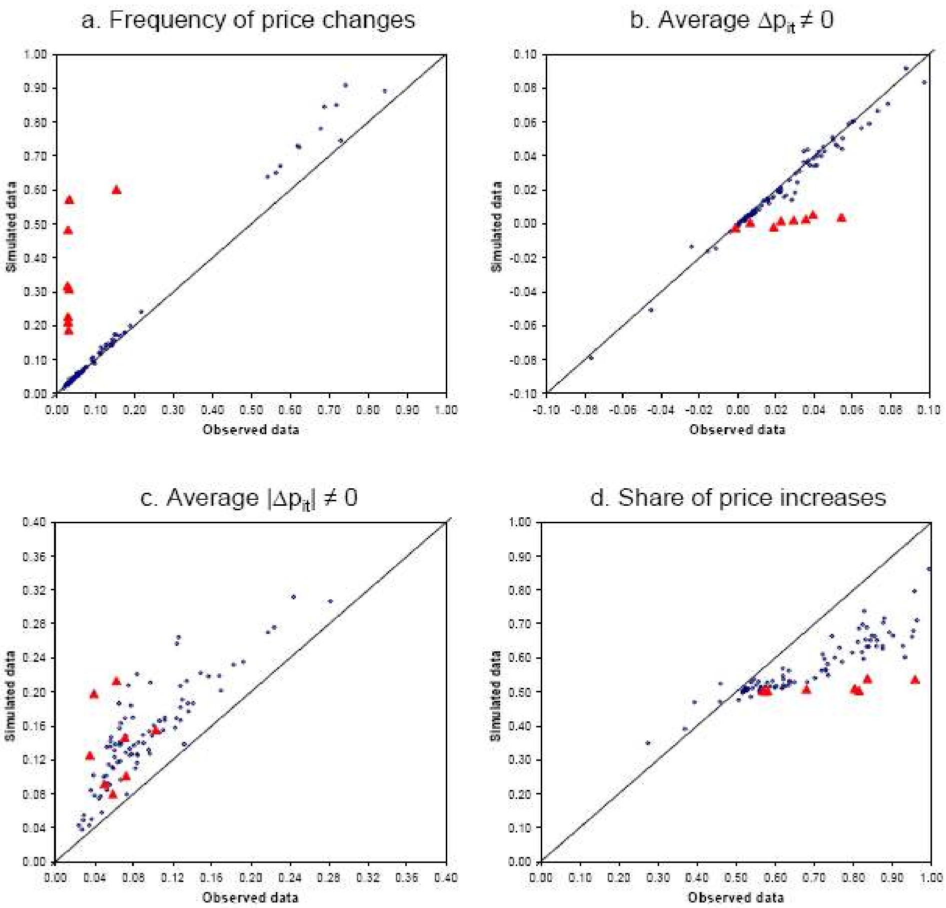

Figure 3 - Characteristics of observed and simulated trajectories

The quality of the fit appears to be less satisfactory regarding the two other characteristics of price changes: their average magnitude and the proportion of price increases, both for Belgium and France. While the proportion of price increases seems to be underestimated in most cases, the magnitude of price changes is overestimated. Since the

products were not considered in the OLS/QML estimation presented in section 4.3

[27]The detailed results are provided in appendix C.

estimated common component appears to fit quite well the specific price indices of each category (see the correlation between f_t and these indices in table A, appendix C), it seems reasonable to conclude that the cause of this problem lies, at least partly, in the idiosyncratic shocks affecting the optimal prices. Indeed, this outcome may result from an overestimation of the size of the idiosyncratic shock and/or from the assumption of pure randomness of the c_{it}'s around their mean. Given this randomness, we may face a number of cases where an observed price increase (resp. a decrease) corresponds, in our model to a situation which would normally induce no price changes (because c_{it} is high and positive), thus implicitly requiring a large positive (resp. negative) shock to make the observed price change likely. Accounting for differences in adjustment costs that are not purely random (such as differences across types of outlets, seasonal variations, etc.) might then be a way to improve our estimates. Regarding the underestimation of the proportion of price increases, one may wonder whether the assumption of no serial correlation in the ε_{it} and the symmetric distribution of the c_{it}'s may explain the underestimation of the proportion of price increases. Indeed, one can observe that for the few products exhibiting a low proportion of price increases, this proportion is overestimated. It might be the case that the symmetry assumed here leads to a bias of the frequency of price increases towards 0.5. A first exploration of this asymmetry issue is provided below.

4.5 Some Extensions of the Empirical Results

4.5.1 Gradual adjustments

As stated in Section 2, several factors, such as the structure of local competition across outlets, the degree of uncertainty in the identification of the shocks to the marginal cost, consumers' inattention, or costly information can motivate a partial adjustment to shocks. However, in order to observe such gradual movements in prices, price changes should be made on a relatively frequent basis. If a firm adjusts its price only once a year, a gradual change might not be sensible. Therefore, a price setting model with partial adjustment should only be estimated for product categories with relatively frequent price changes. For these product categories, the partial adjustment parameter λ introduces an additional source of intrinsic rigidity.

In the following table, we present the estimation results associated with a set of three product categories characterized by relatively frequent price changes (heating oil, oranges and roses). We also present the estimation results for two product categories that in comparison are characterized by less frequent price changes (namely central heating repair tariff and hourly rate of a painter).

Parameters	Heating oil	Oranges	Roses	Central heating	Painter
$\hat{c}$	0.025^{**}	0.075^{**}	0.076^{**}	0.396^{**}	0.144^{**}
$\widehat{\sigma_{\varepsilon}}$	0.052^{**}	0.247^{**}	0.291^{**}	0.074^{**}	0.220^{**}
$\widehat{\sigma_{c}}$	0.010^{**}	0.056^{**}	0.033^{**}	0.190^{**}	0.066^{**}
$\widehat{\sigma_{v}}$	0.044^{**}	0.109^{**}	0.247^{**}	0.151^{**}	0.221^{**}
$\hat{\lambda}$	0.342^{**}	0.395^{**}	0.436^{**}	0.076^{**}	0.864^{**}
Logl	14755.9	−13921.2	−6098.8	−3114.5	−2311.9
$\widehat{\sigma_{\omega}}$	0.097	0.067	0.076	0.004	0.062
$\widehat{\overline{\rho}}$	0.867	0.498	1.038	0.848	0.187

Table 4 - estimation results with gradual adjustment - Belgium
** = significant at the 1% level * = significant at the 5% level

The results are summarized in Table 4. The estimates of λ, the parameter of the partial adjustment, is statistically significant for all five product lines considered, with values that seem eminently sensible for product categories characterized by very frequent price changes. Our estimates indicate that for this kind of products, there is a statistically significant evidence of gradualism in the price setting behavior of firms. This clearly indicates an additional source of extrinsic rigidity. The estimate of λ for "central heating repair tariff" is much smaller, and is in accordance with our prior belief that when a firm adjusts its price rarely, it does it (almost) fully. However, we obtain a very high estimate of λ for an "hourly rate of a plumber" which is difficult to explain from an economic point of view. This last result could be due to the fact that the estimation of a gradual adjustment price setting model on price trajectories that do not contain any price change might be quite problematic. We have conducted some simulations showing that the observation of flat price trajectories biases the estimation of the λ parameter towards one, introducing a high volatility in the unobserved common component.

4.5.2 Asymmetric adjustment costs

As mentioned earlier, our estimates so far are based on the assumption of symmetric adjustment costs. As noted earlier this assumption does not rule out asymmetry in the observed direction of price changes. If the estimated common component, $\hat{f}_t$, is characterized by a positive (negative) trend, our price setting model will generate more price increases (decreases). This is consistent with the argument of Ball and Mankiw (1994).

However, in order to test whether relaxing this assumption could help in capturing the observed degree of asymmetry in the direction of price changes, we have estimated our baseline model introducing different average adjustment cost parameters for price increases (c_U) and for price decreases (c_L).[28] This estimation has been conducted on product categories mainly exhibiting symmetric price changes (e.g. oranges and heating oil) and for product categories that largely show asymmetric price changes (e.g. special beer in a bar, dry cleaning of a shirt). The results are given in Table 5.

	Oranges	Special beer	Heating oil	Dry cleaning (shirt)	Biscuits	Sausage	Cheese (Edam)
$\widehat{c_{up}}$	0.079**	0.543**	0.025**	0.556**	0.226**	0.440**	0.323**
$\widehat{c_{down}}-\widehat{c_{up}}$	0.000	−0.002*	0.001**	−0.004**	0.000	−0.001**	0.000
$\widehat{\sigma_{\varepsilon}}$	0.159**	0.052**	0.036**	0.063**	0.067**	0.110**	0.086**
$\widehat{\sigma_{c}}$	0.063**	0.237**	0.011**	0.251**	0.146**	0.230**	0.174**
$\widehat{\sigma_{u}}$	0.109**	0.151**	0.040**	0.172**	0.189**	0.165**	0.134**
$\widehat{hyper}$	−0.019**	0.000	–	–	−0.036**	−0.108**	−0.020
$\ell(\theta)$	−27381.4	−3076.4	13892.6	−2651.650	−19870.0	−17460.127	−12410.890

Table 5 - Estimation results with asymmetric menu costs - Belgium

** = significant at the 1% level * = significant at the 5% level

The main conclusion emerging from these estimates is that adjustment costs associated with price decreases do not seem to differ much from the adjustment costs associated with price increases. Even if the difference between the two adjustment costs are statistically significant, the difference does not seem to be economically important. Although this conclusion is based on a limited number of cases, it supports the view that asymmetric price changes are more likely to result from a trend in f_t rather than from asymmetric adjustment costs. However, further research is needed to check whether other sources of asymmetry may matter or not.

5 Conclusion

Modern macroeconomics has emphasized the role of price rigidity in the impact of monetary policy on economic activity and inflation dynamics. The slope of the New Keynesian Phillips curve typically depends on intrinsic price rigidity. Most previous empirical literature approximated these intrinsic rigidities by the frequency of price changes. However, in the case of state dependent rules, the frequency of price changes does not only depend on the size of the adjustment costs (intrinsic rigidity), but is also affected by the distribution of shocks that affect outlets (extrinsic rigidity).

[28]It is also possible to introduce asymmetry in the variability of the adjustment costs, but we do not pursue this here.

Following this new strand in theoretical models (see Dotsey, King and Wolman, 1999, and Gertler and Leahy, 2006), we specify a state-dependent (s,S) type model where outlets do not necessarily instantaneously adjust their prices in response to changes in their environment. Since the optimal price targeted by outlets is unobserved, we decompose it into three components: first, a component that is shared across all outlets selling a given fairly homogeneous product. From an economic point of view, this component reflects the average marginal cost augmented with the average outlet-specific desired mark-up associated with this particular product. We model this as a common factor. The second component of the unobserved optimal price is an outlet specific effect, which accounts for product differentiation, local competition conditions, etc.. The third component is an idiosyncratic term, reflecting shocks that may affect the outlet specific optimal price (possibly due to outlet specific demand shocks or unexpected changes in costs, etc.).

This set up involves modeling of the price changes as a non-linear dynamic panel model with unobserved common effects, which allows us to decompose price stickiness into intrinsic and extrinsic components, associated with the variability of the various components of the (unobserved) optimal price. Making use of two large data sets composed of consumer price records used to compute the CPI in Belgium and France, we estimate these different components for a large number of homogenous product categories. Our results show that the now well-documented differences across products in the frequency of price changes do not strictly correspond to differences in terms of adjustment costs; i.e. intrinsic rigidity does not suffice to explain the frequency of price changes. What seems to drive the frequency of price changes is the relative importance of adjustment costs to the size of the shocks, common and/or idiosyncratic.

The high frequency of price changes in the most flexible components of the CPI (energy products and perishable foods) is mainly related to large idiosyncratic and/or common shocks, and not necessarily to small adjustment costs. Conversely, the stickier components of the CPI (durable industrial goods and services) exhibit very low idiosyncratic and common shocks, often in addition to large adjustment costs.

Another important feature of our model is the use of stochastically varying inaction thresholds following Caballero and Engel (2006). This feature helps to explain some of the stylized facts of price setting practices (seasonal pricing, heterogeneity in price stickiness across outlets in terms, synchronization of price changes across and within stores).

Our results also strongly favor the introduction of heterogenous price behaviors in macroeconomic models. Two recent papers examine the implications of heterogeneity of (Calvo) pricing for the New Keynesian Phillips Curve. Using sectoral data on prices and marginal costs, Imbs *et al.* (2006) show that estimates of the NKPC that do not account for industry-level heterogeneity substantially overestimate the backward looking component, and slightly underestimate the role of marginal costs on inflation. In a multi-sector general equilibrium model, Carvalho (2006) shows that under heterogeneous pricing, monetary policy has larger and more persistent real effects than those predicted by single-firm models . In contradiction to the existing view on this issue (Bils and Klenow, 2004, Dhyne *et al.*, 2006), our results indicate that heterogeneity across firms (or product categories) should not necessarily be introduced only through different degrees of nominal/intrinsic rigidity, but also through differences in extrinsic rigidities.

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Appendix A: Mathematical Proofs

Proof of the first part of Lemma 1.

$$E\left[yI(y+a)\right]=\sigma\phi\left(\frac{a+\mu}{\sigma}\right)+\mu\Phi\left(\frac{a+\mu}{\sigma}\right).$$

■

$$\begin{aligned} E\left[yI\left(y+a\right)\right] &= \int_{-a}^{+\infty} y\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy \\ &= \int_{-a}^{+\infty}\frac{y-\mu}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy+\int_{-a}^{+\infty}\frac{\mu}{\sigma}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}dy \end{aligned}$$

Letting $z=(y-\mu)/\sigma$, the above expression becomes

$$\begin{aligned} E\left[yI\left(y+a\right)\right] &= \sigma\int_{-\frac{a+\mu}{\sigma}}^{+\infty} z\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz+\mu\int_{-\frac{a+\mu}{\sigma}}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz \\ &= \sigma\left[-\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}\right]_{-\frac{a+\mu}{\sigma}}^{+\infty}+\mu\int_{-\infty}^{\frac{a+\mu}{b}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz \\ &= \sigma\phi\left(\frac{a+\mu}{\sigma}\right)+\mu\Phi\left(\frac{a+\mu}{\sigma}\right) \end{aligned}$$

Proof of the second part of Lemma 1.

$$E\left[\phi\left(\frac{y+a}{b}\right)\right]=\frac{b}{\sqrt{b^2+\sigma^2}}\phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right)$$

■

$$
\begin{aligned}
E\left[\phi\left(\frac{y+a}{b}\right)\right] &= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y+a}{b}\right)^2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dy \\
&= \frac{1}{\sigma^2\pi} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{\left(\sigma^2+b^2\right)y^2+\left(2a\sigma^2-2b^2\mu\right)y+a^2\sigma^2+b^2\mu^2}{b^2\sigma^2}\right)} dy \\
&= \frac{1}{\sigma^2\pi} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{\left(\sqrt{\sigma^2+b^2}y+A\right)^2-A^2+a^2\sigma^2+b^2\mu^2}{b^2\sigma^2}\right)} dy
\end{aligned}
$$

where $A = \frac{a\sigma^2-\mu b^2}{\sqrt{b^2+\sigma^2}}$.

Let $B = \frac{1}{2}\left(\frac{A^2-a^2\sigma^2-b^2\mu^2}{b^2\sigma^2}\right) = -\frac{1}{2}\frac{(a+\mu)^2}{b^2+\sigma^2}$,

$$
\begin{aligned}
E\left[\phi\left(\frac{y+a}{b}\right)\right] &= \frac{1}{\sigma^2\pi} e^B \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{\left(\sqrt{\sigma^2+b^2}y+A\right)^2}{b^2\sigma^2}\right)} dy \\
&= \frac{1}{\sigma^2\pi} e^B \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\frac{\sigma^2+b^2}{b^2\sigma^2}\left(y+\frac{a\sigma^2-\mu b^2}{b^2+\sigma^2}\right)^2} dy
\end{aligned}
$$

Setting $\omega = \frac{b\sigma}{\sqrt{b^2+\sigma^2}}$ and $\widetilde{\mu} = -\frac{a\sigma^2-\mu b^2}{b^2+\sigma^2}$, we now have

$$
\begin{aligned}
E\left[\phi\left(\frac{y+a}{b}\right)\right] &= \frac{1}{\sigma^2\pi} e^B \int_{-\infty}^{+\infty} e^{-\frac{1}{2\omega^2}(y-\widetilde{\mu})^2} dy \\
&= \frac{1}{\sigma^2\pi} e^B \omega\sqrt{2\pi} = \frac{b}{\sqrt{b^2+\sigma^2}} \frac{1}{\sqrt{2\pi}} e^B \\
&= \frac{b}{\sqrt{b^2+\sigma^2}} \phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right)
\end{aligned}
$$

Proof of the third part of Lemma 1.

$$
E\left(\Phi\left(\frac{y+a}{b}\right)\right) = \Phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right)
$$

■

$$E\left[\Phi\left(\frac{y+a}{b}\right)\right] = \int_{-\infty}^{+\infty}\int_{-\infty}^{\frac{y+a}{b}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}w} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dwdy$$

Stating that $\frac{z+y+a}{b} = w$, the expression above becomes

$$\begin{aligned} E\left[\Phi\left(\frac{y+a}{b}\right)\right] &= \int_{-\infty}^{+\infty}\int_{-\infty}^{0} \frac{1}{b\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z+y+a}{b}\right)^2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dzdy \\ &= \int_{-\infty}^{0} \frac{1}{b} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z+y+a}{b}\right)^2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} dydz \\ &= \int_{-\infty}^{0} \frac{1}{b} E\left[\phi\left(\frac{y+a+z}{b}\right)\right] dz \end{aligned}$$

Using the second part of Lemma 1,

$$\begin{aligned} E\left[\Phi\left(\frac{y+a}{b}\right)\right] &= \int_{-\infty}^{0} \frac{1}{b} \frac{b}{\sqrt{b^2+\sigma^2}} \phi\left(\frac{z+a+\mu}{\sqrt{b^2+\sigma^2}}\right) dz \\ &= \frac{1}{\sqrt{b^2+\sigma^2}} \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z+a+\mu}{\sqrt{b^2+\sigma^2}}\right)^2} dz \end{aligned}$$

Setting $\frac{z+a+\mu}{\sqrt{b^2+\sigma^2}} = \widetilde{z}$,

$$\begin{aligned} E\left[\Phi\left(\frac{y+a}{b}\right)\right] &= \frac{1}{\sqrt{b^2+\sigma^2}} \int_{-\infty}^{\frac{a+\mu}{\sqrt{b^2+\sigma^2}}} \frac{\sqrt{b^2+\sigma^2}}{\sqrt{2\pi}} e^{-\frac{1}{2}\widetilde{z}^2} d\widetilde{z} \\ &= \Phi\left(\frac{a+\mu}{\sqrt{b^2+\sigma^2}}\right) \end{aligned}$$

Proof of the uniqueness of $\tilde{f}_t$ (the non-linear cross section average estimator of f_t). Let

$$z_{it}(f_t) = \frac{d_{it}}{\sqrt{\sigma_c^2 + \sigma_\xi^2}},$$

and

$$\widetilde{\Delta p}_{it} = \frac{\Delta p_{it}}{\sqrt{\sigma_c^2+\sigma_\xi^2}},\ \tilde{\eta}_{it} = \frac{\eta_{it}}{\sqrt{\sigma_c^2+\sigma_\xi^2}},$$
$$\tilde{c} = \frac{c}{\sqrt{\sigma_c^2+\sigma_\xi^2}} \geq 0,\ \delta^2 = \frac{\sigma_\xi^2}{\sigma_c^2+\sigma_\xi^2} < 1,$$

and note that we have

$$\widetilde{\Delta p}_{it} = z_{it}(f_t) + z_{it}(f_t)\left[\Phi\left(z_{it}(f_t)-\tilde{c}\right) - \Phi\left(z_{it}(f_t)+\tilde{c}\right)\right] \tag{10}$$
$$+\delta^2\left[\phi\left(z_{it}(f_t)-\tilde{c}\right) - \phi\left(z_{it}(f_t)+\tilde{c}\right)\right] + \tilde{\eta}_{it}. \tag{11}$$

The cross-sectional average estimate of f_t is now given by the solution of the non-linear equation

$$\Psi(\tilde{f}_t) = \sum_{i=1}^{N} w_{it}\{z_{it}(\tilde{f}_t) + z_{it}(\tilde{f}_t)\left[\Phi\left(z_{it}(\tilde{f}_t)-\tilde{c}\right) - \Phi\left(z_{it}(\tilde{f}_t)+\tilde{c}\right)\right] \tag{12}$$
$$+\delta^2\left[\phi\left(z_{it}(\tilde{f}_t)-\tilde{c}\right) - \phi\left(z_{it}(\tilde{f}_t)+\tilde{c}\right)\right]\} - a_{Nt} \tag{13}$$
$$= 0, \tag{14}$$

where $a_{Nt} = \sum_{i=1}^{N} w_{it}\widetilde{\Delta p}_{it}$.

First it is clear that $\Psi(\tilde{f}_t)$ is a continuous and differentiable function of f_t, and it is now easily seen that

$$\lim_{f_t\to+\infty}\Psi(\tilde{f}_t)\to+\infty \text{ and } \lim_{f_t\to-\infty}\Psi(\tilde{f}_t)\to-\infty.$$

Also the first derivative of $\Psi(f_t)$ is given by[29]

$$\Psi'(\tilde{f}_t) = \frac{1}{\sqrt{\sigma_c^2+\sigma_\xi^2}}\sum_{i=1}^{N} w_{it}q_{it},$$

where

$$q_{it} = 1 + \Phi\left(z_{it}(\tilde{f}_t)-\tilde{c}\right) - \Phi\left(z_{it}(\tilde{f}_t)+\tilde{c}\right) + (1-\delta^2)h(z_{it}(\tilde{f}_t)),$$

[29] Recall that the weights, w_{it}, are non-zero pre-determined constants, and in particular do not depend on f_t.

and

$$h(z_{it}(\tilde{f}_t)) = z_{it}(\tilde{f}_t)\left[\phi\left(z_{it}(\tilde{f}_t) - \tilde{c}\right) - \phi\left(z_{it}(\tilde{f}_t) + \tilde{c}\right)\right].$$

But since $1 - \Phi\left(z_{it}(\tilde{f}_t) + \tilde{c}\right) = \Phi\left(-z_{it}(\tilde{f}_t) - \tilde{c}\right)$, then

$$1 + \Phi\left(z_{it}(\tilde{f}_t) - \tilde{c}\right) - \Phi\left(z_{it}(\tilde{f}_t) + \tilde{c}\right) = \Phi\left(z_{it}(\tilde{f}_t) - \tilde{c}\right) + \Phi\left(-z_{it}(\tilde{f}_t) - \tilde{c}\right) > 0,$$

and it is easily seen that $h(z_{it}(\tilde{f}_t))$ is symmetric, namely $h(z_{it}(\tilde{f}_t)) = h(-z_{it}(\tilde{f}_t))$. Focusing on the non-negative values of $z_{it}(\tilde{f}_t)$ it is easily seen that

$$h(z_{it})) = \frac{z_{it}}{\sqrt{2\pi}}\left[e^{-0.5(z_{it}-\tilde{c})^2} - e^{-0.5(z_{it}+\tilde{c})^2}\right] > 0 \text{ for } \tilde{c} > 0,$$

and by symmetry $h(z_{it})) \geq 0$, for all $\tilde{c} \geq 0$. Hence, $q_{it} > 0$ for all i and t, and $\tilde{c} \geq 0$. Therefore, it also follows that $\Psi'(f_t) > 0$, for all value of $w_{it} \geq 0$ and $c \geq 0$. Thus, by the fixed point theorem, $\Psi(f_t)$ must cut the horizontal axis but only once.

Proof of the consistency of $\tilde{f}_t$ as an estimator of f_t as $N \to \infty$.

Let

$$\begin{aligned}\Psi(f_t) \quad &= \quad \sum_{i=1}^{N} w_{it}\left\{z_{it}(f_t) + z_{it}(f_t)\left[\Phi\left(z_{it}(f_t) - \tilde{c}\right) - \Phi\left(z_{it}(f_t) + \tilde{c}\right)\right]\right. \\ &\qquad \left. + \delta^2\left[\phi\left(z_{it}(f_t) - \tilde{c}\right) - \phi\left(z_{it}(f_t) + \tilde{c}\right)\right]\right\} - a_{Nt},\end{aligned}$$

and note that

$$\Psi(f_t) = -\sum_{i=1}^{N} w_{it}\eta_{it}.$$

Consider now the mean-value expansion of $\Psi(f_t)$ around $\tilde{f}_t$

$$\Psi(f_t) - \Psi(\tilde{f}_t) = \Psi'(\bar{f}_t)(f_t - \tilde{f}_t),$$

where $\bar{f}_t$ lies on the line segment between f_t and $\tilde{f}_t$. Since $\Psi(\tilde{f}_t) = 0$ and $\Psi'(\bar{f}_t) > 0$ for all $\bar{f}_t$ (as established above) we have

$$\tilde{f}_t - f_t = \frac{-\sum_{i=1}^{N} w_{it}\tilde{\eta}_{it}}{\Psi'(\bar{f}_t)}.$$

Recall that $\tilde{\eta}_{it} = \left(\sigma_c^2 + \sigma_\xi^2\right)^{-1/2}\left[\Delta p_{it} - E\left(\Delta p_{it}\left|\mathbf{h}_{it}\right.\right)\right]$, where $\mathbf{h}_{it} = (f_t, \mathbf{x}_{it}, p_{i,t-1})$, and hence $E\left(\tilde{\eta}_{it}\right) = 0$. Further, conditional on f_t and $\mathbf{x}_{it}$, price changes, Δp_{it}, being functions of independent shocks v_i and ε_{it} over i, will be cross sectionally independent. Therefore, η_{it} will also be cross sectionally independent; although they need not be identically distributed even if the underlying shocks, v_i and ε_{it}, are identically distributed over i.

Given the above results we now have (for each t and as $N \to \infty$)

$$\left(\sum_{i=1}^{N} w_{it}^2\right)^{-1/2} \left(\tilde{f}_t - f_t\right) \backsim N\left(0, \vartheta_{\tilde{f}}^2\right),$$

where

$$\vartheta_{\tilde{f}}^2 = \lim_{N\to\infty} \left\{ \frac{\left(\sum_{i=1}^{N} w_{it}^2\right)^{-1} \sum_{i=1}^{N} w_{it}^2 Var(\tilde{\eta}_{it})}{\left[\Psi'(f_t)\right]^2} \right\}.$$

Note that as $N \to \infty$, $\sum_{i=1}^{N} w_{it}\tilde{\eta}_{it} \xrightarrow{p} 0$, and hence $\tilde{f}_t \xrightarrow{p} f_t$, since $\Psi'(f_t) > 0$ for all f_t.It must also be that $\bar{f}_t \xrightarrow{p} f_t$.

In the case where $w_{it} = 1/N$, we have

$$\vartheta_{\tilde{f}}^2 = \lim_{N\to\infty} \left\{ \frac{N^{-1} \sum_{i=1}^{N} Var(\tilde{\eta}_{it})}{\left[\Psi'(f_t)\right]^2} \right\}.$$

It also follows that $\tilde{f}_t - f_t = O_p\left(N^{-1/2}\right)$. ■

Appendix B: Data Sources

The Belgian CPI data set :

The Belgian CPI data set contains monthly individual price reports collected by the Federal Public Service "Economy, SMEs, Self-Employed and Energy" for the computation of the Belgian National and Harmonized Index of Consumer Prices. In its complete version, it covers the 1989:01 - 2005:12 period. Considering the whole sample, would have involved analyzing more than 20,000,000 price records. For this project, we restricted the analysis to the product categories included in the Belgian CPI basket for the base year 1996, and restricted our period of observation to the 1994:07 - 2003:02 period. Our data set covers only the product categories for which the prices are recorded throughout the entire year in a decentralized way, i.e. 65.5%. of the Belgian CPI basket for the base year 1996. The remaining 34.5% relate to product categories that are monitored centrally by the Federal Public Services, such as housing rents, electricity, gas, telecommunications, health care, newspapers and insurance services and to product categories that are not available for sale during the entire year (some fruits and vegetables, winter and summer fees in tennis club). Price reports take into account all types of rebates and promotions, except those relating to the winter and summer sales period, which typically take place in January and July. In addition to the price records, the Belgian CPI data sets provides information on the location of the seller, a seller identifier, the packaging of the product (in order to identify promotions in quantity) and the brand of the product. For all products, the price concept used in this paper correspond to the log of price per unit.

The French CPI data set :

The French CPI data set contains more than 13,000,000 monthly individual price records collected by the INSEE for the computation of the French National and Harmonized Index of Consumer Prices. It covers the period July 1994:07 - February 2003. This data set covers 65.5%. of the French CPI basket. Indeed, the prices of some categories of goods and services are not available in our sample: centrally collected prices - of which major items are car prices and administered or public utility prices (e.g. electricity)- as well as other types of products such as fresh food and rents. At the COICOP 5-digit level, we have access to 128 product categories out of 160 in the CPI. As a result, the coverage rate is above 70% for food and non-energy industrial goods, but closer to 50% in the services, since a large part of services prices are centrally collected, e.g. for transport or administrative or financial services.

Each individual price quote consists of the exact price level of a precisely defined product. What is meant by “product” is a particular product, of a particular brand

and quality, sold in a particular outlet. The individual product identification number allows us to follow the price of a product through time, and to recover information on the type of outlet (hypermarket, supermarket, department store, specialized store, corner shop, service shop, etc.), the category of product and the regional area where the outlet is located (for confidentiality reasons, a more precise location of outlets was not made available to us). The sequences of records corresponding to such defined individual products are referred to as price trajectories. Importantly, if in a given outlet a given product is definitively replaced by a similar product of another brand or of a different quality, a new identification number is created, and a new price trajectory is started. On top of the above mentioned information, the following additional information is recorded : the year and month of the record, a qualitative "type of record" code and (when relevant) the quantity sold. When relevant, division by the indicator of the quantity is used in order to recover a consistent price per unit. The "type of record" code indicates the nature of the price recorded: regular price, sales or rebates, or "pseudo-observation" (a "pseudo-observation" is essentially an observation which has been imputed by the INSEE; see Baudry *et al.* (2004) for more details on the way we have tackled such imputed values to avoid creating "false" price changes).

Confidentiality data restrictions

Due to strong confidentiality restrictions, we are not allowed to provide anyone with the micro price reports underlying this work. However, a data set containing simulated data and the MatLab code of the estimation procedures are available on request (emmanuel.dhyne@nbb.be). A SAS code is also available.

Appendix C - Detailed Results

Description of Tables A and B

Columns (2) to (6) refer to the results obtained by Full ML :

- c represents the estimated value of the average menu cost ;

- sig_e represents the estimated value of σ_ε ;

- sig_c represents the estimated value of σ_c ;

- sig_u represents the estimated value of σ_ν ;

- *Logl* represents the maximized value of the likelihood function ;

Columns (7) and (8) refer to the results associated to the time-series representation of f_t.

- sig_ω represents the estimated value of σ_ω;

- $S(rho_k)$ represents the estimated value of $\overline{\rho} = \sum_{i=1}^{K} \rho_i$

Column (9) present the correlation between f_t and the log of the product category price index.

Columns (10) to (13) provide descriptive statistics of the data set (the average number of observations each month, $Nbar$, the frequency of price changes, $Freq$, the average size of price changes in absolute term, $|Dp|$, and the share of price increases, $\%up$.

Columns (14) to (16) provide averages of the frequency of price changes, $Freq^*$, the average size of price changes in absolute term, $|Dp|^*$, and the share of price increases, $\%up^*$ obtained on the basis of simulated data generated using the estimated structural parameters and the estimated f_t of each product categories. The simulation exercise is replicated 1000 times.

Grey cells indicate product categories for which the model fits the data poorly (low correlation of $f_t$with the log of price index or with $\overline{p}_t$ or poor replication of the data characteristics by simulated data).

Description of Tables C and D

Columns (2) to (8) provide basic statistics describing the estimated f_t :

- σ_f represents the unconditional standard deviation;

- r_i represents the autocorrelation of order i.

Product category	Estimated value of								Observed data				Simulated data		
	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,ln(IP)}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Energy															
Butane	0.007**	0.040**	0.006**	0.215**	6560	0.028	0.880	0.999	128	0.742	0.029	0.539	0.909	0.055	0.530
Gasoline 1000-2000 liters	0.025**	0.036**	0.011**	0.040**	13885	0.063	0.930	1.000	144	0.730	0.073	0.545	0.747	0.080	0.538
Eurosuper (RON95)	0.009**	0.014**	0.002**	0.019**	41334	0.022	0.790	1.000	247	0.720	0.027	0.521	0.771	0.030	0.528
Perisable food															
Paprika pepper	0.046**	0.202**	0.032**	0.117**	-14700	0.145	0.774	0.998	443	0.842	0.282	0.530	0.891	0.305	0.510
Skate (wing)	0.038**	0.141**	0.034**	0.145**	-9162	0.029	0.657	0.985	183	0.688	0.136	0.524	0.845	0.186	0.505
Oranges	0.079**	0.159**	0.063**	0.109**	-27424	0.040	0.734	0.994	447	0.619	0.183	0.528	0.731	0.232	0.505
Carrots	0.114**	0.173**	0.088**	0.125**	-28928	0.085	0.751	0.998	443	0.574	0.224	0.516	0.669	0.275	0.503
Apples, Granny Smith	0.088**	0.126**	0.068**	0.075**	-21021	0.053	0.744	0.998	443	0.564	0.170	0.542	0.649	0.200	0.504
Kiwis	0.141**	0.203**	0.112**	0.135**	-35651	0.046	0.863	0.996	443	0.542	0.244	0.531	0.639	0.310	0.507
Margarine (super)	0.135**	0.046**	0.087**	0.132**	-16650	0.010	0.913	0.885	438	0.189	0.053	0.616	0.196	0.080	0.507
Turkey filet	0.282**	0.098**	0.159**	0.114**	-16682	0.018	0.396	0.959	448	0.154	0.141	0.561	0.172	0.182	0.516
Sirloin	0.166**	0.058**	0.094**	0.096**	-16897	0.011	0.369	0.906	509	0.149	0.082	0.568	0.173	0.107	0.515
Cheese (Gouda type)	0.343**	0.115**	0.190**	0.168**	-23328	0.019	0.833	0.906	491	0.143	0.168	0.558	0.160	0.214	0.509
Full-fat fruit yoghurt	0.276**	0.080**	0.162**	0.195**	-16238	0.011	0.423	0.913	414	0.141	0.090	0.538	0.145	0.140	0.487
Butter	0.171**	0.050**	0.097**	0.105**	-15028	0.012	0.725	0.947	474	0.132	0.067	0.602	0.146	0.092	0.521
Emmentaler	0.285**	0.087**	0.155**	0.138**	-13823	0.021	0.801	0.903	353	0.126	0.124	0.564	0.142	0.165	0.514
Sausage	0.390**	0.117**	0.212**	0.099**	-20864	0.013	0.902	0.983	496	0.113	0.149	0.621	0.137	0.217	0.524
Cheese (Edam type)	0.322**	0.086**	0.173**	0.135**	-12950	0.017	0.805	0.967	334	0.109	0.112	0.635	0.119	0.160	0.531
Belgian waffle	0.400**	0.088**	0.212**	0.230**	-12810	0.019	0.407	0.788	441	0.094	0.112	0.564	0.094	0.159	0.530
Country paté	0.396**	0.098**	0.203**	0.133**	-17209	0.018	0.631	0.960	484	0.090	0.130	0.637	0.100	0.184	0.541
Rice pudding	0.457**	0.075**	0.216**	0.218**	-5724	0.024	0.789	0.924	283	0.053	0.096	0.771	0.054	0.143	0.608
Pastry (carré glacé)	0.391**	0.059**	0.172**	0.103**	-4509	0.019	0.929	0.966	263	0.041	0.095	0.832	0.042	0.123	0.677
Pastry (éclair)	0.444**	0.070**	0.194**	0.101**	-4731	0.031	0.505	0.904	263	0.040	0.105	0.788	0.042	0.148	0.624
Swiss cake	0.506**	0.065**	0.223**	0.267**	-4189	0.021	-0.093	0.932	278	0.036	0.091	0.835	0.034	0.125	0.614
Whole wheat bread	0.129**	0.020**	0.055**	0.140**	-2857	0.013	0.777	0.936	269	0.033	0.037	0.927	0.044	0.049	0.629
Special bread	0.398**	0.031**	0.181**	0.468**	-3046	0.027	0.662	0.773	298	0.028	0.047	0.932	0.029	0.067	0.597
Bread roll	0.583**	0.072**	0.242**	0.157**	-4830	0.017	0.887	0.965	269	0.026	0.128	0.824	0.027	0.152	0.717
Non perishable food															
Frankfurters	0.237**	0.071**	0.154**	0.142**	-17986	0.017	0.775	0.860	369	0.175	0.076	0.587	0.176	0.122	0.512
Biscuits	0.225**	0.067**	0.146**	0.188**	-22331	0.019	0.863	0.985	444	0.175	0.076	0.620	0.175	0.116	0.550
Fruit juice	0.255**	0.080**	0.153**	0.235**	-22172	0.018	0.769	0.952	475	0.162	0.106	0.526	0.167	0.144	0.516
Fishcakes	0.282**	0.081**	0.161**	0.175**	-11923	0.027	0.717	0.913	377	0.143	0.123	0.574	0.145	0.151	0.529
Val de Loire wine	0.310**	0.086**	0.182**	0.216**	-14752	0.007	0.923	0.965	349	0.136	0.101	0.598	0.140	0.149	0.516

Table A - Estimation Results - Belgium

Product category	Estimated value of								Observed data				Simulated data		
	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,ln(IP)}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Ice cream	0.321**	0.090**	0.176**	0.208**	-11697	0.025	0.805	0.961	318	0.126	0.136	0.557	0.133	0.170	0.527
Tinned apricot halves	0.284**	0.076**	0.156**	0.161**	-11960	0.019	0.827	0.939	398	0.118	0.099	0.579	0.125	0.140	0.529
Tinned tomatoes, peeled	0.450**	0.107**	0.252**	0.320**	-16043	0.025	0.662	0.964	457	0.113	0.128	0.621	0.113	0.192	0.542
Tinned peas	0.363**	0.094**	0.196**	0.228**	-14878	0.020	0.860	0.962	465	0.112	0.128	0.594	0.117	0.173	0.536
Tobacco	0.106**	0.012**	0.056**	0.185**	-241	0.006	0.719	0.999	243	0.098	0.035	0.995	0.088	0.040	0.870
Sausage	0.444**	0.112**	0.233**	0.180**	-18770	0.007	0.962	0.998	479	0.093	0.134	0.648	0.105	0.205	0.529
Lemonade	0.431**	0.089**	0.212**	0.183**	-6497	0.024	0.737	0.536	295	0.068	0.106	0.627	0.070	0.157	0.497
Non durable goods															
Roses	0.078**	0.180**	0.034**	0.210**	-7743	0.044	1.190	0.991	160	0.678	0.218	0.523	0.781	0.270	0.515
Chrysanthemums	0.082**	0.152**	0.041**	0.150**	-5542	0.041	0.711	0.988	150	0.622	0.192	0.519	0.725	0.235	0.503
Compact Disc	0.150**	0.064**	0.097**	0.070**	-7501	0.013	0.912	0.952	173	0.217	0.083	0.520	0.240	0.113	0.513
Hair spray	0.102**	0.140**	0.157**	0.165**	-28623	0.005	0.722	0.945	363	**0.154**	**0.063**	0.570	**0.599**	**0.200**	0.503
Cat food	0.2122**	0.066**	0.1208**	0.1619**	-13341	0.019	0.913	0.868	371	0.148	0.097	0.515	0.155	0.122	0.500
Nail polish	0.317**	0.064**	0.171**	0.172**	-7156	0.015	0.873	0.990	255	0.094	0.072	0.726	0.093	0.118	0.618
Water-based paint	0.349**	0.053**	0.182**	0.169**	-6837	0.007	0.951	0.997	217	0.069	0.058	0.860	0.068	0.097	0.643
Oil-based paint	0.400**	0.061**	0.206**	0.192**	-4642	0.005	0.825	0.997	185	0.066	0.061	0.826	0.062	0.104	0.595
Water charge	0.488**	0.067**	0.242**	0.643**	-2316	0.026	0.598	0.839	69	0.059	0.057	0.875	0.056	0.130	0.637
Engine oil	0.575**	0.082**	0.272**	0.246**	-5767	0.004	0.956	0.995	210	0.047	0.079	0.839	0.047	0.151	0.652
Dracaena	0.613**	0.087**	0.282**	0.441**	-3510	0.004	0.770	0.927	131	0.044	0.071	0.637	0.039	0.150	0.533
Dry battery	0.933**	0.129**	0.416**	0.354**	-7859	0.007	0.955	0.982	251	0.040	0.126	0.764	0.038	0.247	0.671
Wool suit	0.405**	0.052**	0.188**	0.224**	-4645	0.002	0.660	0.757	186	0.040	0.039	0.681	0.037	0.086	0.521
Infants' anorak (9 month)	0.148**	0.055**	0.102**	0.187**	-7958	0.004	0.819	-0.627	185	**0.030**	0.073	0.570	**0.221**	0.092	0.510
Men's socks	0.500**	0.068**	0.203**	0.254**	-5611	0.003	0.942	0.991	239	0.030	0.073	0.721	0.025	0.137	0.664
Dress fabric	0.115**	0.058**	0.044**	0.143**	-5869	0.003	0.819	0.991	139	**0.029**	**0.035**	0.803	**0.213**	**0.124**	0.512
Men's T shirt	0.170**	0.087**	0.131**	0.225**	-11079	0.004	0.887	0.941	232	**0.028**	0.103	0.581	**0.312**	0.144	0.505
Color film, 135-24	0.315**	0.045**	0.131**	0.148**	-3902	0.002	0.864	0.537	174	0.027	0.056	0.460	0.027	0.082	0.509
Zip fastener	0.210**	0.022**	0.085**	0.063**	-3486	0.008	0.666	0.977	204	0.024	0.048	0.828	0.023	0.054	0.728
Durable goods															
LaserJet printer	0.4887**	0.1129**	0.3071**	0.221**	-3595	0.042	0.774	0.568	68	0.141	0.084	0.458	0.138	0.197	0.458
VCR, four-head	0.596**	0.096**	0.311**	0.208**	-6071	0.029	0.748	0.986	192	0.078	0.097	0.275	0.074	0.186	0.324
Compact hi-fi system	0.587**	0.089**	0.293**	0.250**	-6255	0.006	0.994	0.993	185	0.062	0.077	0.368	0.059	0.162	0.354
Natural gas heater	0.320**	0.046**	0.160**	0.150**	-4249	0.018	0.653	0.981	165	0.062	0.052	0.861	0.061	0.092	0.662
Calculator	0.727**	0.134**	0.352**	0.305**	-5023	0.007	1.005	0.958	152	0.057	0.124	0.506	0.062	0.240	0.471
Toaster	0.395**	0.059**	0.193**	0.174**	-6618	0.005	0.941	0.871	215	0.056	0.064	0.560	0.051	0.100	0.495
Suitcase	0.554**	0.063**	0.283**	0.186**	-3967	0.008	0.845	0.981	115	0.056	0.061	0.619	0.049	0.102	0.571

Table A - Continued

Product category	Estimated value of								Observed data				Simulated data		
	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,ln(IP)}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Electric coffee machine	0.443**	0.070**	0.219**	0.203**	-6477	0.005	0.900	0.769	225	0.056	0.061	0.568	0.055	0.118	0.499
Children's bicycle	0.458**	0.066**	0.221**	0.158**	-3444	0.020	0.419	0.961	154	0.054	0.066	0.797	0.052	0.124	0.637
Electric fryer	0.553**	0.080**	0.264**	0.221**	-6328	0.003	0.968	0.562	221	0.049	0.066	0.619	0.046	0.135	0.534
Dictionary	0.583**	0.100**	0.259**	0.324**	-3930	0.033	0.659	0.871	162	0.046	0.157	0.394	0.049	0.208	0.516
Bed, slatted base	0.538**	0.065**	0.248**	0.269**	-3797	0.018	0.577	0.848	163	0.040	0.056	0.729	0.036	0.115	0.509
Stainless steel pan	0.609**	0.082**	0.277**	0.365**	-5374	0.004	0.905	0.992	215	0.037	0.067	0.718	0.037	0.143	0.577
Hammer	0.888**	0.093**	0.406**	0.263**	-5173	0.016	0.687	0.964	185	0.036	0.065	0.734	0.032	0.161	0.645
Glass, 4 mm	0.422**	0.055**	0.185**	0.152**	-1590	0.009	0.933	0.992	100	0.035	0.078	0.847	0.036	0.117	0.668
Dining room oak furniture	0.105**	0.125**	0.162**	0.161**	-14540	0.010	0.894	0.860	168	**0.032**	0.040	0.813	**0.566**	0.180	0.513
Spherical glasses	0.641**	0.074**	0.293**	0.219*	-3577	0.007	0.549	0.926	157	0.032	0.056	0.735	0.032	0.123	0.562
Wallet	0.140**	0.047**	0.085**	0.177**	-5672	0.005	0.891	0.978	162	**0.032**	0.050	0.836	**0.182**	0.084	0.564
Torus glasses	0.502**	0.055**	0.223**	0.212**	-4014	0.015	-0.003	0.866	159	0.031	0.055	0.635	0.028	0.097	0.599
Cup and saucer	0.109**	0.086**	0.167**	0.163**	-16952	0.005	0.880	0.974	210	**0.030**	0.071	0.680	**0.469**	0.122	0.516
Services															
School boarding fees	0.118**	0.019**	0.061**	0.100**	-2762	0.005	0.676	0.994	141	0.074	0.024	0.954	0.077	0.040	0.707
Hourly wage, painter	0.261**	0.033**	0.127**	0.167**	-2754	0.010	0.544	0.984	129	0.055	0.040	0.814	0.051	0.069	0.727
Hourly wage, garage mechanic	0.357**	0.049**	0.171**	0.140**	-4289	0.004	0.965	0.999	183	0.053	0.059	0.963	0.052	0.101	0.731
Annual cable subscription	0.133**	0.019**	0.062**	0.068**	-1187	0.013	0.711	0.878	66	0.051	0.029	0.844	0.055	0.047	0.674
Repair of central heating	0.371**	0.068**	0.175**	0.153**	-3142	0.004	0.855	0.995	123	0.051	0.053	0.752	0.059	0.128	0.602
Hourly wage, plumber	0.308**	0.043**	0.148**	0.146	-2826	0.006	0.735	0.997	132	0.051	0.050	0.745	0.050	0.083	0.675
Passport stamp	0.208**	0.026**	0.082**	0.067**	-351	0.033	0.874	0.991	60	0.042	0.132	0.957	0.055	0.138	0.844
Sole meunière	0.429**	0.053**	0.194**	0.205**	-3313	0.019	0.530	0.950	153	0.040	0.066	0.811	0.038	0.106	0.681
Dry cleaning, shirt	0.520**	0.069**	0.232**	0.18	-3934	0.005	0.995	0.996	147	0.036	0.068	0.874	0.035	0.127	0.637
Pepper steak	0.359**	0.041**	0.156**	0.134**	-2705	0.004	0.978	0.996	160	0.034	0.053	0.892	0.033	0.082	0.715
Permanent wave	0.5937**	0.064**	0.266**	0.274**	-4164	0.003	0.919	0.989	198	0.034	0.066	0.901	0.031	0.121	0.699
Domestic service	0.404**	0.045**	0.179**	0.127**	-4669	0.006	0.824	0.980	143	0.033	0.050	0.834	0.032	0.092	0.726
Funeral	0.327**	0.033**	0.145**	0.138**	-2078	0.019	-0.498	0.912	118	0.033	0.037	0.951	0.032	0.074	0.695
School lunch	0.505**	0.062**	0.222**	0.187**	-3612	0.006	0.952	0.997	147	0.033	0.081	0.855	0.033	0.123	0.703
Self-service meal	0.285**	0.030**	0.124**	0.139**	-1713	0.019	0.331	0.573	94	0.033	0.045	0.729	0.028	0.062	0.545
Parking spot in a garage	0.126**	0.037**	0.146**	0.185**	-7994	0.006	0.944	0.960	147	**0.032**	0.059	0.959	**0.290**	0.053	0.568
Wheel balancing	0.756**	0.109**	0.332**	0.278**	-5461	0.003	0.950	0.984	179	0.032	0.075	0.702	0.034	0.193	0.533
Special beer	0.545**	0.054**	0.239**	0.146**	-3426	0.009	0.939	0.992	221	0.030	0.084	0.876	0.028	0.110	0.743
Aperitif	0.486**	0.051**	0.210**	0.191**	-4277	0.006	0.942	0.997	227	0.029	0.084	0.879	0.029	0.111	0.764
Videotape rental	0.639**	0.060**	0.248**	0.240**	-2670	0.005	0.889	0.868	116	0.018	0.085	0.550	0.012	0.103	0.535

Table A - Continued

Product category	Estimated value of								Observed data				Simulated data		
	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,IP}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Energy															
Eurosuper	0.004	0.018	0.003	0.026	183835	0.016	0.792	0.993	1267	0.799	0.020	0.574	0.898	0.027	0.544
Gas-Oil	0.007	0.034	0.005	0.284	41328	0.019	0.796	0.987	505	0.798	0.026	0.570	0.887	0.043	0.534
Perishable food															
Roast-beef	0.225	0.096	0.147	0.196	-100706	0.009	0.742	0.985	1540	0.210	0.100	0.589	0.211	0.157	0.460
Beef burger	0.235	0.095	0.146	0.257	-21591	0.015	0.716	0.942	368	0.195	0.113	0.558	0.194	0.159	0.460
Lamb	0.257	0.117	0.173	0.300	-45846	0.017	0.933	0.994	659	0.233	0.131	0.580	0.237	0.196	0.468
Fresh pork meat	0.278	0.151	0.196	0.203	-71955	0.029	0.909	0.694	915	0.270	0.182	0.549	0.285	0.248	0.449
Ham	0.228	0.130	0.163	0.281	-78612	0.017	0.921	0.976	976	0.287	0.152	0.559	0.297	0.210	0.457
Sausages	0.297	0.128	0.196	0.411	-32303	0.015	0.946	0.889	440	0.215	0.136	0.566	0.214	0.209	0.460
Chicken	0.163	0.093	0.119	0.317	-71243	0.022	0.955	0.961	971	0.257	0.122	0.556	0.319	0.160	0.453
Rabbit/Game	0.123	0.115	0.100	0.105	-14314	0.023	0.870	0.920	204	0.436	0.148	0.536	0.477	0.182	0.469
Creme fraiche	0.160	0.071	0.113	0.312	-14694	0.006	0.971	0.756	231	0.211	0.063	0.578	0.242	0.118	0.489
Milky Desserts	0.140	0.054	0.096	0.237	-12667	0.010	0.900	0.964	226	0.218	0.049	0.579	0.211	0.091	0.492
Cottage cheese	0.153	0.068	0.107	0.327	-26206	0.008	0.950	0.993	423	0.239	0.062	0.583	0.233	0.109	0.494
Processed cheese	0.132	0.066	0.097	0.385	-5277	0.015	0.955	0.978	84	0.275	0.061	0.632	0.269	0.106	0.499
Butter	0.151	0.084	0.111	0.138	-37833	0.007	0.941	0.995	508	0.257	0.074	0.594	0.278	0.130	0.529
Non perishable food															
Rusks and grilled breads	0.217	0.083	0.140	0.222	-7804	0.014	0.880	0.883	129	0.186	0.080	0.592	0.187	0.137	0.508
Flour	0.164	0.067	0.109	0.285	-12644	0.010	0.912	0.969	219	0.213	0.067	0.581	0.208	0.110	0.480
Pasta	0.123	0.126	0.236	0.321	-36192	0.016	0.960	0.793	323	0.178	0.071	0.575	0.529	0.206	0.508
Canned vegetables	0.279	0.094	0.174	0.320	-60997	0.008	0.946	0.946	1007	0.169	0.089	0.567	0.164	0.158	0.493
Sugar	0.126	0.031	0.075	0.096	-7143	0.005	0.894	0.965	193	0.170	0.029	0.713	0.125	0.065	0.559
Chocolate	0.188	0.076	0.130	0.233	-26720	0.010	0.837	0.984	381	0.212	0.063	0.635	0.212	0.126	0.537
Desserts	0.210	0.057	0.127	0.314	-2489	0.021	0.827	0.942	51	0.148	0.055	0.592	0.140	0.104	0.523
Coffee	0.202	0.087	0.142	0.233	-37938	0.011	0.907	0.933	544	0.232	0.077	0.472	0.238	0.150	0.508
Tea	0.181	0.051	0.116	0.248	-5079	0.013	0.639	0.991	92	0.174	0.041	0.678	0.162	0.094	0.530
Fruit juices	0.192	0.072	0.123	0.228	-11853	0.011	0.455	0.920	205	0.191	0.075	0.552	0.190	0.122	0.480
Whisky	0.070	0.037	0.056	0.103	-8223	0.007	0.553	0.437	153	0.303	0.029	0.519	0.294	0.058	0.484
Pet food	0.265	0.083	0.177	0.352	-18344	0.044	1.010	0.913	258	0.180	0.047	0.577	0.183	0.151	0.483
Non durable goods															
Fabrics	0.610	0.120	0.281	0.591	-3847	0.049	-0.597	0.516	124	0.066	0.194	0.546	0.054	0.230	0.433
Men coats	0.317	0.102	0.146	0.405	-2173	0.037	0.179	0.769	61	0.132	0.231	0.506	0.124	0.234	0.428
Men suits	0.333	0.113	0.168	0.355	-1922	0.036	0.709	0.726	45	0.167	0.235	0.492	0.159	0.251	0.417
Men trousers	0.411	0.121	0.207	0.331	-10094	0.031	-0.053	0.873	243	0.119	0.199	0.532	0.107	0.231	0.410

Table B - Estimation Results - France

	Estimated value of								Observed data				Simulated data		
Product category	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,IP}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Skirt	0.457	0.139	0.220	0.508	-2550	0.049	0.445	0.903	60	0.139	0.308	0.511	0.129	0.319	0.402
Dress	0.561	0.164	0.268	0.753	-996	0.094	0.663	0.544	23	0.145	0.391	0.483	0.130	0.403	0.397
Women trousers	0.456	0.128	0.239	0.378	-7394	0.040	0.187	0.856	164	0.119	0.195	0.528	0.109	0.240	0.401
Women jacket	0.451	0.136	0.220	0.491	-2181	0.054	0.739	0.816	51	0.143	0.302	0.515	0.130	0.311	0.402
Children trousers	0.467	0.138	0.247	0.356	-5849	0.037	0.652	0.502	122	0.129	0.212	0.495	0.118	0.256	0.405
Children suits	0.551	0.078	0.255	0.455	-160	0.186	0.191	0.503	6	0.110	0.329	0.453	0.092	0.371	0.422
Men shirts	0.452	0.140	0.231	0.361	-8103	0.033	-0.429	0.824	182	0.138	0.258	0.499	0.128	0.284	0.409
Men socks	0.521	0.102	0.251	0.399	-3021	0.042	0.269	0.536	88	0.071	0.128	0.579	0.057	0.181	0.441
Men sweater	0.527	0.136	0.269	0.625	-8609	0.038	0.234	0.845	196	0.104	0.196	0.544	0.090	0.245	0.410
Women sweater	0.510	0.133	0.244	0.689	-4467	0.056	-0.530	0.686	113	0.101	0.256	0.511	0.090	0.274	0.395
Children sweater	0.535	0.136	0.261	0.528	-3072	0.065	0.774	0.354	75	0.102	0.243	0.513	0.089	0.272	0.413
Babies clothes	0.747	0.124	0.361	0.663	-1328	0.089	0.610	0.279	35	0.079	0.208	0.505	0.062	0.281	0.421
Men shoes	0.526	0.116	0.263	0.449	-7382	0.039	-0.213	0.803	195	0.088	0.161	0.569	0.076	0.215	0.425
Women shoes	0.534	0.134	0.266	0.408	-8893	0.038	0.846	0.518	223	0.105	0.234	0.500	0.094	0.274	0.411
Children shoes	0.585	0.140	0.282	0.346	-3179	0.049	-0.753	0.737	87	0.095	0.244	0.544	0.082	0.285	0.405
Blankets and coverlets	0.392	0.105	0.200	0.569	-6685	0.028	-0.094	0.562	163	0.112	0.157	0.580	0.094	0.187	0.469
Fabrics for furniture	0.463	0.091	0.235	0.489	-5575	0.033	0.093	0.515	145	0.085	0.109	0.619	0.070	0.167	0.476
Batteries	0.309	0.077	0.186	0.277	-15706	0.013	0.714	0.767	299	0.139	0.067	0.536	0.128	0.145	0.501
Car tyres	0.176	0.070	0.122	0.229	-17631	0.013	0.930	0.977	286	0.248	0.071	0.507	0.235	0.130	0.458
Musical disks	0.240	0.083	0.161	0.308	-16969	0.009	0.882	-0.857	277	0.124	0.106	0.554	0.197	0.160	0.444
Blank tapes and disks	0.364	0.086	0.199	0.379	-10519	0.016	0.237	0.560	277	0.105	0.073	0.503	0.089	0.145	0.502
Flowers	0.167	0.086	0.121	0.398	-4269	0.019	-0.674	0.880	64	0.273	0.083	0.527	0.285	0.143	0.503
Children books	0.363	0.060	0.186	0.408	-5134	0.020	0.588	0.949	150	0.076	0.049	0.728	0.063	0.113	0.523
Newspapers	0.100	0.012	0.043	0.036	-703	0.013	0.813	0.961	86	0.050	0.036	0.893	0.048	0.042	0.752
Paper articles	0.511	0.126	0.285	0.498	-11130	0.035	0.925	0.836	217	0.116	0.132	0.598	0.107	0.228	0.498
Leather articles	0.365	0.077	0.188	0.404	-6323	0.031	0.424	0.759	165	0.094	0.095	0.585	0.078	0.146	0.439
Babies apparel	0.324	0.078	0.176	0.334	-2970	0.030	0.027	0.816	65	0.111	0.092	0.626	0.098	0.142	0.461
Durable goods															
box-mattress	0.259	0.104	0.148	0.412	-3811	0.028	0.560	-0.282	72	0.209	0.166	0.558	0.191	0.190	0.444
Armchairs and canapes	0.267	0.097	0.166	0.481	-14930	0.022	0.916	-0.699	249	0.195	0.115	0.556	0.178	0.161	0.460
Washing machine	0.208	0.049	0.113	0.231	-3913	0.017	0.655	0.898	107	0.110	0.061	0.443	0.098	0.120	0.405
Vacuum-cleaner	0.362	0.083	0.190	0.431	-5255	0.026	0.687	0.416	125	0.106	0.092	0.439	0.086	0.146	0.423
Electrical tools	0.327	0.069	0.178	0.436	-5529	0.025	0.821	0.055	126	0.110	0.064	0.538	0.086	0.123	0.462
Bicycles	0.258	0.063	0.146	0.309	-3859	0.026	0.676	0.666	81	0.136	0.070	0.589	0.114	0.118	0.464
Trailor	0.506	0.113	0.319	0.634	-1402	0.063	0.803	-0.110	22	0.162	0.091	0.278	0.146	0.245	0.418

TABLE B - CONTINUED

Product category	Estimated value of								Observed data				Simulated data		
	c	sig_e	sig_c	sig_u	Logl	sig_w	$S(rho_k)$	$r_{ft,IP}$	Nbar	Freq	|Dp|	%up	Freq*	|Dp|*	%up*
Phone set	0.220	0.060	0.126	0.290	-6218	0.020	0.841	0.987	143	0.148	0.082	0.296	0.135	0.115	0.366
TV set	0.243	0.052	0.146	0.281	-460	0.056	0.911	0.912	12	0.167	0.096	0.221	0.153	0.132	0.312
Video camera	0.161	0.029	0.088	0.178	-1240	0.021	0.997	0.957	40	0.101	0.033	0.821	0.088	0.061	0.588
Music instrument	0.468	0.094	0.259	0.815	-9318	0.021	0.564	0.902	179	0.105	0.057	0.663	0.077	0.137	0.509
Electrical razor	0.436	0.093	0.251	0.636	-1972	0.046	0.800	0.777	38	0.127	0.077	0.418	0.103	0.167	0.446
Jewellery	0.373	0.086	0.205	0.325	-16007	0.019	0.977	0.870	342	0.109	0.079	0.511	0.095	0.154	0.459
Services															
Shoe repair	0.285	0.038	0.145	0.185	-2503	0.034	0.856	-0.073	93	0.069	0.044	0.832	0.060	0.100	0.574
Water distribution	0.135	0.034	0.089	0.175	-2021	0.006	0.585	0.908	35	0.212	0.014	0.771	0.155	0.070	0.505
Hourly rate in a garage	0.146	0.031	0.083	0.122	-37002	0.008	0.673	0.980	1205	0.116	0.031	0.836	0.110	0.064	0.632
Car rent	0.443	0.082	0.240	0.363	-3779	0.026	0.644	0.067	94	0.096	0.068	0.569	0.083	0.167	0.492
Urban transports	0.468	0.044	0.244	0.554	-257	0.082	-0.163	0.175	7	0.065	0.049	0.727	0.054	0.168	0.449
Moving services	0.280	0.070	0.162	0.407	-2673	0.035	0.796	0.832	58	0.147	0.088	0.549	0.131	0.140	0.521
Pet care	0.371	0.050	0.186	0.246	-8971	0.016	0.336	0.936	359	0.058	0.040	0.846	0.050	0.096	0.600
cinemas	0.294	0.089	0.175	0.140	-7513	0.032	0.121	0.759	142	0.138	0.089	0.657	0.134	0.150	0.465
monument or museum entrance	1.363	0.486	0.486	0.507	-4899	0.032	0.757	0.890	144	0.038	0.110	0.776	0.051	0.963	0.507
Private high school	0.280	0.035	0.133	0.305	-561	0.014	0.121	0.893	18	0.067	0.031	0.770	0.043	0.079	0.443
Private colleges/universities	0.235	0.036	0.116	0.315	-648	0.015	0.279	0.924	24	0.078	0.027	0.821	0.051	0.064	0.490
classic lunch in a restaurant	0.203	0.102	0.146	0.228	-174744	0.007	0.895	0.670	3271	0.064	0.035	0.769	0.239	0.142	0.522
coffee and hot drinks in bars	0.244	0.038	0.116	0.220	-12013	0.011	0.810	0.985	512	0.059	0.057	0.847	0.054	0.104	0.668
beer in bars	0.255	0.038	0.125	0.189	-8803	0.010	0.859	0.983	349	0.063	0.047	0.818	0.057	0.083	0.609
Non alcoolhic beverage in bars	0.282	0.041	0.133	0.210	-3896	0.014	0.773	0.961	184	0.052	0.052	0.814	0.047	0.086	0.622
Full-board hotel accomodation	0.189	0.086	0.115	0.311	-10061	0.007	0.955	0.985	183	0.159	0.086	0.611	0.208	0.144	0.536
men hairdresser	0.267	0.041	0.128	0.159	-12126	0.009	0.889	0.982	549	0.056	0.038	0.818	0.049	0.082	0.560
women hairdresser	0.298	0.052	0.150	0.239	-11664	0.010	0.842	0.978	409	0.070	0.041	0.799	0.059	0.095	0.543
Watch/clock repair	0.692	0.062	0.304	0.380	-1724	0.073	0.926	-0.639	88	0.036	0.081	0.769	0.027	0.161	0.561
Day-care center	0.437	0.030	0.190	0.145	-749	0.033	0.015	-0.005	46	0.037	0.034	0.699	0.022	0.083	0.489
Home insurance	0.389	0.053	0.191	0.280	-15304	0.013	0.842	0.952	563	0.062	0.048	0.741	0.051	0.120	0.503
Car insurance	0.409	0.117	0.205	0.306	-24775	0.005	0.932	0.661	658	0.071	0.055	0.550	0.092	0.212	0.485

Table B - Continued

Product category	Ξ_f	r1	r2	r3	r4	r6	r12
Energy							
Butane	0.153	0.983	0.959	0.937	0.918	0.890	0.801
Gasoline 1000-2000 liters	0.263	0.973	0.939	0.905	0.867	0.799	0.501
Eurosuper (RON95)	0.114	0.978	0.954	0.935	0.909	0.855	0.692
Perisable food							
Paprika pepper	0.249	0.685	0.288	0.003	-0.131	-0.440	0.715
Skate (wing)	0.072	0.843	0.815	0.764	0.716	0.649	0.830
Oranges	0.111	0.881	0.660	0.423	0.242	0.081	0.745
Carrots	0.179	0.861	0.626	0.399	0.214	0.059	0.231
Apples, Granny Smith	0.140	0.885	0.678	0.515	0.404	0.266	0.612
Kiwis	0.172	0.947	0.862	0.763	0.662	0.551	0.820
Margarine (super)	0.024	0.896	0.830	0.779	0.776	0.748	0.500
Turkey filet	0.046	0.893	0.867	0.872	0.860	0.801	0.677
Sirloin	0.020	0.690	0.757	0.705	0.703	0.647	0.565
Cheese (Gouda type)	0.035	0.709	0.789	0.714	0.755	0.705	0.479
Full-fat fruit yoghurt	0.023	0.828	0.806	0.769	0.771	0.742	0.685
Butter	0.030	0.889	0.873	0.883	0.872	0.841	0.732
Emmentaler	0.037	0.638	0.651	0.761	0.664	0.657	0.491
Sausage	0.062	0.978	0.963	0.946	0.927	0.891	0.777
Cheese (Edam type)	0.050	0.910	0.918	0.908	0.889	0.896	0.845
Belgian waffle	0.027	0.526	0.615	0.502	0.515	0.438	0.387
Country paté	0.063	0.935	0.934	0.936	0.931	0.918	0.884
Rice pudding	0.059	0.852	0.836	0.868	0.864	0.854	0.780
Pastry (carré glacé)	0.076	0.952	0.940	0.937	0.935	0.914	0.915
Pastry (éclair)	0.070	0.829	0.827	0.858	0.799	0.814	0.793
Swiss cake	0.054	0.827	0.859	0.852	0.848	0.860	0.790
Whole wheat bread	0.030	0.870	0.866	0.861	0.851	0.827	0.716
Special bread	0.037	0.576	0.639	0.597	0.619	0.596	0.422
Bread roll	0.080	0.969	0.958	0.960	0.952	0.961	0.937
Non perishable food							
Frankfurters	0.035	0.868	0.796	0.767	0.715	0.656	0.333
Biscuits	0.075	0.968	0.947	0.923	0.903	0.870	0.903
Fruit juice	0.043	0.866	0.849	0.821	0.780	0.748	0.633
Fishcakes	0.046	0.785	0.785	0.742	0.732	0.645	0.385
Val de Loire wine	0.030	0.960	0.962	0.936	0.928	0.892	0.823
Ice cream	0.085	0.950	0.939	0.920	0.902	0.865	0.816
Tinned apricot halves	0.043	0.857	0.847	0.858	0.779	0.765	0.622
Tinned tomatoes, peeled	0.075	0.937	0.913	0.896	0.890	0.831	0.784
Tinned peas	0.062	0.920	0.912	0.905	0.865	0.836	0.715
Tobacco	0.077	0.997	0.994	0.990	0.986	0.980	0.969
Sausage	0.061	0.994	0.990	0.984	0.978	0.966	0.909
Lemonade	0.026	0.124	0.211	0.331	0.359	0.344	0.183
Non durable goods							
Roses	0.139	0.665	0.410	0.209	-0.104	-0.548	0.936
Chrysanthemums	0.126	0.784	0.432	-0.015	-0.425	-0.887	0.914
Compact Disc	0.029	0.860	0.827	0.814	0.796	0.797	0.654
Hair spray	0.024	0.977	0.968	0.949	0.943	0.920	0.841
Cat food	0.028	0.579	0.621	0.577	0.596	0.596	0.395
Nail polish	0.088	0.978	0.970	0.965	0.960	0.969	0.960
Water-based paint	0.074	0.995	0.989	0.983	0.978	0.967	0.920
Oil-based paint	0.055	0.994	0.990	0.985	0.979	0.970	0.953
Water charge	0.080	0.879	0.886	0.890	0.868	0.834	0.811
Engine oil	0.089	0.999	0.998	0.997	0.996	0.994	0.988
Dracaena	0.019	0.969	0.962	0.948	0.946	0.929	0.889
Dry battery	0.130	0.998	0.997	0.995	0.994	0.989	0.977
Wool suit	0.006	0.880	0.803	0.779	0.745	0.642	0.643
Infants' anorak (9 month)	0.015	0.958	0.939	0.917	0.899	0.869	0.823
Men's socks	0.050	0.998	0.995	0.992	0.989	0.982	0.957
Dress fabric	0.027	0.993	0.989	0.986	0.981	0.977	0.956
Men's T shirt	0.017	0.978	0.948	0.919	0.892	0.847	0.705
Color film, 135-24	0.005	0.842	0.835	0.772	0.682	0.624	0.530
Zip fastener	0.034	0.968	0.958	0.951	0.941	0.937	0.901

Table C - Statistical properties of the common component $\widehat{f}_t$ - Belgium

Product category	Ξ_f	r1	r2	r3	r4	r6	r12
Durable goods							
LaserJet printer	0.060	0.625	0.541	0.485	0.493	0.296	-0.171
VCR, four-head	0.177	0.979	0.969	0.964	0.968	0.978	0.974
Compact hi-fi system	0.126	0.999	0.997	0.996	0.994	0.992	0.988
Natural gas heater	0.092	0.979	0.966	0.961	0.957	0.947	0.949
Calculator	0.053	0.991	0.980	0.971	0.961	0.937	0.864
Toaster	0.013	0.935	0.866	0.814	0.744	0.611	0.215
Suitcase	0.046	0.964	0.944	0.930	0.914	0.888	0.833
Electric coffee machine	0.010	0.908	0.837	0.791	0.700	0.589	0.098
Children's bicycle	0.070	0.947	0.922	0.917	0.925	0.916	0.882
Electric fryer	0.017	0.979	0.953	0.928	0.900	0.827	0.585
Dictionary	0.053	0.779	0.594	0.535	0.453	0.303	0.190
Bed, slatted base	0.033	0.815	0.694	0.613	0.643	0.652	0.580
Stainless steel pan	0.034	0.992	0.988	0.981	0.973	0.954	0.896
Hammer	0.069	0.961	0.958	0.943	0.942	0.936	0.916
Glass, 4 mm	0.070	0.991	0.984	0.979	0.970	0.942	0.858
Dining room oak furniture	0.098	0.992	0.983	0.971	0.960	0.939	0.891
Spherical glasses	0.022	0.930	0.887	0.800	0.735	0.740	0.642
Wallet	0.069	0.996	0.991	0.985	0.978	0.965	0.938
Torus glasses	0.027	0.771	0.767	0.617	0.532	0.606	0.504
Cup and saucer	0.068	0.996	0.991	0.986	0.980	0.969	0.944
Services							
School boarding fees	0.044	0.975	0.972	0.968	0.964	0.956	0.986
Hourly wage, painter	0.062	0.981	0.979	0.974	0.969	0.962	0.954
Hourly wage, garage mechanic	0.106	0.999	0.999	0.998	0.998	0.997	0.996
Annual cable subscription	0.029	0.858	0.835	0.779	0.756	0.735	0.674
Repair of central heating	0.059	0.995	0.994	0.990	0.987	0.981	0.972
Hourly wage, plumber	0.057	0.994	0.988	0.984	0.979	0.972	0.961
Passport stamp	1.044	0.959	0.914	0.868	0.821	0.722	0.551
Sole meunière	0.067	0.910	0.903	0.915	0.913	0.890	0.897
Dry cleaning, shirt	0.051	0.996	0.993	0.991	0.989	0.983	0.955
Pepper steak	0.052	0.998	0.996	0.994	0.992	0.988	0.970
Permanent wave	0.072	0.999	0.998	0.997	0.996	0.995	0.993
Domestic service	0.066	0.995	0.994	0.991	0.989	0.986	0.981
Funeral	0.055	0.884	0.881	0.858	0.853	0.892	0.867
School lunch	0.072	0.990	0.984	0.979	0.975	0.972	0.995
Self-service meal	0.025	0.545	0.343	0.289	0.183	0.319	0.402
Parking spot in a garage	0.094	0.997	0.993	0.988	0.982	0.974	0.957
Wheel balancing	0.026	0.991	0.983	0.974	0.966	0.950	0.932
Special beer	0.069	0.988	0.983	0.984	0.981	0.982	0.967
Aperitif	0.076	0.997	0.995	0.994	0.993	0.990	0.977
Videotape rental	0.011	0.868	0.852	0.823	0.758	0.729	0.547

TABLE C - CONTINUED

Product category	÷f	r1	r2	r3	r4	r6	r12
Energy							
Eurosuper	0.091	0.980	0.953	0.929	0.900	0.841	0.650
Gas-Oil	0.133	0.986	0.964	0.942	0.918	0.873	0.671
Perishable food							
Roast-beef	0.054	0.983	0.967	0.951	0.936	0.098	0.956
Beef burger	0.041	0.898	0.901	0.885	0.875	0.207	0.768
Lamb	0.108	0.988	0.977	0.964	0.953	0.433	0.852
Fresh pork meat	0.072	0.919	0.862	0.785	0.708	0.379	0.292
Ham	0.083	0.980	0.963	0.948	0.926	0.266	0.721
Sausages	0.055	0.952	0.934	0.925	0.903	0.372	0.644
Chicken	0.132	0.987	0.972	0.953	0.933	0.840	0.715
Rabbit/Game	0.071	0.945	0.911	0.864	0.827	0.376	0.699
Creme fraiche	0.030	0.980	0.967	0.954	0.933	0.480	0.742
Milky Desserts	0.053	0.981	0.972	0.970	0.966	0.733	0.945
Cottage cheese	0.055	0.987	0.982	0.980	0.970	0.769	0.933
Processed cheese	0.068	0.966	0.964	0.959	0.960	0.881	0.927
Butter	0.054	0.991	0.987	0.985	0.982	0.733	0.938
Non perishable food							
Rusks and grilled breads	0.036	0.878	0.850	0.835	0.839	0.519	0.694
Flour	0.054	0.974	0.972	0.975	0.962	0.786	0.944
Pasta	0.210	0.997	0.991	0.984	0.977	0.935	0.900
Canned vegetables	0.032	0.959	0.954	0.946	0.927	0.559	0.859
Sugar	0.060	0.996	0.993	0.992	0.990	0.739	0.970
Chocolate	0.071	0.988	0.981	0.980	0.980	0.816	0.963
Desserts	0.108	0.963	0.971	0.965	0.964	0.858	0.938
Coffee	0.055	0.939	0.847	0.741	0.641	0.478	0.054
Tea	0.085	0.981	0.982	0.981	0.975	0.961	0.959
Fruit juices	0.034	0.912	0.918	0.897	0.889	0.473	0.871
Whisky	0.008	0.582	0.413	0.386	0.250	-0.078	0.176
Pet food	0.161	0.966	0.931	0.925	0.920	0.915	0.882
Non durable goods							
Fabrics	0.065	0.100	-0.183	0.084	-0.161	-0.089	0.612
Men coats	0.065	0.118	-0.154	-0.094	-0.290	-0.052	0.844
Men suits	0.086	0.271	-0.105	-0.055	-0.132	-0.061	0.858
Men trousers	0.054	0.122	-0.281	-0.141	-0.321	-0.174	0.798
Skirt	0.097	0.138	-0.335	-0.392	-0.381	-0.161	0.828
Dress	0.156	0.414	0.140	0.157	0.172	0.084	0.786
Women trousers	0.059	0.130	-0.244	-0.221	-0.269	-0.061	0.672
Women jacket	0.113	0.284	-0.009	-0.003	-0.008	-0.080	0.794
Children trousers	0.112	0.752	0.645	0.640	0.629	0.436	0.883
Children suits	0.224	0.481	0.392	0.390	0.440	0.356	0.545
Men shirts	0.078	0.059	-0.390	-0.236	-0.403	-0.144	0.897
Men socks	0.043	0.075	-0.050	0.009	0.126	0.051	0.329
Men sweater	0.068	0.273	0.148	0.263	0.133	-0.051	0.825
Women sweater	0.081	0.056	-0.263	-0.106	-0.266	-0.146	0.749
Children sweater	0.091	0.430	0.150	0.147	0.177	0.134	0.704
Babies clothes	0.112	0.083	0.027	0.273	0.107	0.074	0.474
Men shoes	0.057	0.127	-0.126	-0.223	-0.147	-0.072	0.721
Women shoes	0.085	0.317	-0.043	0.008	-0.032	0.065	0.895
Children shoes	0.084	0.126	-0.185	-0.201	-0.236	-0.024	0.795
Blankets and coverlets	0.045	0.186	0.134	0.432	0.203	-0.071	0.792
Fabrics for furniture	0.046	0.548	0.476	0.516	0.461	0.012	0.581
Batteries	0.023	0.762	0.765	0.755	0.740	0.546	0.540
Car tyres	0.053	0.951	0.948	0.936	0.930	0.898	0.840
Musical disks	0.046	0.978	0.952	0.942	0.930	0.896	0.881
Blank tapes and disks	0.019	0.463	0.367	0.404	0.319	0.343	0.202
Flowers	0.058	0.853	0.538	0.205	-0.093	-0.446	0.923
Children books	0.073	0.940	0.939	0.921	0.925	0.915	0.916
Newspapers	0.041	0.919	0.895	0.907	0.900	0.892	0.814
Paper articles	0.077	0.816	0.646	0.633	0.663	0.524	0.722
Leather articles	0.041	0.206	0.169	0.237	0.268	0.571	0.600
Babies apparel	0.051	0.597	0.708	0.640	0.691	0.619	0.580

Table D - Statistical properties of the common component $\widehat{f_t}$ - France

Product category	÷f	r1	r2	r3	r4	r6	r12
Durable goods							
box-mattress	0.037	0.170	0.306	0.123	0.243	0.055	0.574
Armchairs and canapes	0.065	0.886	0.877	0.911	0.864	0.231	0.893
Washing machine	0.035	0.823	0.830	0.819	0.769	0.311	0.687
Vacuum-cleaner	0.032	0.475	0.494	0.502	0.442	0.148	0.420
Electrical tools	0.030	0.430	0.430	0.415	0.412	-0.005	0.286
Bicycles	0.042	0.757	0.718	0.705	0.668	0.088	0.555
Trailor	0.127	0.839	0.802	0.763	0.736	0.697	0.489
Phone set	0.132	0.985	0.984	0.983	0.978	0.976	0.949
TV set	0.226	0.952	0.953	0.956	0.941	0.926	0.886
Video camera	0.106	0.980	0.972	0.964	0.950	0.937	0.902
Music instrument	0.049	0.857	0.821	0.849	0.813	0.817	0.724
Electrical razor	0.085	0.672	0.675	0.690	0.673	0.721	0.565
Jewellery	0.031	0.686	0.701	0.651	0.639	0.656	0.467
Services							
Shoe repair	0.061	0.787	0.797	0.781	0.727	0.244	0.392
Water distribution	0.016	0.825	0.771	0.749	0.676	-0.229	0.570
Hourly rate in a garage	0.094	0.996	0.992	0.990	0.989	0.988	0.980
Car rent	0.047	0.277	0.233	0.302	0.319	0.283	0.226
Urban transports	0.081	-0.147	0.074	0.046	0.058	-0.016	0.067
Moving services	0.149	0.958	0.925	0.894	0.880	0.887	0.913
Pet care	0.046	0.911	0.888	0.864	0.859	0.881	0.875
cinemas	0.041	0.497	0.431	0.421	0.449	0.432	0.341
monument or museum entrance	0.129	0.962	0.959	0.950	0.936	0.923	0.857
Private high school	0.026	0.759	0.736	0.714	0.753	0.712	0.783
Private colleges/universities	0.030	0.812	0.772	0.783	0.718	0.604	0.797
classic lunch in a restaurant	0.025	0.964	0.911	0.858	0.808	0.712	0.417
coffee and hot drinks in bars	0.099	0.992	0.991	0.988	0.985	0.982	0.975
beer in bars	0.067	0.984	0.983	0.978	0.980	0.976	0.963
Non alcoolhic beverage in bars	0.052	0.940	0.933	0.945	0.914	0.914	0.908
Full-board hotel accomodation	0.055	0.982	0.962	0.944	0.938	0.940	0.985
men hairdresser	0.043	0.962	0.953	0.957	0.943	0.956	0.919
women hairdresser	0.049	0.955	0.952	0.944	0.949	0.960	0.941
Watch/clock repair	0.212	0.944	0.910	0.872	0.844	0.765	0.563
Day-care center	0.033	0.046	-0.063	0.119	0.003	0.168	-0.033
Home insurance	0.040	0.910	0.888	0.878	0.851	0.818	0.805
Car insurance	0.022	0.814	0.409	0.209	0.231	0.141	0.062

TABLE D - CONTINUED