



Management

LUXURY GOODS VS NECESSITY GOODS: WHICH GOODS CONTRIBUTE TO ENHANCEMENT OF THE NATIONAL WEALTH?

Yasunori Fujita ^{*1}

^{*1} Keio University, Tokyo, Japan

Abstract

How do luxury goods affect the national wealth? This is a topic that can go back to the seventeenth century. In the present paper, we tackle this problem by examining if the innovation accelerates the generation of luxury goods market. The framework we construct is a model where one firm intends an investment under uncertainty that is expressed by geometric Brownian motion. It is revealed that innovation accelerates the generation of luxury goods market, that is, luxury goods contribute to market generation, as well as enhancement of the national wealth.

Keywords: Luxury Goods; Generation of Market; Optimal Stopping Theory; Value of Waiting.

Cite This Article: Yasunori Fujita. (2018). “LUXURY GOODS VS. NECESSITY GOODS: WHICH GOODS CONTRIBUTE TO ENHANCEMENT OF THE NATIONAL WEALTH?.” *International Journal of Research - Granthaalayah*, 6(2), 157-161. <https://doi.org/10.5281/zenodo.1189221>.

1. Introduction

How do luxury goods affect the national wealth? This is a topic that can go back to the seventeenth century (Mandeville (1714), List (1841), Sombart (1912) and so on). In the present paper, we construct a new model and attempt to tackle this problem by combining the standard microeconomic theory with the optimal stopping theory that has been used to develop strategies on the timing in a stochastic economy since McDonald and Siegel (1986), Dixit (1989), Farzin, Huisman and Kort (1988) etc. revealed the importance of the value of waiting. More precisely, by incorporating utility and cost functions explicitly into the optimal stopping model, as in Fujita (2007), Fujita (2016) etc., we examine if the innovation accelerates the generation of luxury goods market, to enhance the national wealth.

Structure of this paper is as follows. After constructing a basic model to derive the equilibrium profit of the firm in Section 2, section 3 formulates the firm’s objective function. Based on these analyses, in section 4 we demonstrate that the luxury goods contribute to enhancement of the national wealth. Concluding remarks are made in section 5.

2. Basic Model

Let us consider an intertemporal stochastic economy that consists of one household and one firm, for the simplicity of the analysis, where time passes continuously with importance of the future diminishing with discount rate ρ . We assume that the household obtains utility

$u(t) = \frac{A}{-\varepsilon + 1} x(t)^{-\varepsilon + 1} + B$ if she/he consumes $x(t)$ units of the product in period t , where A and B are positive parameters, while ε is a parameter that expresses elasticity of demand, which defines the product as luxury (necessity) one if $\varepsilon > 1$ ($\varepsilon < 1$) as in Varian (2014) etc. Assuming the household takes the price in each period as given, we have the inverse demand function in period t as $p(t) = Ax(t)^{-\varepsilon}$, where $p(t)$ is the price of the product in period t .

As for the firm, on the other hand, we assume that the firm is planning to conduct an investment taking the price in each period as given, and that the market generates only when the firm conducts the investment. We also assume that the firm optimizes the timing of the investment, t^* , as well as the amount of the outputs in each period. As a way of formulation of the uncertainty, we assume variable costs to increase stochastically with time, which the firm incurs after conducting the investment with a fixed cost of K . In the following, we specify the variable costs

when producing $x(t)$ units in period t as $\frac{c}{\varphi(t)} x(t)^\eta$, where c , η and $\varphi(t)$ are parameters that satisfy $c > 0$, $\eta > 1$ and $0 \leq \varphi(t) \leq 1$ with the following geometric Brownian motion,

$$\frac{d\varphi}{\varphi(t)} = \theta dz, \tag{1}$$

with initial value $\varphi(0) < 1$.

Since the firm's profit in period t , $\pi(t)$, is described as $\pi(t) = p(t)x(t) - \frac{c}{\varphi(t)} x(t)^\eta$, we have its first order condition for the profit maximization in period t as $\frac{\partial \pi}{\partial x} = p(t) - \frac{\eta c}{\varphi(t)} x(t)^{\eta-1} = 0$, to yield the

firm's equilibrium outputs of in period t as $x(t) = \left(\frac{\varphi(t)p(t)}{\eta c}\right)^{\frac{1}{\eta-1}}$. By combining this firm's supply function with the above inverse demand function of the household $p(t) = Ax(t)^{-\varepsilon}$, we have the

firm's equilibrium output in period t as $x(t) = \left(\frac{A\varphi(t)}{\eta c}\right)^{\frac{1}{\varepsilon+\eta-1}}$, to obtain the firm's equilibrium profit in period t as

$$\pi(t) = \Lambda \varphi(t)^{\frac{\eta}{\varepsilon+\eta-1}} \text{ where } \Lambda \equiv (\eta-1) \left(\frac{A}{\eta}\right)^{\frac{\eta}{\varepsilon+\eta-1}} c^{\frac{\varepsilon-1}{\varepsilon+\eta-1}}. \tag{2}$$

3. Formulation of The Firm's Objective Function

From Equation (2), we have its first derivative and second derivative with respect to $\varphi(t)$ as $\frac{d\pi}{d\varphi} = \frac{\eta\Lambda}{\varepsilon + \eta - 1} \varphi(t)^{\frac{-\varepsilon+1}{\varepsilon+\eta-1}}$ and $\frac{d^2\pi}{d\varphi^2} = \frac{\eta(-\varepsilon+1)\Lambda}{(\varepsilon + \eta - 1)^2} \varphi(t)^{\frac{-\varepsilon+1}{\varepsilon+\eta-1}-1}$, respectively. Thus, by making use of Ito's lemma, we can express the stochastic process of the firm's profit as

$$\frac{d\pi}{\pi} = \mu dt + \sigma dz, \quad (3)$$

where $\mu = \frac{\eta(-\varepsilon+1)\theta^2}{2(\varepsilon + \eta - 1)^2}$ and $\sigma = \frac{\eta\theta}{\varepsilon + \eta - 1}$ with initial value of $\pi(0) = \Lambda\varphi(0)^{\frac{\eta}{\varepsilon+\eta-1}}$.

By making use of this stochastic process of the firm's profit, let us express the firm's objective function to maximize in period 0, $V = E[\int_{t^*}^{\infty} e^{-\rho t} \pi(t) dt - e^{-\rho t^*} K]$, as a function of φ^* , which we define as the level of φ in period t^* . For this purpose, if we let $G(\pi(0))$ denote the expected value of one unit of the firm's profit in period t^* (i.e., the expected value of $e^{-\rho t^*}$) as a function of its initial profit $\pi(0)$, the general solution to $G(\pi(0))$ is expressed as

$$G(\pi(0)) = \alpha(\pi(0))^{\gamma_1} + \beta(\pi(0))^{\gamma_2}, \quad (4)$$

where $\gamma_1 < 0$ and $\gamma_2 > 0$ are solutions to the characteristic equation $\frac{\sigma^2}{2}x(x-1) + \mu x - \rho = 0$. Since $G(\pi(0))$ satisfies $G(\infty) = 0$ and $G(\pi^*) = 1$ where π^* is defined as the firm's profit in period t^* , it follows that $\alpha = 0$ and $\beta = (\frac{1}{\pi^*})^\gamma$ where $\gamma \equiv \gamma_2 = \frac{\varepsilon + \eta - 1 + \sqrt{(\varepsilon + \eta - 1)^2 + \frac{8\rho}{\theta^2}}}{2\eta}$ for the simplification of notation, which combined with Equation (4) yields

$$G(\pi(0)) = \left(\frac{\pi(0)}{\pi^*}\right)^\gamma. \quad (5)$$

Thus, we can calculate the firm's objective function to maximize in period 0 as $V = \left(\frac{\pi(0)}{\pi^*}\right)^\gamma \left(\frac{\pi^*}{\rho - \mu} - K\right)$, which is rewritten as

$$V = \left(\frac{\varphi(0)}{\varphi^*}\right)^{\frac{\gamma\eta}{\varepsilon+\eta-1}} \left(\frac{1}{\rho - \mu} \Lambda \varphi^{*\frac{\eta}{\varepsilon+\eta-1}} - K\right), \quad (6)$$

by substituting $\pi(0) = \Lambda\varphi(0)^{\frac{\eta}{\varepsilon+\eta-1}}$ and $\pi^* = \Lambda\varphi^{*\frac{\eta}{\varepsilon+\eta-1}}$ into $V = \left(\frac{\pi(0)}{\pi^*}\right)^\gamma \left(\frac{\pi^*}{\rho - \mu} - K\right)$.

4. Optimal Timing of Market Generation

Now, we are ready to determine the optimal timing of the market generation.

Since the model of the present paper is stochastic, the optimal timing of the investment, which is equivalent with the market generation, is expressed by the cut off level of φ^* . Since typical relationship between φ^* and V is depicted as a one-peaked trajectory with the optimal value of φ^* being φ^{*O} on φ^* - V space as in Figure1, by differentiating Equation (6) with respect to φ^* and setting it to zero, we have the firm's optimal cut off level φ^{*O} as

$$\varphi^{*O} = \left\{ \frac{K}{\Lambda} \frac{\rho - \frac{\eta(-\varepsilon+1)\theta^2}{2(\varepsilon+\eta-1)^2}}{1 - \frac{1}{\varepsilon+\eta-1 + \sqrt{(\varepsilon+\eta-1)^2 + \frac{8\rho}{\theta^2}}}} \right\}^{\frac{\varepsilon+\eta-1}{\eta}}, \quad (7)$$

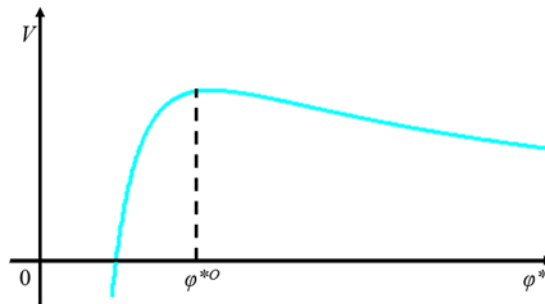


Figure1: Relationship between uncertainty and sum of present value of the profit

Now we are ready to examine how the optimal timing of the market generation is affected by the innovation which we express as a decrease in the cost. It is clear from $\Lambda \equiv (\eta-1) \left(\frac{A}{\eta} \right)^{\frac{\eta}{\varepsilon+\eta-1}} c^{\frac{\varepsilon-1}{\varepsilon+\eta-1}}$ that in accordance with a decrease in c , Λ decreases (increases) if $\varepsilon > 1$ ($\varepsilon < 1$), which means the product is luxury (necessity) one. Since decrease (increase) in Λ increases (reduces) φ^{*O} , which is equivalent to acceleration (postponement) of the market generation, we have the following proposition.

Proposition: Innovation accelerates the generation of the luxury goods market.

This proposition implies that, as Mandeville (1714) showed, luxury goods contribute to enhancement of the national wealth.

5. Conclusions

In the present paper, we examined if innovation accelerates the generation of luxury goods market. The framework we invented was a model that combined the optimal stopping theory with the standard microeconomic theory. It was revealed that innovation accelerates (postpones) the generation of the market where luxury (necessity) goods are traded. The result implies that, as Mandeville (1714) showed, luxury goods contribute to enhancement of the national wealth.

In order to derive explicit results, we made simplifying assumptions on utility function, cost function and stochastic motion. It is necessary to relax those assumptions and construct a general

framework in order to investigate the robustness of the results of the present paper. We also need to test the model of the present paper empirically. We will take up such analyses in our next research.

References

- [1] Mandeville, B. The Fable of the Bees: or, Private Vices, Public Benefits, 1714, Oxford: Clarendon Press, reprinted in F.B. Kaye ed., Mandeville's Fable of the Bees: or, Private Vices, Public Benefits, 1924, Oxford: Clarendon Press.
- [2] List, F. The National System of Political Economy, 1841, reprinted New York: Garland Publishing Co.1974.
- [3] Sombart, W. Liebe, Luxus und Kapitalismus, 1912, Munich: Dunckner & Hum-blot, translated by W.R. Dittmar as: Luxury and Capitalism, 1967, Michigan: The University of Michigan Press.
- [4] McDonald, R. and Siegel, D. The Value of Waiting to Invest, The Quarterly Journal of Economics, 101, 1986, 707- 727.
- [5] Dixit, A.K. Hysteresis, Import Penetration, and Exchange Rate Pass Through, Quarterly Journal of Economics, 104, 1989, 205-228.
- [6] Farzin, Y. H., Huisman, K.J.M. and Kort, P.M. Optimal Timing of Technology Adoption, Journal of Economic Dynamics and Control, 22, 1988, 779-799.
- [7] Fujita, Y. Toward a New Modeling of International Economics: An Attempt to Reformulate an International Trade Model Based on Real Option Theory, Physica A: Statistical Mechanics and its Applications, 383(2), 2007, 507-512.
- [8] Fujita, Y. Backfiring Effect of Uncertain Trade Policy, Modern Economy, 07(05), 2016, 613-618.
- [9] Varian, H. R. Intermediate microeconomics: A modern approach (9th edition), 2014, New York: W.W. Norton & Co.

*Corresponding author.

E-mail address: yfujita@ econ.keio.ac.jp