

## LYAPUNOV–TYPE INEQUALITIES FOR A FRACTIONAL DIFFERENTIAL EQUATION WITH MIXED BOUNDARY CONDITIONS

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*Abstract.* Lyapunov-type inequalities are established for a fractional differential equation under mixed boundary conditions. Using such inequalities, we obtain intervals where certain Mittag-Leffler functions have no real zeros.

*Mathematics subject classification (2010):* 34A08, 34A40, 26D10, 33E12.

*Keywords and phrases:* Lyapunov's inequality, Caputo fractional derivative, mixed boundary conditions, eigenvalue, Mittag-Leffler function.

### REFERENCES

- [1] J.-S. DUAN, Z. WANG AND Y.-L. LIU, X. QIU, *Eigenvalue problems for fractional ordinary differential equations*, *Chaos Solitons Fract.*, **46**, (2013), 46–53.
- [2] R. A. C. FERREIRA, *A Lyapunov-type inequality for a fractional boundary value problem*, *Fract. Calc. Appl. Anal.*, **16**, 4 (2013), 978–984.
- [3] R. A. C. FERREIRA, *On a Lyapunov-type inequality and the zeros of a certain Mittag-Leffler function*, *J. Math. Anal. Appl.*, **412**, 2 (2014), 1058–1063.
- [4] A. A. KILBAS, H. M. SRIVASTAVA AND J. J. TRUJILLO, *Theory and applications of fractional differential equations*, North-Holland Mathematics Studies **204** Elsevier, Amsterdam, The Netherlands, 2006.
- [5] A. M. LIAPUNOV, *Problème général de la stabilité du mouvement*, *Ann. Fac. Sci. Univ. Toulouse.*, **2**, (1907), 203–407.
- [6] F. MAINARDI, *Fractional relaxation-oscillation and fractional diffusion wave phenomena*, *Chaos Solitons Fract.*, **7**, (1996), 1461–77.
- [7] F. MAINARDI AND R. GORENFLO, *On Mittag-Leffler-type functions in fractional evolution processes*, *J Comput Appl Math.*, **118**, (2000), 283–99.
- [8] B. G. PACHPATTE, *Mathematical Inequalities*, North Holland Mathematical Library, 2005.
- [9] I. PODLUBNY, *Fractional differential equations*, Academic, San Diego 1999.
- [10] S. ZHANG, *Positive solutions for boundary-value problems of nonlinear fractional differential equations*, *Electron. J. Differential Equations.*, **36**, (2006), 12 pages.