

## LYAPUNOV–TYPE INEQUALITIES FOR TWO CLASSES OF DIRICHLET QUASILINEAR SYSTEMS

AYDIN TIRYAKI, DEVRİM ÇAKMAK AND MUSTAFA FAHİR AKTAŞ

*Abstract.* In this paper, we establish several new Lyapunov-type inequalities for two classes of Dirichlet quasilinear systems, which almost generalize and extend all related existing results in the literature. As an application, we also obtain sharp lower bounds for the eigenvalues of corresponding systems.

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