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Research Article

M-Matrix-Based Robust Stability and Stabilization Criteria for Uncertain Switched Nonlinear Systems with Multiple Time-Varying Delays

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This paper focuses on the robust stability and the memory feedback stabilization problems for a class of uncertain switched nonlinear systems with multiple time-varying delays. Especially, the considered time delays depend on the subsystem number. Based on a novel common Lyapunov functional, the aggregation techniques, and the Borne and Gentina criterion, new sufficient robust stability and stabilization conditions under arbitrary switching are established. Compared with existing results, the proposed criteria are explicit, simple to use, and obtained without finding a common Lyapunov function for all subsystems through linear matrix inequalities, considered very difficult in this situation. Moreover, compared with the memoryless one, the developed controller guarantees the robust stability of the corresponding closed-loop system with more performance by minimizing the effect of the delays in the system dynamics. Finally, two numerical simulation examples are shown to prove the practical utility and the effectiveness of the proposed theories.

1. Introduction

Switched systems constitute an important class of hybrid systems, which can be described by a family of subsystems and a rule that orchestrates the switching amongst them [1].

Recently, switched systems have attracted considerable attention, and some valuable results have been achieved [1–33]. Among these research topics, stability analysis, stabilization, and control design of switched systems under arbitrary switching are fundamental issues in the design and the analysis of such systems. This kind of switching strategy lies in the fact that the stability of each autonomous or closed-loop subsystem does not necessarily imply the stability of the corresponding switched system. In this framework, it is well known that the existence of a common Lyapunov function (CLF) for all the subsystems through the linear matrix inequalities (LMIs) is a sufficient condition for such systems to be asymptotically stable under arbitrary switching [3]. However, this function is very difficult to find even for switched linear systems [3]. Therefore, this task becomes more and more compiled when switched nonlinear systems are involved [5].

Frequently, to avoid the conservatism related to the existence of a CLF, some attention has been devoted to considering switched systems under restricted switching. Although many interesting results have been proposed for this alternative, such as the dwell time approach [7] and the multiple Lyapunov function [6], stability under arbitrary switching remains more suitable for real systems. In fact, it offers more effectiveness for control design along with stability preserved.

As is well known, time-delay is usually often encountered in many engineering processes, which is considered in many recent studies [4, 13–20, 25–31, 33–37]. Thus, the presence of this phenomenon can affect the dynamic characteristics of systems, and it leads to the degradation of the system performance. Besides, when practical systems with errors or external disturbances are modeling, uncertainties parameters are frequently included. In this context, two types of uncertainties exist in the literature, which are mainly polytopic uncertainties and norm bounded. Indeed, one of the most significant exigencies for a control system is robustness [38, 39]. Therefore, from a practical viewpoint, it is necessary to investigate switched time-varying delay systems with extra uncertain parameters. In this regard, many of the uncertain systems can be approximated by systems with polytopic uncertainties.

In recent years, switched nonlinear time-varying delay systems have received a major interest, and many significant results have been established [4, 13–20, 25–31, 33]. Thus, from the switching strategies, the existing results can be classified into two categories, which are, respectively, restrictive switching and arbitrary switching. In fact, stability analysis and stabilization under restrictive switching have been investigated mainly based on the Lyapunov–Krasovskii functional (LKF) and the average dwell time approach [15]. For example, in [15], the robust stability and the control design problems for switched nonlinear systems have been investigated by using the average dwell time approach. The work in [34] addresses state feedback controllers design for switched nonlinear time-delay systems. Furthermore, the stability analysis of switched nonlinear systems has been investigated in [17] by employing the trajectory-based comparison method.

On the other side, the stability analysis and stabilization of switched time-delay systems under arbitrary switching have been studied based on the common Lyapunov–Krasovskii functional (CLKF) [14] for all the subsystems. Despite the difficulty related to the application of this method for switched nonlinear systems, some results exist for this framework. For instance, in [18], the adaptive control problem for switched nonlinear systems has been presented based on the adaptive backstepping technique and the CLF approach. In addition, in [20], the stabilization problem for switched nonlinear systems has been investigated based on the Metzler matrices. Moreover, the work in [19] deals with the stability analysis of switched nonlinear interconnected systems based on the vector Lyapunov approach and M-matrix theory. The authors in [25, 28] have focused on the stability analysis of switched nonlinear systems by using the aggregation techniques and the M-matrix theory. Furthermore, by including the Takagi–Sugeno (TS) fuzzy model as a powerful approximation tool of the initial nonlinear system, based on the aggregation techniques, algebraic stability criterion for TS Fuzzy switched systems were proposed in [30, 31].

It should be noted that all the aforementioned works for feedback stabilization have considered memoryless state feedback controllers. However, this kind of controllers cannot have an effect on the time-delay systems, since it does not introduce the past state information of the systems. In [26], a memory state feedback controller for time-varying delay switched systems has been considered. Indeed, it has been verified that this kind of controller had better immunity to reduce the influence of delay in system dynamics.

From a practical point of view, switched dynamical systems can be affected by mode depending time-varying delays. However, due to its complexities, this kind of systems is less considered [20, 40].

To the best of our knowledge, the robust stability analysis and the memory state feedback controller design for uncertain switched nonlinear systems with mode depending multiple time-varying delays under arbitrary switching have not been studied yet, which are the subject of this work.

Motivated by this consideration, in this paper, new robust stability criteria and memory feedback controller design under random switching for a class of uncertain switched nonlinear systems with multiple time-varying delays have been established. Indeed, based on a CLF, the aggregation techniques [41], and the Borne–Gentina criterion [41], new robust stability conditions for the considered autonomous systems are given. Besides, the obtained results are extended to develop a memory state feedback controller through the pole assignment control for the closed-loop corresponding switched systems.

The main contributions of this paper are emphasized as follows:

(1) There are no results to address switched nonlinear systems with uncertain parameters and mode-dependent multiple time-varying delays. Out of research interest, novel stability analysis and feedback controller design under arbitrary switching for more general kinds of switched nonlinear systems will be presented.

(2) Compared to the existing criterion for switched systems under arbitrary switching, by using the aggregation techniques the difficulty related to the existence of a CLF through the LMIs approach can be avoided.

(3) Contrary to searching a CLF through the LMIs approach considering a hard task in this investigation, the developed stability and stabilization criteria are explicit and simple to use.

(4) Although there are some studies on memory state feedback control, the memory state feedback controller has not been involved for switched nonlinear systems with multiple time-varying delays. In addition, the developed controller has an explicit form, and it allows stabilizing the resulting closed-loop systems under arbitrary switching without any computations over LMIs constraints.

The rest of the paper is organized as follows: Section 2 gives the problem statement and some definitions. In Section 3, the main results are presented. Section 4 focuses on the application of the main results to switched nonlinear systems modeled by differential equations. In Section 5, some simulation examples are provided to illustrate the effectiveness of the proposed approach. Finally, some conclusions are addressed in Section 6.

Notations. Throughout this paper, I_n is an identity matrix, $\mathfrak{R}^n$ denotes the n-column vectors, and $\|.\|$ denotes the Euclidean norm. In addition, for any given vectors $v = (v_l)_{1\le l\le n}$, $w = (w_l)_{1\le l\le n} \in \mathfrak{R}^n$, the scalar product of vectors u and v is defined as$\langle v, w\rangle = \sum_{l=1}^{n} v_l w_l$. The sign function is defined as $sgn(\varphi) = 1$ (resp. $sgn(\varphi) = -1$) if $\varphi \in \mathbb{R}_+^*$(resp. $\varphi \in \mathbb{R}_-^*$) and $sgn(\varphi) = 0$ if $\varphi = 0$. For a given matrix A, $\lambda(A)$ denotes the set of its eigenvalues and $A^T$and A^{-1} denote its transpose and inverse, respectively. We denote $A^* = (a_{il}^*)_{1\le i,l\le n}$with $a_{il}^* = a_{il}$ if $i = l$ and $a_{il}^* = |a_{il}|$ if $i \ne l$. Finally, the representation $(.)$ denotes $(x(t), t)$.

2. Problem Statement and Preliminaries

2.1. Problem Statement. Consider the following switched nonlinear system with multiple time-varying delays given by

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(\cdot)x(t) + \sum_{l=1}^{L} D_{l,\sigma(t)}(\cdot)x\left(t - r_{l,\sigma(t)}(t)\right) + B_{\sigma(t)}(\cdot)u(t), \\ x(\theta) = \phi(\theta), \quad \theta \in \left[-\max_{l\in \underline{L}}\left(r_{l,\sigma(t)}\right) 0\right], \end{cases} \tag{1}$$

where $x(t) \in \mathfrak{R}^n$ is the state vector at time t, $u(t)$ is the control input, $\sigma(t)$: $\mathfrak{R}_+ \longrightarrow \underline{N} = \{1, \ldots, N\}$ is the switching signal, and $\sigma(t) = i \in \underline{N}$ means that the i^{th} subsystem is active with N being the number of subsystems. $A_i(.)$, $D_{l,i}(.)$, and $B_i(.)$ are matrices which have nonlinear elements with appropriate dimensions, and $\phi(t)$ is the continuous vector valued function specifying the initial state of the system. $r_{l,i}(t)$ denotes the time-varying delay functions which satisfy

$$0 \le r_{l,i}(t) \le \tau, \tag{2}$$

$$\left|\dot{r}_{l,i}(t)\right| \le d < 1, \tag{3}$$

where τ and d are two constant scalars.

Assume that all subsystems are uncertain of polytopic type, which are represented as

$$A_i(.) = \sum_{p=1}^{P} \mu_{ip}(t) A_{ip}(.), \quad i \in \underline{N}, \tag{4}$$

$$D_{l,i}(.) = \sum_{q=1}^{Q} \lambda_{l,iq}(t) D_{l,iq}(.), \tag{5}$$

where $A_{ip}(.)$, $p \in \underline{P}$, and $D_{l,iq}(.)$ $q \in \underline{Q}$ are, respectively, the vertex matrices denoting the extreme points of the polytopes $A_i(.)$ and $D_i(.)$. $P$is the number of the vertex matrices $A_i(.)$, Q is the number of the vertex matrices $D_{l,i}(.)$ and the weighting factors $\mu_{ip}(t)$, $\lambda_{l,iq}(t)$ are polytopic uncertainties parameters belonging to$\mu_{ip}(t)$: $\sum_{p=1}^{P} \mu_{ip}(t) = 1$ $\mu_{ip}(t)$: $\sum_{p=1}^{P} \mu_{ip}$ $(t) = 1$, $\mu_{ip}(t) \ge 0$, and $\lambda_{l,iq}(t)$: $\sum_{q=1}^{Q} \lambda_{l,iq}(t) = 1$, $\lambda_{l,iq}(t) \ge 0$.

2.2. Preliminaries. In the sequel, we introduce some lemmas, definitions, and criteria, which play important roles in deducing our main results.

Lemma 1 (see [40]). *The matrix $A = (a_{ij})_{1\le i,j\le n}$ is called an $M-$ matrix if the following conditions are satisfied:*

(i) $a_{ii} > 0\,(i = 1, \ldots, n)$, $a_{ij} \le 0\,(i \ne j, i, j = 1, \ldots, n)$.

(ii) *All the successive principal minors of A are positive:*

$$\det\begin{pmatrix} a_{11} & \cdots & a_{1i} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ii} \end{pmatrix} > 0, \quad (i = 1, \ldots, n). \tag{6}$$

(iii) *For any positive vector $x = (x_1, \ldots, x_n)^T$, the system of equations $A(.)x$ has a positive solution $x = (x_1, \ldots, x_n)^T$.*

Definition 1. (see [41]). The matrix $T_{mc}(.)$ is said to be the pseudo-overvaluing matrix of the system given by $\dot{x} = A(.)x$, respectively, to the vector norm $p(x) = [|x_1|, \ldots, |x_i|, \ldots, |x_n|]^T$, with $x = [x_1, x_2, \ldots, x_n]^T$, if the next inequality is satisfied:

$$D^+ p(x) \le T_{mc}(.)p(x), \tag{7}$$

where D^+ denotes the right-hand derivative operator.

Assumption 1. In what follows, we assumed that all the nonlinear elements $T_{mc}(.)$ are separated in the last row.

Lemma 2 (see [41]). *If $T_{mc}(.)$is the pseudo-overvaluing matrix of the system: $\dot{x} = A(.)x$, then it verifies the following properties:*

(i) *All the off-diagonal elements of $T_{mc}(.)$ are nonnegative*

(ii) *If the eigenvalues of $T_{mc}(.)$ have negative real parts, then $T_{mc}(.)$ is the opposite of an $M-$ matrix*

(iii) *The main eigenvector $v(t, x(t))$ is related to the main eigenvalue λ_m such that reel(λ_m)= max $(\lambda \in \lambda M_C(.))$ is a constant vector*

Lemma 3 (see [41]). *The application of the Kotelyanski lemma [42] to the pseudo-overvaluing matrix $T_{mc}(.)$is relative to the system: $\dot{x} = A(.)x$; $A(.) = (a_{ij}(.))_{1\le i,j\le n}$ allows deducing the stability of the corresponding system, if $T_{mc}(.)$ is the opposite of an $M-$ matrix, which implies that all the successive principal minors have alternated signs with the first being negative:*

$$a_{1,1} < 0,$$
$$\begin{vmatrix} a_{1,1} & |a_{1,2}| \\ |a_{2,1}| & a_{2,2} \end{vmatrix} > 0, \ldots, (-1)^n \begin{vmatrix} a_{1,1} & |a_{1,2}| & \cdots & |a_{1,n}| \\ |a_{2,1}| & a_{2,2} & \cdots & |a_{2,n}| \\ \vdots & \vdots & \cdots & \vdots \\ |a_{n,1}(.)| & |a_{n,2}(.)| & \cdots & a_{n,n}(.) \end{vmatrix} > 0. \tag{8}$$

3. Main Results

3.1. Stability Analysis. In this section, we investigate sufficient delay-dependent stability conditions for the autonomous system (1).

Theorem 1. *The autonomous system (1) is robustly asymptotically stable under $\sigma(t) = i \in \underline{N}$, if $T_{mc}(.)$ is the opposite of an M − matrix, where*

$$T_{mc}(.) = \max_{\substack{i\in\underline{N}\\ p\in\underline{P}}} \left(A_{ip}(.)\right)^{*} + (1-d)\max_{\substack{i\in\underline{N}\\ q\in\underline{Q}}}\left(\sup_{[.]}\left(\left|\sum_{l=1}^{L} D_{l,iq}(.)\right|\right)\right), \tag{9}$$

and d is given in (3).

Proof. Let $\nu \in \mathfrak{R}^n$ with components $(\nu_h > 0, \forall h = 1, \ldots, n)$ and $x(t) \in \mathfrak{R}^n$.

Define the following Lyapunov functional for the autonomous system (1):

$$V(t) = V_1(t) + V_2(t), \tag{10}$$

with

$$\begin{aligned} V_1(t) &= (1-d)^2\langle |x(t)|, \nu\rangle, \\ V_2(t) &= (1-d)\langle \sum_{l=1}^{L} D_{l,M}\int_{t-r_{l,\sigma(t)}(t)}^{t} |x(s)|\mathrm{d}s, \nu\rangle, \end{aligned} \tag{11}$$

where

$$D_{l,M} = \max_{\substack{i\in\underline{N}\\ q\in\underline{Q}}}\left(\sup_{[.]}\left(\left|\sum_{l=1}^{L} D_{l,iq}(.)\right|\right)\right). \tag{12}$$

The right derivative of $V(t)$ along the trajectory of system (1) yields to

$$\frac{\mathrm{d}^+V(t)}{\mathrm{d}t^+} = \frac{\mathrm{d}^+V_1(t)}{\mathrm{d}t^+} + \frac{\mathrm{d}^+V_2(t)}{\mathrm{d}t^+}, \tag{13}$$

where

$$\begin{aligned} \frac{\mathrm{d}^+V_1(t)}{\mathrm{d}t^+} &= (1-d)^2\langle \frac{\mathrm{d}^+|x(t)|}{\mathrm{d}t^+}, \nu\rangle \\ &= (1-d)^2\langle \operatorname{sgn}(x(t))\frac{\mathrm{d}^+x(t)}{\mathrm{d}t^+}, \nu\rangle, \\ \operatorname{sgn}(x(t)) &= \begin{pmatrix} \operatorname{sgn}(x_1(t)) \\ \ddots \\ \operatorname{sgn}(x_n(t)) \end{pmatrix}. \end{aligned} \tag{14}$$

Then,

$$\begin{aligned} \frac{\mathrm{d}^+V_1(x(t),t)}{\mathrm{d}t^+} &= (1-d)^2\left\langle\left\langle \operatorname{sgn}(x(t))\left(A_{\sigma(t)}(.)x(t) + \sum_{l=1}^{L} D_{l,\sigma(t)}(.)x\left(t-r_{l,\sigma(t)}(t)\right)\right), \nu\right\rangle, \\ &\le (1-d)^2\left\langle\left(\left(A_{mc}(.)\right)^{*}|x(t)| + \left|\sum_{l=1}^{L} D_{l,mc}\right|\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right|\right), \nu, \right\rangle \end{aligned} \tag{15}$$

where $A_{mc}(.) = \max_{\substack{i\in\underline{N}\\ p\in\underline{P}}}((A_{ip}(.))^{*})$.

On the other side, $((\mathrm{d}^+V_2(t))/\mathrm{d}t^+) = (1-d)\langle|\sum_{l=1}^{L} D_{l,mc}|(|x(t)|), \nu\rangle - (1-d)\,(1-\dot r_{l,\sigma(t)}(t))\,\langle|\sum_{l=1}^{L} D_{l,mc}|(|x(t-r_{l,\sigma(t)}(t))|), \nu\rangle$. Therefore, it is easy to see that

$$\begin{aligned} \left(\dot r_{l,\sigma(t)}(t) - 1\right)\left\langle\left|\sum_{l=1}^{L} D_{l,mc}\right|\left(\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right|\right), \nu\right\rangle &\le \left(\left|\dot r_{l,\sigma(t)}(t)\right| - 1\right)\left\langle\left|\sum_{l=1}^{L} D_{l,mc}\right|\left(\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right|\right), \nu\right\rangle, \\ &\le (d-1)\left\langle\left|\sum_{l=1}^{L} D_{l,mc}\right|\left(\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right|\right), \nu\right\rangle, \end{aligned} \tag{16}$$

which implies

$$\frac{\mathrm{d}^+V_2(x(t),t)}{\mathrm{d}t^+} \le (1-d)\left\langle\left|\sum_{l=1}^{L} D_{l,mc}\right|(|x(t)|), \nu\right\rangle - (1-d)^2\left\langle\left|\sum_{l=1}^{L} D_{l,mc}\right|\left(\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right|\right), \nu\right\rangle. \tag{17}$$

From (15) and (17), we obtain

$$\begin{aligned}\frac{d^+V(t)}{dt^+} &< (1-d)^2\left\langle \left((A_{mc}(.))^* |x(t)| \right), \nu \right\rangle + (1-d)^2\left\langle \left(\left|\sum_{l=1}^{L} D_{l,mc}\right| \left|x\left(t-r_{l,\sigma(t)}(t)\right)\right| \right), \nu \right\rangle \\ &+ (1-d)\left\langle \left|\sum_{l=1}^{L} D_{l,mc}\right| \left(|x(t)|\right), \nu \right\rangle - (1-d)^2\left\langle \left|\sum_{l=1}^{L} D_{l,mc}\right| \left(\left|x\left(t-r_{l,\sigma(t)}(t)\right)\right| \right), \nu \right\rangle, \\ &= (1-d)^2\left\langle \left((A_{mc}(.))^* |x(t)| \right), \nu \right\rangle + (1-d)\left\langle \left(\left|\sum_{l=1}^{L} D_{l,mc}\right| |x(t)| \right), \nu \right\rangle.\end{aligned} \tag{18}$$

Since $0 < (1-d) \leq 1$, it becomes

$$\begin{aligned}&\left\langle \left((A_{mc}(.))^* |x(t)| \right), \nu \right\rangle + (1-d)\left\langle \left(\left|\sum_{l=1}^{L} D_{l,mc}\right| |x(t)| \right), \nu \right\rangle \\ &= \left\langle \left((A_{mc}(.))^* + (1-d)\left|\sum_{l=1}^{L} D_{l,mc}\right| \right)|x(t)|, \nu \right\rangle \\ &= \left\langle \left|T_{mc}(.)\right| |x(t)|, \nu \right\rangle,\end{aligned} \tag{19}$$

where is $T_{mc}(.)$ given in (9), knowing that

$$\langle T_{mc}(.)|x(t)|, \nu\rangle = \langle T_{mc}(.)^T \nu, |x(t)|\rangle. \tag{20}$$

The main eigenvector $\nu(t, x(t))$of $T_{mc}(.)$ relative to the main eigenvalue λ_m is constant. We assume that $T_{mc}(.)$ is the opposite of an $M-$ matrix. Therefore, we can find a vector $\omega \in \mathfrak{R}_+^{*\,n}$ $(\omega_h \in \mathfrak{R}_+^*\; h = 1, \ldots, n)$ satisfying the following relation $(-T_{mc}(.))^T \nu = \omega, \forall \nu \in \mathfrak{R}_+^{*n}$.

Thus, it easy to follow that

$$\langle (T_{mc}(.))|x(t)|\rangle, \nu = \langle (T_{mc}(.))^T \nu, |x(t)|\rangle = -\langle \omega, |x(t)|\rangle. \tag{21}$$

Substituting (21) into (19) leads to

$$\frac{d^+V(t)}{dt^+} \leq -\langle \omega, |x(t)|\rangle = -\sum_{h=1}^{n} \omega_h \left|x_h(t)\right|. \tag{22}$$

Therefore, it can be established that $((d^+V(t))/dt^+) < 0$ for all $x(t) \neq 0$.

This completes the proof of Theorem 1. □

Remark 1. Theorem 1 gives the main results of the stability analysis for the autonomous system (1) under $\sigma(t) = i \in \underline{N}$ and all admissible uncertainties (4) and (5). The conditions presented in Theorem 1 will be simplified by applying the Borne–Gentina criterion in Theorem 3 and Corollary 1.

3.2. Memory State Feedback Design. In this section, we consider the following memory state feedback controller:

$$u(t) = -K_{\sigma(t)}(.)x(t) - \sum_{l=1}^{L} L_{l,\sigma(t)}(.)x\left(t - r_{l,\sigma(t)}(t)\right), \tag{23}$$

where $K_i(.)$ and $L_{l,i}(.)$, $i \in \underline{N}$, are nonlinear controller gains to be determined.

The resulting closed-loop switched system composed from (1) and (23) is represented by

$$\begin{cases} \dot{x}(t) = \overline{A}_{\sigma(t)}(.)x(t) + \sum_{l=1}^{L} \overline{D}_{l,\sigma(t)}(.)x\left(t - r_{l,\sigma(t)}(t)\right), \\ x(\theta) = \phi(\theta), \quad \theta \in \left[-\max_{1\leq l\leq L}(\tau_l)0\right], \end{cases} \tag{24}$$

where $\overline{A}_{\sigma(t)}(.) = A_{\sigma(t)}(.) - B_{\sigma(t)}(.)K_{\sigma(t)}(.)$ and $\overline{D}_{l,\sigma(t)}(.) = D_{l,\sigma(t)}(.) - B_{\sigma(t)}(.)L_{l,\sigma(t)}(.)$.

In what follows, we present our result for the memory state feedback control of system (1).

Theorem 2. *System (1) is robustly stabilizable via controller (23) under $\sigma(t) = i \in \underline{N}$, for all admissible uncertainly parameters $\mu_{ip}(t)$ and $\lambda_{iq}(t)$ for each $p \in \underline{P}$ and $q \in \underline{Q}$, such that the closed-loop switched system (5) is robustly asymptotically globally stable, if there exist matrices $K_{ip}(.)$ and $L_{l,iq}(.)$, $l \in \underline{L}$, with appropriate dimensions, satisfying that $T_{mc}(.)$ is the opposite of an $M-$ matrix,where*

$$T_{mc}(.) = \max_{\substack{i\in \underline{N}\\ p\in \underline{P}}} \left(\overline{A}_i(.)\right)^* + (1-d)\max_{\substack{i\in \underline{N}\\ q\in \underline{Q}}} \left(\sum_{l=1}^{L} \sup_{[.]}\left(\left|\overline{D}_{l,i}(.)\right|\right) \right), \tag{25}$$

and d is introduced in (3).

Proof. We assume that there exist matrices $K_{ip}(.)$ and $L_{l,iq}(.)$, $\forall i \in \underline{N}$, $p \in \underline{P}$, $q \in \underline{Q}$, and $l \in \underline{L}$ satisfying that $T_{mc}(.)$ is the opposite of an $M-$ matrix. According to the proof of Theorem (1), system (1) is robustly asymptotically stabilizable via controller (23) under $\sigma(t) = i \in \underline{N}$ and all admissible uncertainties (4) and (5).

The proof of Theorem 2 is completed. □

Remark 2. Theorem 2 gives the main results for the stabilization of the control system (1) via controller (23). In the sequel, by applying the Borne–Gentina criterion, this result will be applied in Theorem 4 to develop a memory feedback controller via the pole assignment control to stabilize the corresponding closed-loop system under $\sigma(t) = i \in \underline{N}$ and all admissible uncertainties (4) and (5).

4. Application to Switched Systems Modeling via Differential Equations

In this subsection, we apply the previously reached results for a class of switched nonlinear systems modeled by a set of differential equations.

Considering a class of uncertain switched nonlinear systems with multiple time-varying delays formed by N subsystems, each subsystem $S_i, i \in \underline{N}$ is given by the following differential equation:

$$\begin{cases} y^{n}(t) + \left(\left(\sum_{p=1}^{P} \mu_{ip}(t) \sum_{h=0}^{n-1} \widetilde{a}_{ip}^{h}(.) y^{(h)}(t) \right) + \sum_{l=1}^{L} \sum_{q=1}^{Q} \lambda_{iq,l}(t) \sum_{h=0}^{n-1} \widetilde{d}_{l,iq}^{h}(.) y^{(h)}\left(t - r_{i,l}(t)\right) \right) = \widetilde{b}_i(.) u(t), \quad 0 \le r_{i,l}(t) \le \tau, \\ y^{(j)}(s) = \phi_h(s), \quad s \in [-\tau\ 0], h = 1, \ldots, n-1, \end{cases} \tag{26}$$

where $y(t) \in \mathfrak{R}^n$, $\widetilde{a}_{ip}^{h}(.)$, and $\widetilde{d}_{iq,l}^{h}(.)$ are nonlinear coefficients, $\forall i \in \underline{N}$, $p \in \underline{P}$, $q \in \underline{Q}$, $l \in \underline{L}$, and $(h = 1, \ldots, n-1)$. $u(t) \in \mathfrak{R}$ is the control input. $r_{i,l}(t)$ denotes the time-varying delays satisfying that $0 \le r_{i,l}(t) \le \tau$ and $|\dot{r}_{i,l}(t)| \le d < 1$ where τ and d are given, respectively, in (2) and (3). $\phi_j(s)(h = 1, \ldots, n-1)$ are the initial conditions on $[-\tau\ 0]$.

Consider the following change of variable:

$$x_{h+1}(t) = \frac{\mathrm{d}y^{(h)}}{\mathrm{d}t^{(h)}}, \quad h = 0, \ldots, n-1. \tag{27}$$

Due to (27), relation (26) becomes

$$\begin{cases} \dot{x}_h(t) = x_{h+1}(t), \\ \dot{x}_n(t) = -\left(\sum_{p=1}^{P} \mu_{ip}(t) \sum_{h=0}^{n-1} \widetilde{a}_{ip}^{h}(.) x_{h+1}(t) + \sum_{l=1}^{L} \sum_{q=1}^{Q} \lambda_{iq,l}(t) \sum_{h=0}^{n-1} \widetilde{d}_{iq,l}^{j}(.) x_{h+1}\left(t - r_{i,l}(t)\right) \right) + \widetilde{B}_i(.) u(t), \quad i \in \underline{N}, \end{cases} \tag{28}$$

or under matrix form, we obtain the following state representation:

$$\begin{cases} \dot{x}(t) = \sum_{p=1}^{P} \mu_{ip}(t) \widetilde{A}_{ip}(.) x(k) + \sum_{l=0}^{L} \sum_{q=1}^{Q} \lambda_{iq,l}(t) \widetilde{D}_{l,iq}(.) x\left(t - r_{l,\sigma(t)}(t)\right) + \widetilde{B}_i(.) u(t), \\ x(s) = \phi(s), \ s \in \left[-\max_{1 \le l \le L}(\tau_l)\ 0 \right], \quad i \in \underline{N}, \end{cases} \tag{29}$$

where $x(t)$ is the state vector, whose components are $x_h(t)$, $h = 1, \ldots, n$.

The vertex matrices $\widetilde{A}_{ip}(.)$, $\widetilde{D}_{l,iq}(.)$, and $\widetilde{B}_i(.)$ are given as follows:

$$\widetilde{A}_{ip}(.) = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -\widetilde{a}^0_{ip}(.) & -\widetilde{a}^1_{ip}(.) & \dots & -\widetilde{a}^{n-1}_{ip}(.) \end{bmatrix}, \quad i \in \underline{N},$$

$$\widetilde{B}_i(.) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \widetilde{b}_i(.) \end{bmatrix}, \quad i \in \underline{N},$$

$$\widetilde{D}_{l,iq}(.) = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ -\widetilde{d}^0_{l,iq}(.) & -\widetilde{d}^1_{l,iq}(.) & \dots & -\widetilde{d}^{n-1}_{l,iq}(.) \end{bmatrix}, \quad i \in \underline{N}, \tag{30}$$

where $\widetilde{a}^h_{ip}(.)$ is a coefficient of the instantaneous characteristic polynomial $G_{\widetilde{A}_{ip}(.)}(s)$ of matrix $\widetilde{A}_{ip}(.)$ given by

$$G_{\widetilde{A}_{ip}(.)}(s) = s^n + \sum_{h=0}^{n-1} \widetilde{a}^h_{ip}(.)s^h, \tag{31}$$

and $\widetilde{d}^h_{l,iq}(.)$ is a coefficient of the instantaneous characteristic polynomial $N_{\widetilde{D}_{l,iq}(.)}(s)$ of matrix $\widetilde{D}_{l,iq}(.)$ defined by

$$N_{\widetilde{D}_{l,iq}(.)}(s) = \sum_{h=0}^{n-1} \widetilde{d}^h_i(.)s^h. \tag{32}$$

Assume that all subsystems are uncertain of polytopic type, which can be described as

$$\begin{aligned} \widetilde{A}_i(.) &= \sum_{p=1}^{P} \mu_{ip}(t)\widetilde{A}_{ip}(.), \quad p \in \underline{P}, i \in \underline{N}, \\ \widetilde{D}_{l,i}(.) &= \sum_{q=1}^{Q} \lambda_{l,iq}(t)\widetilde{D}_{l,iq}(.), \quad q \in \underline{Q}, l \in \underline{L}, i \in \underline{N}. \end{aligned} \tag{33}$$

Considering the switched rule given in (1), the switched control system will be represented as

$$\begin{cases} \dot{x}(t) = \widetilde{A}_{\sigma(t)}(.)x(t) + \sum_{l=1}^{L} \widetilde{D}_{l,\sigma(t)}(.)x\big(t - r_{l,\sigma(t)}(t)\big) + \widetilde{B}_{\sigma(t)}(.)u(t), \\ x(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0], \sigma(t) = i \in \underline{N}. \end{cases} \tag{34}$$

Finally, according to the controller (23), the closed-loop system will be represented by

$$\begin{cases} \dot{x}(t) = \overline{\widetilde{A}}_{\sigma(t)}(.)x(t) + \sum_{l=1}^{L} \overline{\widetilde{D}}_{l,\sigma(t)}(.)x\big(t - r_{l,\sigma(t)}(t)\big), \\ x(s) = \phi(s), \quad s \in [-\tau, 0], \sigma(t) = i \in \underline{N}, \end{cases} \tag{35}$$

with $\overline{\widetilde{A}}_i(.) = \widetilde{A}_i(.) - \widetilde{B}_i(.)K_i(.)$ and $\overline{\widetilde{D}}_{l,i}(.) = \widetilde{D}_{l,i}(.) - \widetilde{B}_i(.)L_{l,i}(.)$.

A change of base for system (35) into the arrow matrix form [31] allows that

$$\begin{cases} \dot{z}(t) = \sum_{p=1}^{P} \mu_{ip}(t)E_{ip}(.)z(t) + \sum_{l=1}^{L}\sum_{q=1}^{Q} \lambda_{l,iq}(t)F_{l,iq}(.)z\big(t - r_{l,i}(t)\big), \\ z(s) = P\phi(s), \quad s \in [-\tau, 0], i \in \underline{N}, \end{cases} \tag{36}$$

where $z(t) = Px(t)$ is the new state vector and P is the corresponding passage matrix given by

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 & 0 \\ \alpha_1 & \alpha_2 & \dots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \dots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \dots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \dots & (\alpha_{n-1})^{n-1} & 1 \end{bmatrix}, \tag{37}$$

with α_j, $j = 1, \dots, n-1$ being distinct arbitrary constant parameters.

The vertex matrices in the arrow form $E_{ip}(.)$ and $F_{l,iq}(.)$ are given by

$$\begin{aligned} E_{ip}(.) &= P^{-1}\overline{\widetilde{A}}_{ip}(.)P = \begin{bmatrix} \alpha_1 & 0 & \dots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & \alpha_{n-1} & \beta_{n-1} \\ \gamma^1_{ip}(.) & \dots & \dots & \gamma^{n-1}_{ip}(.) & \gamma^n_{ip}(.) \end{bmatrix}, \quad i \in \underline{N}, p \in \underline{P}, \\ F_{l,iq}(.) &= P^{-1}\overline{\widetilde{D}}_{l,iq}(.)P = \begin{bmatrix} 0_{n-1,n-1}, \dots, 0_{n-1,1} \\ \delta^1_{l,iq}(.), \dots, \delta^{n-1}_{l,iq}(.)\ \delta^n_{l,iq}(.) \end{bmatrix}, \quad i \in \underline{N}, l \in \underline{L}, q \in \underline{Q}, \end{aligned} \tag{38}$$

with

$$\beta_h = \prod_{\substack{q=1\\ q\neq h}}^{n-1} \left(\alpha_h - \alpha_q\right)^{-1}, \quad \forall h = 1, \dots, n-1. \tag{39}$$

The elements of $E_{ip}(.)$are given as follows:

$$\begin{cases} \gamma_{ip}^{h}(.) = -G_{\overline{\overline{A}}_{ip}(.)}(\alpha_h), & \forall h = 1, \dots, n-1, \\ \gamma_{ip}^{n}(.) = -\overline{\overline{a}}_{ip}^{n-1}(.) - \sum_{h=1}^{n-1} \alpha_h,, \ i \in \underline{N}, p \in \underline{P}, \end{cases} \tag{40}$$

and the elements of $F_{l,iq}(.)$ are

$$\begin{cases} \delta_{l,iq}^{h}(.) = -N_{\overline{\overline{D}}_{l,iq}(.)}(\alpha_h), & \forall h = 1, \dots, n-1, \\ \delta_{l,iq}^{n}(.) = -\overline{\overline{d}}_{l,iq}^{n-1}(.) \end{cases}, i \in \underline{N}, l \in \underline{L}, q \in \underline{Q}. \tag{41}$$

Taking into account the previous relations, the matrix $T_{l,ipq}(.)$ is given by

$$T_{l,ipq}(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & \alpha_{n-1} & |\beta_{n-1}| \\ t_{l,ipq}^{1}(.) & \dots & \dots & t_{l,ipq}^{n-1}(.) & t_{l,ipq}^{n}(.) \end{bmatrix}, \quad i \in \underline{N}, l \in \underline{L}, q \in \underline{Q}, \tag{42}$$

with

$$\begin{cases} t_{l,ipq}^{h}(.) = \left|\gamma_{ip}^{h}(.)\right| + (1-d)\sup_{[.]}\left(\sum_{l=1}^{L}\left|\delta_{l,iq}^{h}(.)\right|\right), & h = 1, \dots, n-1, \\ t_{l,ipq}^{n}(.) = \gamma_{ip}^{n}(.) + (1-d)\sup_{[.]}\left(\left(\sum_{l=1}^{L}\left|\delta_{l,iq}^{n}(.)\right|\right)\right). \end{cases} \tag{43}$$

Finally, the common pseudo-overvaluing matrix $T_{mc}(.)$ of system (35) can be deduced such as

$$T_{mc}(.) = \begin{bmatrix} \alpha_1 & 0 & \dots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & \alpha_{n-1} & |\beta_{n-1}| \\ \overline{t}^{1}(.) & \dots & \dots & \overline{t}^{n-1}(.) & \overline{t}^{n}(.) \end{bmatrix}, \tag{44}$$

where

$$\overline{t}^{h}(.) = \max_{\substack{i\in \underline{N}\\ p\in \underline{P}\\ q\in \underline{Q}}}\left(t_{l,ipq}^{h}(.)\right), \quad \forall h = 1, \dots, n. \tag{45}$$

4.1. Stability Conditions for Continuous-Time Uncertain Switched Nonlinear Systems with Multiple Time-Varying Delays. In this subsection, we give some sufficient stability conditions for the autonomous system (34).

Theorem 3. *The autonomous system (34) is robustly globally asymptotically stable under $\sigma(t) = i \in \underline{N}$ and admissible uncertainties (4) and (5), if there exist $\alpha_h < 0$ $(h = 1, \dots, n-1)$, $\alpha_h \neq \alpha_q, \forall h \neq q$, such that the following condition is satisfied:*

$$-\overline{t}^{n}(.) + \sum_{h=1}^{n-1} \overline{t}^{h}(.)\left|\beta_h\right|\alpha_h^{-1} > 0. \tag{46}$$

Proof. The application of the Borne–Gentina criterion to $T_{mc}(.)$ yields to the following stability conditions for the autonomous system (34):

$$(-1)^h \Delta_h > 0, \quad h = 1, \dots, n-1, \tag{47}$$

where Δ_h is the h^{th} principal minor of $T_{mc}(.)$.

Therefore, for $h = 1, \dots, n-1$, the first condition in Theorem 3 is verified such that $\alpha_h \in \mathfrak{R}_{-}^{*}$.

Finally, for $h = n$, the last condition is verified as follows:

$$(-1)^n \det\left(T_{mc}(.)\right) = (-1)^n \begin{vmatrix} \alpha_1 & 0 & \dots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & \alpha_{n-1} & |\beta_{n-1}| \\ \overline{t}^{1}(.) & \dots & \dots & \overline{t}^{n-1}(.) & \overline{t}^{n}(.) \end{vmatrix}, \tag{48}$$

That is,

$$= (-1)^n \left[\overline{t}^{n}(.)\prod_{q=1}^{n-1}\alpha_q - \sum_{h=1}^{n-1}\left(|\overline{t}^{h}(.)|\left|\beta_h\right|\prod_{\substack{h=1\\ h\neq q}}^{n-1}\alpha_h\right)\right] > 0. \tag{49}$$

The division of this previous condition by $((-1)^{n-1}\prod_{q=1}^{n-1}\alpha_q)$ yields to $-\overline{t}^{n}(.) + \sum_{h=1}^{n-1}\overline{t}^{h}(.)|\beta_h|\alpha_h^{-1} > 0$.

The proof of Theorem 3 is complete. □

Remark 3. If there exist parameters $\alpha_h(h = 1, \dots, n-1)$ satisfying that

$$\begin{aligned} &\beta_h\left(G_{\widetilde{A}_{ip}(.)}(\alpha_h) + (1-d)\sup_{[.]}\left(\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(\alpha_h)\right)\right) \\ &\quad = -\beta_h\left(\gamma_{ip}^{n}(.) + (1-d)\sup_{[.]}\left(\sum_{l=1}^{L}\delta_{l,iq}^{n}(.)\right)\right) < 0, \end{aligned} \tag{50}$$

Theorem 3 can be simplified to Corollary 1.

Corollary 1. *The autonomous system (35) is robustly globally asymptotically stable under $\sigma(t) = i \in \underline{N}$ and admissible uncertainties (4) and (5), if there exist α_h $(h = 1, \ldots, n-1) < 0$ such that $\alpha_h \neq \alpha_q$, $\forall h \neq q$, and the inequalities below are satisfied:*

$$\beta_h \left(G_{\widetilde{A}_{ip}(.)}(s = \alpha_h) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(s = \alpha_h) \right) \right) < 0,$$
$$\beta_h \left(G_{\widetilde{A}_{ip}(.)}(s = 0) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(s = 0) \right) \right) > 0. \tag{51}$$

Proof. If there exist α_h $(h = 1, \ldots, n-1) < 0$ such that

$$\begin{aligned} &\beta_h \left(G_{\widetilde{A}_{ip}(.)}(\alpha_h) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(\alpha_h) \right) \right) \\ &\quad = -\beta_h \left(\gamma_{ip}^{n}(.) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} \delta_{l,iq}^{n}(.) \right) \right) < 0, \end{aligned} \tag{52}$$

$T_{mc}(.)$ will be given by $T_{mc}(.) = \begin{bmatrix} \alpha_1 & 0 & \ldots & 0 & \beta_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & \alpha_{n-1} & \beta_{n-1} \\ t_{l,ipq}^{1}(.) & \ldots & \ldots & t_{l,ipq}^{n-1}(.) & t_{l,ipq}^{n}(.) \end{bmatrix}$, where $\overline{t}_{l,ipq}^{h}(.) = (\gamma_{ip}^{h}(.) + (1-d) \sup_{[.]} (\sum_{l=1}^{L} \delta_{l,iq}^{h}(.)))$, $\forall h = 1, \ldots, n$.

Therefore, the n^{th} principal minor of $T_{mc}(.)$ is calculated as follows:

$$\begin{aligned} \Delta_n &= -\left(\gamma_{ip}^{n}(.) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} \delta_{l,iq}^{n}(.) \right) \right) \\ &\quad + \sum_{h=1}^{n-1} (\alpha_h)^{-1} \left(\gamma_{ip}^{h}(.) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} \delta_{l,iq}^{h}(.) \right) \right) \beta_h, \\ &= \prod_{h=1}^{n-1} (\alpha_h)^{-1} \left(G_{\widetilde{A}_{ip}(.)}(0) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(0) \right) \right). \end{aligned} \tag{53}$$

This implies that $G_{\widetilde{A}_{ip}(.)}(0) + (1-d) \sup_{[.]} (\sum_{l=1}^{L} N_{\widetilde{D}_{l,iq}(.)}(0)) > 0$.

This proof is complete. ☐

4.2. Memory Feedback Stabilization for Uncertain Switched Nonlinear Systems with Time-Varying Delays. In this subsection, a new memory feedback stabilization for the control system (34) via the pole assignment control is given in Theorem 4.

Theorem 4. *Let all n poles $\{p_1, \ldots, p_n\}$ of system (34) be imposed as real, distinct, and negative. Then, the control system (34) is stabilizing via control law (23), such that the corresponding closed-loop, switched system is robustly globally asymptotically stable under $\sigma(t) = i \in \underline{N}$ and admissible uncertainties (4) and (5), if the following conditions are satisfied:*

$$\begin{cases} \overline{t}^{h}(.) = \max\limits_{\substack{i \in \underline{N} \\ p \in \underline{P} \\ q \in \underline{Q}}} \left(\left|\gamma_{ip}^{h}(.)\right| + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} \left|\delta_{l,iq}^{h}(.)\right| \right) \right) = 0, & \forall h = 1, \ldots, n, \\ \overline{t}^{n}(.) = \max\limits_{\substack{i \in \underline{N} \\ p \in \underline{P} \\ q \in \underline{Q}}} \left(\gamma_{ip}^{n}(.) + (1-d) \sup_{[.]} \left(\sum_{l=1}^{L} \left|\delta_{l,iq}^{n}(.)\right| \right) \right) = p_n, \end{cases} \tag{54}$$

where

$$\begin{cases} \beta_h = \prod\limits_{\substack{q=1 \\ q \neq h}}^{n-1} \left(p_h - p_q \right)^{-1}, & \forall h = 1, \ldots, n-1, \\ \gamma_{ip}^{h}(.) = -G_{\overline{\overline{A}}_{ip}}(p_h), & \forall h = 1, \ldots, n-1, \\ \gamma_{ip}^{n}(.) = -\overline{\overline{a}}_{ip}^{(1)}(.) - \sum\limits_{h=1}^{n-1} p_h, \end{cases} \tag{55}$$

$$\begin{cases} \delta_{l,iq}^{h}(.) = -N_{\overline{\overline{D}}_{l,iq}(.)}(p_h), & \forall h = 1, \ldots, n-1, \\ \delta_{l,iq}^{n}(.) = -\overline{\overline{d}}_{l,iq}^{\,n-1}(.) \end{cases} \tag{56}$$

Proof. For $p_h = \alpha_h$ are real and negative $h = 1, \ldots, n-1$, the Borne–Gentina criterion yields to the following stabilization conditions:

$$(-1)^n \begin{vmatrix} p_1 & 0 & \ldots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & p_{n-1} & |\beta_{n-1}| \\ \overline{t}^{1}(.) & \ldots & \ldots & \overline{t}^{n-1}(.) & \overline{t}^{n}(.) \end{vmatrix} > 0. \tag{57}$$

Since the new dynamic of the system permits concluding that overall $\overline{t}^{j}(.) = 0$ for $j = 1, \ldots, n-1$ and $\overline{t}^{n}(.) = p_n \overline{t}^{n}(.) = p_n$, thus (55) becomes

$$T_{mc}(.) = \begin{bmatrix} p_1 & 0 & \dots & 0 & |\beta_1| \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \dots & 0 & p_{n-1} & |\beta_{n-1}| \\ 0 & \dots & \dots & 0 & p_n \end{bmatrix}, \tag{58}$$

and the system is stable since $p_h < 0$ $h = 1, \dots, n$. □

5. Illustrative Examples

In this section, two numerical examples are introduced to demonstrate the theoretical results.

Example 1. Let us consider system (34) with three subsystems, where the randomly switched model is given as

$$\begin{aligned}
A_{11}(.) &= \begin{bmatrix} 0 & 1 \\ -1.5f(.) & 1-\Phi(.) \end{bmatrix}, \\
A_{12}(.) &= \begin{bmatrix} 0 & 1 \\ 1-f(.) & -2\Phi(.) \end{bmatrix}, \\
A_{21}(.) &= \begin{bmatrix} 0 & 1 \\ -f(.) & 1-\Phi(.) \end{bmatrix}, \\
A_{22}(.) &= \begin{bmatrix} 0 & 1 \\ -2f(.) & -3\Phi(.) \end{bmatrix}, \\
A_{31}(.) &= \begin{bmatrix} 0 & 1 \\ & \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -2f(.) & -\Phi(.) \end{bmatrix}, \\
A_{32}(.) &= \begin{bmatrix} 0 & 1 \\ 1-2f(.) & -\Phi(.) \end{bmatrix}, \\
D_{1,11}(.) &= \begin{bmatrix} 0 & 0 \\ -4.5\psi(.) & -3\psi(.) \end{bmatrix}, \\
D_{2,11}(.) &= \begin{bmatrix} 0 & 0 \\ 3.8\psi(.) & -2.7\psi(.) \end{bmatrix}, \\
D_{1,12}(.) &= \begin{bmatrix} 0 & 0 \\ -4\psi(.) & 1.5\psi(.) \end{bmatrix}, \\
D_{12,2}(.) &= \begin{bmatrix} 0 & 0 \\ 2\psi(.) & -3.5\psi(.) \end{bmatrix}, \\
D_{1,21}(.) &= \begin{bmatrix} 0 & 0 \\ -1.3\psi(.) & 0.9\psi(.) \end{bmatrix}, \\
D_{2,21}(.) &= \begin{bmatrix} 0 & 0 \\ -0.4\psi(.) & -1.2\psi(.) \end{bmatrix}, \\
D_{1,22}(.) &= \begin{bmatrix} 0 & 0 \\ -1.8\psi(.) & 1.2\psi(.) \end{bmatrix}, \\
D_{2,22}(.) &= \begin{bmatrix} 0 & 0 \\ -0.7\psi(.) & -2\psi(.) \end{bmatrix}, \\
D_{1,31}(.) &= \begin{bmatrix} 0 & 0 \\ -3\psi(.) & -2.2\psi(.) \end{bmatrix}, \\
D_{2,31}(.) &= \begin{bmatrix} 0 & 0 \\ 0 & -0.4\psi(.) \end{bmatrix}, \\
D_{1,32}(.) &= \begin{bmatrix} 0 & 0 \\ -3\psi(.) & -3\psi(.) \end{bmatrix}, \\
D_{2,32}(.) &= \begin{bmatrix} 0 & 0 \\ 0 & 0.7\psi(.) \end{bmatrix},
\end{aligned} \tag{59}$$

with $f(.)$, $\Phi(.)$, and $\psi(.)$ are general nonlinear functions.

Hence, we suppose that $\psi(.) \in E([-1, 0.2, 0.5])$ and the corresponding delay functions are listed as follows: $r_{1,1}(t) = 0.8 + (1/5)\cos^2(t)$, $r_{2,1}(t) = 1 + (1/5)\cos^2(t)$, $r_{1,2}(t) = 1.2 + (1/8)\cos^2(t)$, $r_{2,2}(t) = 0.5 + (1/8)\cos^2(t)$, $r_{1,3}(t) = 0.6 + (1/6)\cos^2(t)$, and $r_{2,3}(t) = 1 + (1/6)\cos^2(t)$.

From Corollary 1, with $\alpha = -1$, we obtain the following robust stability conditions:

$$\begin{cases} f(.) < -2.39 + \Phi(.) \\ f(.) < -0.65 + 0.5\Phi(.) \\ f(.) > 0.5. \end{cases} \tag{60}$$

The stability domain given by the nonlinear $f(.)$ relative to the nonlinear $\Phi(.)$ is illustrated in Figure 1. For choice $f(.) = 2$, $\Phi(.) = 5.3$, and $\psi(.) = 0.2$, the uncertain parameters

$$\begin{aligned}
&\mu_{11} = \mu_{21} = \mu_{31} = 0.6, \\
&\mu_{12} = \mu_{22} = \mu_{32} = 0.4, \\
&\lambda_{1,11} = \lambda_{2,11} = \lambda_{1,21} = \lambda_{2,21} = \lambda_{1,31} = \lambda_{2,31} = 0.6, \\
&\lambda_{1,12} = \lambda_{2,12} = \lambda_{1,22} = \lambda_{2,22} = \lambda_{1,32} = \lambda_{2,32} = 0.4.
\end{aligned} \tag{61}$$

The initial state vector $\phi(t) = [1\ -4]^T$ and the simulation results are illustrated in Figures 2–4 where the switched signal given in Figure 5 is randomly generated.

Remark 4. From Figures 2 and 3, we observe that the considered system is robustly asymptotically stable under randomly switching and any admissible uncertainties (4) and (5), which proves the effectiveness of the result given in Corollary 1.

Remark 5. The considered system in Example 1 is subject to uncertain complex nonlinear dynamics and mode depending on multiple time-varying delays. However, it is very difficult to find a CLF for the system under consideration in Example 1.

Remark 6. The result given in Corollary 1 can construct an alternative to searching a CLF through the LMIs approach for studying robust stability under arbitrary switching.

Indeed, in [32], the authors introduced a simple linear example without time-delay and uncertainty for which a CLF does not exist.

Example 2. (see [43]). Consider the following switched system given by a set of differential equations represented as

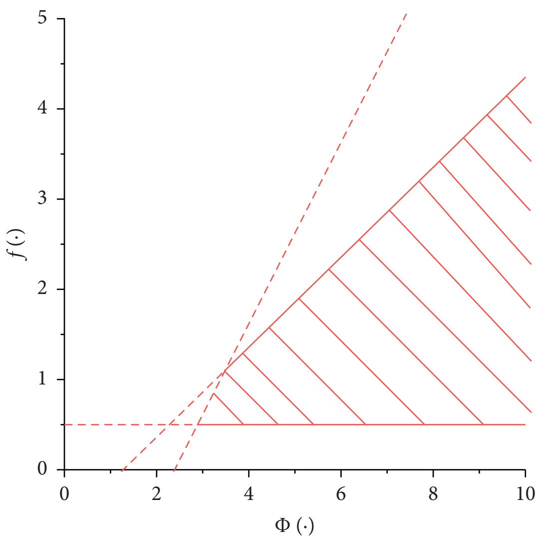

Figure 1: Stability domain for the system in Example 1.

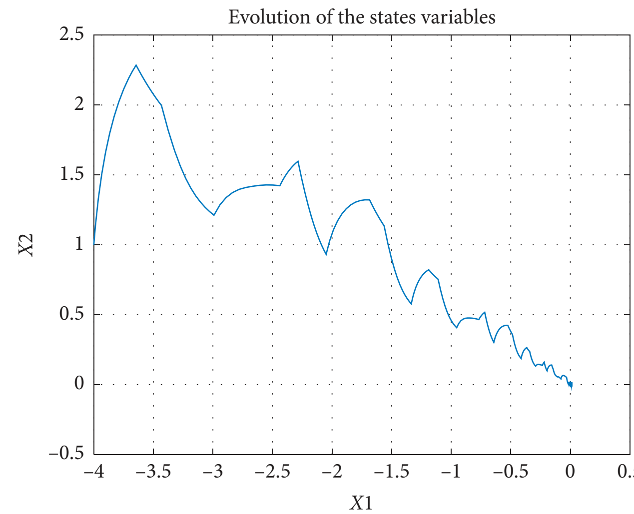

Figure 3: State space of the system in Example 1.

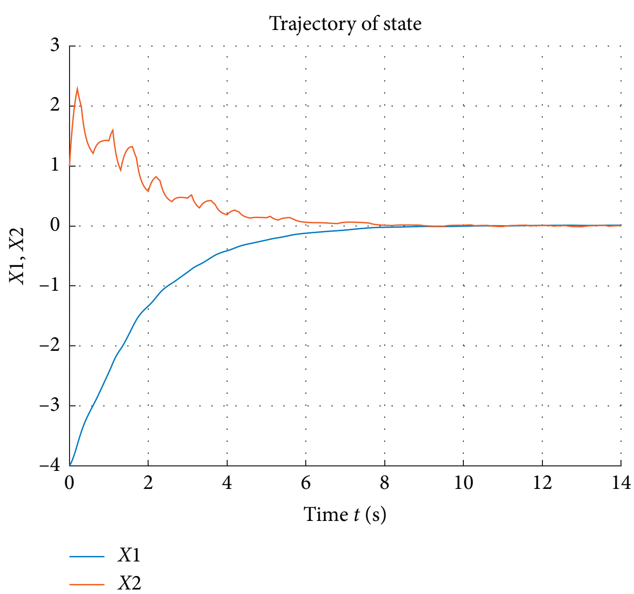

Figure 2: The state responses of the system in Example 1.

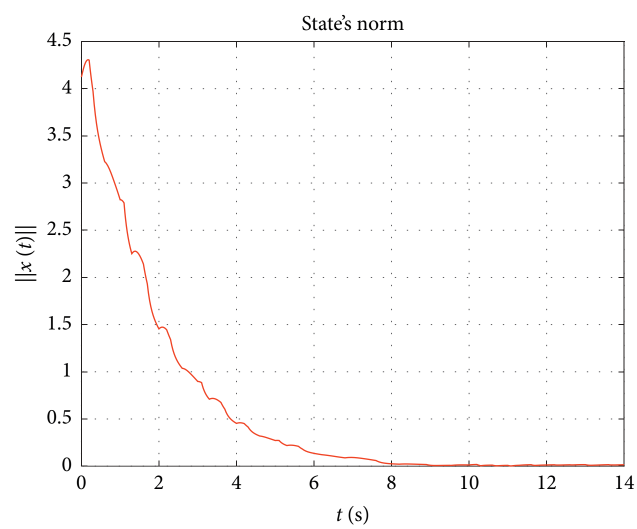

Figure 4: State's norm of the system in Example 1.

$$\begin{aligned}\ddot{x}(t) &+ \sum_{p=1}^{2}\mu_{ip}(t)a_{ip}(.)\dot{x}(t) + \sum_{p=1}^{2}\mu_{ip}(t)\frac{\varphi_{ip}(x(t))}{x(t)}x(t) \\ &+ \sum_{l=1}^{2}\sum_{q=1}^{2}\lambda_{l,iq}(t)b_{l,iq}(.)\dot{x}\big(t-r_{l,i}(t)\big) \\ &+ \sum_{l=1}^{2}\sum_{q=1}^{2}\lambda_{l,iq}(t)c_{l,iq}(.)x\big(t-r_{l,i}(t)\big) + u(t) = 0,\end{aligned} \tag{62}$$

where $a_{ip}(.)$, $b_{l,iq}(.)$, and $c_{l,iq}(.)$ are nonlinear parameters for each $i \in \{1, 2, 3\} p \in \{1, 2\}$, $q \in \{1, 2\}$, and $l \in \{1, 2\}$.

All the subsystems can be represented under matrix representation such as

$$\begin{aligned}\dot{x}(t) = &\sum_{p=1}^{2}\mu_{ip}(t)A_{ip}(.)x(t) + \sum_{l=1}^{2}\sum_{q=1}^{2}\lambda_{l,iq}(t)D_{l,iq}(.)x \\ &\cdot\big(t-r_{l,i}(t)\big) + B_i u(t).\end{aligned} \tag{63}$$

Consider the following controller:

$$u(t) = -\sum_{p=1}^{2}\mu_{ip}(t)K_{ip}(.)x(t) - \sum_{l=1}^{2}\sum_{p=1}^{2}\lambda_{l,iq}(t)L_{l,iq}(.)x\big(t-r_{l,iq}(t)\big), \tag{64}$$

where the gains are $K_{ip}(.) = [K_{ip}^{1}(.)\, K_{ip}^{2}(.)]$ and $L_{l,iq}(.) = [L_{l,iq}^{1}(.)\, L_{l,iq}^{2}(.)]$, for each $i \in \{1, 2, 3\} p \in \{1, 2\}$, $q \in \{1, 2\}$, and $l \in \{1, 2\}$.

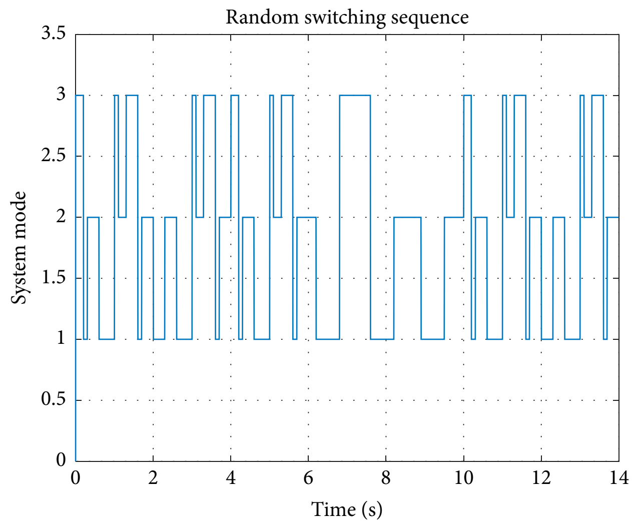

Figure 5: Random switching sequence for the system given in Example 1.

All the closed-loop subsystems will be represented as follows:

$$\dot{x}(t) = \sum_{p=1}^{2} \mu_{ip}(t)\overline{\widetilde{A}}_{ip}(.)x(t) + \sum_{l=1}^{2}\sum_{p=1}^{2} \lambda_{l,iq}(t)\overline{\widetilde{D}}_{l,iq}(.)x\big(t - r_{l,i}(t)\big), \tag{65}$$

where

$$\begin{aligned} \overline{\widetilde{A}}_{ip}(.) &= \begin{pmatrix} 0 & 1 \\ -\dfrac{\varphi_{ip}(x)}{x} - K_{ip}^{1}(.) & -a_{ip}(.) - K_{ip}^{2}(.) \end{pmatrix}, \\ \overline{\widetilde{D}}_{l,iq}(.) &= \begin{pmatrix} 0 & 0 \\ -c_{l,iq}(.) - L_{l,iq}^{1}(.) & -b_{l,iq}(.) - L_{l,iq}^{2}(.) \end{pmatrix}. \end{aligned} \tag{66}$$

The time-varying delay functions are

$$\begin{aligned} r_{1,1}(t) &= 0.8 + \frac{1}{5}\cos^2(t), \\ r_{1,2}(t) &= 0.4 + \frac{1}{8}\cos^2(t), \\ r_{2,1}(t) &= 1 + \frac{1}{5}\cos^2(t), \\ r_{2,2}(t) &= 0.5 + \frac{1}{8}\cos^2(t), \\ r_{1,3}(t) &= 0.6 + \frac{1}{6}\cos^2(t), \\ r_{2,3}(t) &= 0.5 + \frac{1}{6}\cos^2(t). \end{aligned} \tag{67}$$

All the vertex matrices will be represented under the arrow form such as

$$\begin{aligned} E_{11}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{11}^{1}(.) & \gamma_{11}^{2}(.) \end{bmatrix}, \\ E_{12}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{12}^{1}(.) & \gamma_{12}^{2}(.) \end{bmatrix}, \\ E_{21}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{21}^{1}(.) & \gamma_{21}^{2}(.) \end{bmatrix}, \\ E_{22}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{22}^{1}(.) & \gamma_{22}^{2}(.) \end{bmatrix}, \\ E_{31}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{31}^{1}(.) & \gamma_{31}^{2}(.) \end{bmatrix}, \\ E_{32}(.) &= \begin{bmatrix} \alpha & 1 \\ \gamma_{32}^{1}(.) & \gamma_{32}^{2}(.) \end{bmatrix}, \\ F_{1,11}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,11}^{1}(.) & \delta_{1,11}^{2}(.) \end{bmatrix}, \\ F_{2,11}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,11}^{1}(.) & \delta_{2,11}^{2}(.) \end{bmatrix}, \\ F_{1,12}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,12}^{1}(.) & \delta_{1,12}^{2}(.) \end{bmatrix}, \\ F_{2,21}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,21}^{1}(.) & \delta_{2,21}^{2}(.) \end{bmatrix}, \\ F_{1,21}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,21}^{1}(.) & \delta_{1,21}^{2}(.) \end{bmatrix}, \\ F_{2,21}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,21}^{1}(.) & \delta_{2,21}^{2}(.) \end{bmatrix}, \\ F_{1,22}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,22}^{1}(.) & \delta_{1,22}^{2}(.) \end{bmatrix}, \\ F_{2,22}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,22}^{1}(.) & \delta_{2,22}^{2}(.) \end{bmatrix}, \\ F_{1,31}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,31}^{1}(.) & \delta_{1,31}^{2}(.) \end{bmatrix}, \\ F_{2,31}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,31}^{1}(.) & \delta_{2,31}^{2}(.) \end{bmatrix}, \\ F_{1,32}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{1,32}^{1}(.) & \delta_{1,32}^{2}(.) \end{bmatrix}, \\ F_{2,32}(.) &= \begin{bmatrix} 0 & 0 \\ \delta_{2,32}^{1}(.) & \delta_{2,32}^{2}(.) \end{bmatrix}, \end{aligned} \tag{68}$$

where

$$\begin{cases} \gamma_{11}^{1}(.) = -G_{\overline{A}_{11}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{11}(.) + K_{11}^{2}(.)\right) + K_{11}^{1}(.) + \dfrac{\varphi_{11}(x)}{x}\right], \\ \gamma_{11}^{2}(.) = \left(-a_{11}(.) - K_{11}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \gamma_{12}^{1}(.) = -G_{\overline{A}_{12}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{12}(.) + K_{12}^{2}(.)\right) + K_{12}^{1}(.) + \dfrac{\varphi_{12}(x)}{x}\right], \\ \gamma_{12}^{2}(.) = \left(-a_{12}(.) - K_{12}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \gamma_{21}^{1}(.) = -G_{\overline{A}_{21}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{21}(.) + K_{21}^{2}(.)\right) + K_{21}^{1}(.) + \dfrac{\varphi_{21}(x)}{x}\right], \\ \gamma_{21}^{2}(.) = \left(-a_{21}(.) - K_{21}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \gamma_{22}^{1}(.) = -G_{\overline{A}_{22}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{22}(.) + K_{22}^{2}(.)\right) + K_{22}^{1}(.) + \dfrac{\varphi_{22}(x)}{x}\right], \\ \gamma_{22}^{2}(.) = \left(-a_{22}(.) - K_{22}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \gamma_{31}^{1}(.) = -G_{\overline{A}_{31}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{31}(.) + K_{31}^{2}(.)\right) + K_{31}^{1}(.) + \dfrac{\varphi_{31}(x)}{x}\right], \\ \gamma_{31}^{2}(.) = \left(-a_{31}(.) - K_{31}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \gamma_{32}^{1}(.) = -G_{\overline{A}_{32}(.)}(\alpha) = -\left[\alpha^{2} + \alpha\left(a_{32}(.) + K_{32}^{2}(.)\right) + K_{32}^{1}(.) + \dfrac{\varphi_{32}(x)}{x}\right], \\ \gamma_{32}^{2}(.) = \left(-a_{32}(.) - K_{32}^{2}(.) - \alpha\right), \end{cases}$$

$$\begin{cases} \delta_{1,11}^{1}(.) = -N_{\overline{D}_{1,11}(.)}(\alpha) = -\left[\left(b_{1,11}(.) + L_{1,11}^{2}(.)\right)\alpha + c_{1,11}(.) + L_{1,11}^{1}(.)\right], \\ \delta_{1,11}^{2}(.) = -\left[b_{1,11}(.) + L_{1,11}^{2}(.)\right], \end{cases}$$

$$\begin{cases} \delta_{2,11}^{1}(.) = -N_{\overline{D}_{2,11}(.)}(\alpha) = -\left[\left(b_{2,11}(.) + L_{2,11}^{2}(.)\right)\alpha + c_{2,11}(.) + L_{2,11}^{1}(.)\right], \\ \delta_{2,11}^{2}(.) = -\left[b_{2,11}(.) + L_{2,11}^{2}(.)\right], \end{cases}$$

$$\begin{cases} \delta_{1,12}^{1}(.) = -N_{\overline{D}_{1,12}(.)}(\alpha) = -\left[\left(b_{1,12}(.) + L_{1,12}^{2}(.)\right)\alpha + c_{1,12}(.) + L_{1,12}^{1}(.)\right], \\ \delta_{1,12}^{2}(.) = -\left[b_{1,12}(.) + L_{1,12}^{2}(.)\right], \end{cases}$$

$$\begin{cases} \delta^1_{2,12}(.) = -N_{\overline{D}_{2,12}(.)}(\alpha) = -\left[\left(b_{2,12}(.) + L^2_{2,12}(.)\right)\alpha + c_{2,12}(.) + L^1_{2,12}(.)\right], \\ \delta^2_{2,12}(.) = -\left[b_{2,12}(.) + L^2_{2,12}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{1,21}(.) = -N_{\overline{D}_{1,21}(.)}(\alpha) = -\left[\left(b_{1,21}(.) + L^2_{1,21}(.)\right)\alpha + c_{1,21}(.) + L^1_{1,21}(.)\right], \\ \delta^2_{1,21}(.) = -\left[b_{1,21}(.) + L^2_{1,21}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{2,21}(.) = -N_{\overline{D}_{2,21}(.)}(\alpha) = -\left[\left(b_{2,21}(.) + L^2_{2,21}(.)\right)\alpha + c_{2,21}(.) + L^1_{2,21}(.)\right], \\ \delta^2_{2,21}(.) = -\left[b_{2,21}(.) + L^2_{2,21}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{1,22}(.) = -N_{\overline{D}_{1,22}(.)}(\alpha) = -\left[\left(b_{1,22}(.) + L^2_{1,22}(.)\right)\alpha + c_{1,22}(.) + L^1_{1,22}(.)\right], \\ \delta^2_{1,22}(.) = -\left[b_{1,22}(.) + L^2_{1,22}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{2,21}(.) = -N_{\overline{D}_{2,21}(.)}(\alpha) = -\left[\left(b_{2,21}(.) + L^2_{2,21}(.)\right)\alpha + c_{2,21}(.) + L^1_{2,21}(.)\right], \\ \delta^2_{2,21}(.) = -\left[b_{2,21}(.) + L^2_{2,21}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{1,22}(.) = -N_{\overline{D}_{1,22}(.)}(\alpha) = -\left[\left(b_{1,22}(.) + L^2_{1,22}(.)\right)\alpha + c_{1,22}(.) + L^1_{1,22}(.)\right], \\ \delta^2_{1,22}(.) = -\left[b_{1,22}(.) + L^2_{1,22}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{2,22}(.) = -N_{\overline{D}_{2,22}(.)}(\alpha) = -\left[\left(b_{2,22}(.) + L^2_{2,22}(.)\right)\alpha + c_{2,22}(.) + L^1_{2,22}(.)\right], \\ \delta^2_{2,22}(.) = -\left[b_{2,22}(.) + L^2_{2,22}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{1,31}(.) = -N_{\overline{D}_{1,31}(.)}(\alpha) = -\left[\left(b_{1,31}(.) + L^2_{1,31}(.)\right)\alpha + c_{1,31}(.) + L^1_{1,31}(.)\right], \\ \delta^2_{1,31}(.) = -\left[b_{1,31}(.) + L^2_{1,31}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{2,31}(.) = -N_{\overline{D}_{2,31}(.)}(\alpha) = -\left[\left(b_{2,31}(.) + L^2_{2,31}(.)\right)\alpha + c_{2,31}(.) + L^1_{2,31}(.)\right], \\ \delta^2_{2,31}(.) = -\left[b_{2,31}(.) + L^2_{2,31}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{1,32}(.) = -N_{\overline{D}_{1,32}(.)}(\alpha) = -\left[\left(b_{1,32}(.) + L^2_{1,32}(.)\right)\alpha + c_{1,32}(.) + L^1_{1,32}(.)\right], \\ \delta^2_{1,32}(.) = -\left[b_{1,32}(.) + L^2_{1,32}(.)\right], \end{cases}$$
$$\begin{cases} \delta^1_{2,32}(.) = -N_{\overline{D}_{2,32}(.)}(\alpha) = -\left[\left(b_{2,32}(.) + L^2_{2,32}(.)\right)\alpha + c_{2,32}(.) + L^1_{2,32}(.)\right], \\ \delta^2_{2,32}(.) = -\left[b_{2,32}(.) + L^2_{2,32}(.)\right]. \end{cases} \tag{69}$$

For the following pole placement $p_1 = -1$ and $p_2 = -2$, by Theorem 4, we obtain the following robust stabilization conditions:

(i) $\alpha = p_1 = -1 < 0$

(ii) $t^1_{11}(.) = t^1_{21}(.) = t^1_{31}(.) = t^1_{12}(.) = t^1_{22}(.) = t^1_{32}(.) = 0$

(iii) $\max(t^2_{11}(.), t^2_{21}(.), t^2_{31}(.), t^2_{12}(.), t^2_{22}(.), t^2_{32}(.)) = p_2 = -2 < 0$

Condition (iii), when we choose $t^2_{11}(.) = p_2, t^2_{21}(.) = -3$, $t^2_{12}(.) = -3.5, t^2_{22}(.) = -4, t^2_{31}(.) = -4.5$ and $t^2_{32}(.) = -5$ yields to following controller gains:

$$
\begin{aligned}
K_{11}(.) &= \left[2 - \frac{\varphi_{11}(x)}{x} 3 - a_{11}(.)\right], \\
K_{12}(.) &= \left[2 - \frac{\varphi_{12}(x)}{x} 3 - a_{12}(.)\right], \\
K_{21}(.) &= \left[2 - \frac{\varphi_{21}(x)}{x} 3 - a_{21}(.)\right], \\
K_{22}(.) &= \left[2 - \frac{\varphi_{22}(x)}{x} 3 - a_{22}(.)\right], \\
K_{31}(.) &= \left[2 - \frac{\varphi_{31}(x)}{x} 3 - a_{31}(.)\right], \\
K_{32}(.) &= \left[2 - \frac{\varphi_{32}(x)}{x} 3 - a_{32}(.)\right], \\
L_{1,11}(.) &= \left[-c_{1,11}(.) - b_{1,11}(.)\right], \\
L_{2,11}(.) &= \left[-c_{2,11}(.) - b_{2,11}(.)\right], \\
L_{1,12}(.) &= \left[-c_{1,12}(.) - b_{1,12}(.)\right], \\
L_{2,12}(.) &= \left[-c_{2,12}(.) - b_{2,12}(.)\right], \\
L_{1,21}(.) &= \left[-c_{1,21}(.) - b_{1,21}(.)\right], \\
L_{2,21}(.) &= \left[-c_{2,21}(.) - b_{2,21}(.)\right], \\
L_{1,22}(.) &= \left[-c_{1,22}(.) - b_{1,22}(.)\right], \\
L_{2,22}(.) &= \left[-c_{2,22}(.) - b_{2,22}(.)\right], \\
L_{1,31}(.) &= \left[-c_{1,31}(.) - b_{1,31}(.)\right], \\
L_{2,31}(.) &= \left[-c_{2,21}(.) - b_{2,31}(.)\right], \\
L_{1,32}(.) &= \left[-c_{1,32}(.) - b_{1,32}(.)\right], \\
L_{2,32}(.) &= \left[-c_{2,22}(.) - b_{2,32}(.)\right].
\end{aligned} \tag{70}
$$

The simulation results for fixed initial points $\phi(t) = [2 - 1]^T$ are given in Figures 6 –8, respectively, which show the state responses, the state trajectory, and the state's norm of the system given in Example 1 where the switching mode given in Figure 9 is randomly generated.

The simulation results reveal that the state trajectories closed-loop system controlled by the memory state feedback controller are converging to zero, and the closed-loop system is robustly asymptotically stable where the switching signal is randomly generated.

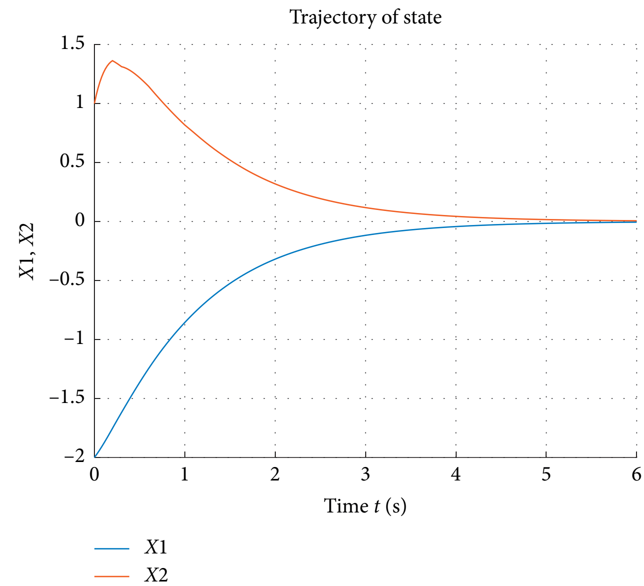

Figure 6: State responses of the closed-loop system in Example 2.

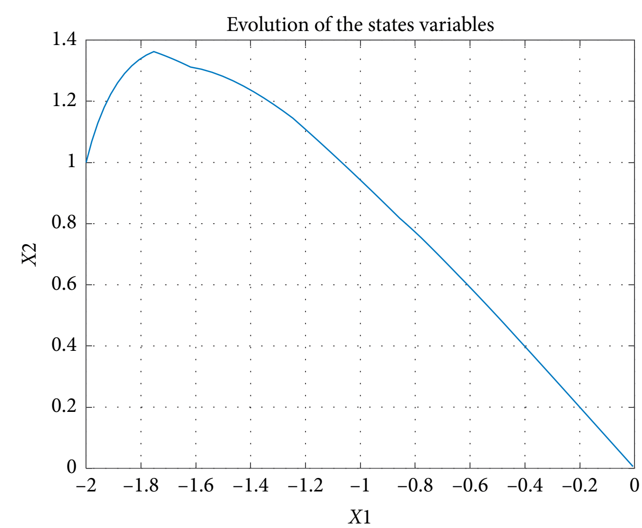

Figure 7: Evaluation of the state's variables of the closed-loop system in Example 2.

Remark 7. The developed memory state feedback controller given in Theorem 4 can reduce the effect of the delays especially for switched systems with multiple time-varying delays and it guaranteed to the considering system more performance and immunity to the delays as well as the uncertainties compared with the memoryless controller.

Remark 8. Form Theorem 4, we obtain the robust stability of a closed-loop system given in Example 2 where the switching signal is randomly generated and for any admissible uncertainties (4) and (5). In fact, the result given in Theorem 4 can be an alternative to find a CLF through the LMI approach.

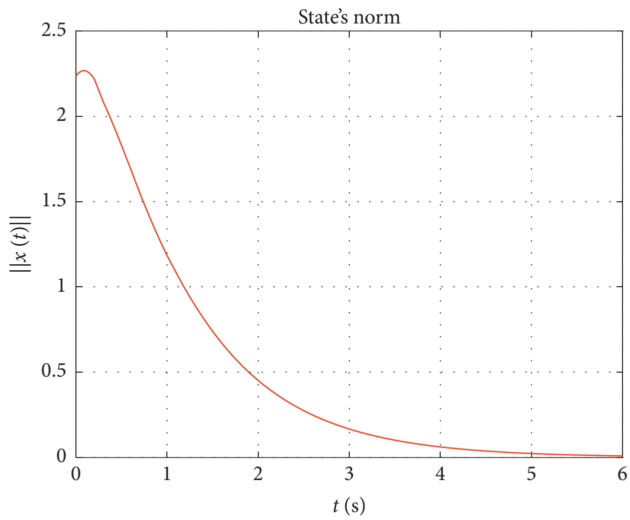

FIGURE 8: State's norm of the closed-loop system in Example 2.

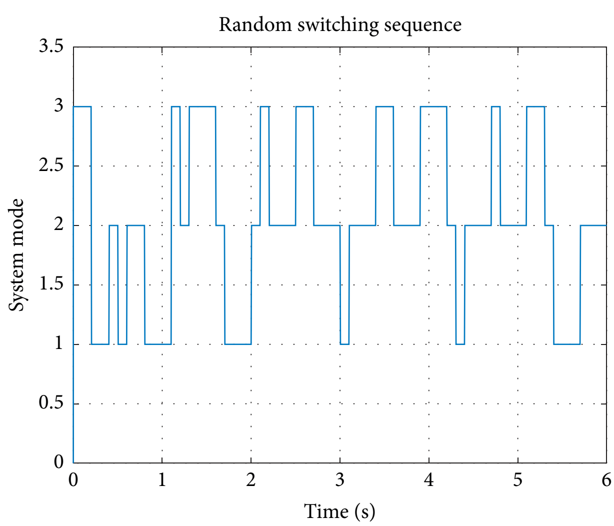

FIGURE 9: Random switching sequence for the system in Example 2.

6. Conclusion

This paper has investigated new robust stability and stabilization criteria under arbitrary switching for a class of uncertain switched nonlinear systems. The systems under consideration are subject to multiple time-varying delays and polytopic-type parameter uncertainty. The proposed results are obtained by using a novel CLF, the Borne–Gentina criterion, and the aggregation techniques. Compared to the existing results in this area, the developed criteria are explicit, are simple to use, and can construct an interesting alternative to find a CLF through the LMI approach, considered a hard task in this case.

Future research will extend the results of this paper to switched stochastic systems with time-varying delays and actuator saturation.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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